Chapter 1

INTRODUCTION TO COMPOSITE MATERIALS

The purpose of the questions in Problem Set 1 is twofold: (1) to gain some new knowledge that is beyond what the book has to offer in order to get students to consult other references and (2) a writing assignment that is always a worthwhile exercise for students. Because the questions are beyond the scope of the book, no responses are given. The ingenious and resourceful students will respond well.

Chapter 2

MACROMECHANICAL BEHAVIOR OF A LAMINA

EXERCISE 2.4.1

SHOW THAT THE DETERMINANT INEQUALITY IN EQ. (2.48) FOR ORTHOTROPIC MATERIALS CORRECTLY REDUCES TO $\nu < 1/2$ FOR ISOTROPIC MATERIALS.

FIRST, THE DETERMINANT INEQUALITY IS

$$\bar{\Delta} = 1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{21}\nu_{32}\nu_{13} > 0 \quad (1)$$

BUT FOR ISOTROPIC MATERIALS

$$\nu_{12} = \nu_{21} = \nu_{23} = \nu_{32} = \nu_{31} = \nu_{13} = \nu \quad (2)$$

SO EQ. (1) REDUCES TO

$$1 - 3\nu^2 - 2\nu^3 > 0 \quad (3)$$

WHICH CAN BE FACTORED TO

$$(1+\nu)(1-\nu-2\nu^2) > 0 \quad (4)$$

OR

$$(1+\nu)(1+\nu)(1-2\nu) > 0 \quad (5)$$

WHEREUPON BECAUSE $(1+\nu)^2 > 0$,

$$1 - 2\nu > 0 \quad (6)$$

OR

$$\nu < \frac{1}{2} \quad (7)$$

(NOTHING CAN BE SAID ABOUT $(1+\nu)^2$ IN EQ. (5), BUT WE KNOW FROM OTHER INEQUALITIES THAT

$$\nu > -1 \quad (8)$$

FROM

$$1 + \nu > 0 \quad (9)$$

SO THE REDUCTION TO ISOTROPIC MATERIALS IS CORRECT.)

EXERCISE 2.4.2

DERIVE EQ. (2.52) FROM THE DETERMINANT INEQUALITY IN EQ. (2.48):

$$\bar{\Delta} = 1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{21}\nu_{32}\nu_{13} > 0 \qquad (1)$$

WHICH IS

$$1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} > 2\nu_{21}\nu_{32}\nu_{13} \qquad (2)$$

OR

$$(1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13})/2 > \nu_{21}\nu_{32}\nu_{13} \qquad (3)$$

NOW SUBSTITUTE THE RECIPROCAL RELATIONS

$$\nu_{ij}/E_i = \nu_{ji}/E_j \qquad (4)$$

TO OBTAIN

$$\left[1 - \nu_{21}^2\left(\frac{E_1}{E_2}\right) - \nu_{32}^2\left(\frac{E_2}{E_3}\right) - \nu_{13}^2\left(\frac{E_3}{E_1}\right)\right]/2 > \nu_{21}\nu_{32}\nu_{13} \qquad (5)$$

BUT SINCE $\nu_{ij}^2 > 0$ AND E_1, E_2, AND $E_3 > 0$, THEN THE BRACKETED QUANTITY IS LESS THAN ONE SO

$$\frac{1}{2} > \left[1 - \nu_{21}^2\left(\frac{E_1}{E_2}\right) - \nu_{32}^2\left(\frac{E_2}{E_3}\right) - \nu_{13}^2\left(\frac{E_3}{E_1}\right)\right]/2 > \nu_{21}\nu_{32}\nu_{13} \qquad (6)$$

WHICH IS EQ. (2.52) AS DESIRED.

FOR ISOTROPIC MATERIALS, EQ. (6) BECOMES

$$\frac{1}{2} > (1 - 3\nu^2)/2 > \nu^3 \qquad (7)$$

OR

$$1 > 1 - 3\nu^2 > 2\nu^3 \qquad (8)$$

OR

$$1 - 3\nu^2 - 2\nu^3 > 0 \qquad (9)$$

WHICH IS SHOWN IN EX. 2.4.1 TO YIELD $\nu < \frac{1}{2}$.

EXERCISE 2.4.3

DERIVE EQ. (2.53) FROM EQ. (2.52) WHICH IS

$$\nu_{21}\nu_{32}\nu_{13} < \left[1-\nu_{21}^2\left(\frac{E_1}{E_2}\right)-\nu_{32}^2\left(\frac{E_2}{E_3}\right)-\nu_{13}^2\left(\frac{E_3}{E_1}\right)\right]/2 < \frac{1}{2} \qquad (1)$$

AND CAN BE WRITTEN AS

$$1-2\nu_{21}\nu_{32}\nu_{13} > 1-\nu_{21}^2\left(\frac{E_1}{E_2}\right)-\nu_{32}^2\left(\frac{E_2}{E_3}\right)-\nu_{13}^2\left(\frac{E_3}{E_1}\right)-2\nu_{21}\nu_{32}\nu_{13} > 0 \qquad (2)$$

OR

$$\left[1-\nu_{32}^2\left(\frac{E_2}{E_3}\right)\right]\left[1-\nu_{13}^2\left(\frac{E_3}{E_1}\right)\right]$$

$$-\nu_{32}^2\nu_{13}^2\left(\frac{E_2}{E_3}\right)\left(\frac{E_3}{E_1}\right)-\nu_{21}^2\left(\frac{E_1}{E_2}\right)-2\nu_{21}\nu_{32}\nu_{13} > 0 \qquad (3)$$

OR

$$\left[1-\nu_{32}^2\left(\frac{E_2}{E_3}\right)\right]\left[1-\nu_{13}^2\left(\frac{E_3}{E_1}\right)\right]$$

$$-\left[\nu_{32}\nu_{13}\left(\frac{E_2}{E_1}\right)^{1/2}+\nu_{21}\left(\frac{E_1}{E_2}\right)^{1/2}\right]^2 > 0 \qquad (4)$$

WHICH IS EQ. (2.53) AS DESIRED.

FOR ISOTROPIC MATERIALS, EQ. (4) BECOMES

$$(1-\nu^2)^2-(\nu^2+\nu)^2 > 0 \qquad (5)$$

OR

$$1-3\nu^2-2\nu^3 > 0 \qquad (6)$$

WHICH IS SHOWN IN EX. 2.4.1 TO YIELD

$$\nu < \frac{1}{2} \qquad (7)$$

EXERCISE 2.4.4

DERIVE EQ. (2.54) FROM EQ. (2.53) WHICH IS

$$\left[1-\nu_{32}^2\left(\frac{E_2}{E_3}\right)\right]\left[1-\nu_{13}^2\left(\frac{E_3}{E_1}\right)\right] - \left[\nu_{21}\left(\frac{E_1}{E_2}\right)^{1/2} + \nu_{32}\nu_{13}\left(\frac{E_2}{E_1}\right)^{1/2}\right]^2 > 0 \tag{1}$$

MULTIPLY EQ. (1) BY E_2/E_1 TO OBTAIN

$$\left[1-\nu_{32}^2\left(\frac{E_2}{E_3}\right)\right]\left[1-\nu_{13}^2\left(\frac{E_3}{E_1}\right)\right]\left(\frac{E_2}{E_1}\right) - \left[\nu_{21} + \nu_{32}\nu_{13}\left(\frac{E_2}{E_1}\right)\right]^2 > 0 \tag{2}$$

WHICH IS OF THE FORM

$$B - (X+A)^2 > 0 \tag{3}$$

WHEREUPON

$$-\sqrt{B} - A < X < +\sqrt{B} - A \tag{4}$$

OR

$$-(A+\sqrt{B}) < X < -(A-\sqrt{B}) \tag{5}$$

WHICH IS

$$-\left\{\nu_{32}\nu_{13}\left(\frac{E_2}{E_1}\right) + \left[1-\nu_{32}^2\left(\frac{E_2}{E_3}\right)\right]^{1/2}\left[1-\nu_{13}^2\left(\frac{E_3}{E_1}\right)\right]^{1/2}\left(\frac{E_2}{E_1}\right)^{1/2}\right\}$$

$$< \nu_{21} <$$

$$-\left\{\nu_{32}\nu_{13}\left(\frac{E_2}{E_1}\right) - \left[1-\nu_{32}^2\left(\frac{E_2}{E_3}\right)\right]^{1/2}\left[1-\nu_{13}^2\left(\frac{E_3}{E_1}\right)\right]^{1/2}\left(\frac{E_2}{E_1}\right)^{1/2}\right\} \tag{6}$$

WHICH IS EQ. (2.54) AS DESIRED.

EXERCISE 2.4.5

SHOW THAT EQ. (2.54),

$$-\left\{\nu_{32}\nu_{13}\left(\frac{E_2}{E_1}\right)+\left[1-\nu_{32}^2\left(\frac{E_2}{E_3}\right)\right]^{1/2}\left[1-\nu_{13}^2\left(\frac{E_3}{E_1}\right)\right]^{1/2}\left(\frac{E_2}{E_1}\right)^{1/2}\right\} < \nu_{21}$$

$$< -\left\{\nu_{32}\nu_{13}\left(\frac{E_2}{E_1}\right)-\left[1-\nu_{32}^2\left(\frac{E_2}{E_3}\right)\right]^{1/2}\left[1-\nu_{13}^2\left(\frac{E_3}{E_1}\right)^{1/2}\left(\frac{E_2}{E_1}\right)^{1/2}\right\} \qquad (1)$$

REDUCES FOR ISOTROPIC MATERIALS TO THE KNOWN BOUNDS ON ν. FIRST, WHEN $\nu_{ij} = \nu$ AND $E_i = E$, EQ. (1) IS

$$-\left[\nu^2 + (1-\nu^2)^{1/2}(1-\nu^2)^{1/2}\right] < \nu$$

$$< -\left[\nu^2 - (1-\nu^2)^{1/2}(1-\nu^2)^{1/2}\right] \qquad (2)$$

OR

$$-1 < \nu < 1-2\nu^2 \qquad (3)$$

OBVIOUSLY, THE LOWER BOUND ON ν IS OBTAINED FROM EQ. (3) BY INSPECTION AS

$$-1 < \nu \qquad (4)$$

HOWEVER, THE UPPER BOUND REQUIRES MANIPULATION. CONSIDER JUST THE UPPER BOUND WRITTEN AS

$$\nu < 1-2\nu^2 \qquad (5)$$

OR

$$0 < 1-\nu-2\nu^2 \qquad (6)$$

OR

$$0 < (1+\nu)(1-2\nu) \qquad (7)$$

SINCE WE KNOW $\nu > -1$ OR $1+\nu > 0$, THEN

$$1-2\nu > 0 \qquad (8)$$

OR

$$\nu < \frac{1}{2} \qquad (9)$$

AS DESIRED.

EXERCISE 2.6.1

DERIVE EQ. (2.82). THE STRESSES IN X-Y COORDINATES IN TERMS OF THOSE IN 1-2 COORDINATES ARE

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \qquad (2.74)$$

SUBSTITUTE THE STRESS-STRAIN LAW, EQ. (2.81):

$$= [T]^{-1}[Q] \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

SUBSTITUTE THE REUTER MATRIX, EQ. (2.77), TO GET THE MODIFIED STRAIN VECTOR:

$$= [T]^{-1}[Q][R] \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12}/2 \end{Bmatrix}$$

TRANSFORM THE STRAINS FROM 1-2 TO X-Y COORDINATES BY THE INVERSE OF EQ. (2.75):

$$= [T]^{-1}[Q][R][T] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy}/2 \end{Bmatrix}$$

FINALLY, USE THE REUTER MATRIX IN THE FORM OF THE INVERSE OF EQ. (2.79) TO GET THE NATURAL STRAIN VECTOR IN THE X-Y COORDINATE SYSTEM:

$$= [T]^{-1}[Q][R][T][R]^{-1} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad (2.82)$$

EXERCISE 26.2

PROVE $[R][T][R]^{-1} = [T]^{-T}$

FIRST,

$$[R][T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} C^2 & S^2 & 2SC \\ S^2 & C^2 & -2SC \\ -SC & SC & C^2-S^2 \end{bmatrix} = \begin{bmatrix} C^2 & S^2 & 2SC \\ S^2 & C^2 & -2SC \\ -2SC & 2SC & 2(C^2-S^2) \end{bmatrix}$$

THEN,

$$[R][T][R]^{-1} = \begin{bmatrix} C^2 & S^2 & 2SC \\ S^2 & C^2 & -2SC \\ -2SC & 2SC & 2(C^2-S^2) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$$

$$= \begin{bmatrix} C^2 & S^2 & SC \\ S^2 & C^2 & -SC \\ -2SC & 2SC & C^2-S^2 \end{bmatrix} = [T]^{-1^T} = [T]^{-T}$$