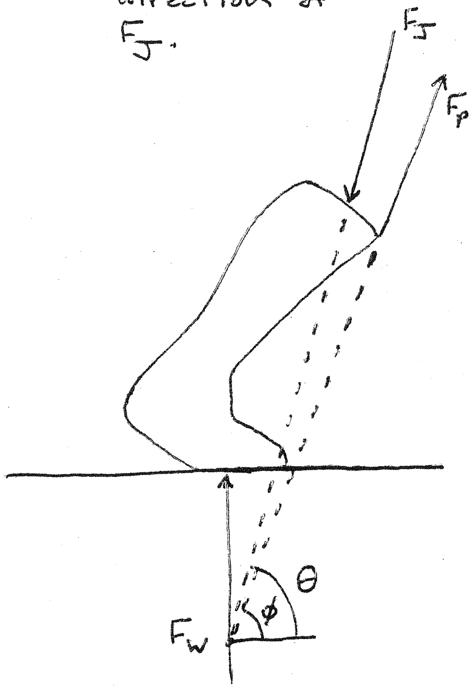
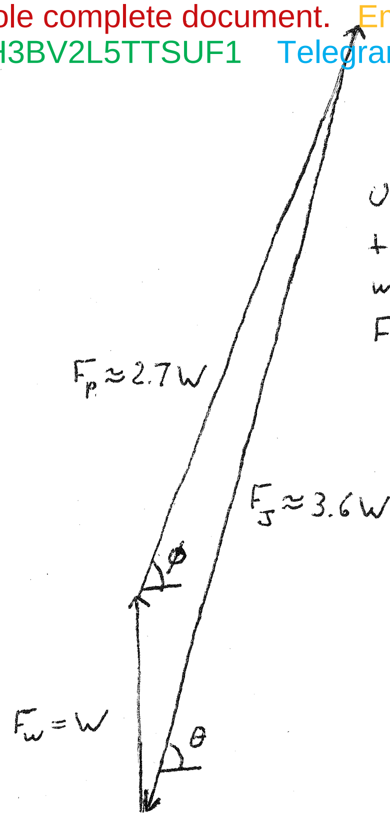
2.1

Find point of concurrency to determine the direction of F_J.



Use a vector diagram to estimate the magnitude of F_J and F_p.



ISBN 0-13-008909-5

2.2

$$\sum M_H = I_H \alpha$$

$$(I_{HAT})_{Hip}\alpha = T_H - d_{HAT} m_{HAT} g$$

$$T_H = (I_{HAT})_{Hip}\alpha + d_{HAT} m_{HAT} g$$

$$(I_{HAT})_{Hip} = (k_{HAT})^2_{Hip} m_{HAT}$$

$$(k_{HAT})_{Hip} = .621 \times .52 \times 1.8 = 0.581\ m, \quad m_{HAT} = .678 \times 70 = 47.46\ kg$$

$$d_{HAT} = .374 \times .52 \times 1.8 = .350\ m$$

$$(I_{HAT})_{Hip} = .581^2 \times 47.46 = 16.02\ kg \cdot m^2$$

$$T_H = 16.02 \times 3 + .350 \times 47.46 \times 9.81 = 48.1 + 162.9 = 211.0\ N \cdot m$$

ISBN 0-13-008909-5

2-3

About the hip

$$\sum M_H = I_H \alpha$$

$$T_H - m_T g d_T - m_{LL} g(d_{LL} + l_T) = (I_T + I_{LL})_H \alpha$$

$$(I_{LL})_H = (I_{LL})_{cm} + (l_T + d_{LL})^2 m_{LL}$$

$$T_H = ((I_T)_H + (I_{LL})_{cm} + (l_T + d_{LL})^2 m_{LL})\alpha + m_T g d_T + m_{LL} g(d_{LL} + l_T)$$

$$T_H = (.4 + .05 + 5(.40 + .20)^2)(2) + (8)(9.81)(.18) + (5)(9.81)(.20 + .40)$$

$$T_H = 48\ N \cdot m$$

For the lower leg

$$\sum F_y = m_{LL}(a_{LL})_y$$

$$F_{Ky} - m_{LL} a_G = m_{LL}(l_T + d_{LL})\alpha$$

$$F_{Ky} = m_{LL}(l_T + d_{LL})\alpha + m_{LL} a_G$$

$$F_{Ky} = 5(.4 + .2)(2) + 5(9.81)$$

$$F_{Ky} = 55\ N$$

About the mass center of the lower leg

$$\sum M_{GL} = I_{GL}\alpha = T_K - d_{LL} F_{Ky}$$

$$T_K = I_{GL}\alpha + d_{LL} F_{Ky}$$

$$T_K = 5(2) + .2(55)$$

$$T_K = 11\ N \cdot m$$

ISBN 0-13-008909-5

2-4

About the hip

$$\sum M_H = I_H \alpha$$

$$T_H + m_T d_T (a_T - a_G) + m_{LL}(l_T + d_{LL})(a_{LL} - a_G) = (I_T + I_{LL})_H \alpha$$

$$a_T = \alpha d_T \qquad a_{LL} = a_K = \alpha l_T$$

$$(I_{LL})_H = (I_{LL})_{cm} + m_{LL}(l_T + d_{LL})^2$$

$$T_H = [I_T + (I_{LL})_{cm} + m_{LL}(l_T + d_{LL})^2]\alpha - m_T d_T(\alpha d_T - a_G) - m_{LL}(l_T + d_{LL})(\alpha l_T - a_G)$$

$$T_H = [.4 + .05 + 5(.4 + .2)^2](3) - (8)(.18)[(3)(.18) - 9.81] - 5(.4 + .2)[(3)(.4) - 9.81]$$

$$T_H = 46\ N \cdot m$$

For the lower leg

$$\sum F_y = m_{LL}(a_{LL})_y \qquad a_K = \alpha l_T$$

$$F_{Ky} = m_{LL}\alpha l_T + m_{LL} a_G = m_{LL}(\alpha l_T + a_G)$$

$$F_{Ky} = 5[3(.4) + 9.81]$$

$$F_{Ky} = 55\ N$$

About the mass center of the lower leg

$$\sum M_{GL} = I_{GL}\alpha = T_K - d_{LL} F_{Ky}$$

$$T_K = I_{GL}\alpha + d_{LL} F_{Ky}$$

$$T_K = .05(3) + .2(55)$$

$$T_K = 11\ N \cdot m$$

ISBN 0-13-008909-5

2.5

moments about the shoulder

$$M_S - m_L g d_L - (m_h + m_b) g (L_L + d_h) = \sum I_{cm}\alpha + \left|\vec{d} \times m\ddot{\vec{r}}\right|$$

RHS of the equation

$$(I_u + I_L)\alpha + d_u m_u a_u + m_L a_L \sqrt{L_u^2 + d_L^2} + (m_h + m_b) a_h \sqrt{L_u^2 + (L_L + d_h)^2}$$

$$a_u = d_u \alpha \quad a_L = \alpha\sqrt{L_u^2 + d_L^2} \quad a_h = \alpha\sqrt{L_u^2 + (L_L + d_h)^2}$$

$$(I_u + I_L)\alpha + d_u^2 m_u \alpha + (L_u^2 + d_L^2) m_L \alpha + \left(L_u^2 + (L_L + d_h)^2\right)(m_h + m_b)\alpha$$

solving for the moment

$$M_S = m_L g d_L + (m_h + m_b) g (L_L + d_h) + \alpha\left[I_u + I_L + d_u^2 m_u + (L_u^2 + d_L^2) m_L + \left(L_u^2 + (L_L + d_h)^2\right)(m_h + m_b)\right]$$

substituting values yields

$$M_S = 12.02\ N \cdot m$$

moments about the elbow

$$M_E - m_L g d_L - (m_h + m_b) g (L_L + d_h) = \sum I_{cm}\alpha + \left|\vec{d} \times m\ddot{\vec{r}}\right|$$

$$M_E - m_L g d_L - (m_h + m_b) g (L_L + d_h) = d_L^2 m_L \alpha + \left(L_L + d_h\right)^2 (m_h + m_b)\alpha$$

$$M_E = m_L g d_L + (m_h + m_b) g (L_L + d_h) + d_L^2 m_L \alpha + \left(L_L + d_h\right)^2 (m_h + m_b)\alpha$$

substituting values yields

$$M_E = 10.66\ N \cdot m$$

$T_h = [(I_T)_H + (I_{LL})_H]\alpha$

$(I_T)_H = (I_T)_{GT} + m_T d_T^{\,2} = 4.2 + .045(8)^2 = 7.08\, lb \cdot sec^2 \cdot in$

$(I_{LL})_H = (I_{LL})_{GL} + m_{LL}(l_T + d_{LL})^2 = 1.4 + .025(18+8)^2 = 18.3\, lb \cdot sec^2 \cdot in$

$T_H = (7.08 + 18.3) \times 20 = 507.6\, in \cdot lb$

$\sum F_x = m_{LL}\ddot{x}_{ll} = F_{Kx}; \quad \sum F_y = m_{LL}\ddot{y}_{ll} = F_{Ky} - m_{LL}g$

$\ddot{x}_{ll} = (l_T + d_{LL})\alpha = (18+8) \times 20 = 520\dfrac{in}{sec^2}; \quad \ddot{y}_{ll} = 0$

$F_{Kx} = m_{LL}\ddot{x}_{ll}\ (1) = .025 \times 520 = 13\, lbf$

$\sum M_{GL} = (I_{LL})_{GL}\ddot{\theta}_{LL} = T_K - a_{LL}F_{Kx}$

$T_K = (I_{LL})_{GL}\ddot{\theta}_{LL} + d_{LL}F_{Kx} = 1.4 \times 20 + 8 \times 13 = 132\, in \cdot lb$

ISBN 0-13-008909-5

2.7

$$T_H = (I_T)_{GT}\alpha + (d_T)m_T a_{GTx} + (l_T + d_{LL})m_{LL}a_{GLx}$$

$$T_H = (I_T)_{GT}\alpha + (d_T)^2 m_T\alpha + (l_T + d_{LL})(l_T)m_{LL}\alpha$$

$$T_H = [(I_T)_{GT} + (d_T)^2 m_T + (l_T + d_{LL})(l_T)m_{LL}]\alpha$$

$$T_H = [4.2 + (8)^2(.045) + (18+8)(18)(.025)]20 = 376\ in\cdot lb$$

For the lower leg

$$\sum F_y = m_{LL}(a_{LL})_y \qquad a_K = \alpha l_T$$

$$F_{Ky} = m_{LL}\alpha l_T + m_{LL}a_G = m_{LL}(\alpha l_T + a_G)$$

$$F_{Ky} = .025[20(18) + 9.81]$$

$$F_{Ky} = 9.25\ lbf$$

About the mass center of the lower leg

$$\sum M_{GL} = I_{GL}\alpha = T_K - d_{LL}F_{Ky}$$

$$T_K = I_{GL}\alpha + d_{LL}F_{Ky}$$

$$T_K = 1.4(20) + 8(9.25)$$

$$T_K = 102\ lb\cdot in$$

ISBN 0-13-008909-5

2-8

By forward difference approximation

$$v_{x1}(t) = \frac{x_1(t+\Delta t) - x_1(t)}{\Delta t}$$

Apply the above to calculate v_{x2}, v_{y1}, v_{y2}

$$\|\vec{\omega}\| = \frac{\|\vec{v}\|}{\|\vec{r}\|}$$

$$\omega(t) = \frac{\sqrt{(v_{x2}(t) - v_{x1}(t))^2 + (v_{y2}(t) - v_{y1}(t))^2}}{\sqrt{(x_2(t) - x_1(t))^2 + (y_2(t) - y_1(t))^2}}$$

$$\alpha(t) = \frac{\omega(t+\Delta t) - \omega(t)}{\Delta t}$$

t, s	**x_1, cm**	**y_1, cm**	**x_2, cm**	**y_2, cm**	**v_{x1}, cm/s**	**v_{y1}, cm/s**	**v_{x2}, cm/s**	**v_{y2}, cm/s**	**ω, rad/s**	**α, rad/s^2**
0.0	131.8	6.1	162.6	6.9	8.0	1.0	-23.0	68.0	**2.396**	**1.987**
0.1	132.6	6.2	160.3	13.7	-2.0	-3.0	-21.0	69.0	**2.595**	**3.110**
0.2	132.4	5.9	158.2	20.6	-6.0	-7.0	-52.0	66.0	**2.906**	**-8.699**
0.3	131.8	5.2	153.0	27.2	-4.0	4.0	-54.0	41.0	**2.036**	
0.4	131.4	5.6	147.6	31.3						

ISBN 0-13-008909-5