 



## Solutions to Exercises in Chapter 3

**3.** For the Black-Scholes Equation (RE-3.1),
Let

$$C = E f(S/E) \quad \text{(RE-3.5)}$$

for some function *f, then*

$$C/E = f(S/E) \quad \text{(RE-3.6)}$$

Apply the change of variables stated in (RE-3.5) to reduce the Black-Scholes Equation (RE-3.1) to its simpler form:

$$\partial C/\partial t + {}^1\!/_2\, \sigma^2 E^2 \partial^2 C/\partial E^2 - rE\partial C/\partial E = 0 \quad \text{(RE-3.7)}$$

which is (RE-.2).

**Proof:**

Since $C = E\, f(S/E)$, from (RE-3.5),

$$\partial C/\partial E = \partial/\partial E\{Ef(S/E)\}$$
$$= f(S/E)\}\partial\{E\}/\partial E + E\partial\{f(S/E)\}/\partial E$$
$$= \{f(S/E)\}.1 + E.S\{(-1/E^2)\partial F/\partial E\}$$
$$= f(S/E) - (S/E)F'(S/E)$$

therefore, upon substituting for $f(S/E)$ from (RE-3.6):

$$\partial C/\partial E = C/E - (S/E)F'(S/E) \quad \text{(RE-3.8)}$$

Again, since $C = E\,f(S/E)$, from (RE-3.4),

$$\partial C/\partial S = (\partial/\partial S)[C]$$
$$= (\partial/\partial S)[Ef(S/E)]$$
$$= E(\partial/\partial S)[f(S/E)]$$
$$= E(1/E)(\partial/\partial S)[f(S/E)]$$
$$= (1)f'(S/E)]$$
$$= f'(S/E)]$$

therefore,

$$\partial C/\partial S = f'(S/E)] \qquad \text{(RE-3.9)}$$

or,

$$\partial C/\partial S = f'(S/E)] = f'$$

hence,

$$S\,\partial C/\partial S = S\,f'(S/E)] = S\,f'$$

Again, from (RE-3.5):

$$C = E\,f(S/E)$$

therefore

$$\partial C/\partial S = (\partial/\partial \mathrm{S})\,[E\,f(S/E)]$$
$$= (1/E)\partial/\partial(S/E)\,[E\,f(S/E)]$$
$$= \partial/\partial(S/E)[f(S/E)]$$
$$= f\,'\,(S/E) \qquad \text{(RE-3.10)}$$

Therefore

$$E\,\partial/\partial E(C) = \partial/\partial E\,[E\,f(S/E)],$$

upon substituting for $C$ from (RE-3.5).

Also, from (RE-3.8):

$$\partial C/\partial E = C/E - (S/E)\,F'(S/E) \qquad \text{(RE-3.11)}$$

Hence,

$$\partial C/\partial E = \partial/\partial E[E\,f(S/E)]$$
$$= f(S/E) \qquad \text{(RE-3.12)}$$

Now,

$$E\,\partial C/\partial E = \mathrm{E}[C/E - (S/E)\,F'(S/E)],\ \text{from (RE} - 3.11)$$
$$= C - SF'(S/E)$$
$$= C - S\,\partial C/\partial S,\ \text{from (RE} - 3.10)$$

Hence,

$$\boldsymbol{S\,\partial C/\partial S = C - E\,\partial C/\partial E} \qquad \text{(RE-3.13)}$$

which is an important ***intermediate*** result.

Similarly, it may be shown that:

$$\boldsymbol{S^2\,\partial^2 C/\partial S^2 = E^2\,\partial^2 C/\partial E^2} \qquad \text{(RE-3.14)}$$

which is a ***final*** result which may be readily obtained as follows:

Since

$$S\,\partial C/\partial S = C - E\,\partial C/\partial E\ ,\ from\ \text{(RE-3.13)}$$

then

$$\partial C/\partial S = C/S - (E/S)\,\partial C/\partial E$$

and

$$\partial^2 C/\partial S^2 = \partial/\partial S\{C/S - (E/S)\,\partial C/\partial E\}$$
$$= \partial/\partial S\{C/S\} - \partial/\partial S\{(E/S)\,(\partial C/\partial E)\}$$
$$= -(C/S^2) + (E/S^2)(\partial C/\partial E)$$

viz.,

$$S^2\,\partial^2 C/\partial S^2 = -C + E(\partial C/\partial E) \qquad \text{(RE-3.15)}$$

Moreover, since

$$\partial C/\partial E \quad = F(S/E),\ \ \text{from (RE-3.12)}$$
$$E\,\partial C/\partial E = E\,F(S/E)$$
$$= C,\ \ \text{from (RE-3.5)}$$

Therefore,

$$\partial C/\partial E = C/E \qquad \text{(RE-3.16)}$$

and

$$(\partial^2 C/\partial E^2) = \partial/\partial E[\partial C/\partial E], \text{ by definition}$$
$$= \partial/\partial E[C/E], \text{ from (RE-3.16)}$$
$$= -(C/E^2) + (1/E)(\partial C/\partial E),$$

by the differentiation of a quotient ruleor,

$$E^2(\partial^2 C/\partial E^2) = -C + E(\partial C/\partial E) \qquad \text{(RE-3.17)}$$

Combining (RE-3.15) and (RE-3.17), one finally obtains (RE-3.14), as required.

And now, upon substituting for the terms $S^2\partial^2 C/\partial S^2$ and $S\partial C/\partial S$, respectively from (RE-3.14) and (RE-3.13), respectively, into the Black-Scholes equation, (RE-3.1), the results is:

$$\partial C/\partial t + ½\, \sigma^2 S^2\, \partial^2 C/\partial S^2 + rS\partial C/\partial S - rC = 0 \qquad \text{(RE-3.1)}$$

viz.,

$$\partial C/\partial t + ½\, \sigma^2(S^2\, \partial^2 C/\partial S^2) + r(S\partial C/\partial S) - rC = 0$$

viz.,

$$\partial C/\partial t + ½\, \sigma^2(E^2\, \partial^2 C/\partial E^2) + r(C - E\partial C/\partial E) - rC = 0$$

viz.,

$$\partial C/\partial t + ½\, \sigma^2(E^2\, \partial^2 C/\partial E^2) - rE\, \partial C/\partial E = 0 \qquad \text{(RE-3.2)}$$

which is (RE-.7), as required.

**4.** Simulation Results Based on the Model BLCOP: **Black-Litterman and Copula Opinion Pooling Frameworks**

In the R domain:

```
>
> install.packages("BLCOP")
The downloaded binary packages are in

C:\Users\Bert\AppData\Local\Temp\Rtmp2jzxqk\
   downloaded_packages
> library(BLCOP)
Loading required package: MASS
Loading required package: quadprog
In addition: Warning messages:
1: package 'BLCOP' was built under R version 3.2.5
> ls("package:BLCOP")
 [1] "addBLViews"                "addCOPViews"
 [3] "assetSet"                  "BLCOPOptions"
```