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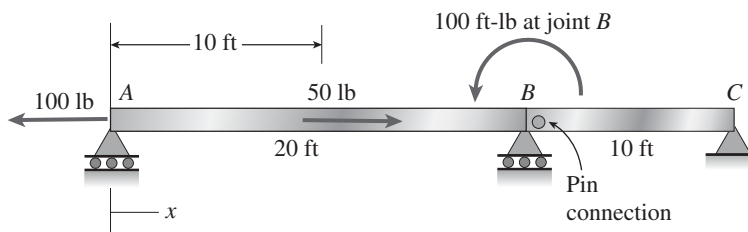
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1

Tension, Compression, and Shear

Statics Review

Problem 1.2-1 Segments AB and BC of beam ABC are pin connected a small distance to the right of joint B (see figure). Axial loads act at A and at mid-span of AB . A concentrated moment is applied at joint B .



- Find reactions at supports A , B , and C .
- Find internal stress resultants N , V , and M at $x = 15$ ft.

Solution 1.2-1

(a) APPLY LAWS OF STATICS

$$\Sigma F_x = 0 \quad C_x = 100 \text{ lb} - 50 \text{ lb} = 50 \text{ lb}$$

$$\text{FBD of } BC \quad \Sigma M_B = 0 \quad C_y = \frac{1}{10 \text{ ft}}(0) = 0$$

$$\text{Entire FBD} \quad \Sigma M_A = 0 \quad B_y = \frac{1}{20 \text{ ft}}(-100 \text{ lb-ft}) = -5 \text{ lb}$$

$$\Sigma F_y = 0 \quad A_y = -B_y = 5 \text{ lb-ft}$$

$$\text{Reactions are } \boxed{A_y = 5 \text{ lb}} \quad \boxed{B_y = -5 \text{ lb}} \quad \boxed{C_x = 50 \text{ lb}} \quad \boxed{C_y = 0}$$

(b) INTERNAL STRESS RESULTANTS N , V , AND M AT $x = 15$ ft

Use FBD of segment from A to $x = 15$ ft

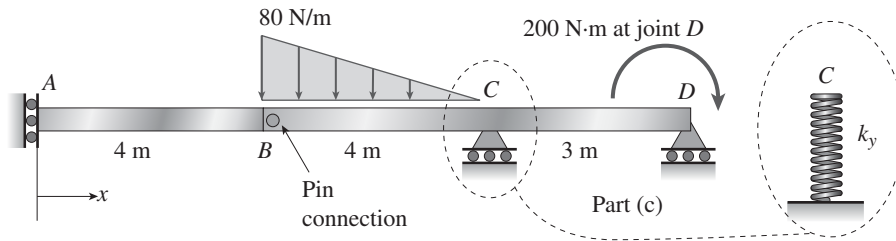
$$\Sigma F_x = 0 \quad \boxed{N = 100 \text{ lb} - 50 \text{ lb} = 50 \text{ lb}}$$

$$\Sigma F_y = 0 \quad \boxed{V = A_y = 5 \text{ lb}}$$

$$\Sigma M = 0 \quad \boxed{M = A_y 15 \text{ ft} = 75 \text{ lb-ft}}$$

Problem 1.2-2 Segments AB and BCD of beam $ABCD$ are pin connected at $x = 4$ m. The beam is supported by a sliding support at A and roller supports at C and D (see figure). A triangularly distributed load with peak intensity of 80 N/m acts on BC . A concentrated moment is applied at joint B .

- (a) Find reactions at supports A , C , and D .
- (b) Find internal stress resultants N , V , and M at $x = 5$ m.
- (c) Repeat parts (a) and (b) for the case of the roller support at C replaced by a linear spring of stiffness $k_y = 200$ kN/m.



Solution 1.2-2

(a) APPLY LAWS OF STATICS

$$\Sigma F_x = 0 \quad A_x = 0$$

FBD of $AB \quad \Sigma M_B = 0 \quad M_A = 0$

Entire FBD $\Sigma M_C = 0 \quad D_y = \frac{1}{3 \text{ m}} \left[200 \text{ N}\cdot\text{m} - \frac{1}{2} (80 \text{ N/m}) 4 \text{ m} \left(\frac{2}{3} \right) 4 \text{ m} \right] = -75.556 \text{ N}$

$$\Sigma F_y = 0 \quad C_y = \frac{1}{2} (80 \text{ N/m}) 4 \text{ m} - D_y = 235.556 \text{ N}$$

Reactions are $M_A = 0 \quad A_x = 0 \quad C_y = 236 \text{ N} \quad D_y = -75.6 \text{ N}$

(b) INTERNAL STRESS RESULTANTS N , V , AND M AT $x = 5$ m

Use FBD of segment from A to $x = 5$ m; ordinate on triangular load at $x = 5$ m is $\frac{3}{4} (80 \text{ N/m}) = 60 \text{ N/m}$.

$$\Sigma F_x = 0 \quad N_x = -A_x = 0$$

$$\Sigma F_y = 0 \quad V = \frac{-1}{2} [(80 \text{ N/m} + 60 \text{ N/m}) 1 \text{ m}] = -70 \text{ N} \quad V = -70 \text{ N} \quad \text{Upward}$$

$$\Sigma M = 0 \quad M = -M_A - \frac{1}{2} (80 \text{ N/m}) 1 \text{ m} \left(\frac{2}{3} 1 \text{ m} \right) - \frac{1}{2} (60 \text{ N/m}) 1 \text{ m} \left(\frac{1}{3} 1 \text{ m} \right) = -36.667 \text{ N}\cdot\text{m}$$

(break trapezoidal load into two triangular loads in moment expression)

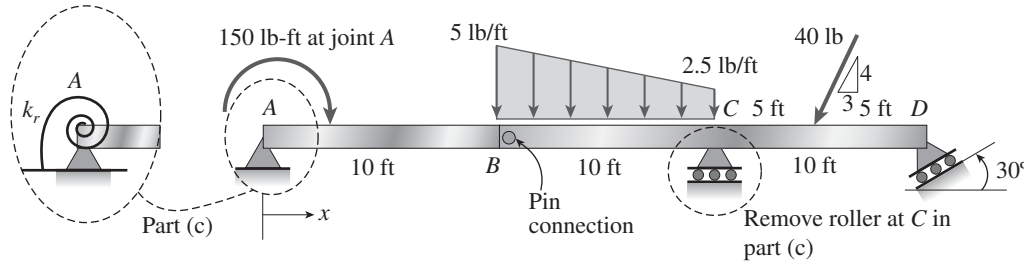
$$M = -36.7 \text{ N}\cdot\text{m} \quad \text{CW}$$

(c) REPLACE ROLLER SUPPORT AT C WITH SPRING SUPPORT

Structure remains statically determinate so all results above in (a) and (b) are unchanged.

Problem 1.2-3 Segments AB and BCD of beam $ABCD$ are pin connected at $x = 10$ ft. The beam is supported by a pin support at A and roller supports at C and D ; the roller at D is rotated by 30° from the x axis (see figure). A trapezoidal distributed load on BC varies in intensity from 5 lb/ft at B to 2.5 lb/ft at C . A concentrated moment is applied at joint A and a 40 lb inclined load is applied at mid-span of CD .

- Find reactions at supports A , C , and D .
- Find the resultant force in the pin connection at B .
- Repeat parts (a) and (b) if a rotational spring ($k_r = 50$ ft-lb/rad) is added at A and the roller at C is removed.



Solution 1.2-3

(a) STATICS

FBD of AB (cut through beam at pin): $\Sigma M_B = 0 \quad A_y = \frac{1}{10 \text{ ft}}(-150 \text{ lb-ft}) = -15 \text{ lb}$

Entire FBD: $\Sigma M_D = 0$

$$C_y = \frac{1}{10 \text{ ft}} \left[\frac{4}{5} 40 \text{ lb}(5 \text{ ft}) + \frac{1}{2} (2.5 \text{ lb/ft}) 10 \text{ ft} \left(10 \text{ ft} + \frac{10 \text{ ft}}{3} \right) + \frac{1}{2} (5 \text{ lb/ft}) 10 \text{ ft} \left(10 \text{ ft} + \frac{2}{3} 10 \text{ ft} \right) - 150 \text{ lb-ft} - A_y 30 \text{ ft} \right] = 104.333 \text{ lb}$$

$$\Sigma F_y = 0 \quad D_y = \frac{4}{5} 40 \text{ lb} + \frac{1}{2} (5 \text{ lb/ft} + 2.5 \text{ lb/ft}) 10 \text{ ft} - A_y - C_y = -19.833 \text{ lb} \quad \text{so} \quad D_x = \frac{-D_y}{\tan(60^\circ)} = 11.451 \text{ lb}$$

$$\Sigma F_x = 0 \quad A_x = \frac{3}{5} 40 \text{ lb} - D_x = 12.549 \text{ lb}$$

$$\boxed{A_x = 12.55 \text{ lb}, A_y = -15 \text{ lb}, C_y = 104.3 \text{ lb}, D_x = 11.45 \text{ lb}, D_y = -19.83 \text{ lb}}$$

(b) USE FBD OF AB ONLY; MOMENT AT PIN IS ZERO

$$F_{Bx} = -A_x \quad F_{Bx} = -12.55 \text{ lb} \quad F_{By} = -A_y \quad F_{By} = 15 \text{ lb} \quad \boxed{\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 19.56 \text{ lb}}$$

(c) ADD ROTATIONAL SPRING AT A AND REMOVE ROLLER AT C ; APPLY EQUATIONS OF STATICAL EQUILIBRIUM

Use FBD of BCD $\Sigma M_B = 0$

$$D_y = \frac{1}{20 \text{ ft}} \left[\frac{1}{2} (2.5 \text{ lb/ft}) 10 \text{ ft} \left(\frac{2}{3} 10 \text{ ft} \right) + \frac{1}{2} (5 \text{ lb/ft}) 10 \text{ ft} \left(\frac{1}{3} 10 \text{ ft} \right) + \frac{4}{5} 40 \text{ lb} (15 \text{ ft}) \right] = 32.333 \text{ lb}$$

$$\text{so} \quad D_x = \frac{-D_y}{\tan(60^\circ)} = -18.668 \text{ lb}$$

Use entire FBD $\Sigma F_y = 0 \quad A_y = \frac{1}{2} (5 \text{ lb/ft} + 2.5 \text{ lb/ft}) 10 \text{ ft} + \frac{4}{5} (40 \text{ lb}) - D_y = 37.167 \text{ lb}$

$$\Sigma F_x = 0 \quad A_x = \frac{3}{5} (40 \text{ lb}) - D_x = 42.668 \text{ lb}$$

4 CHAPTER 1 Tension, Compression, and Shear

Use FBD of AB $\Sigma M_B = 0 \quad M_A = 150 \text{ lb-ft} + A_y(10 \text{ ft}) = 521.667 \text{ lb-ft}$

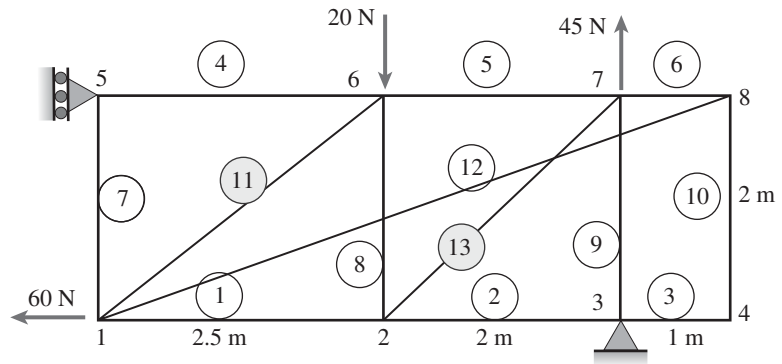
SO REACTIONS ARE $A_x = 42.7 \text{ lb}$ $A_y = 37.2 \text{ lb}$ $M_A = 522 \text{ lb-ft}$ $D_x = -18.67 \text{ lb}$ $D_y = 32.3 \text{ lb}$

RESULTANT FORCE IN PIN CONNECTION AT B

$F_{Bx} = -A_x \quad F_{By} = -A_y$ $\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 56.6 \text{ lb}$

Problem 1.2-4 Consider the plane truss with a pin support at joint 3 and a vertical roller support at joint 5 (see figure).

- (a) Find reactions at support joints 3 and 5.
- (b) Find axial forces in truss members 11 and 13.



Solution 1.2-4

(a) STATICS

$\Sigma F_y = 0 \quad R_{3y} = 20 \text{ N} - 45 \text{ N} = -25 \text{ N}$

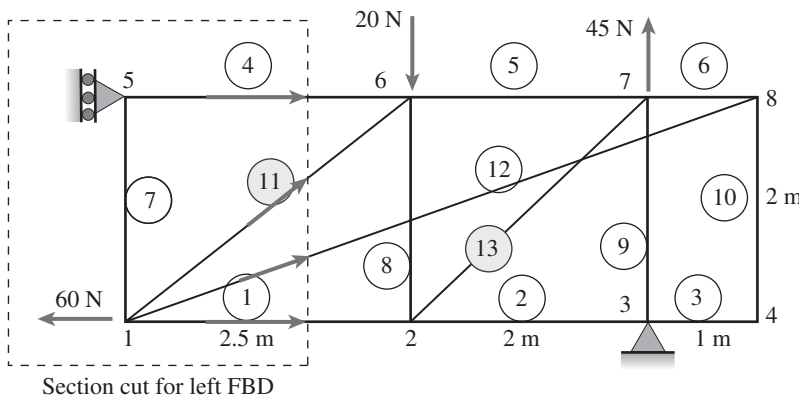
$\Sigma M_3 = 0 \quad R_{5x} = \frac{1}{2 \text{ m}} (20 \text{ N} \times 2 \text{ m}) = 20 \text{ N}$

$\Sigma F_x = 0 \quad R_{3x} = -R_{5x} + 60 \text{ N} = 40 \text{ N}$

(b) MEMBER FORCES IN MEMBERS 11 and 13

Number of unknowns: $m = 13 \quad r = 3 \quad m + r = 16$

Number of equations: $j = 8 \quad 2j = 16 \quad \text{So statically determinate}$



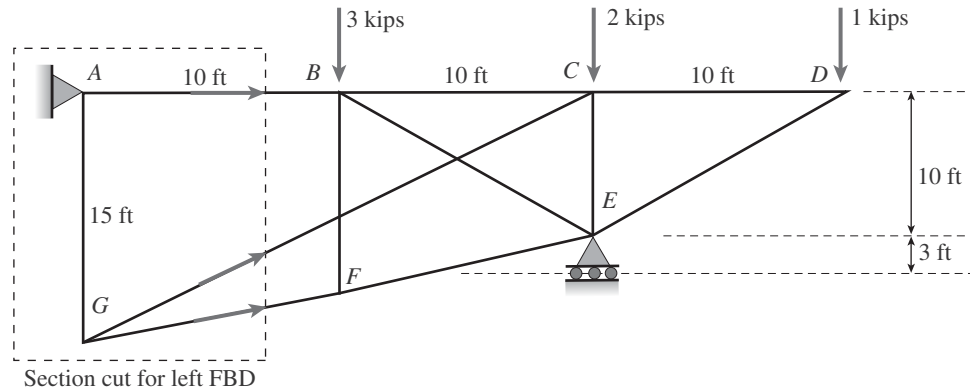
TRUSS ANALYSIS

- (1) $\Sigma F_V = 0$ at joint 4 so $F_{10} = 0$
- (2) $\Sigma F_V = 0$ at joint 8 so $F_{12} = 0$
- (3) $\Sigma F_H = 0$ at joint 5 so $F_4 = -R_{5x} = -20 \text{ N}$
- (4) Cut vertically through 4, 11, 12, and 1; use left FBD; sum moments about joint 2
 $F_{11V} = \frac{1}{2.5 \text{ m}} (R_{5x} - F_4)$ so $F_{11} = 0$
- (5) Sum vertical forces at joint 3; $F_9 = R_{3y}$
 $F_9 = 25 \text{ N}$

(6) Sum vertical forces at joint 7 $F_{13V} = 45 \text{ N} - F_9 = 20 \text{ N}$ $F_{13} = \sqrt{2} F_{13V} = 28.3 \text{ N}$

Problem 1.2-5 A plane truss has a pin support at A and a roller support at E (see figure).

- (a) Find reactions at all supports.
 (b) Find the axial force in truss member FE .



Solution 1.2-5

(a) STATICS

$$\Sigma F_x = 0 \quad A_x = 0$$

$$\Sigma M_A = 0 \quad E_y = \frac{1}{20 \text{ ft}}(3 \text{ k} \times 10 \text{ ft} + 2 \text{ k} \times 20 \text{ ft} + 1 \text{ k} \times 30 \text{ ft}) = 5 \text{ k}$$

$$\Sigma F_y = 0 \quad A_y = 3 \text{ k} + 2 \text{ k} + 1 \text{ k} - E_y = 1 \text{ k}$$

(b) MEMBER FORCE IN MEMBER FE

$$\text{Number of unknowns:} \quad m = 11 \quad r = 3 \quad m + r = 14$$

$$\text{Number of equations:} \quad j = 7 \quad 2j = 14 \quad \text{So statically determinate}$$

TRUSS ANALYSIS

(1) Cut vertically through AB , GC , and GF ; use left FBD; sum moments about C

$$F_{GFx}(15 \text{ ft}) - F_{GFy}(20 \text{ ft}) = A_y(20 \text{ ft}) = 20 \text{ ft-k} \quad F_{GFx} = F_{GF} \frac{10}{\sqrt{2^2 + 10^2}} \quad F_{GFy} = F_{GF} \frac{2}{\sqrt{2^2 + 10^2}}$$

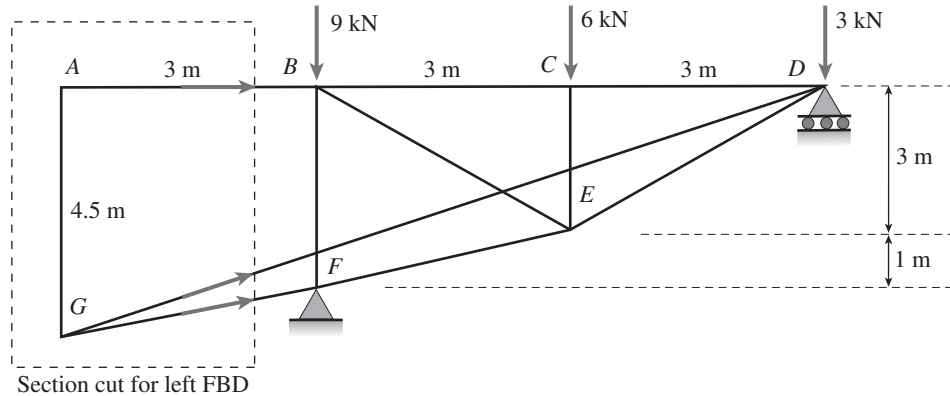
$$\text{so } F_{GF} = \frac{A_y(20 \text{ ft})}{15 \text{ ft} \frac{10}{\sqrt{2^2 + 10^2}} - 20 \text{ ft} \frac{2}{\sqrt{2^2 + 10^2}}} = 1.854 \text{ k} \quad \text{and} \quad F_{GFx} = F_{GF} \frac{10}{\sqrt{2^2 + 10^2}} = 1.818 \text{ k}$$

$$(2) \text{ Sum horizontal forces at joint } F \quad F_{FE} = F_{GFx} = 1.818 \text{ k} \quad F_{FE} = \frac{\sqrt{10^2 + 3^2}}{10} F_{FE} = 1.898 \text{ k}$$

$$\boxed{F_{FE} = 1.898 \text{ k}}$$

Problem 1.2-6 A plane truss has a pin support at F and a roller support at D (see figure).

- (a) Find reactions at both supports.
- (b) Find the axial force in truss member FE .



Solution 1.2-6

(a) STATICS

$$\begin{aligned} \Sigma F_x = 0 & \quad F_x = 0 \\ \Sigma M_F = 0 & \quad D_y = \frac{1}{6 \text{ m}} [3 \text{ kN}(6 \text{ m}) + 6 \text{ kN}(3 \text{ m})] = 6 \text{ kN} \\ \Sigma F_y = 0 & \quad F_y = 9 \text{ kN} + 6 \text{ kN} + 3 \text{ kN} - D_y = 12 \text{ kN} \end{aligned}$$

(b) MEMBER FORCE IN MEMBER FE

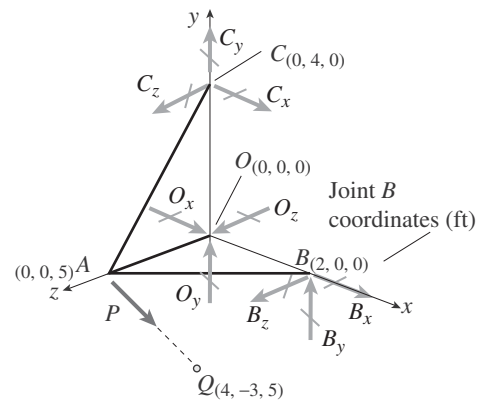
Number of unknowns: $m = 11$ $r = 3$ $m + r = 14$
 Number of equations: $j = 7$ $2j = 14$ So statically determinate

TRUSS ANALYSIS

- (1) Cut vertically through AB , GD , and GF ; use left FBD; sum moments about D to get $F_{GF} = 0$
- (2) Sum horizontal forces at joint F $F_{FE_x} = -F_x = 0$ so $F_{FE} = 0$

Problem 1.2-7 A space truss has three-dimensional pin supports at joints O , B , and C . Load P is applied at joint A and acts toward point Q . Coordinates of all joints are given in feet (see figure).

- (a) Find reaction force components B_x , B_z , and O_z .
- (b) Find the axial force in truss member AC .



Solution 1.2-7

(a) FIND REACTIONS USING STATICS $m = 3$ $r = 9$ $m + r = 12$ $j = 4$ $3j = 12$
 $m + r = 3j$ So truss is statically determinate

$$r_{AQ} = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} \quad r_{OA} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \quad e_{AQ} = \frac{r_{AQ}}{|r_{AQ}|} = \begin{pmatrix} 0.8 \\ -0.6 \\ 0 \end{pmatrix} \quad P_A = P e_{AQ} = \begin{pmatrix} 0.8P \\ -0.6P \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

$\Sigma M = 0$

$$M_O = r_{OA} \times P_A + r_{OC} \times \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} + r_{OB} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 4C_z + 3.0P \\ 4.0P - 2B_z \\ 2B_y - 4C_x \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \quad \text{gives} \quad C_z = \frac{-3}{4}P$$

$$\Sigma M_y = 0 \quad \text{gives} \quad \boxed{B_z = 2P}$$

$\Sigma F = 0$

$$R_O = P_A + \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} = \begin{pmatrix} B_x + C_x + O_x + 0.8P \\ B_y + C_y + O_y - 0.6P \\ O_z + \frac{5P}{4} \end{pmatrix} \quad \text{so} \quad \Sigma M_z = 0 \quad \text{gives} \quad \boxed{O_z = \frac{-5}{4}P}$$

METHOD OF JOINTS Joint O $\Sigma F_x = 0$ $O_x = 0$ $\Sigma F_y = 0$ $O_y = 0$

Joint B $\Sigma F_y = 0$ $B_y = 0$

Joint C $\Sigma F_x = 0$ $C_x = 0$

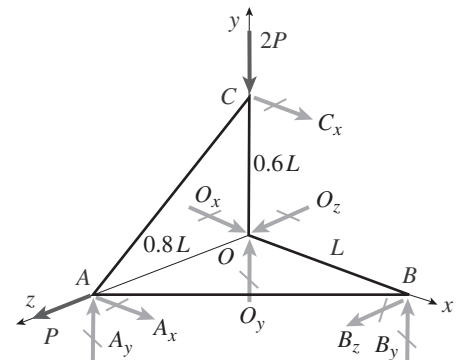
For entire structure $\Sigma F_x = 0$ gives $\boxed{B_x = -0.8P}$ $\Sigma F_y = 0$ $C_y = 0.6P - B_y = O_y$ $C_y = 0.6P$

(b) FORCE IN MEMBER AC

$$\Sigma F_z = 0 \quad \text{at joint C} \quad F_{AC} = \frac{\sqrt{4^2 + 5^2}}{5} |C_z| = \frac{3\sqrt{41}}{20} |P| \quad \boxed{F_{AC} = \frac{3\sqrt{41}}{20} P} \quad \text{tension} \quad \frac{3\sqrt{41}}{20} = 0.96$$

Problem 1.2-8 A space truss is restrained at joints O, A, B, and C, as shown in the figure. Load P is applied at joint A and load 2P acts downward at joint C.

- (a) Find reaction force components A_x , B_y , and B_z in terms of load variable P.
 (b) Find the axial force in truss member AB in terms of load variable P.



Solution 1.2-8

(a) FIND REACTIONS USING STATICS $m = 4$ $r = 8$ $m + r = 12$ $j = 4$ $3j = 12$

$m + r = 3j$ so truss is statically determinate

$$r_{OA} = \begin{pmatrix} 0 \\ 0 \\ 0.8L \end{pmatrix} \quad r_{OB} = \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 0.6L \\ 0 \end{pmatrix} \quad F_A = \begin{pmatrix} A_x \\ A_y \\ P \end{pmatrix} \quad F_B = \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ -2P \\ 0 \end{pmatrix} \quad F_O = \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix}$$

$\Sigma M = 0$

Resultant moment at O

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B + r_{OC} \times F_C = \begin{pmatrix} -0.8A_yL \\ 0.8A_xL - B_zL \\ B_yL - 0.6C_xL \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \quad \text{gives} \quad A_y = 0$$

$\Sigma F = 0$

Resultant force at O

$$R_O = F_O + F_A + F_B + F_C = \begin{pmatrix} A_x + C_x + O_x \\ A_y + B_y + O_y - 2P \\ B_z + O_z + P \end{pmatrix}$$

METHOD OF JOINTS Joint O $\Sigma F_z = 0$ $O_z = 0$

so from $\Sigma F_z = 0$ $B_z = -P$ and $\Sigma M_y = 0$ $A_x = \frac{B_z}{0.8} = -1.25P$

Joint B $\Sigma F_y = 0$ $B_y = 0$

Joint C $\Sigma F_x = 0$ $C_x = 0$

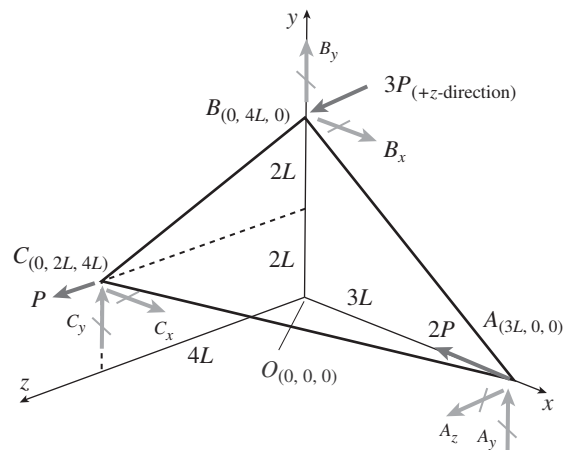
(b) FORCE IN MEMBER AB

$\Sigma F_z = 0$ at joint B $F_{AB} = \frac{\sqrt{(0.8L)^2 + L^2}}{0.8L} |B_z|$ $|B_z| = |P|$ $\frac{\sqrt{(0.8L)^2 + L^2}}{0.8L} = 1.601$

$F_{AB} = 1.601P$ tension

Problem 1.2-9 A space truss is restrained at joints A , B , and C , as shown in the figure. Load $2P$ is applied at in the $-x$ direction at joint A , load $3P$ acts in the $+z$ direction at joint B and load P is applied in the $+z$ direction at joint C . Coordinates of all joints are given in terms of dimension variable L (see figure).

- (a) Find reaction force components A_y and A_z in terms of load variable P .
- (b) Find the axial force in truss member AB in terms of load variable P .



Solution 1.2-9(a) FIND REACTIONS USING STATICS $m = 3$ $r = 6$ $m + r = 9$ $j = 3$ $3j = 9$ $m + r = 3j$ So truss is statically determinate

$$r_{OA} = \begin{pmatrix} 3L \\ 0 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 0 \\ 4L \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 2L \\ 4L \end{pmatrix} \quad F_A = \begin{pmatrix} -2P \\ A_y \\ A_z \end{pmatrix} \quad F_B = \begin{pmatrix} B_x \\ B_y \\ 3P \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ C_y \\ P \end{pmatrix}$$

$\Sigma M = 0$

Resultant moment at O

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B + r_{OC} \times F_C = \begin{pmatrix} 14LP - 4C_yL \\ 4C_xL - 3A_zL \\ 3A_yL - 4B_xL - 2C_xL \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \quad \text{gives} \quad C_y = \frac{14}{4}P$$

$\Sigma F = 0$

Resultant force at O

$$R_O = F_A + F_B + F_C = \begin{pmatrix} B_x + C_x - 2P \\ A_y + B_y + C_y \\ A_z + 4P \end{pmatrix} \quad \text{so} \quad \Sigma F_z = 0 \quad \text{gives} \quad \boxed{A_z = -4.0P}$$

METHOD OF JOINTS

$$\text{Joint A} \quad \Sigma F_z = 0 \quad F_{ACz} = -A_z = 4.0P \quad \text{so} \quad F_{ACy} = \frac{2}{4}F_{ACz} = 2.0P \quad F_{ACx} = \frac{3}{4}F_{ACz} = 3.0P$$

$$\Sigma F_x = 0 \quad F_{ABx} = -2P - F_{ACx} = -3.0P - 2P \quad \text{so} \quad F_{ABy} = \frac{4}{3}F_{ABx} = -4.0P - \frac{8P}{3}$$

$$\Sigma F_y = 0 \quad A_y = -(F_{ABy} + F_{ACy}) = \frac{8P}{3} + 4.0P + -2.0P \quad \boxed{A_y = 4.67P}$$

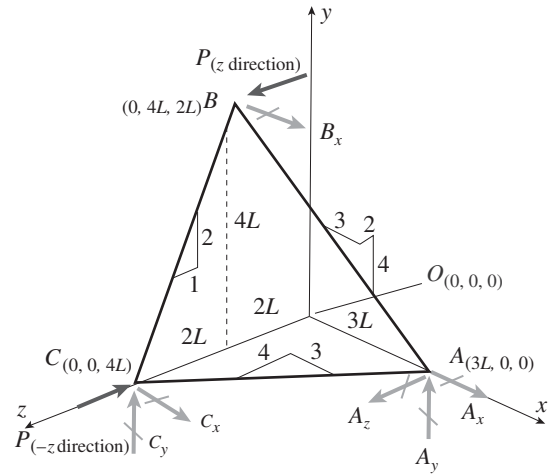
(b) FORCE IN MEMBER AB

$$F_{AB} = \sqrt{F_{ABx}^2 + F_{ABy}^2} \quad F_{AB} = -\sqrt{5^2 + \left(\frac{20}{3}\right)^2}P = -\frac{25P}{3} \quad \frac{25}{3} = 8.33$$

$$\boxed{F_{AB} = -8.33P} \quad \text{compression}$$

Problem 1.2-10 A space truss is restrained at joints A , B , and C , as shown in the figure. Load P acts in the $+z$ direction at joint B and in the $-z$ direction at joint C . Coordinates of all joints are given in terms of dimension variable L (see figure). Let $P = 5$ kN and $L = 2$ m.

- (a) Find the reaction force components A_z and B_x .
- (b) Find the axial force in truss member AB .



Solution 1.2-10

(a) FIND REACTIONS USING STATICS $m = 3$ $r = 6$ $m + r = 9$ $j = 3$ $3j = 9$
 $m + r = 3j$ so truss is statically determinate

$L = 2$ m $P = 5$ kN

$$r_{OA} = \begin{pmatrix} 3L \\ 0 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 0 \\ 4L \\ 2L \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 0 \\ 4L \end{pmatrix} \quad F_A = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \quad F_B = \begin{pmatrix} B_x \\ 0 \\ P \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ C_y \\ -P \end{pmatrix}$$

$\Sigma F = 0$

Resultant force at O $R_O = F_A + F_B + F_C = \begin{pmatrix} A_x + B_x + C_x \\ A_y + C_y \\ A_z \end{pmatrix}$ so $\Sigma F_z = 0$ gives $A_z = 0$

RESULTANT MOMENT AT A

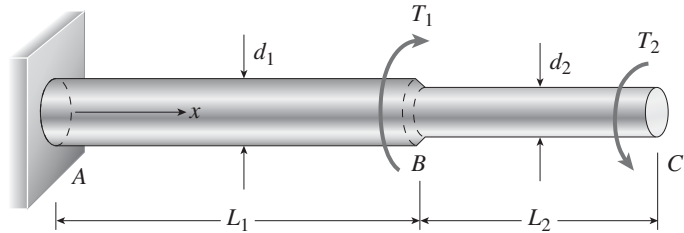
$$r_{AC} = \begin{pmatrix} -3L \\ 0 \\ 4L \end{pmatrix} \quad e_{AC} = \frac{r_{AC}}{|r_{AC}|} = \begin{pmatrix} -0.6 \\ 0 \\ 0.8 \end{pmatrix} \quad r_{AB} = \begin{pmatrix} -3L \\ 4L \\ 2L \end{pmatrix}$$

$$M_A = r_{AB} \times F_B + r_{AC} \times F_C = \begin{pmatrix} 120 \text{ kN} - 24C_y \\ 12B_x + 24C_x \\ -24B_x - 18C_y \end{pmatrix} \quad M_A e_{AC} = -19.2B_x - 72.0 \text{ kN} \quad \text{so } B_x = \frac{-72}{19.2} \text{ kN} = -3.75 \text{ kN}$$

(b) FORCE IN MEMBER AB

Method of joints at B $\Sigma F_x = 0$ $F_{ABx} = -B_x$ $F_{AB} = \frac{\sqrt{29}}{3} F_{ABx} = 6.73 \text{ kN}$

Problem 1.2-11 A stepped shaft ABC consisting of two solid, circular segments is subjected to torques T_1 and T_2 acting in opposite directions, as shown in the figure. The larger segment of the shaft has a diameter of $d_1 = 2.25$ in. and a length of $L_1 = 30$ in.; the smaller segment has a diameter $d_2 = 1.75$ in. and a length $L_2 = 20$ in. The torques are $T_1 = 21,000$ lb-in. and $T_2 = 10,000$ lb-in.



- (a) Find reaction torque T_A at support A.
- (b) Find the internal torque $T(x)$ at two locations: $x = L_1/2$ and $x = L_1 + L_2/2$. Show these internal torques on properly drawn free-body diagrams (FBDs).

Solution 1.2-11

(a) APPLY LAWS OF STATICS $L_1 = 30$ in. $L_2 = 20$ in. $T_1 = 21000$ lb-in. $T_2 = 10000$ lb-in.

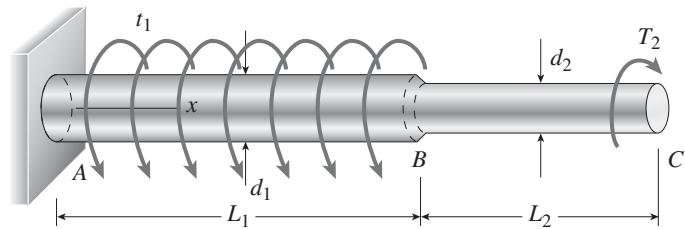
$\Sigma M_x = 0$ $T_A = T_1 - T_2 = 11,000$ lb-in.

(b) INTERNAL STRESS RESULTANT T AT TWO LOCATIONS

Cut shaft at midpoint between A and B at $x = L_1/2$ $\Sigma M_x = 0$ $T_{AB} = -T_A = -11,000$ lb-in.
 (use left FBD)

Cut shaft at midpoint between B and C at $x = L_1 + L_2/2$ $\Sigma M_x = 0$ $T_{BC} = T_2 = 10,000$ lb-in.
 (use right FBD)

Problem 1.2-12 A stepped shaft ABC consisting of two solid, circular segments is subjected to uniformly distributed torque t_1 acting over segment 1 and concentrated torque T_2 applied at C, as shown in the figure. Segment 1 of the shaft has a diameter of $d_1 = 57$ mm and length of $L_1 = 0.75$ m; segment 2 has a diameter $d_2 = 44$ mm and length $L_2 = 0.5$ m. Torque intensity $t_1 = 3100$ N·m/m and $T_2 = 1100$ N·m.



- (a) Find reaction torque T_A at support A.
- (b) Find the internal torque $T(x)$ at two locations: $x = L_1/2$ and $x = L_1 + L_2/2$. Show these internal torques on properly drawn free-body diagrams (FBDs).

Solution 1.2-12

(a) REACTION TORQUE AT A $L_1 = 0.75$ m $L_2 = 0.75$ m $t_1 = 3100$ N·m/m $T_2 = 1100$ N·m

Statics $\Sigma M_x = 0$ $T_A = -t_1 L_1 + T_2 = -1225$ N·m $T_A = -1225$ N·m

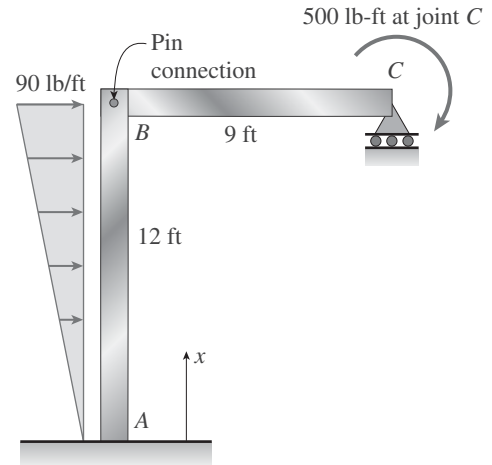
(b) INTERNAL TORSIONAL MOMENTS AT TWO LOCATIONS

Cut shaft between A and B $T_1(x) = -T_A - t_1 x$ $T_1\left(\frac{L_1}{2}\right) = 62.5$ N·m
 (use left FBD)

Cut shaft between B and C $T_2(x) = -T_A - t_1 L_1$ $T_2\left(L_1 + \frac{L_2}{2}\right) = -1100$ N·m
 (use left FBD)

Problem 1.2-13 A plane frame is restrained at joints A and C , as shown in the figure. Members AB and BC are pin connected at B . A triangularly distributed lateral load with peak intensity of 90 lb/ft acts on AB . A concentrated moment is applied at joint C .

- (a) Find reactions at supports A and C .
- (b) Find internal stress resultants N , V , and M at $x = 3 \text{ ft}$ on column AB .



Solution 1.2-13

(a) STATICS

$$\Sigma F_H = 0 \quad A_x = \frac{-1}{2}(90 \text{ lb/ft}) 12 \text{ ft} = -540 \text{ lb}$$

$$\Sigma F_V = 0 \quad A_y + C_y = 0$$

$$\Sigma M_{FBD BC} = 0 \quad C_y = \frac{500 \text{ lb-ft}}{9 \text{ ft}} = 55.6 \text{ lb} \quad A_y = -C_y = -55.6 \text{ lb}$$

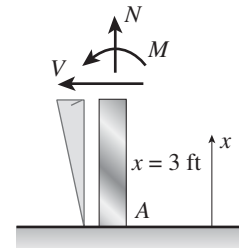
$$\Sigma M_A = 0 \quad M_A = 500 \text{ lb-ft} + \frac{1}{2}(90 \text{ lb/ft}) 12 \text{ ft} \left(\frac{2}{3} 12 \text{ ft}\right) - C_y 9 \text{ ft} = 4320 \text{ lb-ft}$$

(b) INTERNAL STRESS RESULTANTS

$$N = -A_y = 55.6 \text{ lb}$$

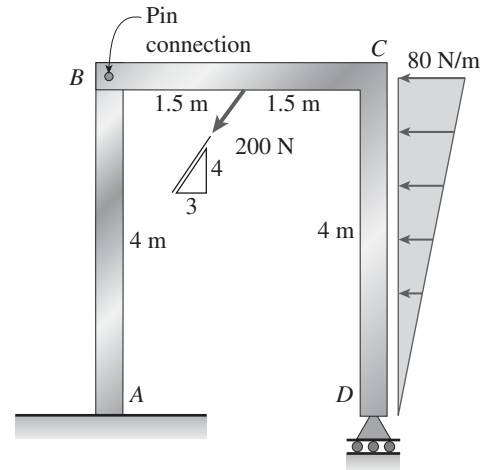
$$V = -A_x - \frac{1}{2} \left(\frac{3}{12} 90 \text{ lb/ft}\right) 3 \text{ ft} = 506 \text{ lb}$$

$$M = -M_A - A_x 3 \text{ ft} - \frac{1}{2} \left(\frac{3}{12} 90 \text{ lb/ft}\right) 3 \text{ ft} \left(\frac{1}{3} 3 \text{ ft}\right) = -2734 \text{ lb-ft}$$



Problem 1.2-14 A plane frame is restrained at joints *A* and *D*, as shown in the figure. Members *AB* and *BCD* are pin connected at *B*. A triangularly distributed lateral load with peak intensity of 80 N/m acts on *CD*. An inclined concentrated force of 200 N acts at the mid-span of *BC*. An inclined concentrated force of 200 N acts at the mid-span of *BC*.

- (a) Find reactions at supports *A* and *D*.
- (b) Find resultant forces in the pins at *B* and *C*.



Solution 1.2-14

(a) STATICS

$$\Sigma F_x = 0 \quad A_x = \frac{3}{5}(200 \text{ N}) + \frac{1}{2}(80 \text{ N/m})4 \text{ m} = 280 \text{ N}$$

$$\Sigma M_{BRHFB} = 0 \quad D_y = \frac{1}{3 \text{ m}} \left[\frac{4}{5}(200 \text{ N})(1.5 \text{ m}) + \frac{1}{2}(80 \text{ N/m})4 \text{ m} \left(\frac{1}{3}4 \text{ m} \right) \right]$$

$$= 151.1 \text{ N} < \text{ use right hand FBD (BCD only)}$$

$$\Sigma F_y = 0 \quad A_y = -D_y + \frac{4}{5}(200 \text{ N}) = 8.89 \text{ N}$$

$$\Sigma M_A = 0 \quad M_A = \frac{4}{5}(200 \text{ N})(1.5 \text{ m}) - \frac{3}{5}(200 \text{ N})(4 \text{ m}) - D_y 3 \text{ m} - \frac{1}{2}(80 \text{ N/m})4 \text{ m} \left(\frac{2}{3}4 \text{ m} \right) = -1120 \text{ N}\cdot\text{m}$$

(b) RESULTANT FORCE IN PIN AT *B*

LEFT HAND FBD (SEE FIGURE)

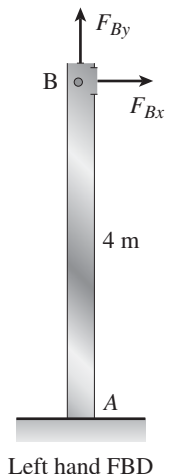
$$F_{Bx} = -A_x = -280 \text{ N} \quad F_{By} = -A_y = -8.89 \text{ N}$$

RIGHT HAND FBD

$$F_{Bx} = \frac{3}{5}(200 \text{ N}) + \frac{1}{2}(80 \text{ N/m})4 \text{ m} = 280 \text{ N}$$

$$F_{By} = \frac{4}{5}(200 \text{ N}) - D_y = 8.89 \text{ N}$$

$$\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 280 \text{ N}$$

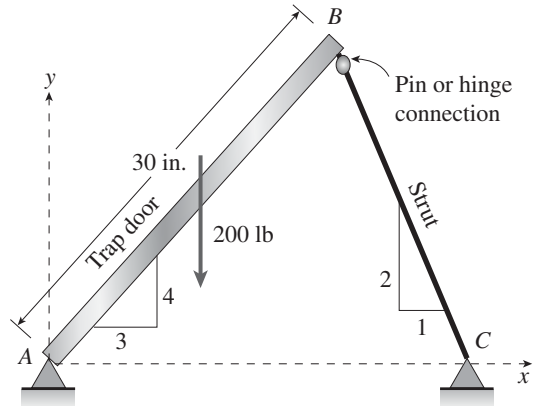


Problem 1.2-15 A 200 lb trap door (AB) is supported by a strut (BC) which is pin connected to the door at B (see figure).

- (a) Find reactions at supports A and C .
- (b) Find internal stress resultants N , V , and M on the trap door at 20 in. from A .

$$L_{BC} = \frac{\frac{4}{5} \cdot 30 \text{ in.}}{\frac{2}{\sqrt{5}}} = 26.833 \text{ in.}$$

$$L_{AC} = \frac{3}{5}(30 \text{ in.}) + \frac{1}{\sqrt{5}} L_{BC} = 30 \text{ in.}$$



Solution 1.2-15

(a) STATICS

$$\begin{aligned} \Sigma M_A = 0 \quad C_y &= \frac{1}{L_{AC}} \left[200 \text{ lb} \left(\frac{1}{2} \right) \left(\frac{3}{5} \right) 30 \text{ in.} \right] = 60 \text{ lb} & C_x &= \frac{-1}{2} C_y = -30 \text{ lb} \\ \Sigma F_x = 0 \quad A_x &= -C_x = 30 \text{ lb} & & \text{(resultant of } C_x \text{ and } C_y \text{ acts along line of strut)} \\ \Sigma F_y = 0 \quad A_y &= 200 \text{ lb} - C_y = 140 \text{ lb} \end{aligned}$$

(b) INTERNAL STRESS RESULTANTS N , V , M (SEE FIGURE)

Distributed weight of door in $-y$ direction $w = \frac{200 \text{ lb}}{30 \text{ in.}} = 6.667 \text{ lb/in.}$

Components of w along and perpendicular to door

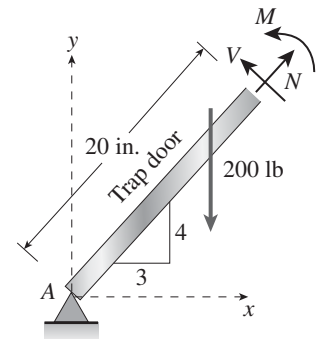
$$w_a = \frac{4}{5} w = 5.333 \text{ lb/in.} \quad w_p = \frac{3}{5} w = 4 \text{ lb/in.}$$

$$N = w_a(20 \text{ in.}) - \frac{3}{5} A_x - \frac{4}{5} A_y = -23.333 \text{ lb}$$

$$V = -w_p(20 \text{ in.}) - \frac{4}{5} A_x + \frac{3}{5} A_y = -20 \text{ lb}$$

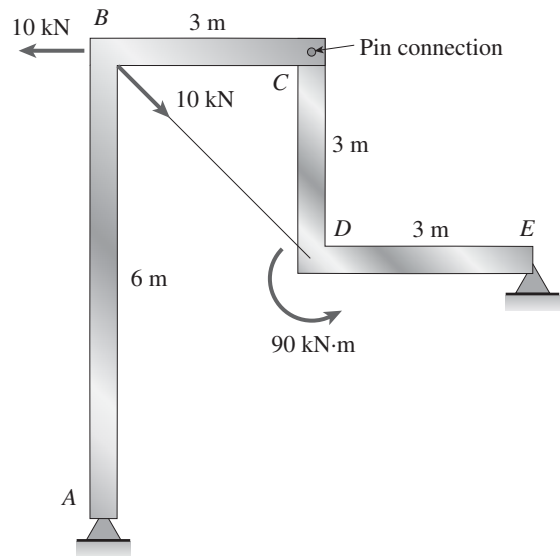
$$M = -w_p(20 \text{ in.}) \frac{20 \text{ in.}}{2} - \frac{4}{5} A_x(20 \text{ in.}) + \frac{3}{5} A_y(20 \text{ in.}) = 33.333 \text{ lb-ft}$$

$N = -23.3 \text{ lb}$ $V = -20 \text{ lb}$ $M = 33.3 \text{ lb-ft}$



Problem 1.2-16 A plane frame is constructed by using a pin connection between segments *ABC* and *CDE*. The frame has pin supports at *A* and *E* and has joint loads at *B* and *D* (see figure).

- (a) Find reactions at supports *A* and *E*.
- (b) Find resultant force in the pin at *C*.



Solution 1.2-16

(a) STATICS

$$\Sigma M_A = 0$$

$$10 \text{ kN}(6 \text{ m}) - 10 \text{ kN}\left(\frac{1}{\sqrt{2}}\right)(6 \text{ m}) + 90 \text{ kN}\cdot\text{m} + E_y(6 \text{ m}) - E_x(3 \text{ m}) = 6E_y \text{ m} - 3E_x \text{ m} + 150 \text{ kN}\cdot\text{m} - 30\sqrt{2} \text{ kN}\cdot\text{m}$$

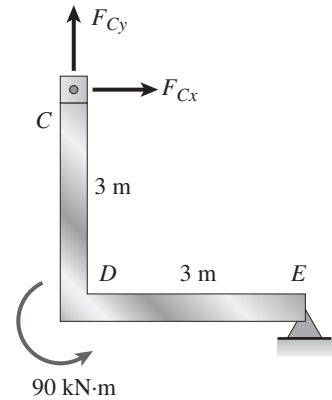
$$\text{so } 6E_y \text{ m} - 3E_x \text{ m} + 150 \text{ kN}\cdot\text{m} - 30\sqrt{2} \text{ kN}\cdot\text{m} = 0$$

$$\text{or } -E_x + 2E_y = \frac{-(150 \text{ kN}\cdot\text{m} - 30\sqrt{2} \text{ kN}\cdot\text{m})}{3 \text{ m}} = -35.858 \text{ kN}$$

$\Sigma M_{CRHFB} = 0$ < right hand FBD (*CDE*) - see figure.

$$(E_x + E_y) 3 \text{ m} = -90 \text{ kN}\cdot\text{m} \quad E_x + E_y = \frac{-90 \text{ kN}\cdot\text{m}}{3 \text{ m}} = -30 \text{ kN}$$

Solving $\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -35.858 \text{ kN} \\ -30 \text{ kN} \end{pmatrix} = \begin{pmatrix} -8.05 \\ -21.95 \end{pmatrix} \text{ kN}$



$$E_x = -8.05 \text{ kN} \quad E_y = -22 \text{ kN}$$

$$\Sigma F_x = 0 \quad A_x = -E_x + 10 \text{ kN} - 10 \text{ kN}\left(\frac{1}{\sqrt{2}}\right) = 10.98 \text{ kN} \quad A_x = 10.98 \text{ kN}$$

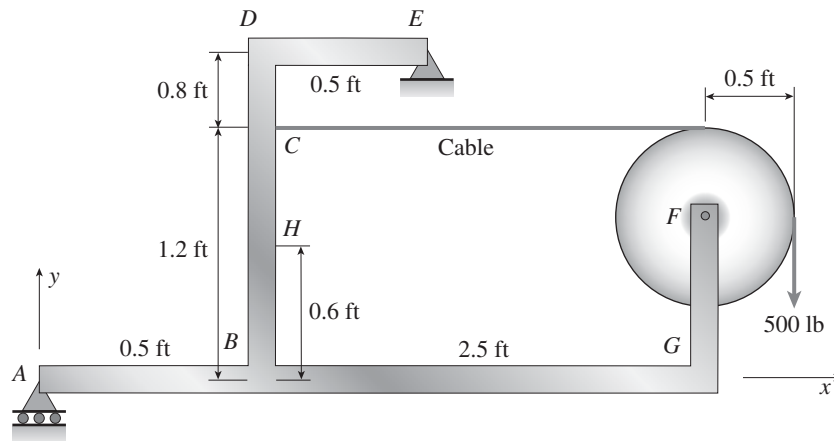
$$\Sigma F_y = 0 \quad A_y = -E_y + 10 \text{ kN}\left(\frac{1}{\sqrt{2}}\right) = 29.07 \text{ kN} \quad A_y = 29.1 \text{ kN}$$

(b) RIGHT HAND FBD $C_x = -E_x = 8.05 \text{ kN} \quad C_y = -E_y = 22 \text{ kN}$

$$\text{Resultant}_C = \sqrt{C_x^2 + C_y^2} = 23.4 \text{ kN}$$

Problem 1.2-17 A plane frame with pin supports at A and E has a cable attached at C , which runs over a frictionless pulley at F (see figure). The cable force is known to be 500 lb.

- (a) Find reactions at supports A and E .
- (b) Find internal stress resultants, N , V , and M at point H .



Solution 1.2-17

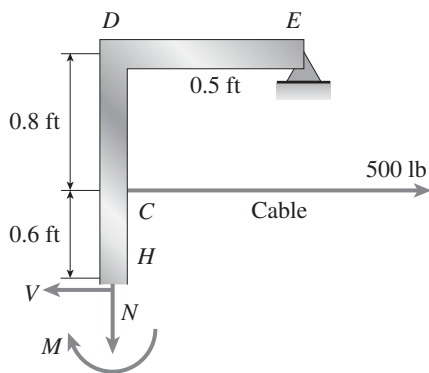
(a) STATICS

$$\Sigma F_x = 0 \quad E_x = 0$$

$$\Sigma M_E = 0 \quad A_y = \frac{1}{1 \text{ ft}}(-500 \text{ lb} \times 2.5 \text{ ft}) = -1250 \text{ lb}$$

$$\Sigma F_y = 0 \quad E_y = 500 \text{ lb} - A_y = 1750 \text{ lb}$$

(b) USE UPPER (SEE FIGURE BELOW) OR LOWER FBD TO FIND STRESS RESULTANTS N , V , AND M AT H



$$\Sigma F_x = 0 \quad V = E_x + 500 \text{ lb} = 500 \text{ lb}$$

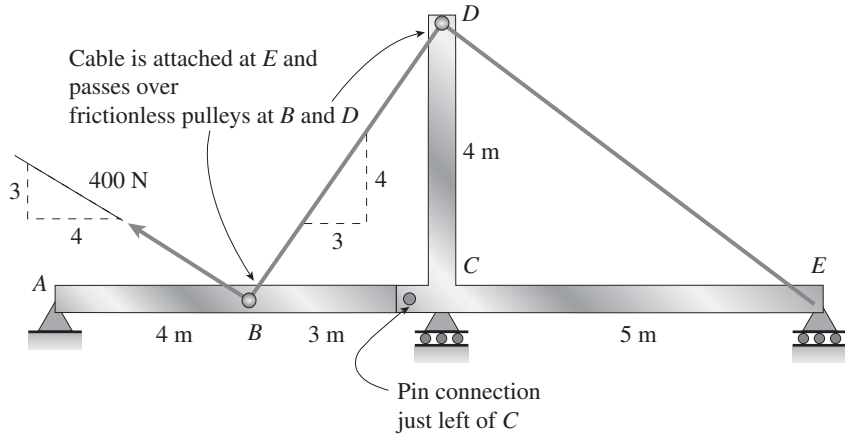
$$\Sigma F_y = 0 \quad N = E_y = 1750 \text{ lb}$$

$$\Sigma M_H = 0$$

$$M = -0.6 \text{ ft}(500 \text{ lb}) - E_x 1.4 \text{ ft} + E_y 0.5 \text{ ft} = 575 \text{ lb-ft}$$

Problem 1.2-18 A plane frame with a pin support at A and roller supports at C and E has a cable attached at E , which runs over frictionless pulleys at D and B (see figure). The cable force is known to be 400 N. There is a pin connection just to the left of joint C .

- Find reactions at supports A , C , and E .
- Find internal stress resultants N , V , and M just to the right of joint C .
- Find resultant force in the pin near C .



Solution 1.2-18

(a) STATICS

$$\sum F_x = 0 \quad A_x = \frac{4}{5}(400 \text{ N}) = 320 \text{ N} \quad \boxed{A_x = 320 \text{ N}}$$

Use left hand FBD (cut through pin just left of C)

$$\sum M_C = 0 \quad A_y = \frac{1}{7 \text{ m}} \left[\left[\frac{-3}{5}(400 \text{ N}) - \frac{4}{5}(400 \text{ N}) \right] (3 \text{ m}) \right] = -240 \text{ N} \quad \boxed{A_y = -240 \text{ N}}$$

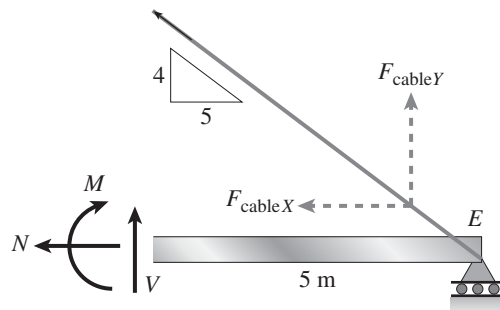
Use entire FBD $\sum M_C = 0 \quad E_y = \frac{1}{5 \text{ m}} \left[A_y(7 \text{ m}) + \left(\frac{3}{5} 400 \text{ N} \right) (3 \text{ m}) \right] = -192 \text{ N} \quad \boxed{E_y = -192 \text{ N}}$

$$\sum F_y = 0 \quad C_y = -A_y - E_y - \frac{3}{5}(400 \text{ N}) = 192 \text{ N} \quad \boxed{C_y = 192 \text{ N}}$$

(b) N , V , AND M JUST RIGHT OF C ; USE RIGHT HAND FBD

$$F_{\text{cable}X} = 400 \text{ N} \left(\frac{5}{\sqrt{4^2 + 5^2}} \right) = 312.348 \text{ N}$$

$$F_{\text{cable}Y} = \frac{4}{5} F_{\text{cable}X} = 249.878 \text{ N}$$



$$\sum F_x = 0 \quad \boxed{N_x = -F_{\text{cable}X} = -312 \text{ N}}$$

$$\sum F_y = 0 \quad \boxed{V = -F_{\text{cable}Y} - E_y = -57.9 \text{ N}}$$

$$\sum M_C = 0 \quad M = (F_{\text{cable}Y} + E_y)(5 \text{ m}) = \boxed{289 \text{ N}\cdot\text{m}}$$

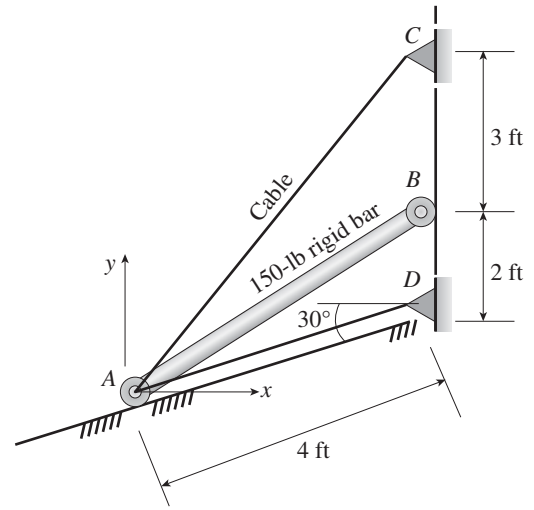
(c) RESULTANT FORCE IN PIN JUST LEFT OF C ; USE LEFT HAND FBD $A_x = 320 \text{ N}$

$$F_{Cx} = -A_x + \left(\frac{4}{5} - \frac{3}{5} \right) 400 \text{ N} = -240 \text{ N} \quad F_{Cy} = -A_y - \left(\frac{3}{5} + \frac{4}{5} \right) 400 \text{ N} = -320 \text{ N}$$

$$\boxed{\text{Res}_C = \sqrt{F_{Cx}^2 + F_{Cy}^2} = 400 \text{ N}}$$

Problem 1.2-19 A 150-lb rigid bar AB , with frictionless rollers at each end, is held in the position shown in the figure by a continuous cable CAD . The cable is pinned at C and D and runs over a pulley at A .

- (a) Find reactions at supports A and B .
- (b) Find the force in the cable.



Solution 1.2-19

(a) STATICS $W = 150$ lb

$$\sum M_A = 0 \quad B_x(4) + W\left(\frac{2\sqrt{3}}{2}\right) = 0 \text{ solve, } B_x = -\frac{75\sqrt{3}}{2}$$

$$\text{so } B_x = -\frac{75\sqrt{3}}{2} = -64.952$$

$$\sum F_x = 0 \quad -A \sin(30^\circ) + B_x + T \cos(30^\circ) + T \cos\left(\arctan\left(\frac{7}{2\sqrt{3}}\right)\right) = 0$$

$$\sum F_y = 0 \quad A \cos(30^\circ) + T \sin(30^\circ) + T \sin\left(\arctan\left(\frac{7}{2\sqrt{3}}\right)\right) = W$$

$$\begin{pmatrix} A \\ T \end{pmatrix} = \begin{pmatrix} -\sin(30^\circ) & \cos(30^\circ) + \cos\left(\arctan\left(\frac{7}{2\sqrt{3}}\right)\right) \\ \cos(30^\circ) & \sin(30^\circ) + \sin\left(\arctan\left(\frac{7}{2\sqrt{3}}\right)\right) \end{pmatrix}^{-1} \begin{pmatrix} -B_x \\ W \end{pmatrix} \quad \begin{pmatrix} A \\ T \end{pmatrix} = \begin{pmatrix} 57.713 \\ 71.634 \end{pmatrix} \text{ lb}$$

SUPPORT REACTIONS

$$\boxed{B_x = -65} \quad \boxed{A = 57.7} \quad \text{Units} = \text{lbs}$$

$$A_x = -A \sin(30^\circ) = -28.9 \text{ lb} \quad A_y = A \cos(30^\circ) = 50 \text{ lb}$$

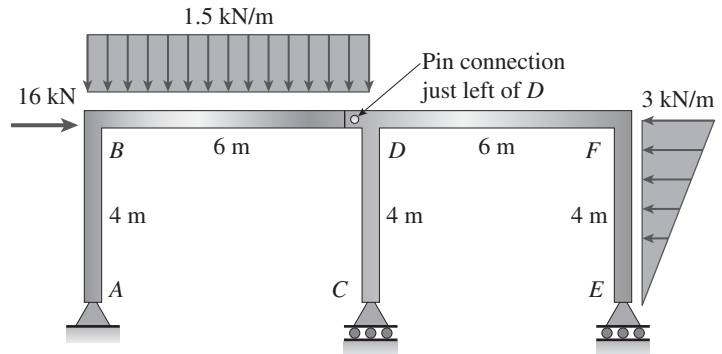
$$\sqrt{A_x^2 + A_y^2} = 57.713$$

(b) CABLE FORCE IS T (LBS) FROM ABOVE SOLUTION

$$\boxed{T = 71.6 \text{ lb}}$$

Problem 1.2-20 A plane frame has a pin support at A and roller supports at C and E (see figure). Frame segments ABD and $CDEF$ are joined just left of joint D by a pin connection.

- Find reactions at supports A , C , and E .
- Find the resultant force in the pin just left of D .



Solution 1.2-20

(a) STATICS

RIGHT-HAND FBD

$$\Sigma M_{\text{pin}} = 0 \quad E_y = \frac{1}{6 \text{ m}} \left[\frac{1}{2} (3 \text{ kN/m}) 4 \text{ m} \left(\frac{1}{3} 4 \text{ m} \right) \right] = 1.333 \text{ kN} \quad \boxed{E_y = 1.333 \text{ kN}}$$

ENTIRE FBD

$$\Sigma M_A = 0 \quad C_y = \frac{1}{6 \text{ m}} \left[-E_y 12 \text{ m} + (16 \text{ kN}) 4 \text{ m} + (1.5 \text{ kN/m}) 6 \text{ m} (3 \text{ m}) - \frac{1}{2} (3 \text{ kN/m}) 4 \text{ m} \left(\frac{2}{3} 4 \text{ m} \right) \right] = 9.833 \text{ kN}$$

$$\boxed{C_y = 9.83 \text{ kN}}$$

$$\Sigma F_y = 0 \quad A_y = -C_y - E_y + (1.5 \text{ kN/m}) 6 \text{ m} = -2.167 \text{ kN} \quad \boxed{A_y = -2.17 \text{ kN}}$$

$$\Sigma F_x = 0 \quad A_x = -16 \text{ kN} + \frac{1}{2} (3 \text{ kN/m}) 4 \text{ m} = -10 \text{ kN} \quad \boxed{A_x = -10 \text{ kN}}$$

(b) RESULTANT FORCE IN PIN; USE EITHER RIGHT HAND OR LEFT HAND FBD (CUT THROUGH PIN EXPOSING PIN FORCES F_{Dx} AND F_{Dy}) THEN SUM FORCES IN x AND y DIRECTIONS FOR EITHER FBD

LHFB:

$$F_{Dx} = -16 \text{ kN} - A_x = -6 \text{ kN}$$

$$F_{Dy} = -A_y + (1.5 \text{ kN/m}) 6 \text{ m} = 11.167 \text{ kN}$$

$$\text{Resultant}_D = \sqrt{F_{Dx}^2 + F_{Dy}^2} = 12.68 \text{ kN}$$

RHFB:

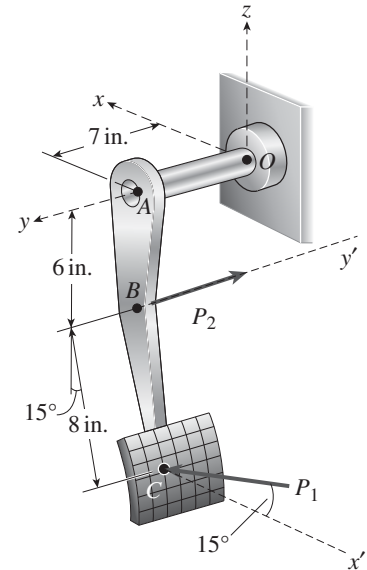
$$F_{Dx} = \frac{1}{2} (3 \text{ kN/m}) 4 \text{ m} = 6 \text{ kN}$$

$$F_{Dy} = -C_y - E_y = -11.167 \text{ kN}$$

$$\boxed{\text{Resultant}_D = 12.68 \text{ kN}}$$

Problem 1.2-21 A special vehicle brake is clamped at O , (when the brake force P_1 is applied—see figure). Force $P_1 = 50$ lb and lies in a plane which is parallel to the xz plane and is applied at C normal to line BC . Force $P_2 = 40$ lb and is applied at B in the $-y$ direction.

- (a) Find reactions at support O .
- (b) Find internal stress resultants N , V , T , and M at the midpoint of segment OA .



Solution 1.2-21

(a) STATICS $P_1 = 50$ lb $P_2 = 40$ lb

$$\Sigma F_x = 0 \quad O_x = -P_1 \cos(15^\circ) = -48.3 \text{ lb} \quad \Sigma F_y = 0 \quad O_y = P_2 = 40 \text{ lb}$$

$$\Sigma F_z = 0 \quad O_z = P_1 \sin(15^\circ) = 12.94 \text{ lb}$$

$$\Sigma M_x = 0 \quad M_{Ox} = P_2(6 \text{ in.}) + P_1 \sin(15^\circ)(7 \text{ in.}) = 331 \text{ lb-in.}$$

$$\Sigma M_y = 0 \quad M_{Oy} = P_1 \sin(15^\circ)(8 \text{ in.} \sin(15^\circ)) + P_1 \cos(15^\circ)(6 \text{ in.} + 8 \text{ in.} \cos(15^\circ))$$

$$M_{Oy} = 690 \text{ lb-in.}$$

$$\Sigma M_z = 0 \quad M_{Oz} = -P_1 \cos(15^\circ)(7 \text{ in.}) = -338 \text{ lb-in.}$$

- (b) INTERNAL STRESS RESULTANTS AT MIDPOINT OF OA

$$N = -O_y = -40 \text{ lb}$$

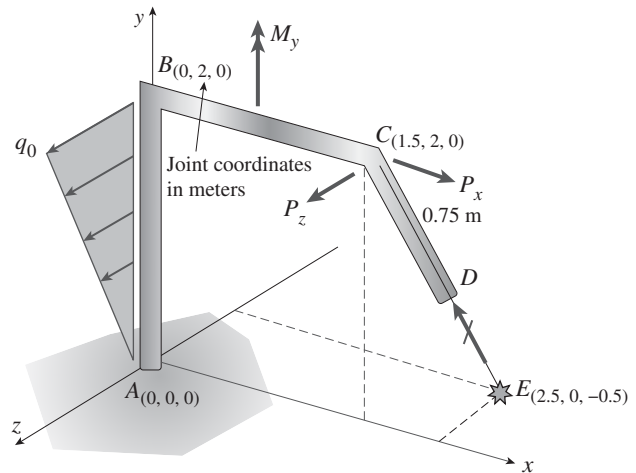
$$V_x = -O_x = 48.3 \text{ lb} \quad V_z = -O_z = -12.94 \text{ lb} \quad V = \sqrt{V_x^2 + V_z^2} = 50 \text{ lb}$$

$$T = -M_{Oy} = -690 \text{ lb-in.}$$

$$M_x = -M_{Ox} = -330.59 \text{ lb-in.} \quad M_z = -M_{Oz} = 338.07 \text{ lb-in.} \quad M = \sqrt{M_x^2 + M_z^2} = 473 \text{ lb-in.}$$

Problem 1.2-22 Space frame $ABCD$ is clamped at A , except it is free to translate in the x -direction. There is also a roller support at D , which is normal to line CDE . A triangularly distributed force with peak intensity $q_0 = 75 \text{ N/m}$ acts along AB in the positive z direction. Forces $P_x = 60 \text{ N}$ and $P_z = -45 \text{ N}$ are applied at joint C and a concentrated moment $M_y = 120 \text{ N}\cdot\text{m}$ acts at the mid-span of member BC .

- (a) Find reactions at supports A and D .
 (b) Find internal stress resultants N , V , T , and M at the mid-height of segment AB .



Solution 1.2-22

FORCES

$$P_x = 60 \text{ N} \quad P_z = -45 \text{ N} \quad M_y = 120 \text{ N}\cdot\text{m} \quad q_0 = 75 \text{ N/m}$$

$$F_C = \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} = \begin{pmatrix} 60 \\ 0 \\ -45 \end{pmatrix} \text{ N} \quad R_A = \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix}$$

VECTOR ALONG MEMBER CD

$$r_{EC} = \begin{bmatrix} 1.5 - 2.5 \\ 2 - 0 \\ 0 - (-0.5) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0.5 \end{bmatrix} \quad |r_{EC}| = 2.291 \quad e_{EC} = \frac{r_{EC}}{|r_{EC}|} = \begin{pmatrix} -0.436 \\ 0.873 \\ 0.218 \end{pmatrix}$$

(a) STATICS (FORCE AND MOMENT EQUILIBRIUM)

$$\Sigma F = 0 \quad \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} + \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = 0 \quad \text{resultant of triangular load: } R_T = \frac{1}{2} q_0 (2 \text{ m}) = 75 \text{ N}$$

$$\text{where } \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = D e_{EC}$$

SOLVING ABOVE THREE EQUATIONS:

$$\Sigma F_x = 0 \quad D_x = -P_x \quad \text{so} \quad D = \frac{-P_x}{e_{EC1}}$$

$$D = 137.477 \text{ N} \quad \boxed{D_x = -60 \text{ N}}$$

$$\Sigma F_y = 0 \quad D_y = e_{EC2} D$$

$$\boxed{D_y = 120 \text{ N}}$$

$$\boxed{A_y = -D_y = -120 \text{ N}}$$

$$\Sigma F_z = 0 \quad D_z = e_{EC3} D$$

$$\boxed{D_z = 30 \text{ N}}$$

$$\sqrt{D_x^2 + D_y^2 + D_z^2} = 137.477 \text{ N}$$

$$\text{so } A_z = -D_z - R_T - P_z$$

$$\boxed{A_z = -60 \text{ N}}$$

$$\Sigma M_A = 0$$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} + r_{AE} \times D + r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} 0 \\ M_y \\ 0 \end{pmatrix} + r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} = 0$$

$$r_{AE} = \begin{pmatrix} 2.5 - 0 \\ 0 - 0 \\ -0.5 - 0 \end{pmatrix} \text{ m} \quad D = \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} \quad D = \begin{pmatrix} -60 \\ 120 \\ 30 \end{pmatrix} \text{ N} \quad |D| = 137.477 \text{ N} \quad r_{AE} \times D = \begin{pmatrix} 60 \\ -45 \\ 300 \end{pmatrix} \text{ N}\cdot\text{m}$$

$$r_{AC} = \begin{pmatrix} 1.5 - 0 \\ 2 - 0 \\ 0 - 0 \end{pmatrix} \text{ m} \quad r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} = \begin{pmatrix} -90 \\ 67.5 \\ -120 \end{pmatrix} \text{ J} \quad r_{cg} = \begin{pmatrix} 0 \\ \frac{2}{3}(2 \text{ m}) \\ 0 \end{pmatrix} \quad r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} = \begin{pmatrix} 100 \\ 0 \\ 0 \end{pmatrix} \text{ N}\cdot\text{m}$$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} = - \left[r_{AE} \times D + r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} 0 \\ M_y \\ 0 \end{pmatrix} + r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} \right] = \begin{pmatrix} -70 \\ -142.5 \\ -180 \end{pmatrix} \text{ N}\cdot\text{m} \quad \boxed{\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} = \begin{pmatrix} -70 \\ -142.5 \\ -80 \end{pmatrix} \text{ N}\cdot\text{m}}$$

(b) RESULTANTS AT MID-HEIGHT OF AB (SEE FBD IN FIGURE BELOW)

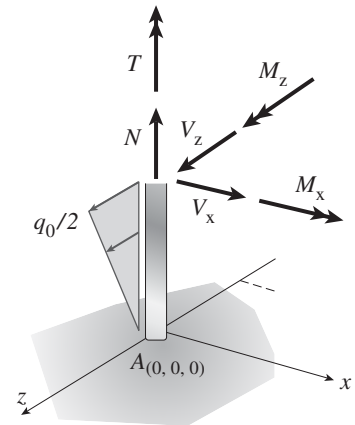
$$\boxed{N = -A_y = 120 \text{ N}} \quad V_x = -D_x - P_x = 0 \text{ N} \quad V_z = -A_z - \frac{1}{2} \frac{q_0}{2} (2\text{m})/2 = 41.25 \text{ N} \quad \boxed{V = V_z = 41.3 \text{ N}}$$

$$\boxed{T = -M_{Ay} = 142.5 \text{ N}\cdot\text{m}} \quad M_x = -M_{Ax} + A_z(1 \text{ m}) + \frac{1}{2} \frac{q_0}{2} 1 \text{ m} \left(\frac{1}{3} 1 \text{ m} \right) = 16.25 \text{ N}\cdot\text{m}$$

$$M_z = -M_{Az} = 180 \text{ N}\cdot\text{m}$$

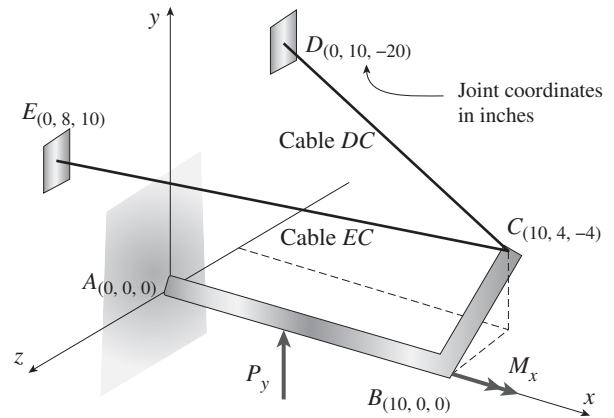
$$M_{\text{resultant}} = \sqrt{M_x^2 + M_z^2} = 180.732 \text{ N}\cdot\text{m}$$

$$\boxed{M_{\text{resultant}} = 180.7 \text{ N}\cdot\text{m}}$$



Problem 1.2-23 Space frame ABC is clamped at A except it is free to rotate at A about the x and y axes. Cables DC and EC support the frame at C . Forces $P_y = -50$ lb is applied at mid-span of AB and a concentrated moment $M_x = -20$ in-lb acts at joint B .

- (a) Find reactions at supports A .
 (b) Find cable tension forces.



Solution 1.2-23

POSITION AND UNIT VECTORS

$$r_{AB} = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix} \quad r_{AP} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \quad r_{AC} = \begin{pmatrix} 10 \\ 4 \\ -4 \end{pmatrix} \quad r_{CD} = \begin{bmatrix} 0 - 10 \\ 10 - 4 \\ -20 - (-4) \end{bmatrix} = \begin{pmatrix} -10 \\ 6 \\ -16 \end{pmatrix} \quad e_{CD} = \frac{r_{CD}}{|r_{CD}|} = \begin{pmatrix} -0.505 \\ 0.303 \\ -0.808 \end{pmatrix}$$

APPLIED FORCE AND MOMENT

$$r_{CE} = \begin{bmatrix} 0 - 10 \\ 8 - 4 \\ 10 - (-4) \end{bmatrix} = \begin{pmatrix} -10 \\ 4 \\ 14 \end{pmatrix} \quad e_{CE} = \frac{r_{CE}}{|r_{CE}|} = \begin{pmatrix} -0.566 \\ 0.226 \\ 0.793 \end{pmatrix}$$

$$P_y = -50 \text{ lb} \quad M_x = -20 \text{ lb-in.}$$

STATICS FORCE AND MOMENT EQUILIBRIUM

First sum moment about point A

$$\sum M_A = 0$$

$$M_A = \begin{pmatrix} 0 \\ 0 \\ M_{Az} \end{pmatrix} + r_{AP} \times \begin{pmatrix} 0 \\ P_y \\ 0 \end{pmatrix} + \begin{pmatrix} M_x \\ 0 \\ 0 \end{pmatrix} + r_{AC} \times (T_D e_{CD} + T_E e_{CE}) = \begin{pmatrix} -2.0203 T_D + 4.0762 T_E - 20.0 \\ 10.102 T_D + -5.6614 T_E \\ M_{Az} + 5.0508 T_D + 4.5291 T_E - 250.0 \end{pmatrix}$$

Solve moment equilibrium equations for moments about x and y axes to get cable tension forces

$$\begin{pmatrix} T_D \\ T_E \end{pmatrix} = \begin{pmatrix} -2.0203 & 4.0762 \\ 10.102 & -5.6614 \end{pmatrix}^{-1} \begin{pmatrix} 20 \\ 0 \end{pmatrix} = \begin{pmatrix} 3.81 \\ 6.79 \end{pmatrix} \text{ lb} \quad (\text{b})$$

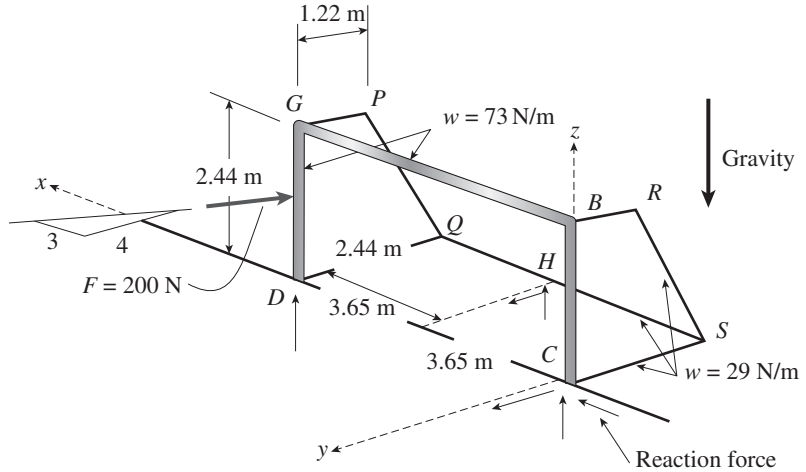
Next, solve moment equilibrium equation about z axis now that cable forces are known

$$M_{Az} = -(5.0508 T_D + 4.5291 T_E - 250.0) = 200 \text{ lb-in.} \quad (\text{a})$$

Finally, use force equilibrium to find reaction forces at point A

$$\sum F = 0 \quad \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = - \begin{pmatrix} 0 \\ P_y \\ 0 \end{pmatrix} - (T_D e_{CD} + T_E e_{CE}) = \begin{pmatrix} 5.77 \\ 47.31 \\ -2.31 \end{pmatrix} \text{ lb}$$

Problem 1.2-24 A soccer goal is subjected to gravity loads (in the $-z$ direction, $w = 73 \text{ N/m}$ for DG , BG , and BC ; $w = 29 \text{ N/m}$ for all other members; see figure) and a force $F = 200 \text{ N}$ applied eccentrically at mid-height of member DG . Find reactions at supports C , D , and H .



Solution 1.2-24

FIND MEMBER LENGTHS

$$L_{QS} = 2(3.65 \text{ m}) = 7.3 \text{ m} \quad L_{RS} = \sqrt{(2.44 \text{ m})^2 + (2.44 \text{ m} - 1.22 \text{ m})^2} = 2.728 \text{ m} \quad L_{PQ} = L_{RS}$$

Assume that soccer goal is supported only at points C , H , and D (see reaction force components in figure)

STATICS SUM MOMENT ABOUT EACH AXIS AND FORCES IN EACH AXIS DIRECTION $F = 200 \text{ N}$

$\Sigma M_x = 0$ TO FIND REACTION COMPONENT H_y :

Find moments about x due to for component F_y and also for distributed weight of each frame component

$$M_{xGP} = \frac{(1.22 \text{ m})^2}{2} (29 \text{ N/m}) \quad M_{xBR} = M_{xGP} \quad M_{xDQ} = \frac{(2.44 \text{ m})^2}{2} (29 \text{ N/m}) \quad M_{xCS} = M_{xDQ}$$

$$M_{xRS} = L_{RS}(29 \text{ N/m}) \left(1.22 \text{ m} + \frac{1.22 \text{ m}}{2} \right) \quad M_{xPQ} = M_{xRS} \quad M_{xQS} = L_{QS}(29 \text{ N/m})(2.44 \text{ m})$$

$$H_z = \frac{1}{2.44 \text{ m}} \left[\frac{4}{5} F \left(\frac{2.44 \text{ m}}{2} \right) + 2M_{xGP} + 2M_{xDQ} + 2M_{xPQ} + M_{xQS} \right] = 498.818 \text{ N} \quad \boxed{H_z = 499 \text{ N}}$$

$\Sigma M_y = 0$ TO FIND REACTION FORCE D_z :

$$M_{yGD} = 2.44 \text{ m} (73 \text{ N/m}) L_{QS} \quad M_{yGP} = 1.22 \text{ m} (29 \text{ N/m}) L_{QS} \quad M_{yDQ} = 2.44 \text{ m} (29 \text{ N/m}) L_{QS}$$

$$M_{yPQ} = L_{RS}(29 \text{ N/m}) L_{QS} \quad M_{yBG} = L_{QS}(73 \text{ N/m}) \frac{L_{QS}}{2} \quad M_{yQS} = L_{QS}(29 \text{ N/m}) \frac{L_{QS}}{2}$$

$$D_z = \frac{1}{L_{QS}} \left[M_{yGD} + M_{yGP} + M_{yDQ} + M_{yPQ} + M_{yBG} + M_{yQS} - H_z \frac{L_{QS}}{2} - \frac{3}{5} F \left(\frac{2.44 \text{ m}}{2} \right) \right] = 466.208 \text{ N}$$

$$\boxed{D_z = 466 \text{ N}}$$

$\Sigma M_z = 0$ TO FIND REACTION FORCE H_y :

$$H_y = \frac{1}{3.65 \text{ m}} \left(\frac{4}{5} F L_{QS} \right) = 320 \text{ N} \quad \boxed{H_y = 320 \text{ N}}$$

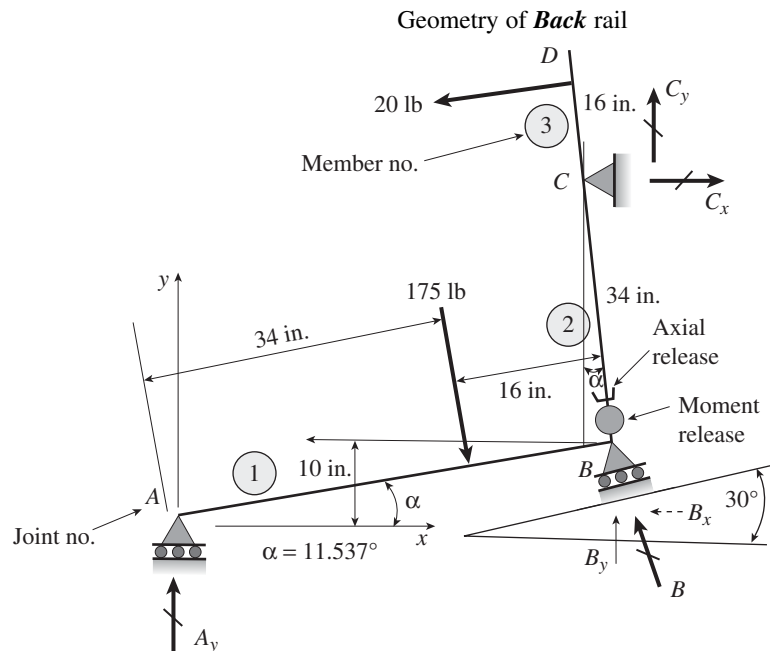
$$\Sigma F_x = 0 \text{ TO FIND REACTION FORCE } C_x: \quad C_x = \frac{3}{5}F = 120 \text{ N}$$

$$\Sigma F_y = 0 \text{ TO FIND REACTION FORCE } C_y: \quad C_y = -H_y + \frac{4}{5}F = -160 \text{ N} \quad C_y = -160 \text{ N}$$

$$\Sigma F_z = 0 \text{ TO FIND REACTION FORCE } C_z:$$

$$C_z = -D_z - H_z + (29 \text{ N/m})(21.22 \text{ m} + 22.44 \text{ m} + 2L_{RS} + L_{QS}) \\ + (73 \text{ N/m})(22.44 \text{ m} + L_{QS}) = 506.318 \text{ N} \quad C_z = 506 \text{ N}$$

Problem 1.2-25 An elliptical exerciser machine (see figure part a) is composed of front and back rails. A simplified plane frame model of the back rail is shown in figure part b. Analyze the plane frame model to find reaction forces at supports A, B, and C for the position and applied loads given in figure part b. Note that there are axial and moment releases at the base of member 2 so that member 2 can lengthen and shorten as the roller support at B moves along the 30° incline. (These releases indicate that the internal axial force N and moment M must be zero at this location.)



Solution 1.2-25

$$\alpha = \arcsin\left(\frac{10}{50}\right) = 11.537^\circ \quad \text{Analysis pertains to this position of exerciser only}$$

STATICS UFBFD (CUT AT AXIAL AND MOMENT RELEASES JUST ABOVE B)

Inclined vertical component of reaction at C = 0 (due to axial release)

Sum moments about moment release to get inclined normal reaction at C

$$C = \frac{20 \text{ lb}(34 \text{ in.} + 16 \text{ in.})}{34 \text{ in.}} = 29.412 \text{ lb} \quad C_x = C \cos(\alpha) = 28.8 \text{ lb}$$

$$C_y = C \sin(\alpha) = 5.88 \text{ lb} \quad \sqrt{C_x^2 + C_y^2} = 29.412 \text{ lb}$$

STATICS LFBFD (CUT THROUGH AXIAL AND MOMENT RELEASES)

Sum moments to find reaction A_y

$$A_y = \frac{175 \text{ lb}(16 \text{ in.})}{(34 \text{ in.} + 16 \text{ in.}) \cos(\alpha)} = 57.2 \text{ lb}$$

STATICS SUM FORCES FOR ENTIRE FBD TO FIND REACTION AT *B*

Sum forces in *x*-direction: $B_x = C_x + 175\text{lb}(\sin(\alpha)) - 20\text{lb}(\cos(\alpha)) = 44.2\text{lb} <$ acts leftward

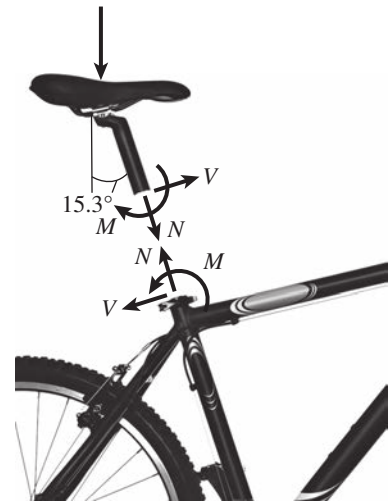
Sum forces in *y*-direction: $B_y = -A_y - C_y + 175\text{lb}(\cos(\alpha)) + 20\text{lb}(\sin(\alpha)) = 112.4\text{lb}$

$B_x = 44.2\text{ lb}$ $B_y = 112.4\text{ lb}$

Resultant reaction force at *B*: $B = \sqrt{B_x^2 + B_y^2} = 120.8\text{ lb}$

Problem 1.2-26 A mountain bike is moving along a flat path at constant velocity. At some instant, the rider (weight = 670 N) applies pedal and hand forces, as shown in the figure part a.

- (a) Find reactions forces at the front and rear hubs. (Assume that the bike is pin supported at the rear hub and roller supported at the front hub).
- (b) Find internal stress resultants *N*, *V*, and *M* in the inclined seat post (see figure part b).



Solution 1.2-26

(a) REACTIONS: SUM MOMENTS ABOUT REAR HUB TO FIND VERTICAL REACTION AT FRONT HUB (FIG. 1)

$$\Sigma M_B = 0$$

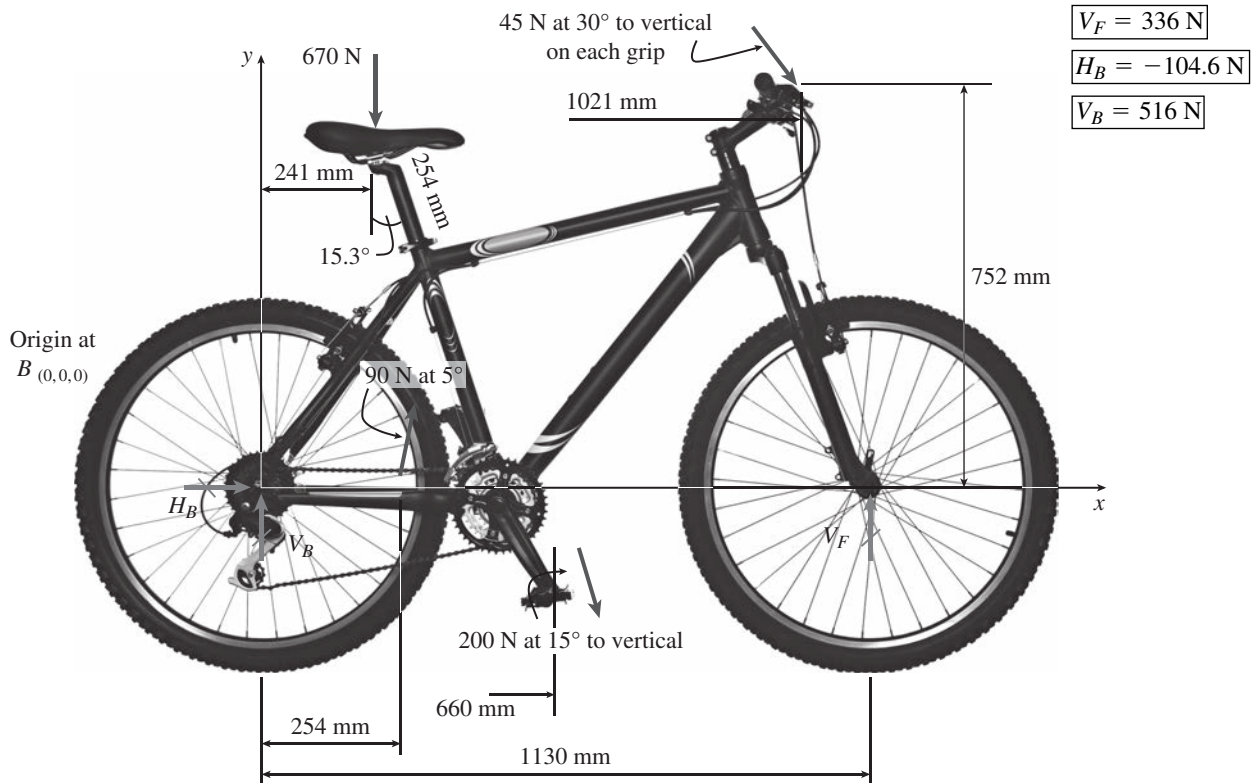
$$V_F = \frac{1}{1130} [670(241) - 90(\cos(5^\circ))254 + 200\cos(15^\circ)660 + 2(45)\cos(30^\circ)1021 + 2(45)\sin(30^\circ)752]$$

$$V_F = 335.945\text{ N}$$

Sum forces to get force components at rear hub

$$\Sigma F_{\text{vert}} = 0 \quad V_B = 670 - 90 \cos(5^\circ) + 200 \cos(15^\circ) + 2(45) \cos(30^\circ) - V_F = 515.525 \text{ N}$$

$$\Sigma F_{\text{horiz}} = 0 \quad H_B = -90 \sin(5^\circ) - 200 \sin(15^\circ) - 2(45) \sin(30^\circ) = -104.608 \text{ N}$$



(b) STRESS RESULTANTS N , V , AND M IN SEAT POST (Fig. 2)

SEAT POST RESULTANTS (FIG. 2)

$$N = -670 \cos(15.3^\circ) = -646.253 \text{ N}$$

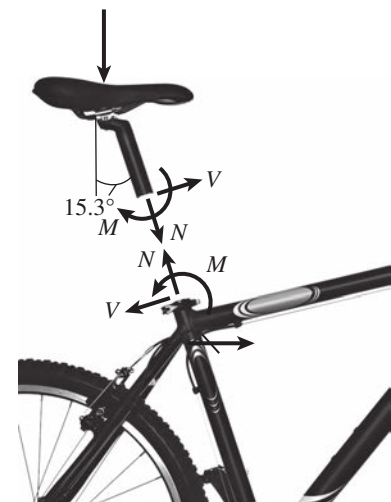
$$N = -646 \text{ N}$$

$$V = 670 \sin(15.3^\circ) = 176.795 \text{ N}$$

$$V = 176.8 \text{ N}$$

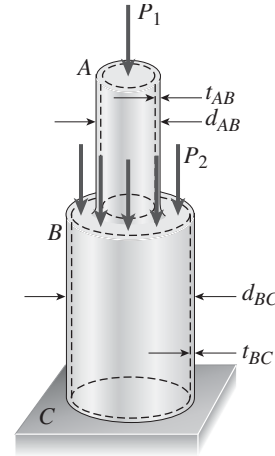
$$M = 670 \sin(15.3^\circ) 254 = 44,905.916 \text{ N}\cdot\text{mm}$$

$$M = 44.9 \text{ N}\cdot\text{m}$$



Normal Stress and Strain

Problem 1.3-1 A hollow circular post *ABC* (see figure) supports a load $P_1 = 1700$ lb acting at the top. A second load P_2 is uniformly distributed around the cap plate at *B*. The diameters and thicknesses of the upper and lower parts of the post are $d_{AB} = 1.25$ in., $t_{AB} = 0.5$ in., $d_{BC} = 2.25$ in., and $t_{BC} = 0.375$ in., respectively.



- (a) Calculate the normal stress σ_{AB} in the upper part of the post.
- (b) If it is desired that the lower part of the post have the same compressive stress as the upper part, what should be the magnitude of the load P_2 ?
- (c) If P_1 remains at 1700 lb and P_2 is now set at 2260 lb, what new thickness of *BC* will result in the same compressive stress in both parts?

Solution 1.3-1

PART (a)

$$P_1 = 1700 \text{ lb} \quad d_{AB} = 1.25 \text{ in.} \quad t_{AB} = 0.5 \text{ in.}$$

$$d_{BC} = 2.25 \text{ in.} \quad t_{BC} = 0.375 \text{ in.}$$

$$A_{AB} = \frac{\pi [d_{AB}^2 - (d_{AB} - 2t_{AB})^2]}{4}$$

$$A_{AB} = 1.178 \text{ in.}^2 \quad \sigma_{AB} = \frac{P_1}{A_{AB}}$$

$$\sigma_{AB} = 1443 \text{ psi} \quad \leftarrow$$

PART (c)

$$P_2 = 2260 \quad \frac{P_1 + P_2}{\sigma_{AB}} = A_{BC}$$

$$\frac{P_1 + P_2}{\sigma_{AB}} = 2.744$$

$$(d_{BC} - 2t_{BC})^2 = d_{BC}^2 - \frac{4}{\pi} \left(\frac{P_1 + P_2}{\sigma_{AB}} \right)$$

PART (b)

$$A_{BC} = \frac{\pi [d_{BC}^2 - (d_{BC} - 2t_{BC})^2]}{4}$$

$$A_{BC} = 2.209 \text{ in.}^2 \quad P_2 = \sigma_{AB} A_{BC} - P_1$$

$$P_2 = 1488 \text{ lbs} \quad \leftarrow$$

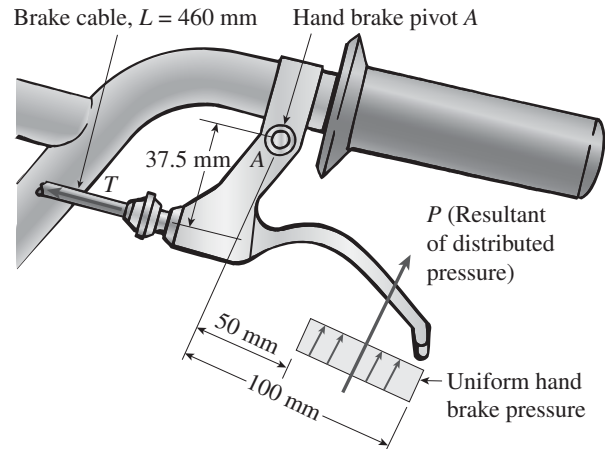
CHECK: $\frac{P_1 + P_2}{A_{BC}} = 1443 \text{ psi}$

$$d_{BC} - 2t_{BC} = \sqrt{d_{BC}^2 - \frac{4}{\pi} \left(\frac{P_1 + P_2}{\sigma_{AB}} \right)}$$

$$t_{BC} = \frac{d_{BC} - \sqrt{d_{BC}^2 - \frac{4}{\pi} \left(\frac{P_1 + P_2}{\sigma_{AB}} \right)}}{2}$$

$$t_{BC} = 0.499 \text{ in.} \quad \leftarrow$$

Problem 1.3-2 A force P of 70 N is applied by a rider to the front hand brake of a bicycle (P is the resultant of an evenly distributed pressure). As the hand brake pivots at A , a tension T develops in the 460-mm long brake cable ($A_e = 1.075 \text{ mm}^2$) which elongates by $\delta = 0.214 \text{ mm}$. Find normal stress σ and strain ε in the brake cable.



Solution 1.3-2

$$P = 70 \text{ N} \quad A_e = 1.075 \text{ mm}^2$$

$$L = 460 \text{ mm} \quad \delta = 0.214 \text{ mm}$$

Statics: sum moments about A to get $T = 2P$

$$\sigma = \frac{T}{A_e} \quad \sigma = 103.2 \text{ MPa} \quad \leftarrow$$

$$\varepsilon = \frac{\delta}{L} \quad \varepsilon = 4.65 \times 10^{-4} \quad \leftarrow$$

$$E = \frac{\sigma}{\varepsilon} = 1.4 \times 10^5 \text{ MPa}$$

NOTE: (E for cables is approximately 140 GPa)