

CHAPTER 1

FOUNDATIONS OF HEAT TRANSFER

PROB. 1.1: Since the flow is steady, the first law (1.31) reduces to

$$\int_{c.s.} \rho \left(h + \frac{1}{2} V^2 + gz \right) \vec{V} \cdot \hat{n} dA = \dot{q}_{c.s.} - \dot{W}_{shear} - \dot{W}_{shaft} - \int_{c.v.} \dot{q}_e d\sigma \quad (1)$$

Assume that all the properties are constant over cross-sections ① and ②.

Thus,

$$\int_{c.s.} \rho \left(h + \frac{1}{2} V^2 + gz \right) \vec{V} \cdot \hat{n} dA = \rho_2 \left(h_2 + \frac{1}{2} V_2^2 + gz_2 \right) V_2 A_2 - \rho_1 \left(h_1 + \frac{1}{2} V_1^2 + gz_1 \right) V_1 A_1$$

The continuity condition yields

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \dot{m}$$

Thus,

$$\int_{c.s.} \rho \left(h + \frac{1}{2} V^2 + gz \right) \vec{V} \cdot \hat{n} dA = \dot{m} \left[\left(h_2 + \frac{1}{2} V_2^2 + gz_2 \right) - \left(h_1 + \frac{1}{2} V_1^2 + gz_1 \right) \right]$$

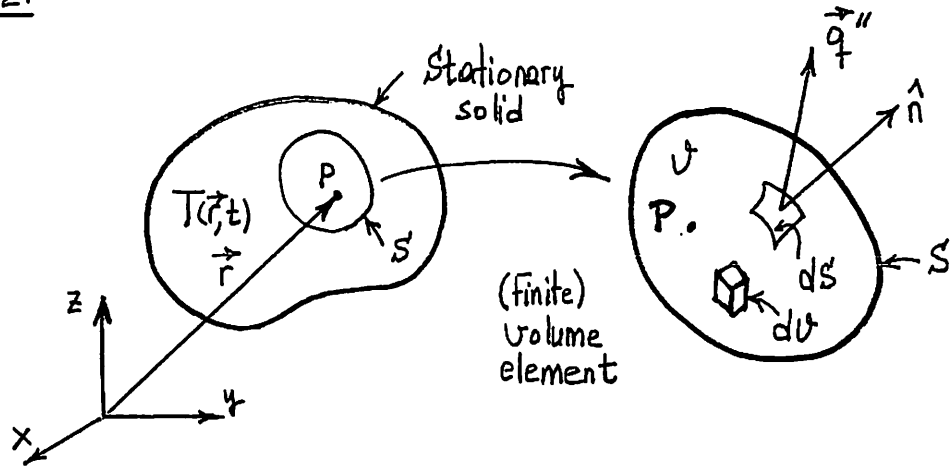
Furthermore, if it is further assumed that the flow is frictionless, then $\dot{W}_{shear} = 0$. In addition,

$$\dot{q}_{c.s.} = \dot{q}, \quad \dot{W}_{shaft} = \dot{W} \quad \text{and} \quad \dot{q}_e = 0$$

Therefore, the steady-flow energy equation (1) can be rewritten as

$$\underline{h_1 + \frac{1}{2} V_1^2 + gz_1 + \frac{\dot{q}}{\dot{m}} = h_2 + \frac{1}{2} V_2^2 + gz_2 + \frac{\dot{W}}{\dot{m}}}$$

PROB. 1.2:



Since $\vec{v} = 0$, $\dot{W}_{\text{shear}} = \dot{W}_{\text{shaft}} = 0$, Eq. (1.30) reduces to

$$\int_v \rho \frac{\partial e}{\partial t} dV = \dot{q}_s + \int_v \dot{q}_e dV \quad (1)$$

$$\int_v \rho \frac{\partial u}{\partial t} dV = \int_v \rho c \frac{\partial T}{\partial t} dV \quad (2)$$

for solids $du = c dT$, $c = \text{specific heat}$

Furthermore,

$$\dot{q}_s = \int_S \vec{q} \cdot \hat{n} dS = \int_v \vec{\nabla} \cdot \vec{q} dV \quad (3)$$

Here, \vec{q} is the heat flux vector at dS , where \hat{n} represents the outward-drawn unit vector normal to dS . Then, by substituting Eqs. (2) and (3) into (1), we obtain

$$\int_v [-\vec{\nabla} \cdot \vec{q} + \dot{q}_e - \rho c \frac{\partial T}{\partial t}] dV = 0$$

Since this volume integral vanishes for every volume element v , its integrand must vanish everywhere, thus yielding

$$\boxed{-\vec{\nabla} \cdot \vec{q} + \dot{q}_e = \rho c \frac{\partial T}{\partial t}}$$

PROB. 1.3: Under the conditions stated

the 1st law gives:

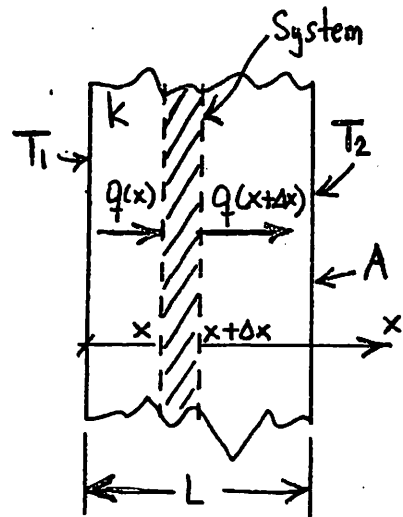
$$q(x) = q(x+\Delta x)$$

As $\Delta x \rightarrow 0$,

$$q(x+\Delta x) = q(x) + \frac{dq}{dx} \Delta x$$

Hence, as $\Delta x \rightarrow 0$, the first law reduces to

$$\frac{dq}{dx} = 0 \text{ or } q(x) = \text{const.}$$



PROB. 1.4:

$$\frac{dq}{dx} = 0 \quad \& \quad q = -kA \frac{dT}{dx} \quad \Rightarrow \quad \frac{d}{dx} \left(-kA \frac{dT}{dx} \right) = 0$$

↑
First law
↑
Fourier's law
↓
 $\frac{d^2T}{dx^2} = 0$

$$\frac{d^2T}{dx^2} = 0 \quad \Rightarrow \quad T(x) = Ax + B$$

$$\left. \begin{aligned} T(0) = T_1 &\Rightarrow B = T_1 \\ T(L) = T_2 &\Rightarrow A = \frac{T_2 - T_1}{L} \end{aligned} \right\}$$

$$\therefore T(x) = T_1 - \frac{T_1 - T_2}{L} x$$

PROB. 1.5: The first law gives (see Prob. 1.3)

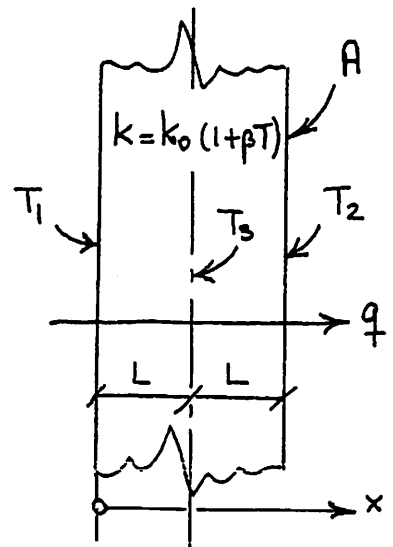
$$q = -kA \frac{dT}{dx} = \text{const.} \Rightarrow k \frac{dT}{dx} = -\frac{q}{A}$$

$$\text{or } k_0(1 + \beta T) dT = -\frac{q}{A} dx$$

$$\therefore [T(x_1) - T(x_2)] + \frac{\beta}{2} [T(x_1)^2 - T(x_2)^2] = -\frac{q}{Ak_0} (x_1 - x_2)$$

$$T_3 - T_1 + \frac{\beta}{2} (T_3^2 - T_1^2) = -\frac{qL}{Ak_0} \quad \text{--- (1)}$$

$$T_2 - T_3 + \frac{\beta}{2} (T_2^2 - T_3^2) = -\frac{qL}{Ak_0} \quad \text{--- (2)}$$

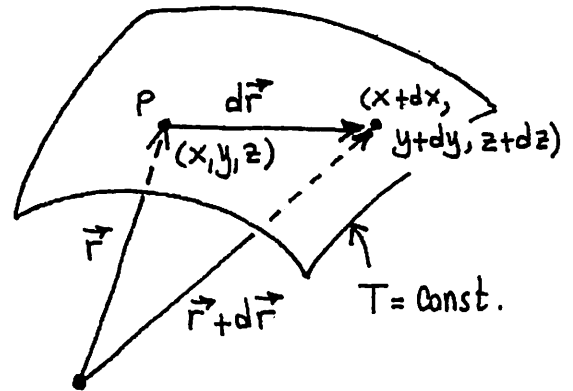


These two equations yield:

$$\left. \begin{aligned} \beta &= -2 \frac{T_1 + T_2 - 2T_3}{T_1^2 + T_2^2 - 2T_3^2} \\ k_0 &= \frac{2(q/A) \times L}{(T_2 - T_1) \left[1 + \frac{\beta}{2} (T_1 + T_2) \right]} \end{aligned} \right\} \Rightarrow k(T) = k_0 [1 + \beta T]$$

PROB. 1.6:

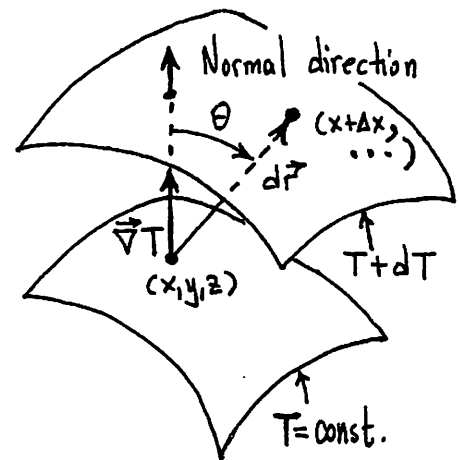
$$\begin{aligned} dT &= \frac{\partial T}{\partial x} dx + \frac{\partial T}{\partial y} dy + \frac{\partial T}{\partial z} dz \\ &= \vec{\nabla} T \cdot (\hat{i} dx + \hat{j} dy + \hat{k} dz) \\ &= \vec{\nabla} T \cdot d\vec{r} \end{aligned}$$



On the $T = \text{Const.}$ surface $dT = 0$. Thus, $\vec{\nabla} T$ is a vector normal to all such $d\vec{r}$. On the other hand, $d\vec{r}$ is a vector tangent to the isothermal surface passing through P . Therefore, $\vec{\nabla} T$ is a vector normal to the isothermal surface.

Again, $dT = \vec{\nabla} T \cdot d\vec{r}$

In order for $dT > 0$, $\cos \theta > 0$; that is, $\vec{\nabla} T$ has to point in the direction of increasing temperature as shown.



PROB. 1.7:

$$\begin{aligned} \vec{\nabla} T \cdot \hat{s} &= \left(\hat{i} \frac{\partial T}{\partial x} + \hat{j} \frac{\partial T}{\partial y} + \hat{k} \frac{\partial T}{\partial z} \right) \cdot \left(\hat{i} \cos \alpha + \hat{j} \cos \beta + \hat{k} \cos \gamma \right) \\ &= \frac{\partial T}{\partial x} \underbrace{\cos \alpha}_{\frac{dx}{ds}} + \frac{\partial T}{\partial y} \underbrace{\cos \beta}_{\frac{dy}{ds}} + \frac{\partial T}{\partial z} \underbrace{\cos \gamma}_{\frac{dz}{ds}} \\ &= \frac{\partial T}{\partial s} \end{aligned}$$

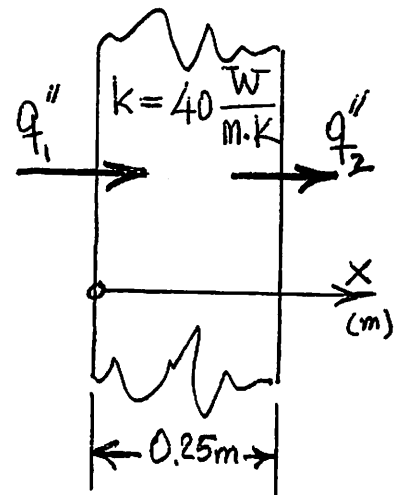
Direction Cosines of \hat{s}

PROB. 1.8:

$$T(^{\circ}\text{C}) = 150 - 400x^2$$

$$\begin{aligned} q_1'' &= -k \left(\frac{\partial T}{\partial x} \right)_{x=0} = -k (-800x)_{x=0} \\ &= 0 \text{ W/m}^2 \end{aligned}$$

$$\begin{aligned} q_2'' &= -k \left(\frac{dT}{dx} \right)_{x=L} = -k (-800L) \\ &= 40 \times 800 \times 0.25 \\ &= 8,000 \text{ W/m}^2 \end{aligned}$$



PROB. 1.9:

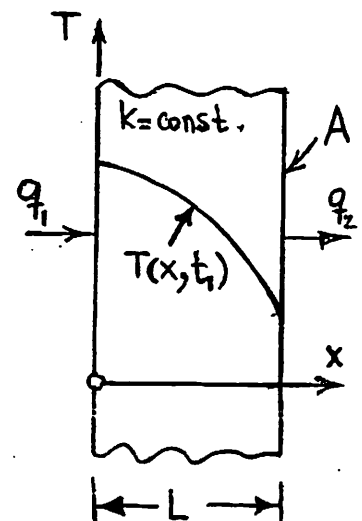
$$q_1 = -kA \left(\frac{\partial T}{\partial x} \right)_{x=0} \quad \text{and} \quad q_2 = -kA \left(\frac{\partial T}{\partial x} \right)_{x=L}$$

$$- \left(\frac{\partial T}{\partial x} \right)_{x=0} < - \left(\frac{\partial T}{\partial x} \right)_{x=L}$$

$$\Downarrow$$

$$q_1 < q_2$$

\therefore The wall is being cooled at t_1 .



PROB.1.10:

$$h = \frac{-k \left(\frac{\partial T}{\partial y} \right)_{y=0}}{T_w - T_\infty} = \frac{-(0.62) \frac{\text{W}}{\text{m}\cdot\text{K}} \times (-80 \times 800) \frac{\text{K}}{\text{m}}}{(100 - 20) \text{K}}$$
$$= 496 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$$

PROB.1.11:

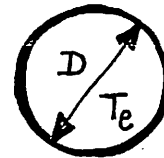
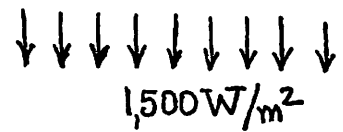
Energy balance yields

$$\alpha_s (1500) \frac{\text{W}}{\text{m}^2} \times (D \times L) \text{ m}^2$$

$$= \epsilon (\pi D L) \text{ m}^2 \times \sigma (T_e^4) \text{ K}^4$$

$$\uparrow$$
$$5.67 \times 10^{-8} \text{ W}/(\text{m}^2\cdot\text{K}^4)$$

$$T_e = \frac{1500}{\pi \times 5.67 \times 10^{-8}} = 303 \text{ K} = 30^\circ \text{C}.$$



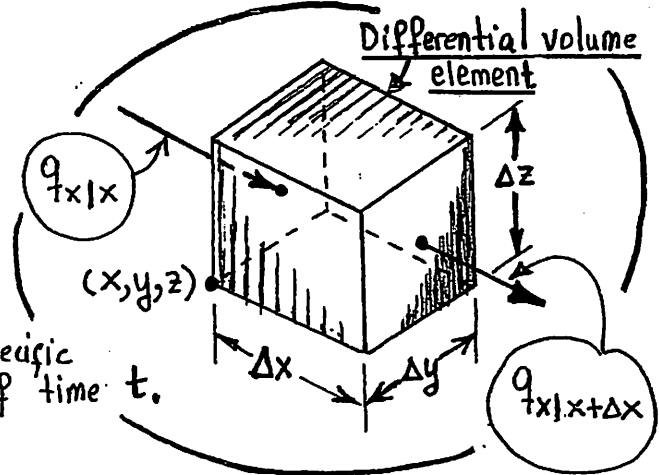
L = Length of cylinder

CHAPTER 2

GENERAL HEAT CONDUCTION EQUATION

PROB. 2.1:

- Homogeneous and isotropic solid, i.e., $k = \text{const.}$
- Let there be no heat sources or sinks, i.e., $\dot{q} = 0.$
- Assume the density ρ and specific heat c are independent of time $t.$



$$\left\{ \begin{array}{l} \text{Net rate of heat conduction} \\ \text{into the c.v. in the} \\ \text{x-direction} \end{array} \right\} = q_{x|x} - q_{x|x+\Delta x} = q_{x|\Delta x} - \left\{ q_{x|x} + \frac{\partial q_{x|x}}{\partial x} \Delta x \right\}$$

$$= - \frac{\partial q_{x|x}}{\partial x} \Delta x$$

where

$$q_{x|x} = -k (\Delta y \Delta z) \frac{\partial T}{\partial x} \leftarrow \begin{array}{l} \text{Rate of heat conduction} \\ \text{in the x-direction} \\ \text{at x into the c.v.} \end{array}$$

$$\therefore q_{x|x} - q_{x|x+\Delta x} = k \frac{\partial^2 T}{\partial x^2} \Delta V, \quad \Delta V = \Delta x \Delta y \Delta z$$

Similarly,

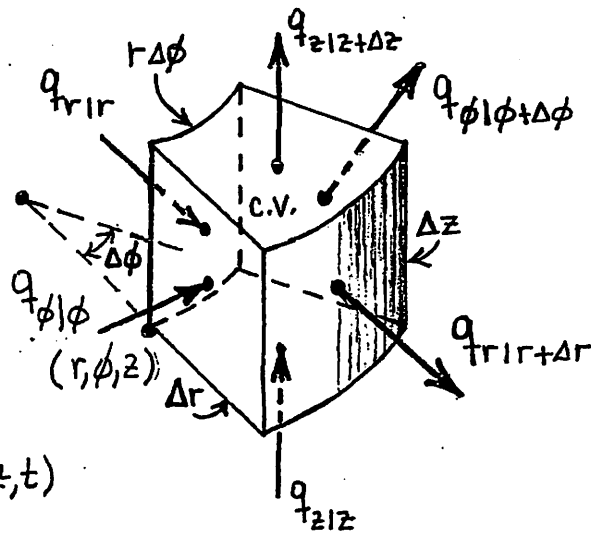
$$q_{y|y} - q_{y|y+\Delta y} = k \frac{\partial^2 T}{\partial y^2} \Delta V \quad \text{and} \quad q_{z|z} - q_{z|z+\Delta z} = k \frac{\partial^2 T}{\partial z^2} \Delta V$$

Thus, energy balance on the c.v. gives

$$k \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right\} \Delta V = \left\{ \begin{array}{l} \text{Net rate of increase} \\ \text{of internal energy} \\ \text{in c.v.} \end{array} \right\} = \frac{\partial}{\partial t} \{ \rho c \Delta V \}$$

$$\therefore \boxed{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} }, \quad \alpha = \frac{k}{\rho c}$$

PROB. 2.2: (a)



$$T = T(r, \phi, z, t)$$

$$\Delta V = r \Delta r \Delta \phi \Delta z$$

$$q_{r|r} - q_{r|r+\Delta r} = - \frac{\partial q_{r|r}}{\partial r} \Delta r = - \frac{\partial}{\partial r} \left[k r \Delta \phi \Delta z \frac{\partial T}{\partial r} \right] \Delta r = \frac{k}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \Delta V$$

$$q_{\phi|\phi} - q_{\phi|\phi+\Delta\phi} = - \frac{\partial q_{\phi|\phi}}{\partial(r\phi)} r \Delta \phi = \frac{\partial}{\partial \phi} \left[k \Delta r \Delta z \frac{\partial T}{\partial(r\phi)} \right] \Delta \phi = \frac{k}{r^2} \frac{\partial^2 T}{\partial \phi^2} \Delta V$$

$$q_{z|z} - q_{z|z+\Delta z} = - \frac{\partial q_{z|z}}{\partial z} \Delta z = \frac{\partial}{\partial z} \left[k r \Delta \phi \Delta r \frac{\partial T}{\partial z} \right] \Delta z = k \frac{\partial^2 T}{\partial z^2} \Delta V$$

$$\therefore k \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right\} \Delta V = \frac{\partial}{\partial t} \left\{ \rho c_p \Delta V \right\}$$

Net rate of heat conduction into c.v. Net rate of increase of internal energy in c.v.

$$\boxed{\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}}, \quad \alpha = \frac{k}{\rho c}$$

PROB. 2.3: General heat conduction equation in the rectangular coordinates:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

a) The cylindrical coordinates:

$$\left. \begin{aligned} x &= r \cos \phi \\ y &= r \sin \phi \\ z &= z \end{aligned} \right\} \begin{aligned} r &= (x^2 + y^2)^{1/2} \\ \phi &= \tan^{-1} \left(\frac{y}{x} \right) \\ z &= z \end{aligned}$$

Thus,

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial T}{\partial \phi} \frac{\partial \phi}{\partial x} = \cos \phi \frac{\partial T}{\partial r} - \frac{\sin \phi}{r} \frac{\partial T}{\partial \phi}$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} &= \frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \right) = \frac{\partial}{\partial r} \left(\frac{\partial T}{\partial x} \right) \frac{\partial r}{\partial x} + \frac{\partial}{\partial \phi} \left(\frac{\partial T}{\partial x} \right) \frac{\partial \phi}{\partial x} \\ &= \cos^2 \phi \frac{\partial^2 T}{\partial r^2} + 2 \frac{\cos \phi \sin \phi}{r^2} \frac{\partial T}{\partial \phi} - 2 \frac{\sin \phi \cos \phi}{r} \frac{\partial^2 T}{\partial r \partial \phi} \\ &\quad + \frac{\sin^2 \phi}{r} \frac{\partial T}{\partial r} + \frac{\sin^2 \phi}{r^2} \frac{\partial^2 T}{\partial \phi^2} \end{aligned}$$

Similarly,

$$\frac{\partial T}{\partial y} = \sin \phi \frac{\partial T}{\partial r} + \frac{\cos \phi}{r} \frac{\partial T}{\partial \phi}$$

$$\begin{aligned} \frac{\partial^2 T}{\partial y^2} &= \sin^2 \phi \frac{\partial^2 T}{\partial r^2} + 2 \frac{\sin \phi \cos \phi}{r} \frac{\partial^2 T}{\partial \phi \partial r} - 2 \frac{\sin \phi \cos \phi}{r^2} \frac{\partial T}{\partial \phi} \\ &\quad + \frac{\cos^2 \phi}{r} \frac{\partial T}{\partial r} + \frac{\cos^2 \phi}{r^2} \frac{\partial^2 T}{\partial \phi^2} \end{aligned}$$

In addition, in both coordinate systems: $\frac{\partial^2 T}{\partial z^2} = \frac{\partial^2 T}{\partial z^2}$

Therefore,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

and the general heat conduction equation in the cylindrical coordinate is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

b) The spherical coordinates:

$$\left. \begin{aligned} x &= r \sin \theta \cos \phi \\ y &= r \sin \theta \sin \phi \\ z &= r \cos \theta \end{aligned} \right\} \begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} \\ \phi &= \tan^{-1} \left(\frac{y}{x} \right) \\ \theta &= \cos^{-1} \frac{z}{[x^2 + y^2 + z^2]^{1/2}} \end{aligned}$$

$$\begin{aligned} \frac{\partial T}{\partial x} &= \frac{\partial T}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial T}{\partial \phi} \frac{\partial \phi}{\partial x} + \frac{\partial T}{\partial \theta} \frac{\partial \theta}{\partial x} \\ &= \sin \theta \cos \phi \frac{\partial T}{\partial r} - \frac{\sin \phi}{r \sin \theta} \frac{\partial T}{\partial \phi} + \frac{\cos \theta \cos \phi}{r} \frac{\partial T}{\partial \theta} \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} &= \frac{\partial}{\partial r} \left(\frac{\partial T}{\partial x} \right) \frac{\partial r}{\partial x} + \frac{\partial}{\partial \phi} \left(\frac{\partial T}{\partial x} \right) \frac{\partial \phi}{\partial x} + \frac{\partial}{\partial \theta} \left(\frac{\partial T}{\partial x} \right) \frac{\partial \theta}{\partial x} \\ &= \sin^2 \theta \cos^2 \phi \frac{\partial^2 T}{\partial r^2} + 2 \frac{\cos \phi \sin \phi}{r^2 \sin^2 \theta} \frac{\partial T}{\partial \phi} - 2 \frac{\sin \phi \cos \phi}{r} \frac{\partial^2 T}{\partial r \partial \phi} \\ &\quad + \frac{\cos \theta}{r^2} \left(-2 \sin \theta \cos^2 \phi + \frac{\sin^2 \phi}{\sin \theta} \right) \frac{\partial T}{\partial \theta} + 2 \frac{\sin \theta \cos \theta \cos^2 \phi}{r} \frac{\partial^2 T}{\partial r \partial \theta} \\ &\quad + \left(\frac{\cos^2 \theta \cos^2 \phi}{r} + \frac{\sin^2 \phi}{r} \right) \frac{\partial T}{\partial r} + \frac{\sin^2 \phi}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \\ &\quad - 2 \frac{\cos \theta \sin \phi \cos \phi}{r^2 \sin \theta} \frac{\partial^2 T}{\partial \phi \partial \theta} + \frac{\cos^2 \theta \cos^2 \phi}{r^2} \frac{\partial^2 T}{\partial \theta^2} \end{aligned}$$

Similarly,

$$\begin{aligned} \frac{\partial T}{\partial y} &= \frac{\partial T}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial T}{\partial \phi} \frac{\partial \phi}{\partial y} + \frac{\partial T}{\partial \theta} \frac{\partial \theta}{\partial y} \\ &= \sin \theta \sin \phi \frac{\partial T}{\partial r} + \frac{\cos \phi}{r \sin \theta} \frac{\partial T}{\partial \phi} + \frac{\cos \theta \sin \phi}{r} \frac{\partial T}{\partial \theta} \end{aligned}$$