12–1.

Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6)\ \text{m/s}^2$, where t is in seconds. What is the particle's velocity when $t = 6$ s, and what is its position when $t = 11$ s?

SOLUTION

$a = 2t - 6$

$dv = a\, dt$

$$\int_0^v dv = \int_0^t (2t - 6)\, dt$$

$v = t^2 - 6t$

$ds = v\, dt$

$$\int_0^s ds = \int_0^t (t^2 - 6t)\, dt$$

$$s = \frac{t^3}{3} - 3t^2$$

When $t = 6\ s$,

$v = 0$ **Ans.**

When $t = 11\ s$,

$s = 80.7$ m **Ans.**

Ans:
$v = 0$
$s = 80.7$ m

12–2.

The acceleration of a particle as it moves along a straight line is given by $a = (4t^3 - 1)\ \text{m/s}^2$, where t is in seconds. If $s = 2$ m and $v = 5$ m/s when $t = 0$, determine the particle's velocity and position when $t = 5$ s. Also, determine the total distance the particle travels during this time period.

SOLUTION

$$\int_5^v dv = \int_0^t (4\,t^3 - 1)\,dt$$

$$v = t^4 - t + 5$$

$$\int_2^s ds = \int_0^t (t^4 - t + 5)\,dt$$

$$s = \frac{1}{5}t^5 - \frac{1}{2}t^2 + 5\,t + 2$$

When $t = 5$ s,

$$v = 625\ \text{m/s} \qquad \textbf{Ans.}$$

$$s = 639.5\ \text{m} \qquad \textbf{Ans.}$$

Since $v \neq 0$ then

$$d = 639.5 - 2 = 637.5\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$v = 625$ m/s
$s = 639.5$ m
$d = 637.5$ m

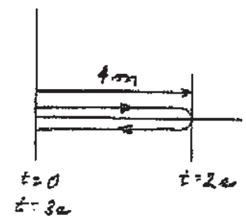
12–3.

The velocity of a particle traveling in a straight line is given by $v = (6t - 3t^2)$ m/s, where t is in seconds. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = 3$ s. How far has the particle traveled during the 3-s time interval, and what is its average speed?

SOLUTION

$v = 6t - 3t^2$

$a = \dfrac{dv}{dt} = 6 - 6t$



At $t = 3$ s

$a = -12 \text{ m/s}^2$ **Ans.**

$ds = v\,dt$

$\int_0^s ds = \int_0^t (6t - 3t^2)dt$

$s = 3t^2 - t^3$

At $t = 3$ s

$s = 0$ **Ans.**

Since $v = 0 = 6t - 3t^2$, when $t = 0$ and $t = 2$ s.

when $t = 2$ s, $s = 3(2)^2 - (2)^3 = 4$ m

$s_T = 4 + 4 = 8$ m **Ans.**

$(v_{sp})_{\text{avg}} = \dfrac{s_T}{t} = \dfrac{8}{3} = 2.67$ m/s **Ans.**

Ans:
$a = -12 \text{ m/s}^2$
$s = 0$
$s_T = 8$ m
$(v_{sp})_{\text{avg}} = 2.67$ m/s

***12–4.**

A particle is moving along a straight line such that its position is defined by $s = (10t^2 + 20)$ mm, where t is in seconds. Determine (a) the displacement of the particle during the time interval from $t = 1$ s to $t = 5$ s, (b) the average velocity of the particle during this time interval, and (c) the acceleration when $t = 1$ s.

SOLUTION

$s = 10t^2 + 20$

(a) $s|_{1\,s} = 10(1)^2 + 20 = 30$ mm

$s|_{5\,s} = 10(5)^2 + 20 = 270$ mm

$\Delta s = 270 - 30 = 240$ mm **Ans.**

(b) $\Delta t = 5 - 1 = 4$ s

$v_{avg} = \dfrac{\Delta s}{\Delta t} = \dfrac{240}{4} = 60$ mm/s **Ans.**

(c) $a = \dfrac{d^2 s}{dt^2} = 20 \text{ mm/s}^2$ (for all t) **Ans.**

Ans:
$\Delta s = 240$ mm
$v_{avg} = 60$ mm/s
$a = 20 \text{ mm/s}^2$

12–5.

A particle moves along a straight line such that its position is defined by $s = (t^2 - 6t + 5)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 6$ s.

SOLUTION

$s = t^2 - 6t + 5$

$v = \dfrac{ds}{dt} = 2t - 6$

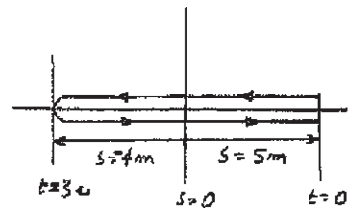
$a = \dfrac{dv}{dt} = 2$

$v = 0$ when $t = 3$

$s|_{t=0} = 5$

$s|_{t=3} = -4$

$s|_{t=6} = 5$



$v_{avg} = \dfrac{\Delta s}{\Delta t} = \dfrac{0}{6} = 0$ **Ans.**

$(v_{sp})_{avg} = \dfrac{s_T}{\Delta t} = \dfrac{9 + 9}{6} = 3\text{ m/s}$ **Ans.**

$a|_{t=6} = 2\text{ m/s}^2$ **Ans.**

Ans:
$v_{avg} = 0$
$(v_{sp})_{avg} = 3\text{ m/s}$
$a|_{t=6\text{ s}} = 2\text{ m/s}^2$

12–6.

A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the distance between the stones another second later.

SOLUTION

$$+\downarrow s = s_1 + v_1 t + \frac{1}{2} a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(9.81)(2)^2$$

$$s_A = 19.62 \text{ m}$$

$$s_A = 0 + 0 + \frac{1}{2}(9.81)(1)^2$$

$$s_B = 4.91 \text{ m}$$

$$\Delta s = 19.62 - 4.91 = 14.71 \text{ m}$$ **Ans.**

Ans:

$\Delta s = 14.71$ m

12–7.

A bus starts from rest with a constant acceleration of 1 m/s^2. Determine the time required for it to attain a speed of 25 m/s and the distance traveled.

SOLUTION

***Kinematics*:**

$v_0 = 0$, $v = 25$ m/s, $s_0 = 0$, and $a_c = 1$ m/s^2.

$(\xrightarrow{+})$ $\quad v = v_0 + a_c t$

$25 = 0 + (1)t$

$t = 25$ s **Ans.**

$(\xrightarrow{+})$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$25^2 = 0 + 2(1)(s - 0)$

$s = 312.5$ m **Ans.**

Ans:

$t = 25$ s

$s = 312.5$ m

***12–8.**

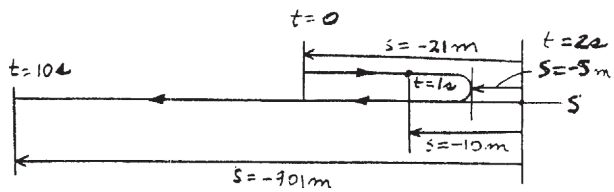
A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

SOLUTION

$$v = 12 - 3t^2 \qquad \textbf{(1)}$$

$$a = \frac{dv}{dt} = -6t\big|_{t=4} = -24 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\int_{-10}^{s} ds = \int_{1}^{t} v\,dt = \int_{1}^{t} \left(12 - 3t^2\right) dt$$



$$s + 10 = 12t - t^3 - 11$$

$$s = 12t - t^3 - 21$$

$$s|_{t=0} = -21$$

$$s|_{t=10} = -901$$

$$\Delta s = -901 - (-21) = -880 \text{ m} \qquad \textbf{Ans.}$$

From Eq. (1):

$v = 0$ when $t = 2s$

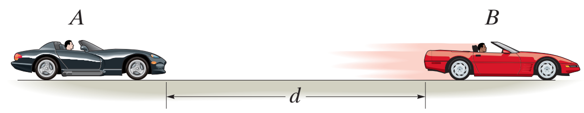
$$s|_{t=2} = 12(2) - (2)^3 - 21 = -5$$

$$s_T = (21 - 5) + (901 - 5) = 912 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$a = -24$ m/s^2
$\Delta s = -880$ m
$s_T = 912$ m

12–9.

When two cars A and B are next to one another, they are traveling in the same direction with speeds v_A and v_B, respectively. If B maintains its constant speed, while A begins to decelerate at a_A, determine the distance d between the cars at the instant A stops.



SOLUTION

Motion of car A:

$$v = v_0 + a_c t$$

$$0 = v_A - a_A t \qquad t = \frac{v_A}{a_A}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = v_A^2 + 2(-a_A)(s_A - 0)$$

$$s_A = \frac{v_A^2}{2a_A}$$

Motion of car B:

$$s_B = v_B t = v_B\left(\frac{v_A}{a_A}\right) = \frac{v_A v_B}{a_A}$$

The distance between cars A and B is

$$s_{BA} = |s_B - s_A| = \left|\frac{v_A v_B}{a_A} - \frac{v_A^2}{2a_A}\right| = \left|\frac{2v_A v_B - v_A^2}{2a_A}\right|$$ **Ans.**

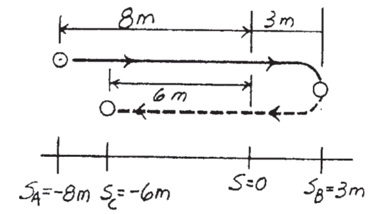
Ans:

$$S_{BA} = \left|\frac{2v_A v_B - v_A^2}{2a_A}\right|$$

12–10.

A particle travels along a straight-line path such that in 4 s it moves from an initial position $s_A = -8$ m to a position $s_B = +3$ m. Then in another 5 s it moves from s_B to $s_C = -6$ m. Determine the particle's average velocity and average speed during the 9-s time interval.

SOLUTION



Average Velocity: The displacement from A to C is $\Delta s = s_C - S_A = -6 - (-8) = 2$ m.

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{2}{4 + 5} = 0.222 \text{ m/s} \qquad \textbf{Ans.}$$

Average Speed: The distances traveled from A to B and B to C are $s_{A \to B} = 8 + 3 = 11.0$ m and $s_{B \to C} = 3 + 6 = 9.00$ m, respectively. Then, the total distance traveled is $s_{Tot} = s_{A \to B} + s_{B \to C} = 11.0 + 9.00 = 20.0$ m.

$$(v_{sp})_{avg} = \frac{s_{Tot}}{\Delta t} = \frac{20.0}{4 + 5} = 2.22 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_{avg} = 0.222$ m/s
$(v_{sp})_{avg} = 2.22$ m/s

12–11.

Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

SOLUTION

$v = v_1 + a_c t$

$120 = 70 + 6000(t)$

$t = 8.33(10^{-3})\text{ hr} = 30\text{ s}$ **Ans.**

$v^2 = v_1^2 + 2\,a_c(s - s_1)$

$(120)^2 = 70^2 + 2(6000)(s - 0)$

$s = 0.792\text{ km} = 792\text{ m}$ **Ans.**

Ans:
$t = 30\text{ s}$
$s = 792\text{ m}$

***12–12.**

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s^2, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use a numerical method to evaluate the integral.

SOLUTION

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

$$v = 1.29 \text{ m/s}$$ **Ans.**

Ans:
$v = 1.29$ m/s

12–13.

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)\ \text{m/s}^2$, where t is in seconds. If $s = 1\ \text{m}$ and $v = 2\ \text{m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6\ \text{s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION

$a = 2t - 1$

$dv = a\,dt$

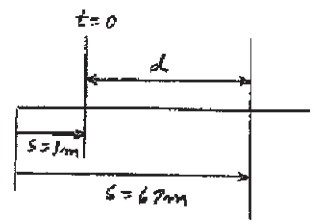
$$\int_2^v dv = \int_0^t (2t - 1)dt$$

$v = t^2 - t + 2$

$dx = v\,dt$

$$\int_t^s ds = \int_0^t (t^2 - t + 2)dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$$



When $t = 6\ \text{s}$

$v = 32\ \text{m/s}$ **Ans.**

$s = 67\ \text{m}$ **Ans.**

Since $v \neq 0$ for $0 \leq t \leq 6\ \text{s}$, then

$d = 67 - 1 = 66\ \text{m}$ **Ans.**

Ans:
$v = 32\ \text{m/s}$
$s = 67\ \text{m}$
$d = 66\ \text{m}$

12–14.

A train starts from rest at station A and accelerates at $0.5\ \text{m/s}^2$ for 60 s. Afterwards it travels with a constant velocity for 15 min. It then decelerates at $1\ \text{m/s}^2$ until it is brought to rest at station B. Determine the distance between the stations.

SOLUTION

Kinematics: For stage (1) motion, $v_0 = 0$, $s_0 = 0$, $t = 60$ s, and $a_c = 0.5\ \text{m/s}^2$. Thus,

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(60^2) = 900\ \text{m}$$

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$v_1 = 0 + 0.5(60) = 30\ \text{m/s}$$

For stage (2) motion, $v_0 = 30\ \text{m/s}$, $s_0 = 900$ m, $a_c = 0$ and $t = 15(60) = 900$ s. Thus,

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_2 = 900 + 30(900) + 0 = 27\,900\ \text{m}$$

For stage (3) motion, $v_0 = 30\ \text{m/s}$, $v = 0$, $s_0 = 27\,900$ m and $a_c = -1\ \text{m/s}^2$. Thus,

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$0 = 30 + (-1)t$$

$$t = 30\ \text{s}$$

$$\overset{+}{\rightarrow} \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_3 = 27\,900 + 30(30) + \frac{1}{2}(-1)(30^2)$$

$$= 28\,350\ \text{m} = 28.4\ \text{km}$$ **Ans.**

Ans:
$s = 28.4$ km

12–15.

A particle is moving along a straight line such that its velocity is defined as $v = (-4s^2)$ m/s, where s is in meters. If $s = 2$ m when $t = 0$, determine the velocity and acceleration as functions of time.

SOLUTION

$$v = -4s^2$$

$$\frac{ds}{dt} = -4s^2$$

$$\int_2^s s^{-2}\,ds = \int_0^t -4\,dt$$

$$-s^{-1}\Big|_2^s = -4t\Big|_0^t$$

$$t = \frac{1}{4}(s^{-1} - 0.5)$$

$$s = \frac{2}{8t + 1}$$

$$v = -4\left(\frac{2}{8t + 1}\right)^2 = -\frac{16}{(8t + 1)^2} \text{ m/s}$$ **Ans.**

$$a = \frac{dv}{dt} = \frac{16(2)(8t + 1)(8)}{(8t + 1)^4} = \frac{256}{(8t + 1)^3} \text{ m/s}^2$$ **Ans.**

Ans:

$$v = \frac{16}{(8t + 1)^2} \text{ m/s}$$

$$a = \frac{256}{(8t + 1)^3} \text{ m/s}^2$$

***12–16.**

Determine the time required for a car to travel 1 km along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate at 1.5 m/s^2 and decelerate at 2 m/s^2.

SOLUTION

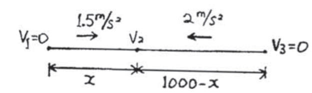
Using formulas of constant acceleration:

$v_2 = 1.5\, t_1$

$x = \frac{1}{2}(1.5)(t_1^2)$

$0 = v_2 - 2\, t_2$

$1000 - x = v_2 t_2 - \frac{1}{2}(2)(t_2^2)$



Combining equations:

$t_1 = 1.33\, t_2; \quad v_2 = 2\, t_2$

$x = 1.33\, t_2^2$

$1000 - 1.33\, t_2^2 = 2\, t_2^2 - t_2^2$

$t_2 = 20.702$ s; $\qquad t_1 = 27.603$ s

$t = t_1 + t_2 = 48.3$ s **Ans.**

Ans:

$t = 48.3$ s

12–17.

A particle is moving with a velocity of v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -kv^3$, where k is a constant, determine its velocity and position as functions of time.

SOLUTION

$$a = \frac{d\nu}{dt} = -\mathrm{k}\nu^3$$

$$\int_{\nu 0}^{\nu} \nu^{-3}\, d\nu = \int_0^t -k\, dt$$

$$-\frac{1}{2}(\nu^{-2} - \nu_0^{-2}) = -kt$$

$$\nu = \left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{-\frac{1}{2}}$$ **Ans.**

$$ds = \nu\, dt$$

$$\int_0^s ds = \int_0^t \frac{dt}{\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}}}$$

$$s = \left.\frac{2\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}}}{2k}\right|_0^t$$

$$s = \frac{1}{k}\left[\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}} - \frac{1}{\nu_0}\right]$$ Ans.

Ans:

$$v = \left(2kt + \frac{1}{v_0^2}\right)^{-1/2}$$

$$s = \frac{1}{k}\left[\left(2kt + \frac{1}{v_0^2}\right)^{1/2} - \frac{1}{v_0}\right]$$

12–18.

A particle is moving along a straight line with an initial velocity of 6 m/s when it is subjected to a deceleration of $a = (-1.5v^{1/2})$ m/s^2, where v is in m/s. Determine how far it travels before it stops. How much time does this take?

SOLUTION

Distance Traveled: The distance traveled by the particle can be determined by applying Eq. 12–3.

$$ds = \frac{vdv}{a}$$

$$\int_0^s ds = \int_{6\text{ m/s}}^{v} \frac{v}{-1.5v^{\frac{1}{2}}}\,dv$$

$$s = \int_{6\text{ m/s}}^{v} -0.6667\,v^{\frac{1}{2}}\,dv$$

$$= \left(-0.4444v^{\frac{3}{2}} + 6.532\right)\text{ m}$$

When $v = 0$, $\quad s = -0.4444\left(0^{\frac{3}{2}}\right) + 6.532 = 6.53\text{ m}$ **Ans.**

Time: The time required for the particle to stop can be determined by applying Eq. 12–2.

$$dt = \frac{dv}{a}$$

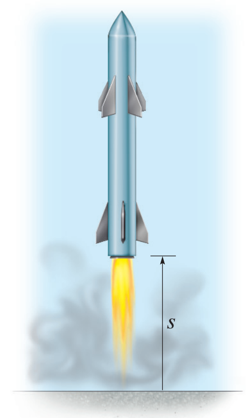
$$\int_0^t dt = -\int_{6\text{ m/s}}^{v} \frac{dv}{1.5v^{\frac{1}{2}}}$$

$$t = -1.333\left(v^{\frac{1}{2}}\right)\Big|_{6\text{ m/s}}^{v} = \left(3.266 - 1.333v^{\frac{1}{2}}\right)\text{s}$$

When $v = 0$, $\quad t = 3.266 - 1.333\left(0^{\frac{1}{2}}\right) = 3.27\text{ s}$ **Ans.**

Ans:
$s = 6.53$ m
$t = 3.27$ s

12–19. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)\ \text{m/s}^2$, where s is in meters. Determine the rocket's velocity when $s = 2$ km and the time needed to reach this attitude. Initially, $v = 0$ and $s = 0$ when $t = 0$.



SOLUTION

$$b = 6\ \text{m/s}^2 \qquad c = 0.02\ \text{s}^{-2} \qquad s_{p1} = 2000\ \text{m}$$

$$a_p = b + c s_p = v_p \frac{dv_p}{ds_p}$$

$$\int_0^{v_p} v_p\, dv_p = \int_0^{s_p} (b + c s_p)\, ds_p$$

$$\frac{{v_p}^2}{2} = b s_p + \frac{c}{2} {s_p}^2$$

$$v_p = \frac{ds_p}{dt} = \sqrt{2b s_p + c {s_p}^2} \qquad v_{p1} = \sqrt{2b s_{p1} + c {s_{p1}}^2} \qquad v_{p1} = 322.49\ \text{m/s} \quad \textbf{Ans.}$$

$$t = \int_0^{s_p} \frac{1}{\sqrt{2b s_p + c {s_p}^2}}\, ds_p \qquad t_1 = \int_0^{s_{p1}} \frac{1}{\sqrt{2b s_p + c {s_p}^2}}\, ds_p \qquad t_1 = 19.27\ \text{s} \quad \textbf{Ans.}$$

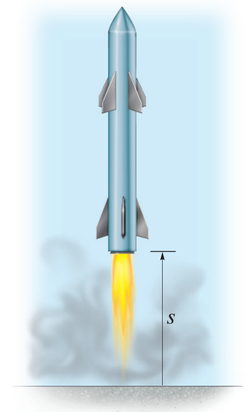
Ans:

$v_{p1} = 322.49\ \dfrac{\text{m}}{\text{s}}$

$t_1 = 19.27\ \text{s}$

***12–20.**

The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)\ \text{m/s}^2$, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100$ m. Initially, $v = 0$ and $s = 0$ when $t = 0$.



SOLUTION

$$a\,ds = v\,dv$$

$$\int_0^s (6 + 0.02\,s)\,ds = \int_0^v v\,dv$$

$$6\,s + 0.01\,s^2 = \frac{1}{2}v^2$$

$$v = \sqrt{12\,s + 0.02\,s^2}$$

$$ds = v\,dt$$

$$\int_0^{100} \frac{ds}{\sqrt{12\,s + 0.02\,s^2}} = \int_0^t dt$$

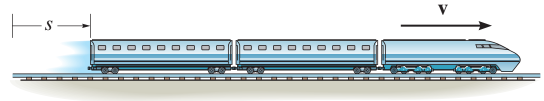
$$\frac{1}{\sqrt{0.02}}\ln\left[\sqrt{12s + 0.02s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}}\right]_0^{100} = t$$

$t = 5.62$ s **Ans.**

Ans:
$t = 5.62$ s

12–21.

When a train is traveling along a straight track at 2 m/s, it begins to accelerate at $a = (60\,v^{-4})$ m/s^2, where v is in m/s. Determine its velocity v and the position 3 s after the acceleration.



SOLUTION

$$a = \frac{dv}{dt}$$

$$dt = \frac{dv}{a}$$

$$\int_0^3 dt = \int_2^v \frac{dv}{60v^{-4}}$$

$$3 = \frac{1}{300}\left(v^5 - 32\right)$$

$$v = 3.925 \text{ m/s} = 3.93 \text{ m/s}$$ **Ans.**

$$a\,ds = v\,dv$$

$$ds = \frac{v\,dv}{a} = \frac{1}{60}v^5\,dv$$

$$\int_0^s ds = \frac{1}{60}\int_2^{3.925} v^5\,dv$$

$$s = \frac{1}{60}\left(\frac{v^6}{6}\right)\Bigg|_2^{3.925}$$

$$= 9.98 \text{ m}$$ **Ans.**

Ans:

$v = 3.93$ m/s

$s = 9.98$ m

12–22.

The acceleration of a particle along a straight line is defined by $a = (2t - 9)\ \text{m/s}^2$, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

SOLUTION

$a = 2t - 9$

$$\int_{10}^{v} dv = \int_{0}^{t} (2t - 9)\, dt$$

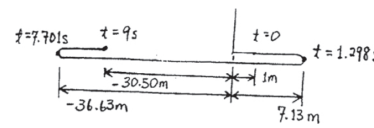
$v - 10 = t^2 - 9\,t$

$v = t^2 - 9\,t + 10$

$$\int_{1}^{s} ds = \int_{0}^{t} (t^2 - 9t + 10)\, dt$$

$$s - 1 = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t$$

$$s = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t + 1$$



Note when $v = t^2 - 9\,t + 10 = 0$:

$$t = 1.298 \text{ s and } t = 7.701 \text{ s}$$

When $t = 1.298$ s, $\quad s = 7.13$ m

When $t = 7.701$ s, $\quad s = -36.63$ m

When $t = 9$ s, $\quad s = -30.50$ m

(a) $\quad s = -30.5$ m **Ans.**

(b) $\quad s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

$\quad s_{Tot} = 56.0$ m **Ans.**

(c) $\quad v = 10$ m/s **Ans.**

Ans:
(a) $s = -30.5$ m
(b) $s_{\text{Tot}} = 56.0$ m
(c) $v = 10$ m/s

12–23.

If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s^2, where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \to \infty$).

SOLUTION

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81}\left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)}\right]$$

$$9.81t = 50\ln\left(\frac{1 + 0.01v}{1 - 0.01v}\right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \qquad \textbf{(1)}$$

a) When $t = 5$ s, then, from Eq. (1)

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5 \text{ m/s} \qquad \textbf{Ans.}$$

b) If $t \to \infty$, $\frac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \to 1$. Then, from Eq. (1)

$$v_{max} = 100 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
(a) $v = 45.5$ m/s
(b) $v_{max} = 100$ m/s

***12–24.**

A sandbag is dropped from a balloon which is ascending vertically at a constant speed of 6 m/s. If the bag is released with the same upward velocity of 6 m/s when $t = 0$ and hits the ground when $t = 8$ s, determine the speed of the bag as it hits the ground and the altitude of the balloon at this instant.

SOLUTION

$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$h = 0 + (-6)(8) + \frac{1}{2}(9.81)(8)^2$$

$$= 265.92 \text{ m}$$

During $t = 8$ s, the balloon rises

$$h' = vt = 6(8) = 48 \text{ m}$$

Altitude $= h + h' = 265.92 + 48 = 314$ m **Ans.**

$(+\downarrow) \qquad v = v_0 + a_c t$

$v = -6 + 9.81(8) = 72.5$ m/s **Ans.**

Ans:
$h = 314$ m
$v = 72.5$ m/s

12–25.

A particle is moving along a straight line such that its acceleration is defined as $a = (-2v)\ \text{m/s}^2$, where v is in meters per second. If $v = 20$ m/s when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

SOLUTION

$$a = -2v$$

$$\frac{dv}{dt} = -2v$$

$$\int_{20}^{v} \frac{dv}{v} = \int_0^t -2\, dt$$

$$\ln \frac{v}{20} = -2t$$

$$v = \left(20e^{-2t}\right)\ \text{m/s}$$ **Ans.**

$$a = \frac{dv}{dt} = \left(-40e^{-2t}\right)\ \text{m/s}^2$$ **Ans.**

$$\int_0^s ds = v\, dt = \int_0^t (20e^{-2t})dt$$

$$s = -10e^{-2t}\big|_0^t = -10\left(e^{-2t} - 1\right)$$

$$s = 10\left(1 - e^{-2t}\right)\ \text{m}$$ **Ans.**

Ans:
$v = \left(20e^{-2t}\right)$ m/s
$a = \left(-40e^{-2t}\right)$ m/s^2
$s = 10\left(1 - e^{-2t}\right)$ m