Chapter 1 Solutions

Problem 1.3-1

$$L = 14ft \qquad q_0 = 12\frac{lbf}{ft} \qquad P = 50lbf \qquad M_0 = 300lbf \cdot ft$$

Reactions

$$\Sigma F_x = 0 \qquad B_x = \frac{3}{5} \cdot P = 30 \cdot lbf$$

$$\Sigma M_A = 0 \qquad B_y = \frac{1}{L} \cdot \left[-M_0 + \left(\frac{1}{2} \cdot q_0\right) \cdot L \cdot \left(\frac{2 \cdot L}{3}\right) + \frac{4}{5} \cdot P \cdot \left(L + \frac{L}{2}\right)\right] = 94.571 \cdot lbf$$

$$\Sigma F_y = 0 \qquad A_y = \left(\frac{1}{2} \cdot q_0\right) \cdot L + \frac{4}{5} \cdot P - B_y = 29.429 \cdot lbf$$

N, V and M at midspan of AB - LHFB is used below

$$N_{mid} = 0$$

$$V_{mid} = A_y - \frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} = 8.429 \cdot lbf$$

$$M_{mid} = -M_0 + A_y \cdot \frac{L}{2} - \frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} \cdot \left(\frac{1}{3} \cdot \frac{L}{2}\right) = -143 \cdot lbf \cdot ft$$

Problem 1.3-2

$$L = 4m \qquad q_0 = 160\frac{N}{m} \qquad P = 200 \cdot N \qquad M_0 = 380N \cdot m$$

Reactions

$$\Sigma F_x = 0 \qquad B_x = \frac{-3}{5} \cdot P = -120\,N$$

$$\Sigma M_A = 0 \qquad B_y = \frac{1}{L} \cdot \left[M_0 + \left(\frac{1}{2} \cdot q_0\right) \cdot L \cdot \left(\frac{L}{3}\right) - \frac{4}{5} \cdot P \cdot \left(L + \frac{L}{2}\right)\right] = -38.333 \cdot N$$

$$\Sigma F_y = 0 \qquad A_y = \left(\frac{1}{2} \cdot q_0\right) \cdot L - \frac{4}{5} \cdot P - B_y = 198.333 \cdot N$$

N, V and M at midspan of AB - LHFB is used below

$$N_{mid} = 0$$

$$V_{mid} = A_y - \frac{1}{2} \cdot \left(\frac{q_0}{2} + q_0\right) \cdot \frac{L}{2} = -41.667 \cdot N$$

$$M_{mid} = M_0 + A_y \cdot \frac{L}{2} - \frac{1}{2} \cdot q_0 \cdot \frac{L}{2} \cdot \left(\frac{2}{3} \cdot \frac{L}{2}\right) - \frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} \cdot \left(\frac{1}{3} \cdot \frac{L}{2}\right) = 510 \cdot N \cdot m$$

Check using RHFB

$$N_{mid} = B_x + \frac{3}{5} \cdot P = 0\,N \qquad V_{mid} = \frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} - B_y - \frac{4}{5} \cdot P = -41.667\,N$$

$$M_{mid} = \frac{-1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} \cdot \left(\frac{1}{3} \cdot \frac{L}{2}\right) + B_y \cdot \frac{L}{2} + \frac{4}{5} \cdot P \cdot \left(\frac{L}{2} + \frac{L}{2}\right) = 510 \cdot N \cdot m$$

Problem 1.3-3

(a) APPLY LAWS OF STATICS

$\Sigma F_x = 0 \qquad C_x = 100\text{ lb} - 50\text{ lb} = 50\text{ lb}$

FBD of $BC \qquad \Sigma M_B = 0 \qquad C_y = \dfrac{1}{10\text{ ft}}(0) = 0$

Entire FBD $\qquad \Sigma M_A = 0 \qquad B_y = \dfrac{1}{20\text{ ft}}(-100\text{ lb-ft}) = -5\text{ lb}$

$\Sigma F_y = 0 \qquad A_y = -B_y = 5\text{ lb-ft}$

Reactions are $\boxed{A_y = 5\text{ lb}}$ $\boxed{B_y = -5\text{ lb}}$ $\boxed{C_x = 50\text{ lb}}$ $\boxed{C_y = 0}$

(b) INTERNAL STRESS RESULTANTS N, V, AND M AT $x = 15$ ft

Use FBD of segment from A to $x = 15$ ft

$\Sigma F_x = 0 \qquad \boxed{N = 100\text{ lb} - 50\text{ lb} = 50\text{ lb}}$

$\Sigma F_y = 0 \qquad \boxed{V = A_y = 5\text{ lb}}$

$\Sigma M = 0 \qquad \boxed{M = A_y 15\text{ ft} = 75\text{ lb-ft}}$

Problem 1.3-4

(a) APPLY LAWS OF STATICS

$\Sigma F_x = 0 \qquad A_x = 0$

FBD of $AB \qquad \Sigma M_B = 0 \qquad M_A = 0$

Entire FBD $\qquad \Sigma M_C = 0 \qquad D_y = \frac{1}{3\text{ m}}\left[200\text{ N}\cdot\text{m} - \frac{1}{2}(80\text{ N/m})\,4\text{ m}\left(\frac{2}{3}\right)4\text{ m}\right] = -75.556\text{ N}$

$\Sigma F_y = 0 \qquad C_y = \frac{1}{2}(80\text{ N/m})\,4\text{ m} - D_y = 235.556\text{ N}$

Reactions are $\boxed{M_A = 0} \qquad A_x = 0 \qquad \boxed{C_y = 236\text{ N}} \qquad \boxed{D_y = -75.6\text{ N}}$

(b) INTERNAL STRESS RESULTANTS N, V, AND M AT $x = 5$ m

Use FBD of segment from A to $x = 5$ m; ordinate on triangular load at $x = 5$ m is $\frac{3}{4}(80\text{ N/m}) = 60\text{ N/m}$.

$\Sigma F_x = 0 \qquad N_x = -A_x = 0$

$\Sigma F_y = 0 \qquad V = \frac{-1}{2}[(80\text{ N/m} + 60\text{ N/m})\,1\text{ m}] = -70\text{ N} \qquad \boxed{V = -70\text{ N}}$ Upward

$\Sigma M = 0 \qquad M = -M_A - \frac{1}{2}(80\text{ N/m})\,1\text{ m}\left(\frac{2}{3}1\text{ m}\right) - \frac{1}{2}(60\text{ N/m})\,1\text{ m}\left(\frac{1}{3}1\text{ m}\right) = -36.667\text{ N}\cdot\text{m}$

(break trapezoidal load into two triangular loads in moment expression)

$\boxed{M = -36.7\text{ N}\cdot\text{m}}$ CW

(c) REPLACE ROLLER SUPPORT AT C WITH SPRING SUPPORT

Structure remains statically determinate so all results above in (a) and (b) are unchanged.

Problem 1.3-5

(a) STATICS

FBD of AB (cut through beam at pin): $\Sigma M_B = 0 \qquad A_y = \dfrac{1}{10\text{ ft}}(-150\text{ lb-ft}) = -15\text{ lb}$

Entire FBD: $\Sigma M_D = 0$

$$C_y = \frac{1}{10\text{ ft}}\left[\frac{4}{5}40\text{ lb}(5\text{ ft}) + \frac{1}{2}(2.5\text{ lb/ft})10\text{ ft}\left(10\text{ ft} + \frac{10\text{ ft}}{3}\right) + \frac{1}{2}(5\text{ lb/ft})10\text{ ft}\left(10\text{ ft} + \frac{2}{3}10\text{ ft}\right)\right.$$
$$\left. - 150\text{ lb-ft} - A_y\,30\text{ ft}\right] = 104.333\text{ lb}$$

$$\Sigma F_y = 0 \quad D_y = \frac{4}{5}40\text{ lb} + \frac{1}{2}(5\text{ lb/ft} + 2.5\text{ lb/ft})\,10\text{ ft} - A_y - C_y = -19.833\text{ lb} \quad \text{so} \quad D_x = \frac{-D_y}{\tan(60^\circ)} = 11.451\text{ lb}$$

$$\Sigma F_x = 0 \quad A_x = \frac{3}{5}40\text{ lb} - D_x = 12.549\text{ lb}$$

$$\boxed{A_x = 12.55\text{ lb}, A_y = -15\text{ lb}, C_y = 104.3\text{ lb}, D_x = 11.45\text{ lb}, D_y = -19.83\text{ lb}}$$

(b) USE FBD OF AB ONLY; MOMENT AT PIN IS ZERO

$$F_{Bx} = -A_x \qquad F_{Bx} = -12.55\text{ lb} \qquad F_{By} = -A_y \qquad F_{By} = 15\text{ lb} \qquad \boxed{\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 19.56\text{ lb}}$$

(c) ADD ROTATIONAL SPRING AT A AND REMOVE ROLLER AT C; APPLY EQUATIONS OF STATICAL EQUILIBRIUM

Use FBD of BCD $\Sigma M_B = 0$

$$D_y = \frac{1}{20\text{ ft}}\left[\frac{1}{2}(2.5\text{ lb/ft})\,10\text{ ft}\left(\frac{2}{3}10\text{ ft}\right) + \frac{1}{2}(5\text{ lb/ft})\,10\text{ ft}\left(\frac{1}{3}10\text{ ft}\right) + \frac{4}{5}40\text{ lb}\,(15\text{ ft})\right] = 32.333\text{ lb}$$

so $\qquad D_x = \dfrac{-D_y}{\tan(60^\circ)} = -18.668\text{ lb}$

Use entire FBD $\qquad \Sigma F_y = 0 \qquad A_y = \dfrac{1}{2}(5\text{ lb/ft} + 2.5\text{ lb/ft})\,10\text{ ft} + \dfrac{4}{5}(40\text{ lb}) - D_y = 37.167\text{ lb}$

$\qquad \Sigma F_x = 0 \qquad A_x = \dfrac{3}{5}(40\text{ lb}) - D_x = 42.668\text{ lb}$

Use FBD of AB $\qquad \Sigma M_B = 0 \qquad M_A = 150\text{ lb-ft} + A_y 10\text{ ft} = 521.667\text{ lb-ft}$

SO REACTIONS ARE $\boxed{A_x = 42.7\text{ lb}}$ $\boxed{A_y = 37.2\text{ lb}}$ $\boxed{M_A = 522\text{ lb-ft}}$ $\boxed{D_x = -18.67\text{ lb}}$ $\boxed{D_y = 32.3\text{ lb}}$

RESULTANT FORCE IN PIN CONNECTION AT B

$$F_{Bx} = -A_x \qquad F_{By} = -A_y \qquad \boxed{\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 56.6\text{ lb}}$$

Problem 1.3-6

(a) STATICS

$\Sigma F_y = 0 \qquad R_{3y} = 20\text{ N} - 45\text{ N} = -25\text{ N}$

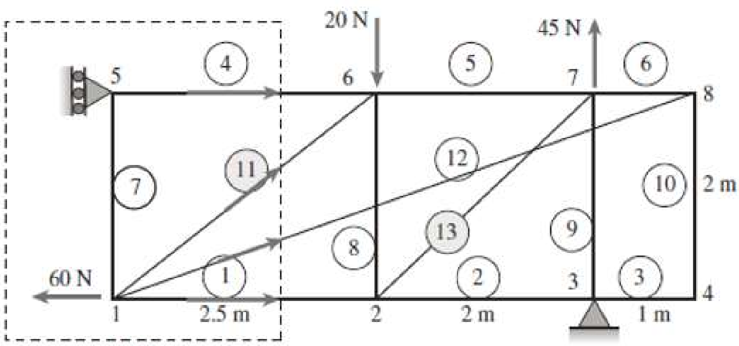
$\Sigma M_3 = 0 \qquad R_{5x} = \frac{1}{2\text{ m}}(20\text{ N} \times 2\text{ m}) = 20\text{ N}$

$\Sigma F_x = 0 \qquad R_{3x} = -R_{5x} + 60\text{ N} = 40\text{ N}$

(b) MEMBER FORCES IN MEMBERS 11 and 13

Number of unknowns: $m = 13 \qquad r = 3 \qquad m + r = 16$

Number of equations: $j = 8 \qquad 2j = 16$ So statically determinate



Section cut for left FBD

TRUSS ANALYSIS

(1) $\Sigma F_V = 0$ at joint 4 so $F_{10} = 0$

(2) $\Sigma F_V = 0$ at joint 8 so $F_{12} = 0$

(3) $\Sigma F_H = 0$ at joint 5 so $F_4 = -R_{5x} = -20\text{ N}$

(4) Cut vertically through 4, 11, 12, and 1; use left FBD; sum moments about joint 2

$F_{11V} = \frac{1}{2.5\text{ m}}(R_{5x} - F_4)$ so $\boxed{F_{11} = 0}$

(5) Sum vertical forces at joint 3; $F_9 = -R_{3y}$

$F_9 = 25\text{ N}$

(6) Sum vertical forces at joint 7 $\qquad F_{13V} = 45\text{ N} - F_9 = 20\text{ N} \qquad \boxed{F_{13} = \sqrt{2}F_{13V} = 28.3\text{ N}}$

Problem 1.3-7

(a) Statics

$\Sigma F_x = 0 \qquad A_x = 0$

$\Sigma M_A = 0 \qquad E_y = \frac{1}{20\text{ ft}}(3\text{ k} \times 10\text{ ft} + 2\text{ k} \times 20\text{ ft} + 1\text{ k} \times 30\text{ ft}) = 5\text{ k}$

$\Sigma F_y = 0 \qquad A_y = 3\text{ k} + 2\text{ k} + 1\text{ k} - E_y = 1\text{ k}$

(b) Member force in member *FE*

Number of unknowns: $m = 11 \qquad r = 3 \qquad m + r = 14$

Number of equations: $j = 7 \qquad 2j = 14$ So statically determinate

Truss analysis

(1) Cut vertically through *AB*, *GC*, and *GF*; use left FBD; sum moments about *C*

$$F_{GFx}(15\text{ ft}) - F_{GFy}(20\text{ ft}) = A_y(20\text{ ft}) = 20\text{ ft-k} \qquad F_{GFx} = F_{GF}\frac{10}{\sqrt{2^2 + 10^2}} \qquad F_{GFy} = F_{GF}\frac{2}{\sqrt{2^2 + 10^2}}$$

$$\text{so} \quad F_{GF} = \frac{A_y(20\text{ ft})}{15\text{ ft}\frac{10}{\sqrt{2^2 + 10^2}} - 20\text{ ft}\frac{2}{\sqrt{2^2 + 10^2}}} = 1.854\text{ k} \quad \text{and} \quad F_{GFx} = F_{GF}\frac{10}{\sqrt{2^2 + 10^2}} = 1.818\text{ k}$$

(2) Sum horizontal forces at joint *F* $\qquad F_{FEx} = F_{GFx} = 1.818\text{ k} \qquad F_{FE} = \frac{\sqrt{10^2 + 3^2}}{10}F_{FEx} = 1.898\text{ k}$

$$\boxed{F_{FE} = 1.898\text{ k}}$$

Problem 1.3-8

(a) STATICS

$\Sigma F_x = 0 \qquad F_x = 0$

$\Sigma M_F = 0 \qquad D_y = \frac{1}{6\text{ m}}\left[3\text{ kN}(6\text{ m}) + 6\text{ kN}(3\text{ m})\right] = 6\text{ kN}$

$\Sigma F_y = 0 \qquad F_y = 9\text{ kN} + 6\text{ kN} + 3\text{ kN} - D_y = 12\text{ kN}$

(b) MEMBER FORCE IN MEMBER *FE*

Number of unknowns: $m = 11 \qquad r = 3 \qquad m + r = 14$

Number of equations: $j = 7 \qquad 2j = 14$ So statically determinate

TRUSS ANALYSIS

(1) Cut vertically through *AB*, *GD*, and *GF*; use left FBD; sum moments about *D* to get $F_{GF} = 0$

(2) Sum horizontal forces at joint *F* $\quad F_{FEx} = -F_x = 0 \quad$ so $\boxed{F_{FE} = 0}$

Problem 1.3-9

$c = 8\,\text{ft} \quad P = 20\,\text{kip}$

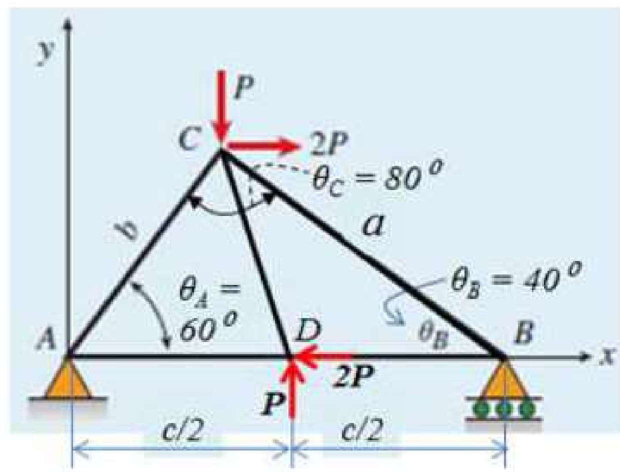
$a = \dfrac{\sin(60\text{deg})}{\sin(80\text{deg})} \cdot c = 7.035 \cdot \text{ft} \qquad b = \dfrac{\sin(40\text{deg})}{\sin(80\text{deg})} \cdot c = 5.222 \cdot \text{ft}$

$\Sigma M_A = P \cdot \dfrac{c}{2} - P \cdot b \cdot \cos(60\text{deg}) - 2P \cdot b \cdot \sin(60\text{deg}) + B_y \cdot c = 0$

$B_y = \dfrac{P \cdot b \cdot \cos(60\text{deg}) + 2P \cdot b \cdot \sin(60\text{deg}) - P \cdot \dfrac{c}{2}}{c} = 19.137 \cdot \text{kip}$

$A_y = -B_y = -19.137 \cdot \text{kip}$

$A_x = 0$



Joint A

$F_{AC} = \dfrac{-A_y}{\sin(60\text{deg})} = 22.098 \cdot \text{kip}$

$F_{AD} = -F_{AC} \cdot \cos(60\text{deg}) - A_x = -11.049 \cdot \text{kip}$

Joint B

$F_{BC} = \dfrac{-B_y}{\sin(40\text{deg})} = -29.772 \cdot \text{kip}$

$F_{BD} = -F_{BC} \cdot \cos(40\text{deg}) = 22.807 \cdot \text{kip}$

$CD = \sqrt{b^2 + \left(\dfrac{c}{2}\right)^2 - 2 \cdot b \cdot \dfrac{c}{2} \cdot \cos(60\text{deg})} = 4.731 \cdot \text{ft}$

$ACD = \text{asin}\left(\dfrac{\sin(60\text{deg})}{CD} \cdot \dfrac{c}{2}\right) = 47.077 \cdot \text{deg}$

Joint D

$$F_{DC} = \frac{-P}{\cos(90deg - 72.923deg)} = -20.922 \cdot kip$$

$$180deg - 60deg - ACD = 72.923 \cdot deg$$

$$BCD = asin\left(\frac{\sin(40deg)}{CD} \cdot \frac{c}{2}\right) = 32.923 \cdot deg$$

$$\frac{a \cdot \sin(BCD)}{\sin(ACD)} = 5.222 \cdot ft$$

$$ACD + BCD = 80 \cdot deg$$

Problem 1.3-10

Geometry $b = 3\text{m}$ $P = 80\text{kN}$

$$a = \sin(60\text{deg})\cdot\left(\frac{b}{\sin(40\text{deg})}\right) = 4.042\,\text{m}$$

$$L_{AB} = \sin(80\text{deg})\cdot\left(\frac{b}{\sin(40\text{deg})}\right) = 4.596\,\text{m}$$

$$L_{DB} = \sqrt{\left(\frac{b}{2}\right)^2 + L_{AB}^{\ 2} - 2\cdot\left(\frac{b}{2}\right)\cdot(L_{AB})\cdot\cos(60\text{deg})} = 4.06\,\text{m}$$

$$\frac{L_{DB}}{\sin(60\text{deg})} = \frac{\frac{b}{2}}{\sin(DBA)} \quad \text{so} \quad DBA = \text{asin}\left(\frac{\frac{b}{2}}{L_{DB}}\cdot\sin(60\text{deg})\right) = 18.662\cdot\text{deg}$$

and $CBD = 40\text{deg} - DBA = 21.338\cdot\text{deg}$ $ADB = 180\text{deg} - 60\text{deg} - DBA = 101.338\cdot\text{deg}$

$CDB = 180\text{deg} - ADB = 78.662\cdot\text{deg}$

Reactions

$\Sigma F_x = 0$ $A_x = -2\cdot P + 2\cdot P = 0\,\text{N}$

$$\Sigma M_A = 0 \quad B_y = \frac{1}{L_{AB}}\cdot\left[-2\cdot P\cdot\left(\frac{b}{2}\cdot\sin(60\text{deg})\right) + P\cdot(b\cdot\cos(60\text{deg})) + 2\cdot P\cdot(b\cdot\sin(60\text{deg}))\right] = 71.329\cdot\text{kN}$$

$\Sigma F_y = 0$ $A_y = P - B_y = 8.671\cdot\text{kN}$

MoJ to find member forces

Joint A $AD = \dfrac{-A_y}{\sin(60\text{deg})} = -10.013\cdot\text{kN}$ $AB = -A_x - AD\cdot\cos(60\text{deg}) = 5.006\cdot\text{kN}$

Joint D - sum forces normal to & along line ADC $DB = \dfrac{2\cdot P\cdot\sin(60\text{deg})}{\cos(90\text{deg} - CDB)} = 141.322\cdot\text{kN}$

$$DC = AD + 2\cdot P\cdot(\cos(60\text{deg})) - DB\cdot\cos(CDB) = 42.204\cdot\text{kN}$$

Joint C $CB = \dfrac{1}{\cos(40\text{deg})}\cdot(-2\cdot P + DC\cdot\cos(60\text{deg})) = -181.319\cdot\text{kN}$

Joint B check $-AB - DB\cdot\cos(DBA) - CB\cdot\cos(40\text{deg}) = 0\,\text{N}$

$$DB\cdot\sin(DBA) + CB\cdot\sin(40\text{deg}) + B_y = 0\,\text{N}$$

Problem 1.3-11

Reactions $\quad c = 8ft \quad P = 20kip$

$A_x = 0 \qquad A_y = -19.137kip \qquad B_y = -A_y$

AC: MoS - cut through AC and AD, use LHFB

$\Sigma M_D = 0 \qquad -A_y \cdot \frac{c}{2} - AC \cdot \sin(60deg) \cdot \frac{c}{2} = 0$

$$AC = \frac{-A_y}{\sin(60deg)} = 22.098 \cdot kip$$

BD: MoS - cut through BC andf BD, use RHFB $\qquad b = \frac{\sin(40deg)}{\sin(80deg)} \cdot c = 5.222 \cdot ft$

$\Sigma M_C = 0 \qquad B_y \cdot (c - b \cdot \cos(60deg)) - BD \cdot (b \cdot \sin(60deg)) = 0 \qquad BD = \frac{B_y \cdot (c - b \cdot \cos(60deg))}{b \cdot \sin(60deg)} = 22.807 \cdot kip$

Problem 1.3-12

Reactions $\quad b = 3m \quad P = 80kN$

$A_x = 0 \quad A_y = 8.671kN \quad B_y = 71.329kN$

AB: MoS - cut through AD and AB, use LHFB

$$\Sigma M_D = 0 \qquad AB\cdot\frac{b}{2}\cdot\sin(60deg) + A_x\cdot\frac{b}{2}\cdot\sin(60deg) - A_y\cdot\frac{b}{2}\cdot\cos(60deg) = 0$$

$$AB = \frac{-\left(A_x\cdot\frac{b}{2}\cdot\sin(60deg) - A_y\cdot\frac{b}{2}\cdot\cos(60deg)\right)}{\left(\frac{b}{2}\cdot\sin(60deg)\right)} = 5.006\cdot kN$$

DC: MoS - cut through DC and CB, use upper FBD $\qquad a = \sin(60deg)\cdot\left(\frac{b}{\sin(40deg)}\right) = 4.042\,m$

$$DC_x = DC\cdot\cos(60deg) \qquad DC_y = DC\cdot\sin(60deg)$$

$$\Sigma M_B = 0 \qquad -\left(-DC_x + 2\cdot P\right)\cdot(a\cdot\sin(40deg)) + \left(DC_y + P\right)\cdot(a\cdot\cos(40deg)) = 0$$

$$-(-DC\cdot\cos(60deg) + 2\cdot P)\cdot(a\cdot\sin(40deg)) + (DC\cdot\sin(60deg) + P)\cdot(a\cdot\cos(40deg)) = 0$$

Collect and simplify, solve for DC

$$DC = \frac{1.0\cdot(80.0\cdot kN\cdot\cos(40.0\cdot deg) - 160.0\cdot kN\cdot\sin(40.0\cdot deg))}{\cos(60.0\cdot deg)\cdot\sin(40.0\cdot deg) + \sin(60.0\cdot deg)\cdot\cos(40.0\cdot deg)} = 42.204\cdot kN$$

Problem 1.3-13

(a) FIND REACTIONS USING STATICS $\quad m = 3 \quad r = 9 \quad m + r = 12 \quad j = 4 \quad 3j = 12$

$m + r = 3j$ So truss is statically determinate

$$r_{AQ} = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} \quad r_{OA} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \quad e_{AQ} = \frac{r_{AQ}}{|r_{AQ}|} = \begin{pmatrix} 0.8 \\ -0.6 \\ 0 \end{pmatrix} \quad P_A = P e_{AQ} = \begin{pmatrix} 0.8P \\ -0.6P \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

$\Sigma M = 0$

$$M_O = r_{OA} \times P_A + r_{OC} \times \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} + r_{OB} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 4C_z + 3.0P \\ 4.0P - 2B_z \\ 2B_y - 4C_x \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \text{ gives } C_z = \frac{-3}{4}P$$

$\Sigma M_y = 0$ gives $\boxed{B_z = 2\,P}$

$\Sigma F = 0$

$$R_O = P_A + \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} = \begin{pmatrix} B_x + C_x + O_x + 0.8P \\ B_y + C_y + O_y + -0.6P \\ O_z + \frac{5P}{4} \end{pmatrix} \quad \text{so} \quad \Sigma M_z = 0 \text{ gives } \boxed{O_z = \frac{-5}{4}P}$$

METHOD OF JOINTS $\quad$ Joint O $\quad \Sigma F_x = 0 \quad O_x = 0 \quad \Sigma F_y = 0 \quad O_y = 0$

Joint B $\quad \Sigma F_y = 0 \quad B_y = 0$

Joint C $\quad \Sigma F_x = 0 \quad C_x = 0$

For entire structure $\quad \Sigma F_x = 0$ gives $\boxed{B_x = -0.8P} \quad \Sigma F_y = 0 \quad C_y = 0.6P - B_y = O_y \quad C_y = 0.6P$

(b) FORCE IN MEMBER AC

$\Sigma F_z = 0$ at joint C $\quad F_{AC} = \dfrac{\sqrt{4^2 + 5^2}}{5}|C_Z| = \dfrac{3\sqrt{41}\,|P|}{20} \quad \boxed{F_{AC} = \dfrac{3\sqrt{41}}{20}P}$ tension $\quad \dfrac{3\sqrt{41}}{20} = 0.96$

Problem 1.3-14

(a) FIND REACTIONS USING STATICS $m = 4 \quad r = 8 \quad m + r = 12 \quad j = 4 \quad 3j = 12$

$m + r = 3j$ so truss is statically determinate

$$r_{OA} = \begin{pmatrix} 0 \\ 0 \\ 0.8L \end{pmatrix} \quad r_{OB} = \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 0.6L \\ 0 \end{pmatrix} \quad F_A = \begin{pmatrix} A_x \\ A_y \\ P \end{pmatrix} \quad F_B = \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} \quad F_c = \begin{pmatrix} C_x \\ -2P \\ 0 \end{pmatrix} \quad F_O = \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix}$$

$\Sigma M = 0$

Resultant moment at O

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B + r_{OC} \times F_C = \begin{pmatrix} -0.8A_yL \\ 0.8A_xL - B_zL \\ B_yL - 0.6C_xL \end{pmatrix} \quad \textit{so} \quad \Sigma M_x = 0 \text{ gives } A_y = 0$$

$\Sigma F = 0$

Resultant force at O

$$R_O = F_O + F_A + F_B + F_C = \begin{pmatrix} A_x + C_x + O_x \\ A_y + B_y + O_y - 2P \\ B_z + O_z + P \end{pmatrix}$$

METHOD OF JOINTS Joint O $\quad \Sigma F_z = 0 \quad O_z = 0$

so from $\Sigma F_z = 0$ $\boxed{B_z = -P}$ and $\Sigma M_y = 0$ $\boxed{A_x = \frac{B_z}{0.8} = -1.25P}$

Joint B $\quad \Sigma F_y = 0$ $\boxed{B_y = 0}$

Joint C $\quad \Sigma F_x = 0 \quad C_x = 0$

(b) FORCE IN MEMBER AB

$$\Sigma F_z = 0 \text{ at joint } B \quad F_{AB} = \frac{\sqrt{(0.8L)^2 + L^2}}{0.8L}|B_z| \quad |B_z| = |P| \quad \frac{\sqrt{(0.8L)^2 + L^2}}{0.8L} = 1.601$$

$\boxed{F_{AB} = 1.601P}$ tension

Problem 1.3-15

(a) FIND REACTIONS USING STATICS $m = 3$ $r = 6$ $m + r = 9$ $j = 3$ $3j = 9$

$m + r = 3j$ So truss is statically determinate

$$r_{OA} = \begin{pmatrix} 3L \\ 0 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 0 \\ 4L \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 2L \\ 4L \end{pmatrix} \quad F_A = \begin{pmatrix} -2P \\ A_y \\ A_z \end{pmatrix} \quad F_B = \begin{pmatrix} B_x \\ B_y \\ 3P \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ C_y \\ P \end{pmatrix}$$

$\Sigma M = 0$

Resultant moment at O

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B + r_{OC} \times F_C = \begin{pmatrix} 14LP - 4C_yL \\ 4C_xL - 3A_zL \\ 3A_yL - 4B_xL - 2C_xL \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \quad \text{gives} \quad C_y = \frac{14}{4}P$$

$\Sigma F = 0$

Resultant force at O

$$R_O = F_A + F_B + F_C = \begin{pmatrix} B_x + C_x - 2P \\ A_y + B_y + C_y \\ A_z + 4P \end{pmatrix} \quad \text{so} \quad \Sigma F_z = 0 \quad \text{gives} \quad \boxed{A_z = -4.0P}$$

METHOD OF JOINTS

Joint A $\quad \Sigma F_z = 0 \quad F_{ACz} = -A_z = 4.0P \quad$ so $\quad F_{ACy} = \frac{2}{4}F_{ACz} = 2.0P \quad F_{ACx} = \frac{3}{4}F_{ACz} = 3.0P$

$\Sigma F_x = 0 \quad F_{ABx} = -2P - F_{ACx} = -3.0P - 2P \quad$ so $\quad F_{ABy} = \frac{4}{3}F_{ABx} = -4.0P - \frac{8P}{3}$

$\Sigma F_y = 0 \quad A_y = -(F_{ABy} + F_{ACy}) = \frac{8P}{3} + 4.0P + -2.0P \quad \boxed{A_y = 4.67P}$

(b) FORCE IN MEMBER AB

$$F_{AB} = \sqrt{F_{ABx}{}^2 + F_{ABy}{}^2} \qquad F_{AB} = -\sqrt{5^2 + \left(\frac{20}{3}\right)^2}\,P = -\frac{25P}{3} \qquad \frac{25}{3} = 8.33$$

$\boxed{F_{AB} = -8.33P}$ compression

Problem 1.3-16

(a) FIND REACTIONS USING STATICS $\quad m = 3 \quad r = 6 \quad m + r = 9 \quad j = 3 \quad 3j = 9$

$m + r = 3j$ so truss is statically determinate

$L = 2\text{ m} \quad P = 5\text{ kN}$

$$r_{OA} = \begin{pmatrix} 3L \\ 0 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 0 \\ 4L \\ 2L \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 0 \\ 4L \end{pmatrix} \quad F_A = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \quad F_B = \begin{pmatrix} B_x \\ 0 \\ P \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ C_y \\ -P \end{pmatrix}$$

$\Sigma F = 0$

Resultant force at O $\quad R_O = F_A + F_B + F_C = \begin{pmatrix} A_x + B_x + C_x \\ A_y + C_y \\ A_z \end{pmatrix}$ so $\Sigma F_z = 0$ gives $A_z = 0$

RESULTANT MOMENT AT A

$$r_{AC} = \begin{pmatrix} -3L \\ 0 \\ 4L \end{pmatrix} \quad e_{AC} = \frac{r_{AC}}{|r_{AC}|} = \begin{pmatrix} -0.6 \\ 0 \\ 0.8 \end{pmatrix} \quad r_{AB} = \begin{pmatrix} -3L \\ 4L \\ 2L \end{pmatrix}$$

$$M_A = r_{AB} \times F_B + r_{AC} \times F_C = \begin{pmatrix} 40\cdot\text{kN}\cdot\text{m} - 8\cdot C_y\cdot\text{m} \\ 4\cdot B_x\cdot\text{m} + 8\cdot C_x\cdot\text{m} \\ -8\cdot B_x\cdot\text{m} - 6\cdot C_y\cdot\text{m} \end{pmatrix} \quad M_A e_{AC} = -\,6.4\,B_x - 24.0\text{ kN} \quad \text{so} \quad \boxed{B_x = \frac{-24}{6.4}\text{kN} = -3.75\text{ kN}}$$

(b) FORCE IN MEMBER AB

Method of joints at B $\quad \Sigma F_x = 0 \quad F_{ABx} = -B_X \quad \boxed{F_{AB} = \frac{\sqrt{29}}{3}F_{ABx} = 6.73\text{ kN}}$

Problem 1.3-17

(a) APPLY LAWS OF STATICS $L_1 = 30$ in. $L_2 = 20$ in. $T_1 = 21000$ lb-in. $T_2 = 10000$ lb-in.

$\Sigma M_x = 0$ $\boxed{T_A = T_1 - T_2 = 11{,}000 \text{ lb-in.}}$

(b) INTERNAL STRESS RESULTANT T AT TWO LOCATIONS

Cut shaft at midpoint between A and B at $x = L_1/2$ (use left FBD) $\Sigma M_x = 0$ $\boxed{T_{AB} = -T_A = -11{,}000 \text{ lb-in.}}$

Cut shaft at midpoint between B and C at $x = L_1 + L_2/2$ (use right FBD) $\Sigma M_x = 0$ $\boxed{T_{BC} = T_2 = 10{,}000 \text{ lb-in.}}$

Problem 1.3-18

(a) REACTION TORQUE AT A $\quad L_1 = 0.75\text{ m} \quad L_2 = 0.75\text{ m} \quad t_1 = 3100\text{ N·m/m} \quad T_2 = 1100\text{ N·m}$

Statics $\quad \Sigma M_x = 0 \quad T_A = -t_1 L_1 + T_2 = -1225\text{ N·m} \quad \boxed{T_A = -1225\text{ N·m}}$

(b) INTERNAL TORSIONAL MOMENTS AT TWO LOCATIONS

Cut shaft between A and B (use left FBD) $\quad T_1(x) = -T_A - t_1 x \quad \boxed{T_1\left(\frac{L_1}{2}\right) = 62.5\text{ N·m}}$

Cut shaft between B and C (use left FBD) $\quad T_2(x) = -T_A - t_1 L_1 \quad \boxed{T_2\left(L_1 + \frac{L_2}{2}\right) = -1100\text{ N·m}}$

Problem 1.3-19

(a) Statics

$\Sigma F_H = 0 \qquad A_x = \frac{-1}{2}(90\ \text{lb/ft})\,12\ \text{ft} = -540\ \text{lb}$

$\Sigma F_V = 0 \qquad A_y + C_y = 0$

$\Sigma M_{\text{FBD}BC} = 0 \qquad C_y = \frac{500\ \text{lb-ft}}{9\ \text{ft}} = 55.6\ \text{lb} \qquad A_y = -C_y = -55.6\ \text{lb}$

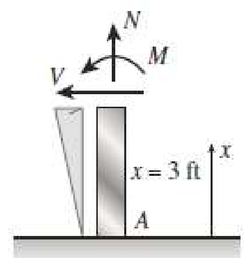
$\Sigma M_A = 0 \qquad M_A = 500\ \text{lb-ft} + \frac{1}{2}(90\ \text{lb/ft})\,12\ \text{ft}\left(\frac{2}{3}12\ \text{ft}\right) - C_y\,9\ \text{ft} = 4320\ \text{lb-ft}$

(b) Internal stress resultants

$N = -A_y = 55.6\ \text{lb}$

$V = -A_x - \frac{1}{2}\left(\frac{3}{12}\,90\ \text{lb/ft}\right)3\ \text{ft} = 506\ \text{lb}$

$M = -M_A - A_x 3\ \text{ft} - \frac{1}{2}\left(\frac{3}{12}\,90\ \text{lb/ft}\right)3\ \text{ft}\left(\frac{1}{3}3\ \text{ft}\right) = -2734\ \text{lb-ft}$



Problem 1.3-20

(a) STATICS

$\Sigma F_x = 0 \qquad A_x = \frac{3}{5}(200\text{ N}) + \frac{1}{2}(80\text{ N/m})\,4\text{ m} = 280\text{ N}$

$\Sigma M_{BRHFB} = 0 \quad D_y = \frac{1}{3\text{ m}}\left[\frac{4}{5}(200\text{ N})(1.5\text{ m}) + \frac{1}{2}(80\text{ N/m})\,4\text{ m}\left(\frac{1}{3}4\text{ m}\right)\right]$

$= 151.1\text{ N}$ < use right hand FBD (*BCD* only)

$\Sigma F_y = 0 \qquad A_y = -D_y + \frac{4}{5}(200\text{ N}) = 8.89\text{ N}$

$\Sigma M_A = 0 \qquad M_A = \frac{4}{5}(200\text{ N})(1.5\text{ m}) - \frac{3}{5}(200\text{ N})(4\text{ m}) - D_y\,3\text{ m} - \frac{1}{2}(80\text{ N/m})\,4\text{ m}\left(\frac{2}{3}4\text{ m}\right) = -1120\text{ N·m}$

(b) RESULTANT FORCE IN PIN AT *B*

LEFT HAND FBD (SEE FIGURE)

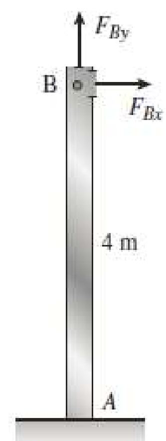
$F_{Bx} = -A_x = -280\text{ N} \qquad F_{By} = -A_y = -8.89\text{ N}$

RIGHT HAND FBD

$F_{Bx} = \frac{3}{5}(200\text{ N}) + \frac{1}{2}(80\text{ N/m})\,4\text{ m} = 280\text{ N}$

$F_{By} = \frac{4}{5}(200\text{ N}) - D_y = 8.89\text{ N}$

$\boxed{\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 280\text{ N}}$



Left hand FBD

Problem 1.3-21

$$L = 14\text{ft} \quad q_o = 12\frac{\text{lbf}}{\text{ft}} \quad P = 50\text{lbf} \quad M_o = 300\text{lbf}\cdot\text{ft}$$

$$\Sigma M_D = -M_o + \frac{1}{2}\cdot q_o\cdot L\cdot\frac{L}{3} - \frac{4}{5}\cdot P\cdot\frac{L}{2} + \frac{3}{5}P\cdot L - \frac{1}{2}\cdot q_o\cdot L\cdot\frac{2L}{3} - A_y\cdot L = 0$$

$$A_y = \frac{-M_o + \frac{1}{2}\cdot q_o\cdot L\cdot\frac{L}{3} - \frac{4}{5}\cdot P\cdot\frac{L}{2} + \frac{3}{5}P\cdot L - \frac{1}{2}\cdot q_o\cdot L\cdot\frac{2L}{3}}{L} = -39.429\cdot\text{lbf}$$

$$D_y = -A_y + \frac{1}{2}\cdot q_o\cdot L + \frac{4}{5}\cdot P = 163.429\cdot\text{lbf}$$

$$D_x = \frac{-1}{2}\cdot q_o\cdot L + \frac{3}{5}\cdot P = -54\cdot\text{lbf}$$

$$V_{midAB} = A_y - \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{q_o}{2} = -60.429\cdot\text{lbf} \qquad N_{mid} = 0$$

$$M_{midAB} = M_o + A_y\cdot\frac{L}{2} - \frac{1}{2}\cdot\frac{q_o}{2}\cdot\frac{L}{2}\cdot\frac{\frac{L}{2}}{3} = -25\cdot\text{lbf}\cdot\text{ft}$$

Problem 1.3-22

$$L = 4m \qquad q_0 = 160 \cdot \frac{N}{m} \qquad P = 200N \qquad M_0 = 380 \cdot N \cdot m$$

Reactions

$\Sigma F_x = 0 \qquad A_x = 2 \cdot \left(\frac{3}{5} \cdot P\right) - \frac{1}{2} \cdot q_0 \cdot L = -80\,N$

$\Sigma M_A = 0 \qquad D_y = \frac{1}{L} \cdot \left[M_0 + \frac{4}{5} \cdot P \cdot \frac{L}{2} + \frac{4}{5} \cdot P \cdot \frac{3 \cdot L}{2} - \frac{1}{2} \cdot q_0 \cdot L \cdot \left(\frac{L}{3}\right)\right] = 308.333\,N$

$\Sigma F_y = 0 \qquad A_y = -D_y + 2 \cdot \left(\frac{4}{5} \cdot P\right) = 11.667\,N$

Column BD internal forces and moment at mid-height - cut through column, use lower FBD (D on your left)

$N_{mid} = -D_y = -308.333\,N \qquad V_{mid} = \frac{-1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} = -80\,N \qquad M_{mid} = -\left(\frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2}\right) \cdot \left(\frac{1}{3} \cdot \frac{L}{2}\right) = -53.333 \cdot N \cdot m$

Problem 1.3-23

$$L_{BC} = \frac{\frac{4}{5}\cdot 30in}{\frac{2}{\sqrt{5}}} = 26.833\cdot in \qquad L_{AC} = \frac{3}{5}\cdot(30in) + \frac{1}{\sqrt{5}}\cdot L_{BC} = 30\cdot in$$

Part (a) - statics

$\Sigma M_A = 0$ $\qquad C_y = \frac{1}{L_{AC}}\cdot\left(200lb\cdot\frac{1}{2}\cdot\frac{3}{5}\cdot 30in\right) = 60\,lbf$ $\qquad C_x = \frac{-1}{2}\cdot C_y = -30\,lbf$

(resultant of C_x and C_y acts along line of strut)

$\Sigma F_x = 0$ $\qquad A_x = -C_x = 30\,lbf$

$\Sigma F_y = 0$ $\qquad A_y = 200lb - C_y = 140\,lbf$

Part (b) - internal stress resultants N, V, M

distributed weight of door in -y dir. $\qquad w = \frac{200lb}{30in} = 6.667\cdot\frac{lb}{in}$

components of w along and perpendicular to door

$$w_a = \frac{4}{5}\cdot w = 5.333\cdot\frac{lb}{in} \qquad w_p = \frac{3}{5}\cdot w = 4\cdot\frac{lb}{in}$$

$$N_x = w_a\cdot(20in) - \frac{3}{5}\cdot A_x - \frac{4}{5}\cdot A_y = -23.333\,lbf$$

$$V_x = -w_p\cdot(20in) - \frac{4}{5}\cdot A_x + \frac{3}{5}\cdot A_y = -20\,lbf$$

$$M_x = -w_p\cdot(20in)\cdot\frac{20in}{2} - \frac{4}{5}\cdot A_x\cdot(20in) + \frac{3}{5}\cdot A_y\cdot(20in) = 400\cdot lb\cdot in \qquad M_x = 33.333\,lb\cdot ft$$

$\boxed{N_x = -23.3\,lbf}$ $\qquad \boxed{V_x = -20\,lbf}$ $\qquad \boxed{M_x = 33.3\cdot lb\cdot ft}$

Problem 1.3-24

(a) Statics

$\Sigma M_A = 0$

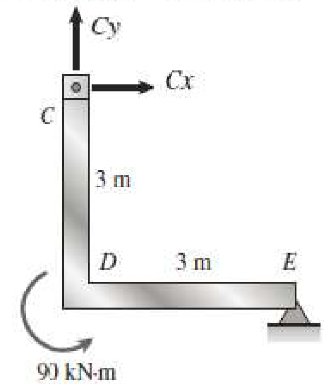
$$10\text{ kN}(6\text{ m}) - 10\text{ kN}\left(\frac{1}{\sqrt{2}}\right)(6\text{ m}) + 90\text{ kN}\cdot\text{m} + E_y(6\text{ m}) - E_x 3\text{ m} = 6E_y\text{m} - 3E_x\text{m} + 150\text{ kN}\cdot\text{m} - 30\sqrt{2}\text{ kN}\cdot\text{m}$$

so $\quad 6E_y\text{m} - 3E_x\text{m} + 150\text{ kN}\cdot\text{m} - 30\sqrt{2}\text{ kN}\cdot\text{m} = 0$

or $\quad -E_x + 2E_y = \dfrac{-(150\text{ kN}\cdot\text{m} - 30\sqrt{2}\text{ kN}\cdot\text{m})}{3\text{ m}} = -35.858\text{ kN}$

$\Sigma M_{CRHFB} = 0$ < right hand FBD (CDE) - see figure.

$(E_x + E_y)3\text{ m} = -90\text{ kN}\cdot\text{m} \qquad E_x + E_y = \dfrac{-90\text{ kN}\cdot\text{m}}{3\text{ m}} = -30\text{ kN}$



Solving $\quad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -35.858\text{ kN} \\ -30\text{ kN} \end{pmatrix} = \begin{pmatrix} -8.05 \\ -21.95 \end{pmatrix}\text{kN}$

$\boxed{E_x = -8.05\text{ kN}}$ $\quad \boxed{E_y = -22\text{ kN}}$

$\Sigma F_x = 0 \qquad A_x = -E_x + 10\text{ kN} - 10\text{ kN}\left(\frac{1}{\sqrt{2}}\right) = 10.98\text{ kN} \qquad \boxed{A_x = 10.98\text{ kN}}$

$\Sigma F_y = 0 \qquad A_y = -E_y + 10\text{ kN}\left(\frac{1}{\sqrt{2}}\right) = 29.07\text{ kN} \qquad \boxed{A_y = 29.1\text{ kN}}$

(b) Right hand FBD $\qquad C_x = -E_x = 8.05\text{ kN} \qquad C_y = -E_y = 22\text{ kN}$

$\boxed{\text{Resultant}_C = \sqrt{C_x^2 + C_y^2} = 23.4\text{ kN}}$