CHAPTER 1

SOLUTION (1.1)

We have

$$A = 50 \times 75 = 3.75(10^{-3})\ \text{m}^2, \theta = 50^o, \text{ and } \sigma_x = P/A.$$

Equations (1.11), with $\theta = 50^o$:

$$\sigma_{x'} = 700(10^3) = \sigma_x \cos^2 50^o = 0.413\sigma_x = 110.18P$$

or $P = 6.35$ kN

and

$$|\tau_{x'y'}| = 560(10^3) = \sigma_x \sin 50^o \cos 50^o = 0.492\sigma_x = 131.2P$$

Solving

$$P = 4.27\ \text{kN} = P_{all}$$ ▲

SOLUTION (1.2)

Normal stress is

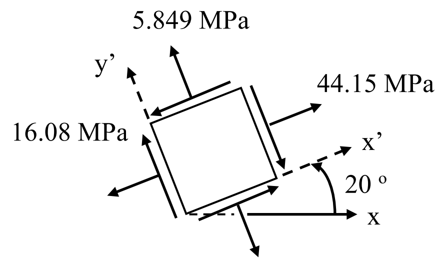
$$\sigma_x = \frac{P}{A} = \frac{125(10^3)}{0.05 \times 0.05} = 50\ \text{MPa}$$

(a) Equations (1.11), with $\theta = 20^o$:

$$\sigma_{x'} = 50\cos^2 20^o = 44.15\ \text{MPa}$$

$$\tau_{x'y'} = -50\sin 20^o \cos 20^o = -16.08\ \text{MPa}$$ ▲

$$\sigma_{y'} = 50\cos^2(20^o + 90^o) = 5.849\ \text{MPa}$$

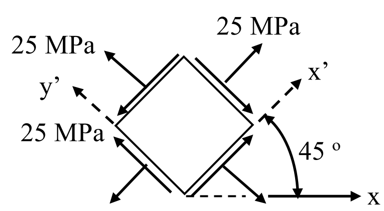


(b) Equations (1.11), with $\theta = 45^o$:

$$\sigma_{x'} = 50\cos^2 45^o = 25\ \text{MPa}$$

$$\tau_{x'y'} = -50\sin 45^o \cos 45^o = -25\ \text{MPa}$$ ▲

$$\sigma_{y'} = 50\cos^2(45^o + 90^o) = 25\ \text{MPa}$$



SOLUTION (1.3)

From Eq. (1.11a),

$$\sigma_x = \frac{\sigma_{x'}}{\cos^2\theta} = \frac{-75}{\cos^2 30^o} = -100 \text{ MPa}$$

For $\theta = 50^o$, Eqs. (1.11) give then

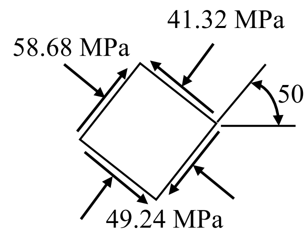
$$\sigma_{x'} = -100\cos^2 50^o = -41.32 \text{ MPa} \qquad \blacktriangle$$

$$\tau_{x'y'} = -(-100)\sin 50^o \cos 50^o$$
$$= 49.24 \text{ MPa}$$

Similarly, for $\theta = 140^o$:

$$\sigma_{x'} = -100\cos^2 140^o = -58.68 \text{ MPa} \qquad \blacktriangle$$

$$\tau_{x'y'} = -49.24 \text{ MPa}$$



SOLUTION (1.4)

Refer to Fig. 1.6c. Equations (1.11) by substituting the double angle-trigonometric relations, or Eqs. (1.18) with $\sigma_y = 0$ and $\tau_{xy} = 0$, become

$$\sigma_{x'} = \tfrac{1}{2}\sigma_x + \tfrac{1}{2}\sigma_x \cos 2\theta \qquad \text{and} \qquad |\tau_{x'y'}| = \tfrac{1}{2}\sigma_x \sin 2\theta$$

or

$$20 = \tfrac{P}{2A}(1+\cos 2\theta) \qquad \text{and} \qquad 10 = \tfrac{P}{2A}\sin 2\theta$$

The foregoing lead to

$$2\sin 2\theta - \cos 2\theta = 1 \qquad \text{(a)}$$

By introducing trigonometric identities, Eq. (a) becomes

$4\sin\theta\cos\theta - 2\cos^2\theta = 0$ or $\tan\theta = 1/2$. Hence

$$\theta = 26.56^o \qquad \blacktriangleleft$$

Thus,

$$20 = \tfrac{P}{2(1300)}(1+0.6)$$

gives

$$P = 32.5 \text{ kN} \qquad \blacktriangleleft$$

It can be shown that use of Mohr's circle yields readily the same result.

SOLUTION (1.5)

Equations (1.12):

$$\sigma_1 = \frac{P}{A} = \frac{-150(10^3)}{\frac{\pi}{4}(50)^2} = -76.4 \text{ MPa} \qquad \blacktriangleleft$$

$$\tau_{max} = \frac{P}{2A} = 38.2 \text{ MPa} \qquad \blacktriangleleft$$

SOLUTION (1.6)

Shaded transverse area:

$$A = 2at = 2(10)(75) = 1.5(10^3)\ \text{mm}^2$$

Metal is capable of supporting the load

$$P = \sigma A = 90(10^6)(1.5\times10^{-3}) = 135\ \text{kN}$$

Apply Eqs. (1.11):

$$\sigma_{x'} = 25(10^6) = \frac{P}{1.5(10^{-3})}(\cos^2 55^o), \quad P = 114\ \text{kN}$$

$$\tau_{x'y'} = 12(10^6) = -\frac{P}{1.5(10^{-3})}\sin 55^o \cos 55^o, \quad P = 38.3\ \text{kN}$$

Thus,

$$P_{all} = 38.3\ \text{kN}$$ ▲

SOLUTION (1.7)

Use Eqs. (1.11):

$$\sigma_{x'} = 20(10^6) = \frac{P}{1.5(10^{-3})}(\cos^2 40^o), \quad P = 51.1\ \text{kN}$$

$$\tau_{x'y'} = 8(10^6) = -\frac{P}{1.5(10^{-3})}\sin 40^o \cos 40^o, \quad P = 24.4\ \text{kN}$$

Thus,

$$P_{all} = 24.4\ \text{kN}$$ ▲

SOLUTION (1.8)

$$A = 15\times30 = 450\ \text{mm}^2$$

Apply Eqs. (1.11):

$$\sigma_{x'} = \frac{120(10^3)}{450\times10^{-6}}(\cos^2 40^o) = 156\ \text{MPa}$$ ◀

$$\tau_{x'y'} = -\frac{120(10^3)}{450\times10^{-6}}\sin 40^o \cos 40^o = -131\ \text{MPa}$$

SOLUTION (1.9)

We have $A = 450(10^{-6})\ \text{m}^2$. Use Eqs. (1.11):

$$\sigma_{x'} = \frac{-100(10^3)}{450\times10^{-6}}(\cos^2 60^o) = -55.6\ \text{MPa}$$ ◀

$$\tau_{x'y'} = -\frac{-100(10^3)}{450\times10^{-6}}\sin 60^o \cos 60^o = 96.2\ \text{MPa}$$

SOLUTION (1.10)

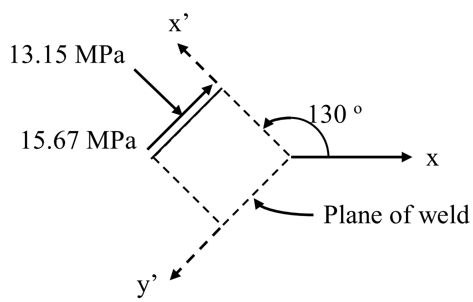
$$\theta = 40^o + 90^o = 130^o$$

$$\sigma_x = \tfrac{P}{A} = -\tfrac{150(10^3)}{\pi(0.08^2 - 0.07^2)} = -31.83 \text{ MPa}$$

Equations (1.11):

$$\sigma_{x'} = -31.83\cos^2 130^o = -13.15 \text{ MPa}$$ ▲

$$\tau_{x'y'} = 31.83\sin 130^o \cos 130^o = -15.67 \text{ MPa}$$ ▲



SOLUTION (1.11)

Use Eqs. (1.14),

$$(2x) + (-2xy) + (x) + F_x = 0$$

$$(-y^2) + (-2yz + x) + (0) + F_y = 0$$

$$(z - 4xy) + (0) + (-2z) + F_z = 0$$

Solving, we have (in MN/m^3):

$$F_x = -3x + 2xy \qquad F_y = -x + y^2 + 2yz \qquad F_z = 4xy + z \quad \text{(a)}$$ ◄

Substituting x=-0.01 m, y=0.03 m, and z=0.06 m, Eqs. (a) yield the following values

$$F_x = 29.4 \text{ kN}/\text{m}^3 \qquad F_y = 14.5 \text{ kN}/\text{m}^3 \qquad F_z = 58.8 \text{ kN}/\text{m}^3$$

Resultant body force is thus

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = 67.32 \text{ kN}/\text{m}^3$$ ◄

SOLUTION (1.12)

Equations (1.14):

$$-2c_1y - 2c_1y + 0 + 0 = 0, \qquad 4c_1y \neq 0$$

$$0 + c_3z + 0 + 0 = 0, \qquad c_3z \neq 0$$

$$0 + 0 + 0 + 0 = 0$$

◄

No. Eqs. (1.14) are not satisfied.

SOLUTION (1.13)

(a) No. Eqs. (1.14) are not satisfied. ◀

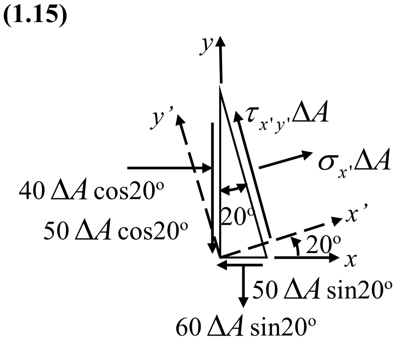
(b) Yes. Eqs. (1.14) are satisfied. ◀

SOLUTION (1.14)

Eqs. (1.14) for the given stress field yield: ◀

$$F_x = F_y = F_z = 0$$

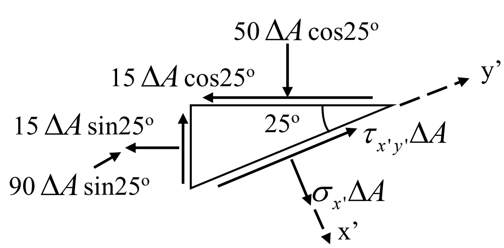
SOLUTION (1.15)



$$\sum F_{x'} = 0: \quad \sigma_{x'}\Delta A + 40\cos^2 20^o - 60\Delta A \sin^2 20^o$$
$$-2(50\Delta A \sin 20^o \cos 20^o) = 0$$
$$\sigma_{x'} = -35.32 + 7.02 + 32.14 = 3.8 \text{ MPa}$$ ◀

$$\sum F_{y'} = 0: \quad \tau_{x'y'}\Delta A - 40\Delta A \sin 20^o \cos 20^o$$
$$-60\Delta A \sin 20^o \cos 20^o - 50\Delta A \cos^2 20^o$$
$$+50\Delta A \sin^2 20^o = 0$$
$$\tau_{x'y'} = 12.86 + 19.28 + 44.15 - 5.85 = 70.4 \text{ MPa}$$ ◀

SOLUTION (1.16)



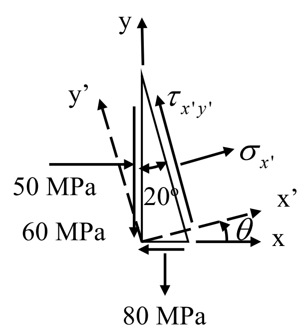
$$\sum F_{x'} = 0: \quad \sigma_{x'}\Delta A + 50\Delta A \cos^2 25^o$$
$$-90\Delta A \sin^2 25^o - 2(15\Delta A \sin 25^o \cos 25^o) = 0$$
$$\sigma_{x'} = -41.07 + 16.07 + 11.49 = -13.5 \text{ MPa}$$ ◀

(CONT.)

1.16 (CONT.)

$$\sum F_{y'} = 0: \quad \tau_{x'y'}\Delta A - 50\Delta A \sin 25^o \cos 25^o$$
$$-90\Delta A \sin 25^o \cos 25^o - 15\Delta A \cos^2 25^o + 15\Delta A \sin^2 25^o = 0$$
$$\tau_{x'y'} = 19.15 + 34.47 + 12.32 - 2.68 = 63.3 \text{ MPa}$$ ▲

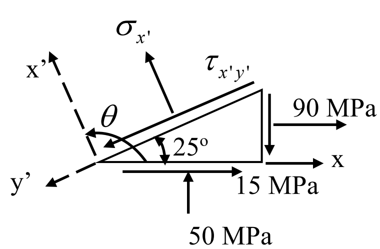
SOLUTION (1.17)



θ=20°

$$\sigma_{x'} = \frac{1}{2}(-40+60) + \frac{1}{2}(-40-60)\cos 40^o + 50\sin 40^o$$
$$= 10 - 38.3 + 32.1 = 3.8 \text{ MPa}$$ ▲
$$\tau_{x'y'} = -\frac{1}{2}(-40-60)\sin 40^o + 50\cos 40^o$$
$$= 32.14 + 38.3 = 70.4 \text{ MPa}$$ ▲

SOLUTION (1.18)



θ=115°

$$\sigma_{x'} = \frac{1}{2}(90-50) + \frac{1}{2}(90+50)\cos 230^o - 15\sin 230^o$$
$$= 20 - 45 + 11.5 = -13.5 \text{ MPa}$$ ▲
$$\tau_{x'y'} = -\frac{1}{2}(90+50)\sin 230^o - 15\cos 230^o$$
$$= 53.62 + 9.64 = 63.3 \text{ MPa}$$ ▲

SOLUTION (1.19)

Transform from $\theta = 40^o$ to $\theta = 0$. For convenience in computations, Let

$$\sigma_x = -160 \text{ MPa}, \quad \sigma_y = -80 \text{ MPa}, \quad \tau_{xy} = 40 \text{ MPa} \text{ and } \theta = -40^o$$

Then

$$\sigma_{x'} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{1}{2}(-160 - 80) + \frac{1}{2}(-160 + 80)\cos(-80^o) + 40\sin(-80^o)$$

$$= -166.3 \text{ MPa}$$ ▲

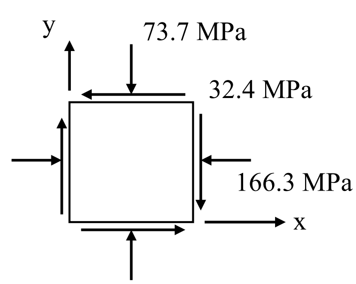
$$\tau_{x'y'} = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{1}{2}(-160 + 80)\sin(-80^o) + 40\cos(-80^o)$$

$$= -32.4 \text{ MPa}$$ ▲

So $\quad \sigma_{y'} = \sigma_x + \sigma_y - \sigma_{x'} = -160 - 80 + 166.3 = -73.7 \text{ MPa}$ ▲

For $\theta = 0^o$:



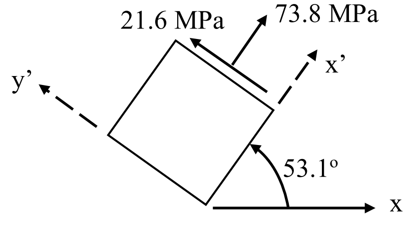
SOLUTION (1.20)

$$\theta = \tan^{-1}\frac{4}{3} = 53.1^o$$

$$\sigma_{x'} = \frac{45+90}{2} + \frac{45-90}{2}\cos 106.2^o$$

$$= 67.5 + 6.28 = 73.8 \text{ MPa}$$ ◄

$$\tau_{x'y'} = -\frac{45-90}{2}\sin 106.2^o = 21.6 \text{ MPa}$$



SOLUTION (1.21)

$$\tau_{xy} = 0 \qquad \theta = 70^o$$

(**a**) $\tau_{x'y'} = -30 = -\dfrac{\sigma - 60}{2}\sin 140^o \qquad \sigma = 153.3 \text{ MPa}$ ▲

(**b**) $\sigma_{x'} = 80 = \dfrac{\sigma + 60}{2} + \dfrac{\sigma - 60}{2}\cos 140^o \qquad \sigma = 231 \text{ MPa}$ ▲

SOLUTION (1.22)

Equations(1.18) with $\theta = 60^o$, $\sigma_x = 110$ MPa, $\sigma_y = 0$, $\tau_{xy} = 50$ MPa give

$$\sigma_{x'} = \tfrac{1}{2}(110) + \tfrac{1}{2}(110)\cos 120^o + 50\sin 120^o = 70.8 \text{ MPa}$$
$$\tau_{x'y'} = -\tfrac{1}{2}(110)\sin 120^o + 50\cos 120^o = -72.6 \text{ MPa}$$
$$\sigma_{y'} = \tfrac{1}{2}(110) - \tfrac{1}{2}(110)\cos 120^o - 50\sin 120^o = 39.2 \text{ MPa}$$

SOLUTION (1.23)

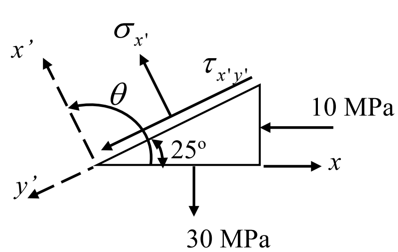
Equations(1.18) with $\theta = 30^o$, $\sigma_x = 110$ MPa, $\sigma_y = 0$, $\tau_{xy} = 50$ MPa result in

$$\sigma_{x'} = \tfrac{1}{2}(110) + 55\cos 60^o + 50\sin 60^o = 125.8 \text{ MPa}$$
$$\tau_{x'y'} = -\tfrac{1}{2}(110)\sin 60^o + 50\cos 60^o = -22.6 \text{ MPa}$$
$$\sigma_{y'} = \tfrac{1}{2}(110) - 55\cos 60^o - 50\sin 60^o = -15.8 \text{ MPa}$$

SOLUTION (1.24)

We have

$$\theta = 25 + 90 = 115^o$$
$$\sigma_x = -10 \text{ MPa}$$
$$\sigma_y = 30 \text{ MPa}$$
$$\tau_{xy} = 0$$



(**a**) $\sigma_{x'} = \dfrac{1}{2}(\sigma_x + \sigma_y) + \dfrac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta$

$$= \frac{1}{2}(-10 + 30) + \frac{1}{2}(-10 - 30)\cos 230^o = 22.86 \text{ MPa}$$ ◄

Thus,

$$\sigma_w = \sigma_{x'} = 22.86 \text{ MPa}$$

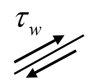
(CONT.)

(b) $\tau_{x'y'} = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta$

$= -\frac{1}{2}(-10-30)\sin 230^o = -15.32 \text{ MPa}$

So

$\tau_w = \tau_{x'y'} = -15.32 \text{ MPa}$ ▲



SOLUTION (1.25)

(a) $\sigma_1 = 80 = \frac{0+50}{2} + \sqrt{(\frac{0-50}{2})^2 + \tau^2}$

$\tau = 49 \text{ MPa}$ ▲

(b) $\tau_{\max} = \sqrt{(\frac{-50}{2})^2 + 49^2} = 55 \text{ MPa}$

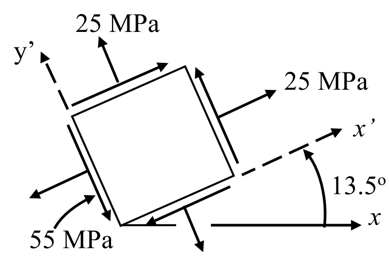
$\sigma' = \frac{50}{2} = 25 \text{ MPa}$ ▲

$2\theta_s = \tan^{-1}[-\frac{0-50}{2(49)}] = 27^o$

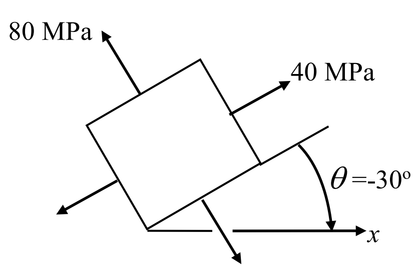
$\tau_{x'y'} = \frac{50}{2}\sin 27^o + 49\cos 27^o = 55 \text{ MPa}$

Thus,

$\theta_s' = 13.5^o$ ▲



SOLUTION (1.26)

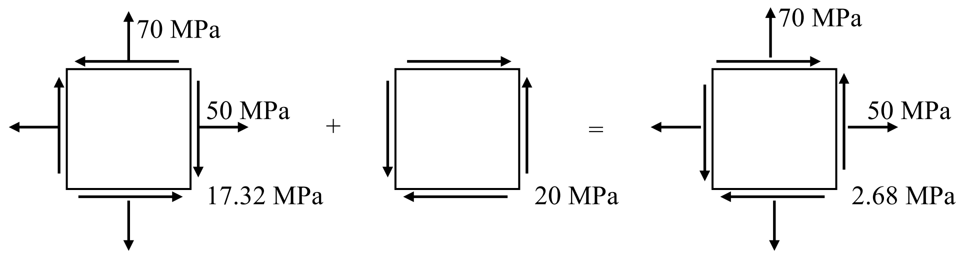


(CONT.)

$$\sigma_x = \frac{40+80}{2} + \frac{40-80}{2}\cos(-60^o) = 60 - 10 = 50 \text{ MPa}$$

$$\sigma_y = 60 + 10 = 70 \text{ MPa}$$

$$\tau_{xy} = -\frac{40-80}{2}\sin(-60^o) = -17.32 \text{ MPa}$$



$$\sigma_{1,2} = \frac{50+70}{2} \pm \sqrt{(\frac{50-70}{2})^2 + 2.68^2} = 60 \pm 10.35$$

$$\sigma_1 = 70.35 \text{ MPa} \qquad \sigma_2 = 49.65 \text{ MPa}$$

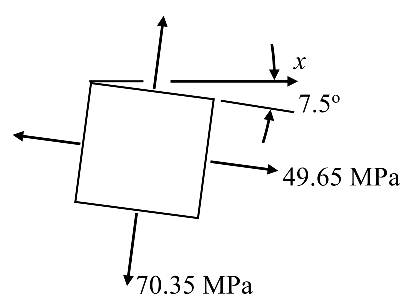
$$2\theta_p = \tan^{-1}[\frac{2(2.68)}{50-70}] = -15^o$$

$$\sigma_{x'} = 60 + \frac{50-70}{2}\cos(-15^o) + 2.68\sin(-15^o)$$

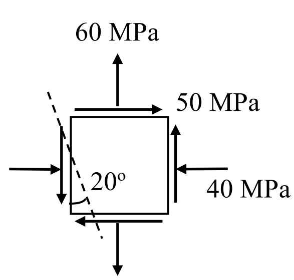
$$= 60 - 9.66 - 0.694 = 49.65 \text{ MPa}$$

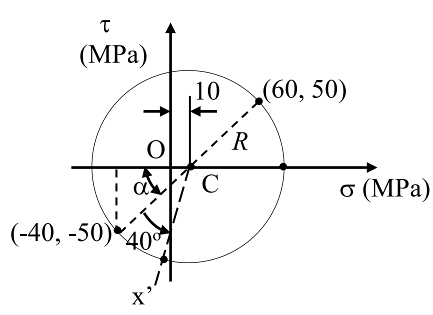
Thus,

$$\theta_p'' = -7.5^o$$



SOLUTION (1.27)



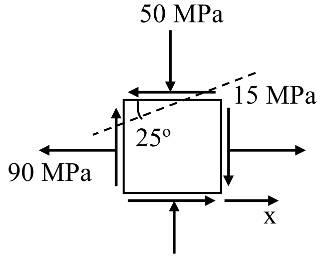


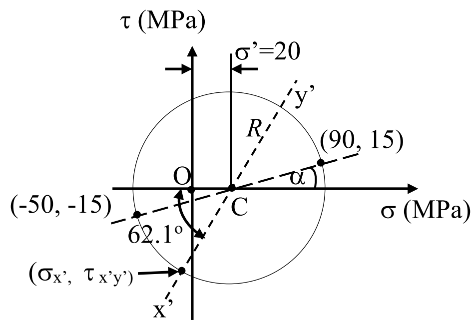
(CONT.)

$$\alpha = \tan^{-1}\frac{50}{50} = 45^o$$
$$R = (50^2 + 50^2)^{\frac{1}{2}} = 70.7$$
$$\tau_{x'y'} = \sin 85^o(70.7) = 70.4 \text{ MPa}$$
$$\sigma_{x'} = 10 - \cos 85^o(70.7) = 3.84 \text{ MPa}$$

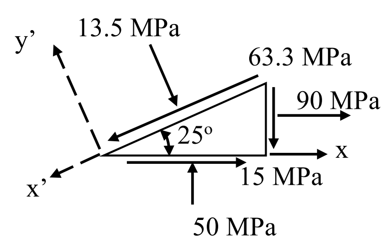
▲

SOLUTION (1.28)



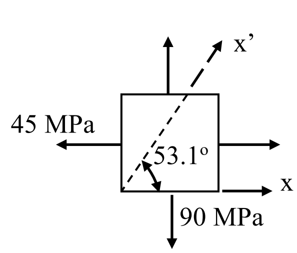


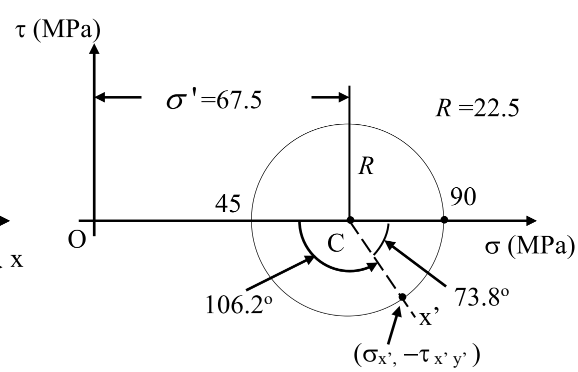
$$\alpha = \tan^{-1}\frac{15}{70} = 12.1^o$$
$$R = (15^2 + 70^2)^{\frac{1}{2}} = 71.6$$
$$\tau_{x'y'} = 71.6\sin 62.1^o = 63.3 \text{ MPa}$$
$$\sigma_{x'} = -71.6\cos 62.1^o + 20$$
$$= -13.5 \text{ MPa}$$



▲

SOLUTION (1.29)





(CONT.)

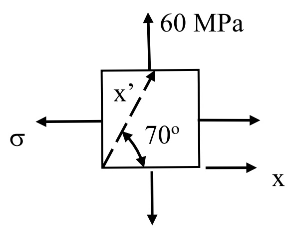
$$\tau_{x'y'} = 22.5 \sin 73.8^o = 21.6 \text{ MPa}$$

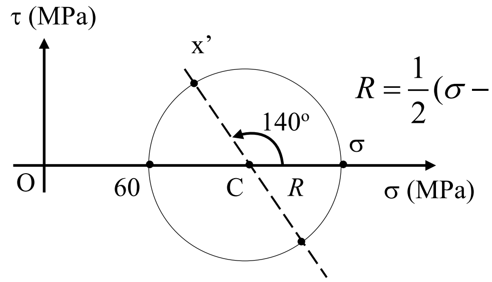
$$\sigma_{x'} = 67.5 + 22.5 \cos 73.8^o = 73.8 \text{ MPa}$$

Sketch of results is as shown in solution of Prob. 1.20.

SOLUTION (1.30)

(a)





$$R = \frac{1}{2}(\sigma - 60)$$

$$\tau_{x'y'} = -30 = \frac{\sigma - 60}{2} \sin(-40^o); \qquad \sigma = 153.3 \text{ MPa}$$

(b) $$\sigma_{x'} = 80 = 60 + \frac{\sigma - 60}{2}[1 - \cos(-40^o)]$$

$$\sigma = 231 \text{ MPa}$$

SOLUTION (1.31)

(a) From Mohr's circle, Fig. (a):

$$\sigma_1 = 121 \text{ MPa} \qquad \sigma_2 = -71 \text{ MPa} \qquad \tau_{\max} = 96 \text{ MPa}$$

$$\theta_p' = -19.3^o \qquad \theta_s' = 25.7^o$$

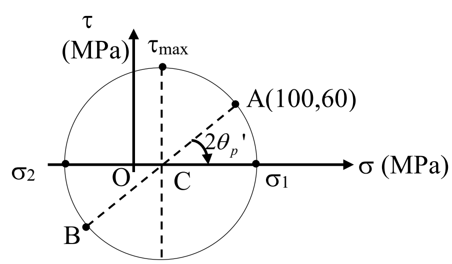


Figure (a)

By applying Eq. (1.20):

$$\sigma_{1,2} = \tfrac{50}{2} \pm \left[\tfrac{22,500}{4} + 3600\right]^{\frac{1}{2}} = 25 \pm 96$$

or $$\sigma_1 = 121 \text{ MPa} \qquad \sigma_2 = -71 \text{ MPa}$$

Using Eq. (1.19):

$$\tan 2\theta_p = -\tfrac{12}{15} = -0.8$$

$$\theta_p' = -19.3^o \qquad \theta_s' = 25.7^o$$

(CONT.)

(b) From Mohr's circle, Fig. (b):

$$\sigma_1 = 200 \text{ MPa} \qquad \sigma_2 = -50 \text{ MPa} \qquad \tau_{max} = 125 \text{ MPa}$$

$$\theta_p' = 26.55^o \qquad \theta_s' = 71.55^o$$

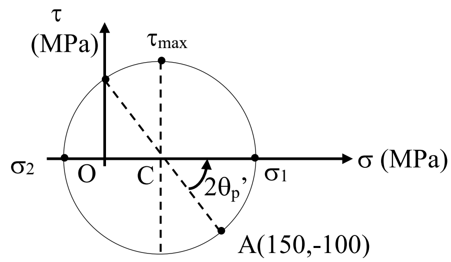


Figure (b)

Through the use of Eq. (1.20),

$$\sigma_{1,2} = 75 \pm \left[\tfrac{22,500}{4} + 10,000\right]^{\frac{1}{2}} = 75 \pm 125$$

or

$$\sigma_1 = 200 \text{ MPa} \qquad \sigma_2 = -50 \text{ MPa}$$

Using Eq. (1.19), $\tan 2\theta_p = 4/3$:

$$\theta_p' = 26.57^o \qquad \theta_s' = 71.57^o$$

SOLUTION (1.32)

Referring to Mohr's circle, Fig. 1.15:

$$\sigma_{x'} = \tfrac{\sigma_1+\sigma_2}{2} + \tfrac{\sigma_1-\sigma_2}{2}\cos 2\theta \qquad \text{(a)}$$

$$\sigma_{y'} = \tfrac{\sigma_1+\sigma_2}{2} - \tfrac{\sigma_1-\sigma_2}{2}\cos 2\theta$$

$$\tau_{x'y'} = \tfrac{\sigma_1-\sigma_2}{2}\sin 2\theta \qquad \text{(b)}$$

From Eqs. (a),

$$\sigma_{x'} + \sigma_{y'} = \sigma_1 + \sigma_2$$

By using $\cos^2 2\theta + \sin^2 2\theta = 1$, and Eqs. (a) and (b), we have

$$\sigma_{x'} \cdot \sigma_{y'} - \tau^2_{x'y'} = \sigma_1 \cdot \sigma_2 = const.$$

SOLUTION (1.33)

We have

$$\tan 2\theta_p = \tfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \tfrac{2(-70)}{50-(-190)} = -0.583$$

$$2\theta_p = -30.24^o \qquad \text{and } \theta_p = -15.12^o$$

(CONT.)

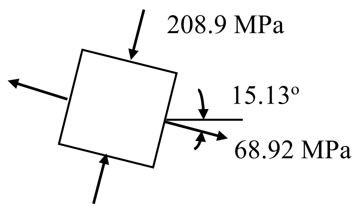
1.33 (CONT.)

Equations (1.18):

$$\sigma_{x'} = \tfrac{50-190}{2} + \tfrac{50+190}{2}\cos(-30.26^o) - 70\sin(-30.26^o)$$

$$= -70 + 103.65 + 35.275 = 68.93 \text{ MPa} = \sigma_1$$ ▲

$$\sigma_{y'} = \sigma_x + \sigma_y - \sigma_{x'} = -208.9 \text{ MPa} = \sigma_2$$



SOLUTION (1.34)

$$\tau_{max} = \sqrt{(\tfrac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$$

Substituting the given values

$$140^2 = \left(\tfrac{60+100}{2}\right)^2 + \tau_{xy}^2$$

or

$$\tau_{xy,max} = 114.89 \text{ MPa}$$ ▲

SOLUTION (1.35)

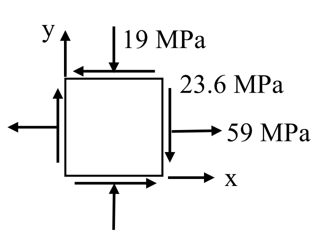
Transform from $\theta = 60^o$ to $\theta = 0^o$ with $\sigma_{x'} = -20$ MPa, $\sigma_{y'} = 60$ MPa,

$\tau_{x'y'} = -22$ MPa, and $\theta = -60^o$. Use Eqs. (1.18):

$$\sigma_x = \tfrac{-20+60}{2} + \tfrac{-20-60}{2}\cos 2(-60^o) - 22\sin 2(-60^o) = 59 \text{ MPa}$$

$$\sigma_y = \sigma_{x'} + \sigma_{y'} - \sigma_x = -19 \text{ MPa}$$ ▲

$$\tau_{xy} = -23.6 \text{ MPa}$$



SOLUTION (1.36)

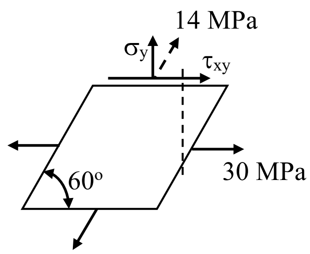


Figure (a)

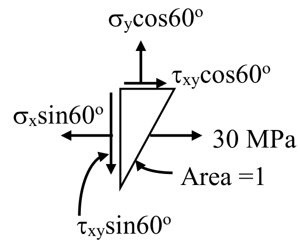


Figure (b)

(a) Figure (a):

$$\sigma_y = 14\sin 60^o = 12.12 \text{ MPa}$$ ▲

$$\tau_{xy} = 14\cos 60^o = 7 \text{ MPa}$$

Figure (b):

$$\sum F_y = 12.12\cos 60^o - \tau_{xy}\sin 60^o = 0$$

or

$$\tau_{xy} = 7 \text{ MPa} \text{ (as before)}$$

$$\sum F_x = -\sigma_x \sin 60^o + 30 + 7\cos 60^o = 0$$

or

$$\sigma_x = 38.68 \text{ MPa}$$ ▲

(b) Equation (1.20) is therefore:

$$\sigma_{1,2} = \tfrac{38.68+12.12}{2} \pm \left[\left(\tfrac{38.68-12.12}{2}\right)^2 + 7^2\right]^{\frac{1}{2}}$$ ▲

or $\sigma_1 = 40.41 \text{ MPa}, \qquad \sigma_2 = 10.39 \text{ MPa}$

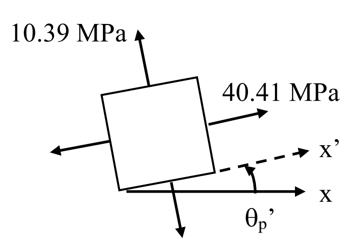
Also,

$$\theta_p = \tfrac{1}{2}\tan^{-1}\tfrac{2(7)}{38.68-12.12} = 13.9^o$$

Note: Eq. (1.18a) gives, $\sigma_{x'} = 40.41 \text{ MPa}$.

Thus,

$$\theta_p' = 13.9^o$$ ▲



SOLUTION (1.37)

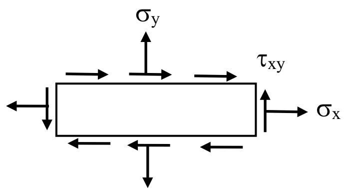


Figure (a)

Figure (a):

$$\sigma_x = 100\cos 45^o = 70.7 \text{ MPa}$$
$$\sigma_y = 100\sin 45^o = 70.7 \text{ MPa}$$
$$\tau_{xy} = 100\cos 45^o = 70.7 \text{ MPa}$$

Now, Eqs. (1.18) give (Fig. b):

$$\sigma_{x'} = 70.7 + 0 + 70.7\sin 240^o = 9.47 \text{ MPa}$$
$$\tau_{x'y'} = -0 + 70.7\cos 240^o = -35.35 \text{ MPa}$$
$$\sigma_{y'} = 70.7 - 0 - 70.7\sin 240^o = 131.9 \text{ MPa}$$

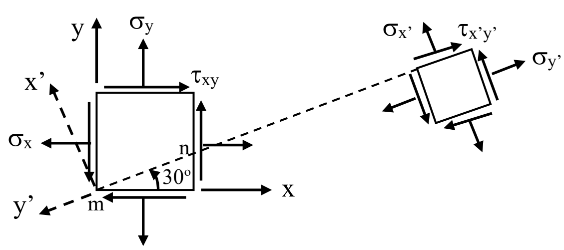


Figure (b)

SOLUTION (1.38)

$$\sigma_y = -70\sin 30^o = -35 \text{ MPa}$$
$$\tau_{xy} = 70\cos 30^o = 60.6 \text{ MPa}$$

(a) Figure (a):

$$\sum F_x = -150 + 0.5\sigma_x + 60.6(0.866) = 0$$

or $\sigma_x = 195 \text{ MPa}$

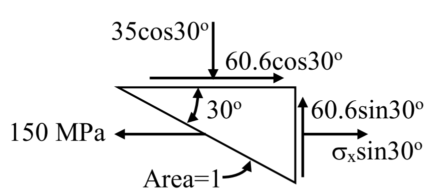


Figure (a)

(CONT.)

(b) Equation (1.20):

$$\sigma_{1,2} = \tfrac{195-35}{2} \pm \left[(\tfrac{195+35}{2})^2 + 60.6^2 \right]^{\frac{1}{2}}$$

or $\sigma_1 = 210 \text{ MPa} \qquad \sigma_2 = -50 \text{ MPa}$ ▲

Also,

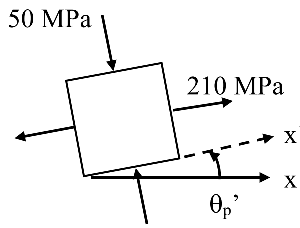
$$\theta_p = \tfrac{1}{2} \tan^{-1} \tfrac{2(60.6)}{195+35} = 13.89^o$$

Equation (1.18a):

$$\sigma_{x'} = 80 + 115 \cos 2(13.89^o) + 60.6 \sin 2(13.89^o) = 210 \text{ MPa}$$

Thus,

$$\theta_p' = 13.89^o$$ ▲



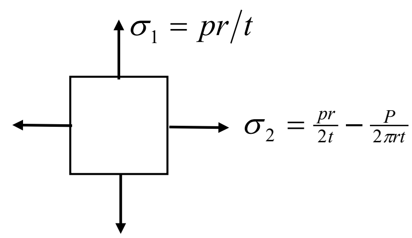
SOLUTION (1.39)

For pure shear, $\sigma_1 = -\sigma_2$:

$$\tfrac{pr}{t} = -\tfrac{pr}{2t} + \tfrac{P}{2\pi rt}$$

from which

$$P = 3\pi pr^2$$ ◄



SOLUTION (1.40)

Table D.4:

$$A = 2\pi rt$$
$$J = 2\pi r^3 t$$

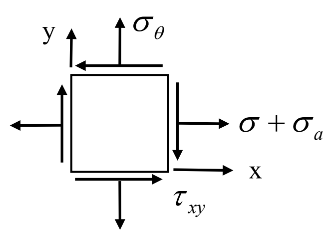


Figure (a)

Stresses are (Fig. a):

$$\sigma = \tfrac{-P}{A} = \tfrac{-30(10^3)\pi}{2\pi(0.12)(0.005)} - 25 \text{ MPa}$$

$$\sigma_a = \tfrac{pr}{2t} = \tfrac{4(10^6)120}{2(5)} = 48 \text{ MPa}$$

$$\sigma_\theta = 2\sigma_a = 96 \text{ MPa}$$

$$\tau_{xy} = \tfrac{-Tr}{J} = \tfrac{-10\pi(10^3)}{2\pi(0.12^2)(0.005)} = -69.4 \text{ MPa}$$

(CONT.)