Solutions to Exercises

Chapter 1

Exercise 1. Use Eq. (1.6) and take the resistivity value from Table 1.2. Also, the wire has diameter 1 mm $= 10^{-3}$ m so the area is $A = \pi r^2 = \pi(\frac{10^{-3}\ \text{m}}{2})^2 = 7.85 \times 10^{-3}\ \text{m}^2$. Finally, the length of the wire is 1 m, so $R = \rho\frac{L}{A} = (100 \times 10^{-8}\ \Omega \cdot \text{m})(\frac{1\ \text{m}}{7.85\times10^{-3}\ \text{m}^2}) = 1.27\ \Omega$.

Exercise 2. Equation (1.7) gives $P = I^2R$ for resistors. Thus $I_{max} = \sqrt{(P/R)}$. For the 10 W resistor we obtain $I_{max} = \sqrt{(10\ \text{W}/10\ \text{k}\Omega)} = 0.032$ A and for the quarter-watt resistor $I_{max} = \sqrt{(0.25\ \text{W}/10\ \text{k}\Omega)} = 0.005$ A.

Exercise 3. Equation (1.8) gives $P = V^2/R$ for resistors. For the 100 Ω resistor we obtain $P = (100\ \text{V})^2/100\ \Omega = 100$ W while for the 100 kΩ resistor we obtain $P = (100\ \text{V})^2/100\ \text{k}\Omega = 0.1$ W.

Exercise 4. The current through R_3 is the same as the current supplied by the battery. To find this current, we first find the total resistance of the circuit. The parallel combination of R_1 and R_2 is $\frac{R_1R_2}{R_1+R_2} = \frac{6\cdot5}{6+5} = 2.73\ \Omega$. This resistance is then in series with R_3 so $R_{tot} = 3\ \Omega + 2.73\ \Omega = 5.73\ \Omega$ and $I_3 = V_1/R_{tot} = 5\ \text{V}/5.73\ \Omega = 0.873$ A.

Exercise 5. Building on the result of the previous problem, the voltage across R_1 and R_2 is $V_1 - I_3R_3 = 5\ \text{V} - (0.873\ \text{A})(3\ \Omega) = 2.38$ V. Therefore, $I_2 = (2.38\ \text{V})/(6\ \Omega) = 0.397$ A and $I_1 = (2.38\ \text{V})/(5\ \Omega) = 0.476$ A.

Exercise 6. When the meter is connected, the input resistance of the meter R_m will be in parallel with the 1 kΩ resistor (call this resistor R_2). The resulting circuit will still be a voltage divider, but R_2 is replaced by the parallel combination of R_2 and R_m, $R_2//R_m = \frac{R_2R_m}{R_2+R_m}$. The voltage across this resistance is then given by the voltage divider equation [Eq. (1.21)]:

$$V_{out} = V_{in}\frac{R_2//R_m}{R_1 + (R_2//R_m)} = V_{in}\frac{R_2R_m}{R_1R_2 + R_1R_m + R_2R_m}$$

where we have called the 2 kΩ resistor R_1. Putting in the numbers:

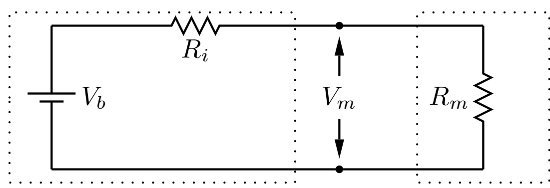
$$R_m = 100\ \Omega: \quad V_{out} = 3\ \text{V}\frac{(1\ \text{k}\Omega)(100\ \Omega)}{(2\ \text{k}\Omega)(1\ \text{k}\Omega) + (2\ \text{k}\Omega)(100\ \Omega) + (1\ \text{k}\Omega)(100\ \Omega)} = 0.130\ \text{V}$$

$$R_m = 1\ \text{k}\Omega: \quad \text{V}_{\text{out}} = 3\ \text{V}\frac{(1\ \text{k}\Omega)(1\ \text{k}\Omega)}{(2\ \text{k}\Omega)(1\ \text{k}\Omega) + (2\ \text{k}\Omega)(1\ \text{k}\Omega) + (1\ \text{k}\Omega)(1\ \text{k}\Omega)} = 0.600\ \text{V}$$

$$R_m = 50\ \text{k}\Omega: \quad \text{V}_{\text{out}} = 3\ \text{V}\frac{(1\ \text{k}\Omega)(50\ \text{k}\Omega)}{(2\ \text{k}\Omega)(1\ \text{k}\Omega) + (2\ \text{k}\Omega)(50\ \text{k}\Omega) + (1\ \text{k}\Omega)(50\ \text{k}\Omega)} = 0.987\ \text{V}$$

$$R_m = 1\ \text{M}\Omega: \quad \text{V}_{\text{out}} = 3\ \text{V}\frac{(1\ \text{k}\Omega)(1\ \text{M}\Omega)}{(2\ \text{k}\Omega)(1\ \text{k}\Omega) + (2\ \text{k}\Omega)(1\ \text{M}\Omega) + (1\ \text{k}\Omega)(1\ \text{M}\Omega)} = 0.999\ \text{V}$$

Exercise 7. When the voltmeter is attached the circuit appears as shown below.



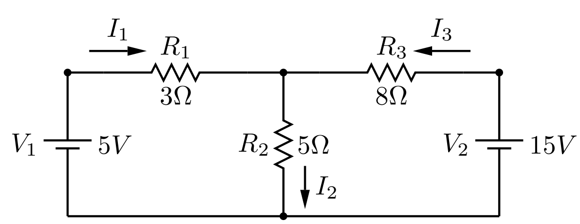
This is a voltage divider so $V_m = V_b \frac{R_m}{R_i + R_m}$. Solving for R_i gives $R_i = R_m \frac{V_b}{V_m} - R_m = (1000\ \Omega)\frac{1.5\ \text{V}}{0.9\ V} - 1000\ \Omega = 667\ \Omega$.

Exercise 8. Referring to the diagram for Exercise 7, we have $V_m = V_b \frac{R_m}{R_m + R_i} = (1.5\ \text{V})\frac{1\ \text{M}\Omega}{1\ \text{M}\Omega + 667\ \Omega} = 1.499\ \text{V}$.

Exercise 9. Start by combining the 10 Ω and 5 Ω resistors in series to form a 15 Ω resistor. This, in parallel with the existing 15 Ω resistor gives a 7.5 Ω resistor. This resistor is in series with the 3 Ω and 2 Ω, giving a total of $3 + 7.5 + 2 = 12.5\ \Omega$ across the terminals.

Exercise 10. From the result of Exercise 9 we know that the current supplied by the battery is $I_1 = V_{bat}/R_{tot} = 25\ \text{V}/12.5\ \Omega = 2\ \text{A}$. This means the voltage across the 15 Ω resistor is $V_{15} = 25\ \text{V} - \text{I}_1(3\ \Omega + 2\ \Omega) = 15\ \text{V}$. This is also the voltage across the series combination of the 10 Ω and 5 Ω resistors. Thus the current through the 10 Ω resistor is $V_{15}/(10\ \Omega + 5\ \Omega) = 1\ \text{A}$.

Exercise 11. Assign current names and directions as shown below. For the left and right



halves of the circuit, KVL gives

$$\text{a)} \quad \text{V}_1 - \text{I}_1\text{R}_1 - \text{I}_2\text{R}_2 = 0$$

$$\text{b)} \quad \text{V}_2 - \text{I}_3\text{R}_3 - \text{I}_2\text{R}_2 = 0 \quad \rightarrow \quad \text{I}_3 = \frac{\text{V}_2 - \text{I}_2\text{R}_2}{\text{R}_3}$$

and KCL gives $I_1 + I_3 = I_2$ or $I_1 = I_2 - I_3$. Using this last result in a) gives

$$V_1 - (I_2 - I_3)R_1 - I_2 R_2 = 0$$