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CHAPTER 2



Solution Manual

Exact

2.1. (a) $d\Omega = \sin \theta \, d\theta \, d\phi$

$$\Omega_A = \int_{45^\circ}^{60^\circ} \int_{\pi/4}^{\pi/3} d\Omega = \int_{\pi/4}^{\pi/3} \int_{\pi/6}^{\pi/3} \sin \theta \, d\theta \, d\phi$$

$$= (\phi) \Big|_{\pi/4}^{\pi/3} (-\cos \theta) \Big|_{\pi/6}^{\pi/3}$$

$$= \left(\frac{\pi}{3} - \frac{\pi}{4} \right) (-0.5 + 0.866)$$

$$\Omega_A = \left(\frac{\pi}{12} \right) (0.366) = 0.09582 \text{ sterads}$$

$$\Omega_A = \begin{cases} 0.09582 \text{ sterads} \\ 0.09582 \left(\frac{180}{\pi} \right) \left(\frac{180}{\pi} \right) = 314.5585 \text{ (degrees)}^2 \end{cases}$$

Approximate

$$\Omega_A \simeq \left(\frac{\pi}{3} - \frac{\pi}{4} \right) \left(\frac{\pi}{3} - \frac{\pi}{6} \right)$$

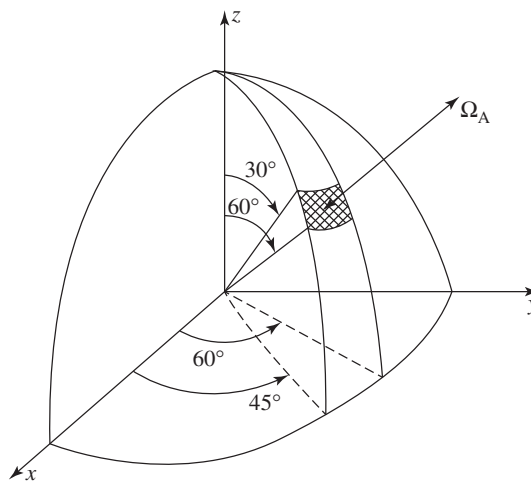
$$\simeq \left(\frac{\pi}{12} \right) \left(\frac{\pi}{6} \right) = \frac{\pi^2}{72}$$

$$\Omega_A \simeq 0.13708 \text{ sterads}$$

$$\Omega_A \simeq (60 - 45)(60 - 30)$$

$$\simeq 450 \text{ (degrees)}^2 \text{ or error of}$$

$$\left(\frac{450 - 314.5585}{314.5585} \right) \times 100 = 43.06\%$$



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$$\begin{aligned} \text{(b) } D_0 &= \frac{4\pi}{\Omega_A(\text{sterads})} = \frac{4\pi}{0.09582} = 131.1456 \text{ (dimensionless)} \\ &= 10 \log_{10}(131.1456) = 21.1775 \text{ dB} \end{aligned}$$

or

$$D_0 = \frac{4\pi \left(\frac{180}{\pi}\right) \left(\frac{180}{\pi}\right)}{\Omega_A(\text{degrees})^2} = 131.1456 \text{ (dimensionless)} = 21.1775 \text{ dB}$$

$$D_0 = \begin{cases} 131.1456 \text{ (dimensionless)} \\ 21.1775 \text{ (dB)} \end{cases}$$

2.2. $\underline{\mathcal{W}} = \underline{\mathcal{E}} \times \underline{\mathcal{H}} = \text{Re} [\underline{E}e^{j\omega t}] \times \text{Re} [\underline{H}e^{j\omega t}]$

Using the identity $\text{Re} [\underline{A}e^{j\omega t}] = \frac{1}{2} [\underline{A}e^{j\omega t} + \underline{A}^* e^{-j\omega t}]$

The instant Poynting vector can be written as

$$\begin{aligned} \underline{\mathcal{W}} &= \left\{ \frac{1}{2} [\underline{E}e^{j\omega t} + \underline{E}^* e^{-j\omega t}] \right\} \times \left\{ \frac{1}{2} [\underline{H}e^{j\omega t} + \underline{H}^* e^{-j\omega t}] \right\} \\ &= \frac{1}{2} \left\{ \frac{1}{2} [\underline{E} \times \underline{H}^* + \underline{E}^* \times \underline{H}] + \frac{1}{2} [\underline{E} \times \underline{H}e^{j2\omega t} + \underline{E}^* \times \underline{H}^* e^{-j2\omega t}] \right\} \\ &= \frac{1}{2} \left\{ \frac{1}{2} [\underline{E} \times \underline{H}^* + (\underline{E} \times \underline{H}^*)^*] + \frac{1}{2} [\underline{E} \times \underline{H}e^{j2\omega t} + (\underline{E} \times \underline{H}e^{j2\omega t})^*] \right\} \end{aligned}$$

Using the above identity again, but this time in reverse order, we can write that

$$\underline{\mathcal{W}} = \frac{1}{2} [\text{Re}(\underline{E} \times \underline{H}^*)] + \frac{1}{2} [\text{Re}(\underline{E} \times \underline{H}e^{j2\omega t})]$$

2.3. (a) $\underline{W}_{\text{rad}} = \frac{1}{2} \text{Re}[\underline{E} \times \underline{H}^*] = \frac{E^2}{2\eta} \hat{a}_r = \frac{5^2}{2(120\pi)} \hat{a}_r = 0.03315 \hat{a}_r \text{ Watts/m}^2$

$$\begin{aligned} \text{(b) } P_{\text{rad}} &= \oint_s \underline{W}_{\text{rad}} \cdot d\mathbf{s} = \int_0^{2\pi} \int_0^\pi (0.03315)(r^2 \sin \theta \, d\theta \, d\phi) \\ &= \int_0^{2\pi} \int_0^\pi (0.03315)(100)^2 \sin \theta \, d\theta \, d\phi \\ &= 2\pi(0.03315)(100)^2 \int_0^\pi \sin \theta \, d\theta = 2\pi(0.03315)(100)^2 \cdot (2) \\ &= 4165.75 \text{ Watts} \end{aligned}$$

2.4. (a) $U(\theta) = \cos \theta$

$$U(\theta_h) = 0.5 = \cos \theta_h \Rightarrow \theta_h = \cos^{-1}(0.5) = 60^\circ$$

$$\Rightarrow \Theta_h = 2(60^\circ) = 120^\circ = \frac{2\pi}{3} \text{ rads.}$$

$$U(\theta_n) = 0 = \cos \theta_n \Rightarrow \theta_n = \cos^{-1}(0) = 90^\circ$$

$$\Rightarrow \Theta_n = 2(90^\circ) = 180^\circ = \pi \text{ rads.}$$

(b) $U(\theta) = \cos^2 \theta$

$$U(\theta_h) = 0.5 = \cos^2 \theta_h \Rightarrow \theta_h = \cos^{-1}(0.5)^{1/2} = 45^\circ$$

$$\Rightarrow \Theta_h = 2(45^\circ) = 90^\circ = \pi/2 \text{ rads}$$

$$U(\theta_n) = 0 = \cos^2 \theta_n \Rightarrow \theta_n = \cos^{-1}(0) = 90^\circ$$

$$\Rightarrow \Theta_n = 2(90^\circ) = 180^\circ = \pi \text{ rads}$$

(c) $U(\theta) = \cos(2\theta)$

$$U(\theta_h) = 0.5 = \cos(2\theta_h) \Rightarrow \theta_h = \frac{1}{2} \cos^{-1}(0.5) = 30^\circ$$

$$\Rightarrow \Theta_h = 2(30^\circ) = 60^\circ = \pi/3 \text{ rads}$$

$$U(\theta_n) = 0 = \cos(2\theta_n) \Rightarrow \theta_n = \frac{1}{2} \cos^{-1}(0) = 45^\circ$$

$$\Rightarrow \Theta_n = 2(45^\circ) = 90^\circ = \pi/2 \text{ rads}$$

(d) $U(\theta) = \cos^2(2\theta)$

$$U(\theta_h) = 0.5 = \cos^2(2\theta_h) \Rightarrow \theta_h = \frac{1}{2} \cos^{-1}(0.5)^{1/2} = 22.5^\circ$$

$$\Rightarrow \Theta_h = 2(22.5^\circ) = 45^\circ = \frac{\pi}{4} \text{ rads}$$

$$U(\theta_n) = 0 = \cos^2(2\theta_n) \Rightarrow \theta_n = \frac{1}{2} \cos^{-1}(0) = 45^\circ$$

$$\Rightarrow \Theta_n = 2(45^\circ) = 90^\circ = \pi/2 \text{ rads}$$

(e) $U(\theta) = \cos(3\theta)$

$$U(\theta_h) = \cos(3\theta_h) = 0.5 \Rightarrow \theta_h = \frac{1}{3} \cos^{-1}(0.5) = 20^\circ$$

$$\Rightarrow \Theta_h = 2(20^\circ) = 40^\circ = 0.698 \text{ rads}$$

$$U(\theta_n) = \cos(3\theta_n) = 0 \Rightarrow \theta_n = \frac{1}{3} \cos^{-1}(0) = 30^\circ$$

$$\Rightarrow \Theta_n = 2(30^\circ) = 60^\circ = \pi/3 \text{ rads}$$

(f) $U(\theta) = \cos^2(3\theta)$

$$U(\theta_h) = 0.5 = \cos^2(3\theta_h) \Rightarrow \theta_h = \frac{1}{3} \cos^{-1}(0.5)^{1/2} = 15^\circ$$

$$\Rightarrow \Theta_h = 2(15^\circ) = 30^\circ = \pi/6 \text{ rads}$$

$$U(\theta_n) = 0 = \cos^2(3\theta_n) \Rightarrow \theta_n = \frac{1}{3} \cos^{-1}(0) = 30^\circ$$

$$\Rightarrow \Theta_n = 2(30^\circ) = 60^\circ = \pi/3 \text{ rads}$$

2.5. Using the results of Problem 2.4 and a nonlinear solver to find the half power beamwidth of the radiation intensity represented by the transcendental functions, we have that:

$$(a) U(\theta) = \cos \theta \cos(2\theta) \Rightarrow \begin{cases} \text{HPBW} = 55.584^\circ \\ \text{FNBW} = 90^\circ \end{cases}$$

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$$(b) U(\theta) = \cos^2 \theta \cos^2(2\theta) \Rightarrow \begin{cases} \text{HPBW} = 40.985^\circ \\ \text{FNBW} = 90^\circ \end{cases}$$

$$(c) U = \cos \theta \cos(3\theta) \Rightarrow \begin{cases} \text{HPBW} = 38.668^\circ \\ \text{FNBW} = 60^\circ \end{cases}$$

$$(d) U = \cos^2 \theta \cos^2(3\theta) \Rightarrow \begin{cases} \text{HPBW} = 28.745^\circ \\ \text{FNBW} = 60^\circ \end{cases}$$

$$(e) U = \cos(2\theta) \cos(3\theta) \Rightarrow \begin{cases} \text{HPBW} = 34.942^\circ \\ \text{FNBW} = 60^\circ \end{cases}$$

$$(f) U = \cos^2(2\theta) \cos^2(3\theta) \Rightarrow \begin{cases} \text{HPBW} = 25.583^\circ \\ \text{FNBW} = 60^\circ \end{cases}$$

$$2.6. (a) D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi(200 \times 10^{-3})}{0.9(125.66 \times 10^{-3})} = 22.22 = 13.47 \text{ dB}$$

$$G_0 = \epsilon_{cd} D_0 = 0.9(22.22) = 20 = 13.01 \text{ dB}$$

$$(b) D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi(200 \times 10^{-3})}{(125.66 \times 10^{-3})} = 20 = 13.01 \text{ dB}$$

$$G_0 = \epsilon_{cd} D_0 = 0.9 \cdot (20) = 18 = 12.55 \text{ dB}$$

$$2.7. \boxed{U = B_0 \cos^2 \theta}$$

$$(a) P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi/2} U \sin \theta \, d\theta = 2\pi B_0 \int_0^{\pi/2} \cos^2 \theta \sin \theta \, d\theta$$

$$= 2\pi B_0 \int_0^{\pi/2} \cos^2 \theta \, d(-\cos \theta)$$

$$P_{\text{rad}} = -2\pi B_0 \left. \frac{\cos^3 \theta}{3} \right|_0^{\pi/2} = -2\pi B_0 \left[\frac{-1}{3} \right] = \frac{2\pi}{3} B_0 = 10 \Rightarrow B_0 = \frac{15}{\pi}$$

$$U = \frac{15}{\pi} \cos^2 \theta \Rightarrow W_{\text{rad}} \Big|_{\max} = \frac{U}{r^2} \Big|_{\max} = \frac{15}{\pi} \frac{\cos^2 \theta}{r^2} \Big|_{\max}$$

$$= \frac{15}{\pi(10^3)^2} = 4.7746 \times 10^{-6} \text{ Watts/m}^2 @ \theta = 0^\circ$$

$$W_{\text{rad}} \Big|_{\max} = 4.7746 \times 10^{-6} \text{ Watts/m}^2 @ \theta = 0^\circ$$

$$(b) \Omega_A \text{ (exact)} = \int_0^{2\pi} \int_0^\pi U_n \cos^2 \theta \sin \theta \, d\theta \, d\phi$$

$$\Omega_A \text{ (exact)} = \frac{2\pi}{3} \text{ steradians} = 2.0944 \text{ sterads} = 6,875.51 \text{ (degrees)}^2$$

$$U = 0.5 = \cos^2 \theta_h \Rightarrow \theta_h = \cos^{-1}(0.5)^{1/2} = 45^\circ$$

$$\Rightarrow \Theta_h = 2(45^\circ) = 90^\circ = \pi/2 \text{ rads}$$

$$\Omega_A \left(\frac{\text{Kraus}'}{\text{approx}} \right) = \Theta_h^2 = (\pi/2)^2 = \frac{\pi^2}{4} = 2.4674 \text{ sterads} = 8,099.997 \text{ (degrees)}^2$$

$$(c) \quad D_0 \text{ (exact)} = \frac{4\pi}{\Omega_A \text{ (exact)}} = \frac{4\pi}{2\pi/3} = 6 = 7.782 \text{ dB}$$

$$D_0 \text{ (approx/Kraus')} = \frac{4\pi}{\Omega_A \text{ (approx)}} = \frac{4\pi}{\pi^2/4} = \frac{16}{\pi} = 5.093 = 7.0697$$

(d) G_0 Assuming lossless antenna ($P_{in} = P_{rad}$)

$$G_0 \text{ (exact)} = D_0 \text{ (exact)} = 6 = 7.782 \text{ dB}$$

$$G_0 \text{ (approx)} = D_0 \text{ (approx)} = 5.093 = 7.0697 \text{ dB}$$

$$\boxed{U = B_0 \cos^3 \theta}$$

$$(a) \quad P_{rad} = -2\pi B_0 \left(-\frac{1}{4}\right) = \frac{\pi}{2} B_0 = 10 \Rightarrow B_0 = 20/\pi$$

$$W_{rad} \Big|_{\max} = \frac{20}{\pi} \frac{1}{\pi^2} = \frac{20}{\pi} \times 10^{-6} = 6.366 \times 10^{-6} \text{ Watts/m}^2$$

(b) $\Omega_A \text{ (exact)} = (\pi/2) = 1.5708 \text{ sterads}$

$$U = 0.5 = \cos^3 \theta_h \Rightarrow \theta_h \cos^{-1}(0.5)^{1/3} = 37.467^\circ$$

$$\Rightarrow \Theta_h = 2(37.467^\circ) = 74.934^\circ = 1.30785 \text{ rads}$$

$$\Omega_A \text{ (approx)} = (1.30785)^2 = 1.71 \text{ sterads}$$

(c) $D_0 \text{ (exact)} = 4\pi/(\pi/2) = 8 = 9.031 \text{ dB}$

$$D_0 \text{ (approx)} = \frac{4\pi}{1.71} = 7.347 = 8.66 \text{ dB}$$

(d) Assuming lossless antenna \Rightarrow Gain = Directivity (see part c)

2.8. $\underline{E}_a = \hat{a}_\theta E_a \sin^{1.5} \theta \frac{e^{-jkr}}{r} \Rightarrow U_n = (\sin^{1.5} \theta)^2 = \sin^3 \theta$ **Normalized U_n**

$$(a) \quad D_0 = \frac{4\pi U_{\max}}{P_{rad}}, U_{\max} = U_n \Big|_{\max} = \sin^3 \theta \Big|_{\theta=\theta_{\max}} = 1, \theta_{\max} = 90^\circ$$

$$P_{rad} = \int_0^{2\pi} \int_0^\pi U_n \sin \theta \, d\theta \, d\phi = \int_0^{2\pi} \int_0^\pi \sin^3 \theta \sin \theta \, d\theta \, d\phi = 2\pi \int_0^\pi \sin^4 \theta \, d\theta$$

$$= 2\pi \left[-\frac{\sin^3 \theta \cos \theta}{4} \Big|_0^\pi + \frac{3}{4} \int_0^\pi \sin^2 \theta \, d\theta \right] = 2\pi \left[\frac{3}{4} \left(\frac{1}{2} - \frac{1}{4} \sin(2\theta) \right) \Big|_0^\pi \right]$$

$$= 2\pi \left[\frac{3}{4} \left(\frac{\pi}{2} \right) \right] = \frac{3\pi^2}{4}$$

$$D_0 = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi(1)}{3\pi^2/4} = \frac{16}{3\pi} = \boxed{1.698 = 2.298 \text{ dB}}$$

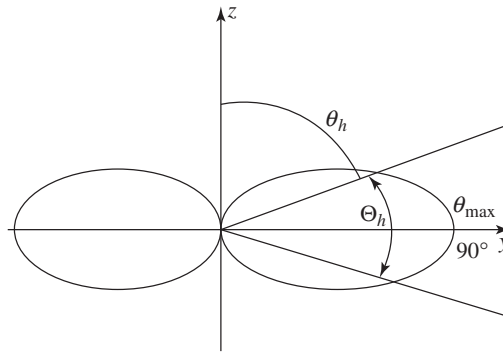
(b) $U_n = \sin^3 \theta, U_{n\max} = 1, \theta_{\max} = 90^\circ$

$$U_n \Big|_{\theta=\theta_h} = 0.5 = \sin^3 \theta_h \Rightarrow \theta_h = [\sin^{-1}(0.5^4)^3] = \sin^{-1}(0.794) = 52.533^\circ$$

$$\text{HPBW} = \Theta_h = 2(\theta_{\max} - \theta_h) = 2(90 - 52.533)$$

$$\boxed{\Theta_h = 2(37.467) = 74.934^\circ}$$

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(c) Because pattern is **omnidirectional**:

$$D_0(\text{McDonald}) = \frac{101}{\text{HPBW} - 0.0027(\text{HPBW})^2} = \frac{101}{74.934 - 0.0027(74.934)^2}$$

$$D_0(\text{McDonald}) = \frac{101}{74.934 - 15.161} = \frac{101}{59.773} = \boxed{1.690 = 10 \log_{10}(1.690) = 2.278}$$

(d) Because pattern is **omnidirectional**:

$$D_0(\text{Pozar}) = -172.4 + 191 \sqrt{0.818 + \frac{1}{\text{HPBW}}} = -172.4 + 191 \sqrt{0.818 + \frac{1}{74.934}}$$

$$= -172.4 + 191(0.912) = -172.4 + 174.150 = 1.750$$

$$P_0(\text{Pozar}) = \boxed{1.750 = 10 \log_{10}(1.750) = 2.431 \text{ dB}}$$

(e) Computer Program Directivity: $\boxed{D_0 = 1.693 = 2.2864 \text{ dB}}$

Input parameters:

The lower bound of theta in degrees = 0
 The upper bound of theta in degrees = 180
 The lower bound of phi in degrees = 0
 The upper bound of phi in degrees = 360

Output parameters:

Radiated power (watts) = 7.4228
 Directivity (dimensionless) = 1.6930
 Directivity (dB) = 2.2864

2.9. $U(\theta, \phi) = \cos^n(\theta) \quad 0 \leq \theta \leq \pi/2, 0 \leq \phi \leq 2\pi$

(a) $U_n(\theta_n, \phi) = 0.5 = \cos^n(5^\circ) = [\cos(5^\circ)]^n = (0.99619)^n$

$$0.5 = (0.99619)^n$$

$$\log_{10}(0.5) = \log_{10}[(0.99619)^n] = n \log_{10}(0.99619) = n(-0.00166)$$

$$-0.30103 = -0.00166n$$

$$\boxed{n = 181.34}$$

(b) $U(\theta, \phi) = \cos^{181.34}(\theta); U_{\max} = 1, \theta = 0^\circ$

$$\begin{aligned}
 P_{\text{rad}} &= \int_0^{2\pi} \int_0^{\pi/2} U(\theta, \phi) \sin \theta \, d\theta \, d\phi = 2\pi \int_0^{\pi/2} \cos^{181.34}(\theta) \sin \theta \, d\theta \\
 &= 2\pi \left[-\frac{\cos^{182.34}(\theta)}{182.34} \right]_0^{\pi/2} = \left[-0 + \frac{1}{182.34} \right] 2\pi = \frac{2\pi}{182.34} = 0.03446 \\
 D_0 &= \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi(1)}{2\pi} (182.34) = 2(182.34) = 364.68
 \end{aligned}$$

$$D_0 = 364.68 = 25.62 \text{ dB}$$

(c) *Kraus' Approximation (2.27):*

$$D_0 \simeq \frac{41,253}{\Theta_{1d}\Theta_{2d}} = \frac{41,253}{(10)(10)} = 412.53 = 26.15 \text{ dB}$$

$$D_0 \simeq 412.53 = 26.15 \text{ dB}$$

(d) *Tai & Pereira (2.30b):*

$$D_0 \simeq \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72,815}{2(10)^2} = \frac{72,815}{200} = 364.075 = 25.61 \text{ dB}$$

$$D_0 \simeq 364.075 = 25.61 \text{ dB}$$

2.10.

$$U(\theta, \phi) = \begin{cases} 1 & 0^\circ \leq \theta \leq 20^\circ \\ 0.342 \csc(\theta) & 20^\circ \leq \theta \leq 60^\circ \\ 0 & 60^\circ \leq \theta \leq 180^\circ \end{cases} \quad 0^\circ \leq \phi \leq 360^\circ$$

$$\begin{aligned}
 P_{\text{rad}} &= \int_0^{2\pi} \int_0^\pi U(\theta, \phi) \sin \theta \, d\theta \, d\phi \\
 &= 2\pi \left[\int_0^{20^\circ} \sin \theta \, d\theta + \int_{20^\circ}^{60^\circ} 0.342 \csc(\theta) \times \sin \theta \, d\theta \right] \\
 &= 2\pi \left\{ -\cos \theta \Big|_0^{\pi/9} + 0.342 \cdot \theta \Big|_{\pi/9}^{\pi/3} \right\} \\
 &= 2\pi \left\{ \left[-\cos \left(\frac{\pi}{9} \right) + 1 \right] + 0.342 \left(\frac{\pi}{3} - \frac{\pi}{9} \right) \right\} \\
 &= 2\pi \left\{ [-0.93969 + 1] + 0.342\pi \left(\frac{2}{9} \right) \right\} \\
 &= 2\pi \{0.06031 + 0.23876\} = 1.87912 \\
 D_0 &= \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi(1)}{1.87912} = 6.68737 = 8.25255 \text{ dB}
 \end{aligned}$$

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2.11. (a) $D_0 \simeq \frac{41,253}{\Theta_{1d}\Theta_{2d}} = \frac{41,253}{30(35)} = 39.29 = 15.94 \text{ dB}$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$

(b) $D_0 \simeq \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72,815}{(30)^2 + (35)^2} = 34.27 = 15.35 \text{ dB}$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$

2.12. $D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}}$

(a) $U = \sin \theta \sin \phi$ for $0 \leq \theta \leq \pi, 0 \leq \phi \leq \pi$

$U|_{\max} = 1$ and it occurs when $\theta = \phi = \pi/2$.

$$P_{\text{rad}} = \int_0^\pi \int_0^\pi U \sin \theta \, d\theta \, d\phi = \int_0^\pi \sin \phi \, d\phi \int_0^\pi \sin^2 \theta \, d\theta = 2 \left(\frac{\pi}{2} \right) = \pi.$$

Thus $D_0 = \frac{4\pi(1)}{\pi} = 4 = 6.02 \text{ dB}$

The half-power beamwidths are equal to

$$\text{HPBW (az.)} = 2[90^\circ - \sin^{-1}(1/2)] = 2(90^\circ - 30^\circ) = 120^\circ$$

$$\text{HPBW (el.)} = 2[90^\circ - \sin^{-1}(1/2)] = 2(90^\circ - 30^\circ) = 120^\circ$$

In a similar manner, it can be shown that for the following:

(b) $U = \sin \theta \sin^2 \phi \Rightarrow D_0 = 5.09 = 7.07 \text{ dB}$

$$\text{HPBW (el.)} = 120^\circ, \text{HPBW (az.)} = 90^\circ$$

(c) $U = \sin \theta \sin^3 \phi \Rightarrow D_0 = 6 = 7.78 \text{ dB}$

$$\text{HPBW (el.)} = 120^\circ, \text{HPBW (az.)} = 74.93^\circ$$

(d) $U = \sin^2 \theta \sin \phi \Rightarrow D_0 = 12\pi/8 = 4.71 = 6.73 \text{ dB}$

$$\text{HPBW (el.)} = 90^\circ, \text{HPBW (az.)} = 120^\circ$$

(e) $U = \sin^2 \theta \sin^2 \phi \Rightarrow D_0 = 6 = 7.78 \text{ dB}$

$$\text{HPBW (az.)} = \text{HPBW (el.)} = 90^\circ$$

(f) $U = \sin^2 \theta \sin^3 \phi \Rightarrow D_0 = 9\pi/4 = 7.07 = 8.49 \text{ dB}$

$$\text{HPBW (el.)} = 90^\circ, \text{HPBW (az.)} = 74.93^\circ$$

2.13. $U = \sin \theta \cos^2 \phi, \quad 0 \leq \theta \leq 180^\circ, \quad 90^\circ \leq \phi \leq 270^\circ$

(a) $D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}}, \quad U_{\max} = \sin \theta \cos^2 \phi \Big|_{\substack{\theta=90^\circ \\ \phi=180^\circ}} = 1$

$$\begin{aligned}
 P_{\text{rad}} &= \int_{\pi/2}^{3\pi/2} \int_0^{\pi} U(\theta, \phi) \sin \theta \, d\theta \, d\phi = \int_{\pi/2}^{3\pi/2} \int_0^{\pi} \sin \theta \cos^2 \phi \sin \theta \, d\theta \, d\phi \\
 &= \int_{\pi/2}^{3\pi/2} \cos^2 \phi \, d\phi \int_0^{\pi} \sin^2 \theta \, d\theta \\
 P_{\text{rad}} &= \left(\frac{\pi}{2}\right) \left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} \\
 D_0 &= \frac{4\pi(1)}{\pi^2/4} = \frac{16}{\pi} = 5.09296 = 10 \log_{10}(5.09296) = 7.0697 \text{ dB}
 \end{aligned}$$

$$D_0(\text{exact}) = 5.09296(\text{dim}) = 7.0697 \text{ dB}$$

(b) Azimuth (Horizontal) Principal Plane ($\theta = 90^\circ$):

$$\begin{aligned}
 U(\theta = 90^\circ) &= \sin \theta \cos^2 \phi|_{\theta=90^\circ} = \cos^2 \phi \\
 U_h &= \cos^2 \phi|_{\phi=\phi_h} = 0.5 \Rightarrow \phi_h = \cos^{-1}(\pm\sqrt{0.5}) = \cos^{-1}(\pm 0.707) = 135^\circ \\
 \Phi_h(az) &= 2(180 - 135) = 2(45^\circ) = 90^\circ \\
 \Phi_h(az) &= 90^\circ
 \end{aligned}$$

(c) Elevation (vertical) Principal plane ($\phi = 180^\circ$):

$$\begin{aligned}
 U(\phi = 180^\circ) &= \sin \theta \cos^2 \phi|_{\phi=180^\circ} = \sin \theta \\
 U_h &= \sin \theta|_{\theta=\theta_h} = 0.5 \Rightarrow \theta_h = \sin^{-1}(0.5) = 30^\circ \\
 \Theta_h &= 2(90^\circ - 30^\circ) = 2(60) = 120^\circ \\
 \Theta_h(\text{elev}) &= 120^\circ
 \end{aligned}$$

(d) **Either:** $D_0(\text{Kraus}) = \frac{41,253}{\Phi_h \Theta_h} = \frac{41,253}{90^\circ(120^\circ)} = 3.8197 = 5.82 \text{ dB}$

$$D_0(\text{Kraus}) = 3.8197 \text{ dim} = 5.82 \text{ dB}$$

or:

$$D_0(\text{Tai \& Pereira}) = \frac{72,815}{\Phi_h^2 + \Theta_h^2} = \frac{72,815}{(90^\circ)^2 + (120^\circ)^2} = \frac{72,815}{25,500} = 3.236$$

$$D_0(\text{T\&P}) = 3.236 \text{ dim} = 5.1 \text{ dB}$$

2.14. Using the half-power beamwidths found in Problem 2.12, the directivity for each intensity using Kraus' and Tai & Pereira's formulas is given by

$$\begin{aligned}
 U &= \sin \theta \sin \phi; \\
 \text{(a) } D_0 &\simeq \frac{41253}{\Theta_{1d} \Theta_{2d}} = \frac{41253}{120(120)} = 2.86 = 4.57 \text{ dB}
 \end{aligned}$$

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$$(b) D_0 \simeq \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72,815}{(120)^2 + (120)^2} = 2.53 = 4.03 \text{ dB}$$

$$U = \sin \theta \sin^2 \phi;$$

$$(a) D_0 \simeq 3.82 = 5.82 \text{ dB}$$

$$(b) D_0 \simeq 3.24 = 5.10 \text{ dB}$$

$$U = \sin \theta \sin^3 \phi;$$

$$(a) D_0 \simeq 4.59 = 6.62 \text{ dB}$$

$$(b) D_0 \simeq 3.64 = 5.61 \text{ dB}$$

$$U = \sin^2 \theta \sin \phi;$$

$$(a) D_0 \simeq 3.82 = 5.82 \text{ dB}$$

$$(b) D_0 \simeq 3.24 = 5.10 \text{ dB}$$

$$U = \sin^2 \theta \sin^2 \phi;$$

$$(a) D_0 \simeq 5.09 = 7.07 \text{ dB}$$

$$(b) D_0 \simeq 4.49 = 6.53 \text{ dB}$$

$$U = \sin^2 \theta \sin^3 \phi;$$

$$(a) D_0 \simeq 6.12 = 7.87 \text{ dB}$$

$$(b) D_0 \simeq 5.31 = 7.25 \text{ dB}$$

$$2.15. (a) D_0 = \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi}{(1.5064)^2} = 5.5377 = 7.433 \text{ dB}$$

$$(b) D_0 = \frac{32 \ln(2)}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{32 \ln(2)}{(1.5064)^2 + (1.5064)^2} = 4.88725 = 6.8906 \text{ dB}$$

$$2.16. (a) D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{U_{\max}}{U_0}$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi U \sin \theta \, d\theta \, d\phi = 2\pi \int_0^\pi U \sin \theta \, d\theta = 2\pi \left\{ \int_0^{30^\circ} \sin \theta \, d\theta \right. \\ \left. + \int_{30^\circ}^{60^\circ} (0.5) \sin \theta \, d\theta + \int_{60^\circ}^{90^\circ} (0.1) \sin \theta \, d\theta \right\}$$

$$= 2\pi \left\{ (-\cos \theta) \Big|_0^{30^\circ} + \left(-\frac{\cos \theta}{2}\right) \Big|_{30^\circ}^{60^\circ} + (-0.1 \cos \theta) \Big|_{60^\circ}^{90^\circ} \right\}$$

$$= 2\pi \left\{ (-0.866 + 1) + \left(\frac{-0.5 + 0.866}{2}\right) + \left(\frac{-0 + 0.5}{10}\right) \right\}$$

$$P_{\text{rad}} = 2\pi \{-0.866 + 1 - 0.25 + 0.433 + 0.05\} = 2\pi(0.367)$$

$$= 0.734\pi = 2.3059$$

$$D_0 = \frac{1(4\pi)}{2.3059} = 5.4496 = 7.3636 \text{ dB}$$

$$(b) D_0(\text{dipole}) = 1.5 = 1.761 \text{ dB}$$

$$D_0(\text{above dipole}) = (7.3636 - 1.761) \text{ dB} = 5.6026 \text{ dB}$$

$$D_0(\text{above dipole}) = \frac{5.45}{1.5} = 3.633 = 5.603 \text{ dB}$$

2.17. (a)
$$P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi U(\theta, \phi) \sin \theta \, d\theta \, d\phi = \int_0^{2\pi} \sin^2 \phi \, d\phi \int_0^{\pi/2} \cos^4 \theta \sin \theta \, d\theta$$

$$= (\pi) \left(\frac{1}{5}\right) = \frac{\pi}{5}$$

$$U_{\text{max}} = U(\theta = 0^\circ, \phi = \pi/2) = 1$$

$$D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi}{(\pi/5)} = 20 = 13.0 \text{ dB}$$

(b) Elevation Plane: θ varies, ϕ fixed

\Rightarrow Choose $\phi = \pi/2$.

$$U(\theta, \phi = \pi/2) = \cos^4 \theta, \quad 0 \leq \theta \leq \pi/2.$$

$$\cos^4 \left[\frac{\text{HPBW}(\text{el.})}{2} \right] = \frac{1}{2}$$

$$\text{HPBW}(\text{el.}) = 2 \cos^{-1} \{ \sqrt{0.5} \}^{1/2} = 65.5^\circ$$

2.18. (a)
$$P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi U(\theta, \phi) \sin \theta \, d\theta \, d\phi = 2\pi$$

$$\cdot \left\{ \int_0^{30^\circ} \sin \theta \, d\theta + \int_{30^\circ}^{90^\circ} \frac{\cos \theta \sin \theta}{0.866} \, d\theta \right\}$$

$$= 2\pi \left\{ \int_0^{\pi/6} \sin \theta \, d\theta + \int_{\pi/6}^{\pi/2} \frac{1}{0.866} \cos \theta \sin \theta \, d\theta \right\}$$

$$= 2\pi \left\{ -\cos \theta \Big|_0^{\pi/6} + \frac{1}{0.866} \left(-\frac{\cos^2 \theta}{2} \right) \Big|_{\pi/6}^{\pi/2} \right\} = 2\pi[-0.866 + 1 + 0.433]$$

$$P_{\text{rad}} = 3.5626$$

$$D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi(1)}{3.5626} = 3.5273 = 5.4745 \text{ dB}$$

(b)
$$U = \frac{\cos(\theta)}{0.866} = 0.5 \Rightarrow \cos \theta = 0.5(0.866) = 0.433, \theta = \cos^{-1}(0.433) = 64.34^\circ$$

$$\Theta_{1r} = 2(64.34) = 128.68^\circ = 2.246 \text{ rad} = \Theta_{2r}$$

$$D_0 \simeq \frac{4\pi}{\Theta_{1r} \Theta_{2r}} = \frac{4\pi}{(2.246)^2} = 2.4912 = 3.9641 \text{ dB}$$

2.19. (a) 35 dB
 (b)
$$20 \log_{10} \left| \frac{E_{\text{max}}}{E_s} \right| = 35, \log_{10} \left| \frac{E_{\text{max}}}{E_s} \right| = \frac{35}{20} = 1.75$$

$$\left| \frac{E_{\text{max}}}{E_s} \right| = 10^{1.75} = 56.234$$

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2.20. (a) $U = \sin \theta, U_{\max} = 1, P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi U \sin \theta \, d\theta \, d\phi$

$$= \int_0^{2\pi} \int_0^\pi \sin^2 \theta \, d\theta \, d\phi = \pi^2$$

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi}{\pi^2} = \frac{4}{\pi} = 1.2732$$

(b) HPBW = $120^\circ, 2\pi/3$
 The directivity based on (2-33a) is equal to,

$$D_0 = \frac{101}{120^\circ - 0.0027(120^\circ)^2} = 1.2451$$

while that based on (2-33b) is equal to,

$$D_0 = -172.4 + 191 \sqrt{0.818 + \frac{1}{120^\circ}} = 1.2245$$

(c) Computer Program: $D_0 = 1.2732$

2.21. (a) $U = \sin^3 \theta, U_{\max} = 1, P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi \sin^4 \theta \, d\theta \, d\phi = \frac{3}{4}\pi^2$

$$D_0 = \frac{4\pi}{\frac{3}{4}\pi^2} = \frac{16}{3\pi} = 1.6976$$

(b) HPBW = 74.93°
 From (2-33a), $D_0 = \frac{101}{(74.93^\circ) - 0.0027(74.93^\circ)^2} = 1.68971$

From (2-33b), $D_0 = -172.4 + 191 \sqrt{0.818 + \frac{1}{74.93^\circ}} = 1.75029$

(c) Computer program: $D_0 = 1.693$
 The value of $D_0 = 1.693$ is similar to that of (4-91) or 1.643

2.22. (a) $U = J_1^2(ka \sin \theta),$
 $a = \lambda/10, ka \sin \theta = \frac{\pi}{5} \sin \theta. \text{ HPBW} = 93.10^\circ$
 From (2-33a): $D_0 = 101 / [(93.10) - 0.0027(93.10)^2] = 1.449120$
 From (2-33b): $D_0 = -172.4 + 191 \sqrt{0.818 + \frac{1}{93.10}} = 1.477271$

$a = \lambda/20, ka \sin \theta = \frac{\pi}{10} \sin \theta, \text{ HPBW} = 91.10^\circ$

From (2-33a), $D_0 = 1.47033$; From (2-33b), $D_0 = 1.502$

(b) $a = \frac{\lambda}{10}: P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi J_1^2(ka \sin \theta) \cdot \sin \theta \, d\theta \, d\phi = 0.7638045$

$$U_{\max} = 0.0893, D_0 = \frac{4\pi(0.0893)}{0.7638045} = 1.469193$$

$$a = \frac{\lambda}{20}: P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi J_1^2(\pi/10 \sin \theta) \sin \theta d\theta d\phi = 0.202604$$

$$U_{\text{max}} = 0.0240714, D_0 = \frac{4\pi(0.0240714)}{0.202604} = 1.49257$$

If the radius of loop is smaller than $\lambda/20$, the directivity approaches 1.5.

2.23. Using the numerical techniques, the directivity for each intensity of Prob. 2.12, with 10° uniform divisions is equal to for $U = \sin \theta \sin \phi$:

(a) Midpoint: $D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}}$

$$U_{\text{max}} = 1: P_{\text{rad}} = \frac{\pi}{18} \left(\frac{\pi}{18} \right) \sum_{j=1}^{18} \sin \phi_j \sum_{i=1}^{18} \sin^2 \theta_i$$

$$\theta_i = \frac{\pi}{36} + (i-1)\frac{\pi}{18}, i = 1, 2, 3, \dots, 18$$

$$\phi_j = \frac{\pi}{36} + (j-1)\frac{\pi}{18}, j = 1, 2, 3, \dots, 18$$

$$P_{\text{rad}} = \left(\frac{\pi}{18} \right)^2 (11.38656)(8.9924) = 3.119$$

$$D_0 = \frac{4\pi(1)}{3.119} = 4.03 = 6.05 \text{ dB}$$

(b) Trailing edge of each division:

$$\text{Trailing edge: } \theta_i = i(\pi/18), i = 1, 2, 3, \dots, 18$$

$$\phi_j = j(\pi/18), j = 1, 2, 3, \dots, 18$$

$$P_{\text{rad}} = \left(\frac{\pi}{18} \right)^2 (11.25640)(8.96985) = 3.076$$

$$D_0 = \frac{4\pi(1)}{3.119} = 4.09 = 6.11 \text{ dB}$$

In a similar manner:

$$U = \sin \theta \sin^2 \phi;$$

(a) $P_{\text{rad}} = 2.463 \Rightarrow D_0 = 5.10 = 7.07 \text{ dB}$

(b) $P_{\text{rad}} = 2.451 \Rightarrow D_0 = 5.13 = 7.10 \text{ dB}$

$$U = \sin \theta \sin^3 \phi;$$

(a) $P_{\text{rad}} = 2.092 \Rightarrow D_0 = 6.01 = 7.79 \text{ dB}$

(b) $P_{\text{rad}} = 2.086 \Rightarrow D_0 = 6.02 = 7.80 \text{ dB}$

$$U = \sin^2 \theta \sin \phi;$$

(a) $P_{\text{rad}} = 2.469 \Rightarrow D_0 = 4.74 = 6.76 \text{ dB}$

(b) $P_{\text{rad}} = 2.618 \Rightarrow D_0 = 4.80 = 6.81 \text{ dB}$

$$U = \sin^2 \theta \sin^2 \phi;$$

(a) $P_{\text{rad}} = 2.092 \Rightarrow D_0 = 6.01 = 7.79 \text{ dB}$

(b) $P_{\text{rad}} = 2.086 \Rightarrow D_0 = 6.02 = 7.80 \text{ dB}$

$$U = \sin^2 \theta \sin^3 \phi;$$

(a) $P_{\text{rad}} = 1.777 \Rightarrow D_0 = 7.07 = 8.49 \text{ dB}$

(b) $P_{\text{rad}} = 1.775 \Rightarrow D_0 = 7.08 = 8.50 \text{ dB}$

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2.24. Using the computer program **Directivity** of Chapter 2, the directivities for each radiation intensity of Problem 2.12 are equal to:

- (a) $U = \sin \theta \sin \phi; P_{\text{rad}} = 3.1318$
 $U_{\text{max}} = 1; D_0 = \frac{4\pi \cdot U_{\text{max}}}{3.1318} = 4.0125 \Rightarrow 6.034 \text{ dB}$
- (b) $U = \sin \theta \sin^2 \phi; P_{\text{rad}} = 2.4590$
 $U_{\text{max}} = 1; D_0 = \frac{4\pi \cdot 1}{2.4590} = 5.110358 \Rightarrow 7.0845 \text{ dB}$
- (c) $U = \sin \theta \sin^3 \phi; P_{\text{rad}} = 2.0870$
 $U_{\text{max}} = 1; D_0 = \frac{4\pi \cdot 1}{2.0870} = 6.02124 \Rightarrow 7.80 \text{ dB}$
- (d) $U = \sin^2 \theta \sin \phi; P_{\text{rad}} = 2.6579$
 $U_{\text{max}} = 1; D_0 = \frac{4\pi \cdot 1}{2.6579} = 4.72793 \Rightarrow 6.746 \text{ dB}$
- (e) $U = \sin^2 \theta \sin^2 \phi; P_{\text{rad}} = 2.0870$
 $U_{\text{max}} = 1; D_0 = \frac{4\pi \cdot 1}{2.0870} = 6.02126 \Rightarrow 7.7968 \text{ dB}$
- (f) $U = \sin^2 \theta \sin^3 \phi; P_{\text{rad}} = 1.7714$
 $U_{\text{max}} = 1; D_0 = \frac{4\pi \cdot 1}{1.7714} = 7.09403 \Rightarrow 8.5089 \text{ dB}$

2.25. (a) $E|_{\text{max}} = \cos \left[\frac{\pi}{4}(\cos \theta - 1) \right] |_{\text{max}} = 1$ at $\theta = 0^\circ$.

$$0.707E_{\text{max}} = 0.707 \cdot (1) = \cos \left[\frac{\pi}{4}(\cos \theta_1 - 1) \right]$$

$$\frac{\pi}{4}(\cos \theta_1 - 1) = \pm \frac{\pi}{4} \Rightarrow \theta_1 = \begin{cases} \cos^{-1}(2) = \text{does not exist} \\ \cos^{-1}(0) = 90^\circ = \frac{\pi}{2} \text{ rad.} \end{cases}$$

$$\Theta_{1r} = \Theta_{2r} = 2 \left(\frac{\pi}{2} \right) = \pi$$

$$D_0 \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi}{\pi^2} = \frac{4}{\pi} = 1.273 = 1.049 \text{ dB}$$

(b) Using the computer program **Directivity** of Chapter 2

$$D_0 = 2.00789 = 3.027 \text{ dB}$$

Since the pattern is not very narrow, the answer obtained using Kraus' approximate formula is not as accurate.

2.26. (a) $E|_{\text{max}} = \cos \left[\frac{\pi}{4}(\cos \theta + 1) \right] |_{\text{max}} = 1$ at $\theta = \pi$.

$$0.707 = \cos \left[\frac{\pi}{4}(\cos \theta_1 + 1) \right]$$

$$\frac{\pi}{4}(\cos \theta_1 + 1) = \pm \frac{\pi}{4} \Rightarrow \theta_1 = \begin{cases} \cos^{-1}(-2) \rightarrow \text{does not exist.} \\ \cos^{-1}(0) \rightarrow 90^\circ \rightarrow \frac{\pi}{2} \text{ rad} \end{cases}$$

$$\Theta_{1r} = \Theta_{2r} = 2 \left(\frac{\pi}{2} \right) = \pi$$

$$D_0 \simeq \frac{4\pi}{\pi^2} = \frac{4}{\pi} = 1.273 = 1.049 \text{ dB}$$

(b) Computer Program **Directivity**:

$$D_0 = 2.00789 = 3.027 \text{ dB}$$

2.27. (a)
$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi/2} U_0 \sin(\pi \sin \theta) \sin \theta \, d\theta \, d\phi = 2\pi U_0 \frac{\pi}{2} J_1(\pi) = U_0 \pi^2 J_1(\pi)$$

$$D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi U_0}{U_0 \pi^2 J_1(\pi)} = \frac{4}{\pi} \frac{1}{J_1(\pi)} = 4.4735$$

$$\frac{\pi}{2} J_1(\pi) = 0.447$$

(b) Computer program **Directivity**:

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi/2} U_0 \sin(\pi \sin \theta) \sin \theta \, d\theta \, d\phi = 2\pi(0.447)$$

$$D_0 = 4.4735$$

2.28.
$$E_\phi = C_0 \sin^{1.5} \theta \frac{e^{-jkr}}{r}$$

(a)
$$U_n = |E_\phi|^2 = e_0^2 \sin^3 \theta, \Rightarrow U_n|_{\text{max}} = C_0^2$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi U \sin \theta \, d\theta \, d\phi = 2\pi \int_0^\pi C_0^2 \sin^3 \theta \sin \theta \, d\theta = C_0^2 (2\pi) \int_0^\pi \sin^4 \theta \, d\theta$$

$$\int_0^\pi \sin^4 \theta \, d\theta = -\frac{\sin^3 \theta \cos \theta}{4} \Big|_0^\pi + \frac{4-3}{4} \int_0^\pi \sin^2 \theta \, d\theta = \frac{3}{4} \int_0^\pi \sin^2 \theta \, d\theta$$

$$= \frac{3}{4} \left[\frac{\theta}{2} - \frac{1}{4} \sin(2\theta) \right]_0^\pi = \frac{3}{4} \left(\frac{\pi}{2} \right) = \frac{3\pi}{8}$$

$$P_{\text{rad}} = 2\pi C_0^2 \left(\frac{3\pi}{8} \right) = \frac{3\pi^2}{4} C_0^2$$

$$D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi C_0^2}{\frac{3\pi^2}{4} C_0^2} = \frac{16}{3\pi} = 1.69765 = 2.298 \text{ dB}$$

$$D_0 = 1.69765 = 2.298 \text{ dB}$$

(b)
$$U_n = C_0 \sin^3 \theta, U_n|_{\text{max}} = C_0^2 \text{ at } \theta = 90^\circ, U_n|_{\theta=\theta_h} = 0.5C_0^2 = \sin^3 \theta_h C_0^2$$

$$\sin^3 \theta_h = 0.5, \theta_h = \sin^{-1}(0.5)^{1/3} = \sin^{-1}(0.7937) = 52.5327^\circ$$

$$\Theta_h = 2(90^\circ - 52.5327^\circ) = 74.935^\circ$$

$$D_0(\text{McDonald}) = \frac{101}{74.935 - 0.0027(74.935)^2} = \frac{101}{59.7738} = 1.6897 = 2.278 \text{ dB}$$

$$D_0(\text{McDonald}) = 1.6897 \text{ dimensionless} = 2.278 \text{ dB}$$

$$D_0(\text{Pozar}) = -172.4 + 191 \sqrt{0.818 + \frac{1}{74.935^\circ}} = -172.4 + 191(0.91178)$$

$$D_0(\text{Pozar}) = -172.4 + 174.1502 = 1.7502 \text{ dimensionless} = 2.431 \text{ dB}$$

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2.29. (a) Using the computer program **Directivity** of Chapter 2.

$$D_0 = 14.0707 \text{ dimensionless} = 11.48 \text{ dB}$$

$$(b) U|_{\max} = \left[\frac{\sin(\pi \sin \theta)}{\pi \sin \theta} \right]_{\max}^2 = 1 \quad \text{when } \theta = 0^\circ.$$

$$U = \frac{1}{2} U_{\max} = \frac{1}{2}(1) = \left[\frac{\sin(\pi \sin \theta_1)}{\pi \sin \theta_1} \right]^2$$

Iteratively we obtain $\theta_1 = 26.3^\circ$. Therefore

$$\Theta_{1d} = \Theta_{2d} = 2(26.3^\circ) = 52.6^\circ.$$

and $D_0 \simeq \frac{41,253}{(52.6)^2} = 14.91 \text{ dimensionless} = 11.73 \text{ dB}$ using the Kraus' formula

(c) For Tai and Pereira's formula

$$D_0 = \frac{72,815}{2 \cdot \Theta_{1d}^2} = \frac{72,815}{2(52.6)^2} = 13.16 \text{ dimensionless} = 11.19 \text{ dB}$$

2.30. $U = \frac{1}{2\eta} |E|^2 = \frac{1}{2\eta} \sin^2 \theta \cos^2 \phi \Rightarrow U_{\max} = \frac{1}{2\eta}$

$$(a) P_{\text{rad}} = 2 \cdot \int_0^{\pi/2} \int_0^\pi \frac{1}{2\eta} \sin^2 \theta \cos^2 \phi \, d\theta \, d\phi = \frac{1}{\eta} \left(\frac{\pi}{4} \right) \left(\frac{\pi}{2} \right) = \frac{\pi^2}{8\eta}$$

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi \left(\frac{1}{2\eta} \right)}{\frac{\pi^2}{8\eta}} = \frac{16}{\pi} = 5.09 = 7.07 \text{ dB}$$

$$(b) U_{\max} = \frac{1}{2\eta} \text{ at } \theta = \pi/2, \phi = 0$$

In the elevation plane through the maximum $\phi = 0$ and $U = \frac{1}{2\eta} \sin^2 \theta$, the 3-dB point occurs when

$$U = 0.5U_{\max} = 0.5 \left(\frac{1}{2\eta} \right) = \frac{1}{2\eta} \sin^2 \theta_1 \Rightarrow \theta_1 = \sin^{-1}(0.5) = 30^\circ$$

Therefore $\Theta_{1d} = 2(90 - 30) = 120^\circ$

In the azimuth plane through the maximum $\theta = \pi/2$ and $U = \frac{1}{2\eta} \cos^2 \phi$, the 3-dB point

occurs when $U = 0.5U_{\max} = 0.5 \left(\frac{1}{2\eta} \right) = \frac{1}{2\eta} \cos^2 \theta_1 \Rightarrow \phi_1 = \cos^{-1}(0.707) = 45^\circ$

$$\Theta_{2d} = 2(90^\circ - 45^\circ) = 90^\circ$$

Therefore using Kraus' formula: $D_0 \simeq \frac{41,253}{120(90)} = 3.82 \text{ dimensionless} = 5.82 \text{ dB}$

(c) Using Tai and Pereira's formula:

$$D_0 \simeq \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72,815}{(120)^2 + (90)^2} = 3.24 \text{ dimensionless} = 5.10 \text{ dB}$$

(d) Using the computer program **Directivity** of Chapter 2.

$$D_0 = 5.16425 = 7.13 \text{ dB}$$

$$2.31. \quad U = \left[\frac{J_1(ka \sin \theta)}{\sin \theta} \right]^2 = (ka)^2 \left[\frac{J_1(ka \sin \theta)}{ka \sin \theta} \right]^2 = U_0 \left[\frac{J_1(ka \sin \theta)}{ka \sin \theta} \right]^2$$

(a) $U_{\max} = U_0 \left(\frac{1}{2} \right)^2 = \frac{U_0}{4}$ and it occurs when $ka \sin \theta = 0 \Rightarrow \theta = 0^\circ$. The 3-dB point is obtained using

$$U = \frac{1}{2} U_{\max} = \frac{U_0}{8} = U_0 \left[\frac{J_1(ka \sin \theta)}{ka \sin \theta} \right]^2 \Rightarrow \frac{J_1(ka \sin \theta)}{ka \sin \theta} = 0.3535$$

with the aid of the $J_1(x)/x$ of Appendix V.

$$x = ka \sin \theta_1 = 1.61 \Rightarrow \theta_1 = \sin^{-1}(1.61/2\pi) = 14.847^\circ \\ \Rightarrow \Theta_{1r} = 29.694^\circ$$

(b) Since $\Theta_{1r} = \Theta_{2r} = 29.694^\circ$, the directivity using Kraus' formula is equal to

$$D_0 \simeq \frac{41,253}{(29.694)^2} = 46.79 \text{ dimensionless} = 16.70 \text{ dB}$$

$$2.32. \quad G_0 = 16 \text{ dB} \Rightarrow 16 = 10 \log_{10} G_0 \text{ (dimensionless)} \Rightarrow G_0 \text{ (dimensionless)} = 10^{1.6} = 39.81$$

$$r = 100 \text{ meters} = 10,000 \text{ cm} = 10^4 \text{ cm}$$

$$P_{\text{rad}} = e_{cd} P_{in} = (1) P_{in} = 8 \text{ watts}$$

$$f = 1,900 \text{ MHz} \Rightarrow \lambda = 30 \times 10^9 / 1.9 \times 10^9 = 15.789 \text{ cm}$$

$$(a) \quad W_0 = \frac{P_{\text{rad}}}{4\pi r^2} = \frac{8}{4\pi(10^4)^2} = \frac{8}{4\pi \times 10^8} \\ = \frac{2}{\pi} \times 10^{-8} = 0.6366 \times 10^{-8} \text{ Watts/cm}^2$$

$$W_0 = 0.6366 \times 10^{-8} = 6.366 \times 10^{-9} \text{ Watts/cm}^2$$

$$W_{\max} = W_0 G_0 \text{ (dim)} = 6.366 \times 10^{-9} (39.81) = 253.438 \times 10^{-9}$$

$$\boxed{W_{\max} = 253.438 \times 10^{-9} \text{ Watts/cm}^2}$$

(b) $D_0(\lambda/4 \text{ monopole}) = 1.643$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 = \frac{\lambda^2}{4\pi} (1.643) = \frac{1.643(15.789)^2}{4\pi} = 32.5938 \text{ cm}^2$$

$$A_{em} = 32.5938 \text{ cm}^2$$

$$P(\text{received}) = W_{\max} A_{em} = (253.438 \times 10^{-9})(32.5938)$$

$$\boxed{P(\text{received}) = 8.2606 \times 10^{-6} \text{ Watts}}$$

2.33. (a) Linear because $\Delta\phi = 0$.

(b) Linear because $\Delta\phi = 0$.

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(c) Circular because

1. $E_x = E_y$
 2. $\Delta\phi = \pi/2$.
- CCW because E_y leads E_x . AR = 1, $\tau = 90^\circ$

(d) Circular because

1. $E_x = E_y$
 2. $\Delta\phi = -\pi/2$
- CW because E_y lags E_x . AR = 1, $\tau = 90^\circ$

(e) Elliptical because $\Delta\phi$ is not odd multiples of $\pi/2$. CCW because E_y leads E_x .
 AR = OA/OB

Letting $E_x = E_y = E_0$

$$\left. \begin{aligned} \text{OA} &= E_0[0.5(1 + 1 + \sqrt{2})]^{1/2} = 1.30656E_0 \\ \text{OB} &= E_0[0.5(1 + 1 + \sqrt{2})]^{1/2} = 0.541196E_0 \end{aligned} \right\} \Rightarrow \text{AR} = \frac{1.30656}{0.541196} = 2.414$$

$$\begin{aligned} \tau &= 90^\circ - \frac{1}{2} \tan^{-1} \left[\frac{2(1) \cos(45^\circ)}{1 - 1} \right] = 90^\circ - \frac{1}{2} \tan^{-1} \left(\frac{1.414}{0} \right) \\ &= 90^\circ - \frac{1}{2}(90^\circ) = 45^\circ \end{aligned}$$

(f) Elliptical because $\Delta\phi$ is not odd multiples of $\pi/2$. CW because E_y lags E_x .

$$\left. \begin{aligned} \text{From above } \text{OA} &= 1.30656E_0 \\ \text{OB} &= 0.541196E_0 \end{aligned} \right\} \Rightarrow \text{AR} = \frac{1.30656}{0.541196} = 2.414$$

$$\text{From above } \tau = 90^\circ - \frac{1}{2}(90^\circ) = 45^\circ$$

(g) Elliptical because

1. $E_x \neq E_y$
 2. $\Delta\phi$ is not zero or multiples of π .
- CCW because E_y leads E_x .

$$\left. \begin{aligned} \text{OA} &= E_y \left\{ \frac{1}{2} [0.25 + 1 + 0.75] \right\}^{1/2} = E_y \\ \text{OB} &= E_y \left\{ \frac{1}{2} [0.25 + 1 - 0.75] \right\}^{1/2} = 0.5E_y \end{aligned} \right\} \Rightarrow \text{AR} = \frac{1}{0.5} = 2$$

$$\tau = 90^\circ - \frac{1}{2} \tan^{-1} \left(\frac{0}{-0.75} \right) = 90^\circ - \frac{1}{2}(180^\circ) = 0^\circ$$

(h) Elliptical because

1. $E_x \neq E_y$
 2. $\Delta\phi$ is not zero or multiples of π .
- CCW because E_y lags E_x .

$$\left. \begin{aligned} \text{From above } \text{OA} &= E_y \\ \text{OB} &= 0.5E_y \end{aligned} \right\} \Rightarrow \text{AR} = \frac{1}{0.5} = 2$$

$$\tau = 90^\circ - \frac{1}{2}(180^\circ) = 0^\circ$$

2.34. $\mathcal{E}_x(z, t) = \text{Re}[E_x e^{j(\omega t + kz + \phi_x)}] = E_x \cos(\omega t + kz + \phi_x)$

$\mathcal{E}_y(z, t) = \text{Re}[E_y e^{j(\omega t + kz + \phi_y)}] = E_y \cos(\omega t + kz + \phi_y)$

where E_x and E_y are real positive constants.

Choosing $z = 0$ and letting $\Delta\phi = \phi_y - \phi_x = \phi_y - 0 = \phi$

$$\mathcal{E}_x(t) = E_x \cos(\omega t) \quad (1)$$

$$\mathcal{E}_y(t) = E_y \cos(\omega t + \phi)$$

and

$$\mathcal{E}(t) = \sqrt{\mathcal{E}_x^2 + \mathcal{E}_y^2} = \sqrt{E_x^2 \cos^2(\omega t) + E_y^2 \cos^2(\omega t + \phi)} \quad (2)$$

The maximum and minimum values of (2) are the major and minor axes of the polarization ellipse. Squaring (2) and using the half-angle identity, (2) can be written as

$$\mathcal{E}^2(t) = \frac{1}{2} \{E_x^2 + E_y^2 + E_x^2 \cos(2\omega t) + E_y^2 \cos^2[2(\omega t + \phi)]\} \quad (3)$$

Since E_x and E_y are constants, the maximum and minimum values of (3) occur when $f(t) = E_x^2 \cos(2\omega t) + E_y^2 \cos[2(\omega t + \phi)]$ is maximum or minimum. These are found by differentiating (4) and setting it equal to zero. Thus

$$\frac{df}{d(2\omega t)} = -E_x^2 \sin(2\omega t) - E_y^2 \sin[2(\omega t + \phi)] = 0 \quad (4)$$

or

$$\begin{aligned} E_x^2 \sin(2\omega t) &= -E_y^2 \sin[2(\omega t + \phi)] \\ &= -E_y^2 \{ \sin 2\omega t \cos 2\phi + \cos 2\omega t \sin 2\phi \} \end{aligned} \quad (5)$$

Dividing (5) by $\cos(2\omega t)$ yields

$$E_x^2 \tan(2\omega t) = -E_y^2 [\tan(2\omega t) \cos(2\phi) + \sin(2\phi)]$$

or

$$\tan(2\omega t) = \frac{-E_y^2 \sin(2\phi)}{E_x^2 + E_y^2 \cos(2\phi)}$$

from which we obtain that

$$\cos(2\omega t) = \frac{E_x^2 + E_y^2 \cos(2\phi)}{\pm \rho} \quad (6)$$

$$\cos(2\omega t + 2\phi) = \frac{E_y^2 + E_x^2 \cos(2\phi)}{\pm \rho} \quad (7)$$

where

$$\rho = \sqrt{E_x^4 + E_y^4 + 2E_x^2 E_y^2 \cos(2\phi)} \quad (8)$$

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Substituting (6)–(8) into (3) yields

$$\mathcal{E}^2 = \frac{1}{2} \left[E_x^2 + E_y^2 \pm \frac{1}{\rho} (\rho^2) \right]$$

whose maximum value is

$$\mathcal{E}_{\max} = \text{OA} = \left\{ \frac{1}{2} [E_x^2 + E_y^2 + (E_x^4 + E_y^4 + 2E_x^2 E_y^2 \cos 2\phi)^{1/2}] \right\}^{1/2}$$

$$\mathcal{E}_{\max} = \text{OB} = \left\{ \frac{1}{2} [E_x^2 + E_y^2 - (E_x^4 + E_y^4 + 2E_x^2 E_y^2 \cos 2\phi)^{1/2}] \right\}^{1/2}$$

The tilt angle τ can be obtained by expanding (1) and writing the two as

$$\frac{\mathcal{E}_x^2}{E_x^2} - \frac{2\mathcal{E}_x \mathcal{E}_y \cos \phi}{E_x E_y} + \frac{\mathcal{E}_y^2}{E_y^2} = \sin^2 \phi \quad (9)$$

which is the equation of a tilted ellipse. Choosing a coordinate system whose principal axes coincide with the major and minor axes of the tilted ellipse, we can write that

$$\begin{aligned} \mathcal{E}_x &= \mathcal{E}'_x \sin(z) - \mathcal{E}'_y \cos(z) \\ \mathcal{E}_y &= \mathcal{E}'_x \cos(z) + \mathcal{E}'_y \sin(z) \end{aligned} \quad (10)$$

where \mathcal{E}'_x and \mathcal{E}'_y are the new field values along the new principal axes x', y', z' . Substituting (10) into (9) yields

$$\frac{2\mathcal{E}'_x \mathcal{E}'_y \cos(z) \sin(z)}{E_x^2} - \frac{2\mathcal{E}'_x \mathcal{E}'_y \cos(z) \sin(z)}{E_y^2} - \frac{2\mathcal{E}'_x \mathcal{E}'_y \cos \phi}{E_x E_y} (\sin^2 z - \cos^2 z) = 0$$

which when solved for the tilt angle τ reduces to

$$\tan \left[2 \left(\frac{\pi}{2} - \tau \right) \right] = \frac{2E_x E_y \cos \phi}{E_x^2 - E_y^2}$$

or

$$\tau = \frac{\pi}{2} - \frac{1}{2} \tan^{-1} \left(\frac{2E_x E_y \cos \phi}{E_x^2 - E_y^2} \right)$$

For more details on the tilt angle derivation, see J.D. Kraus, *Antennas*, McGraw-Hill, 1950, pp. 464–476.

2.35. (a) $\hat{\rho}_w = \hat{a}_x \cos \phi_1 + \hat{a}_y \sin \phi_1$

$$\hat{\rho}_a = \hat{a}_x \cos \phi_2 + \hat{a}_y \sin \phi_2$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |(\hat{a}_x \cos \phi_1 + \hat{a}_y \sin \phi_1) \cdot (\hat{a}_x \cos \phi_2 + \hat{a}_y \sin \phi_2)|^2$$

$$= |\cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2|^2 = |\cos(\phi_1 - \phi_2)|^2$$

$$\begin{aligned}
 \text{(b)} \quad \hat{\rho}_w &= \hat{a}_x \sin \theta_1 \cos \phi_1 + \hat{a}_y \sin \theta_1 \sin \phi_1 + \hat{a}_z \cos \theta_1 \\
 \hat{\rho}_a &= \hat{a}_x \sin \theta_2 \cos \phi_2 + \hat{a}_y \sin \theta_2 \sin \phi_2 + \hat{a}_z \cos \theta_2 \\
 \text{PLF} &= |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |\sin \theta_1 \cos \phi_1 \sin \theta_2 \cos \phi_2 + \sin \theta_1 \sin \phi_1 \sin \theta_2 \cdot \sin \phi_2 \\
 &\quad + \cos \theta_1 \cdot \cos \theta_2|^2 \\
 \text{PLF} &= |\sin \theta_1 \cdot \sin \theta_2 (\cos \phi_1 \cdot \cos \phi_2 + \sin \phi_1 \sin \phi_2) + \cos \theta_1 \cos \theta_2|^2 \\
 \text{PLF} &= |\sin \theta_1 \sin \theta_2 \cos(\phi_1 - \phi_2) + \cos \theta_1 \cos \theta_2|^2
 \end{aligned}$$

2.36. Assuming electric field is x -polarized

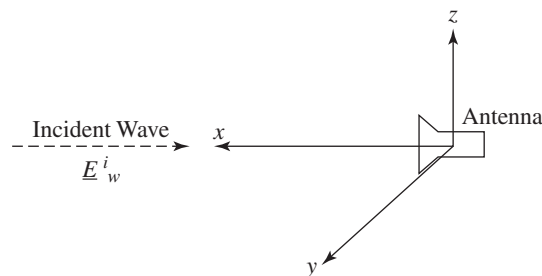
$$\begin{aligned}
 \text{(a)} \quad \underline{E}_w &= \hat{a}_x E_1 e^{-jkz} \Rightarrow \hat{\rho}_w = \hat{a}_x \\
 \underline{E}_a &= (\hat{a}_\theta - j\hat{a}_\phi) E_0 f(r, \theta, \phi) \Rightarrow \hat{\rho}_a = \left(\frac{\hat{a}_\theta - j\hat{a}_\phi}{\sqrt{2}} \right) \\
 \text{PLF} &= |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \frac{1}{2} |\hat{a}_x \cdot \hat{a}_\theta - j\hat{a}_x \cdot \hat{a}_\phi|^2 \\
 \text{since } \hat{a}_\theta &= \hat{a}_x \cos \theta \cos \phi + \hat{a}_y \cos \theta \sin \phi - \hat{a}_z \sin \theta \\
 \hat{a}_\phi &= -\hat{a}_x \sin \phi + \hat{a}_y \cos \phi \\
 \text{PLF} &= \frac{1}{2} (\cos^2 \theta \cos^2 \phi + \sin^2 \phi)
 \end{aligned}$$

(b) when $\underline{E}_a = (\hat{a}_\theta + j\hat{a}_\phi) E_0 f(r, \theta, \phi)$, PLF is also

$$\text{PLF} = \frac{1}{2} (\cos^2 \theta \cos^2 \phi + \sin^2 \phi)$$

A more general, but also more complex, expression can be derived when the incident electric field is of the form $\underline{E}_w = (a\hat{a}_x + b\hat{a}_y)e^{-jkz}$ where a, b are real constants. It can be shown (using the same procedure) that

$$\text{PLF} = \frac{1}{\sqrt{2(a^2 + b^2)}} [(a \cos \theta \cos \phi + b \sin \theta \sin \phi)^2 + (a \sin \phi - b \cos \phi)^2]^{1/2}$$



- 2.37. (a) $\underline{E}_w = E_0(j\hat{a}_y + 3\hat{a}_z)e^{+jkx}$
1. **Elliptical polarization; AR = $\frac{3}{1} = 3$; Left Hand (CCW)**
 - a. 2 components orthogonal to direction of propagation
 - b. Not of same magnitude
 - c. 90° phase difference between them
 - d. y component is leading the z component or z component is lagging the y component

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(b) $\underline{E}_a = E_a(\hat{a}_y + 2\hat{a}_z)e^{-jkx}$

1. **Linear polarization; AR = ∞; No rotation**

- a. 2 components orthogonal to direction of propagation.
- b. Not of same magnitude
- c. 0° phase difference between them,

(c) $PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2$

$$\underline{E}_w = E_0(j\hat{a}_y + 3\hat{a}_z)e^{+jkx} = E_0 \underbrace{\left(\frac{j\hat{a}_y + 3\hat{a}_z}{\sqrt{10}}\right)}_{\hat{\rho}_w} \sqrt{10}e^{+jkx}$$

$$\hat{\rho}_w = \left(\frac{j\hat{a}_y + 3\hat{a}_z}{\sqrt{10}}\right)$$

$$\underline{E}_a = E_a(\hat{a}_y + 2\hat{a}_z)e^{-jkx} = E_0 \underbrace{\left(\frac{\hat{a}_y + 2\hat{a}_z}{\sqrt{5}}\right)}_{\hat{\rho}_a} \sqrt{5}e^{-jkx}$$

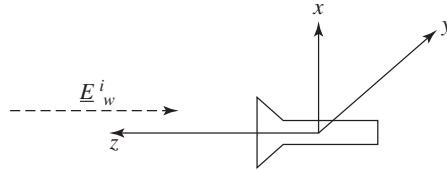
$$\hat{\rho}_a = \left(\frac{\hat{a}_y + 2\hat{a}_z}{\sqrt{5}}\right)$$

$$PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \frac{(j\hat{a}_y + 3\hat{a}_z)}{\sqrt{10}} \cdot \frac{(\hat{a}_y + 2\hat{a}_z)}{\sqrt{5}} \right|^2 = \frac{|j + 6|^2}{50} = \frac{37}{50}$$

$$PLF = \frac{37}{50} = \boxed{0.740 = -1.31 \text{ dB}}$$

2.38. $\underline{E}_w^i = (\hat{a}_x + j\hat{a}_y)E_0e^{+jkz}$

$$\underline{E}_a = (\hat{a}_x + 2\hat{a}_y)E_1 \frac{e^{-jkr}}{r} \Big|_{\substack{\theta = 0^\circ \\ z \text{ axis}}} = (\hat{a}_x + 2\hat{a}_y)E_1 \frac{e^{-jkz}}{z}$$



(a) $\underline{E}_w^i = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}\right) \sqrt{2}E_0e^{+jkz}$

Circular: 2 components, same amplitude, 90° phase difference

(b) **Clockwise** (y component is leading the x component)

(c) $\underline{E}_a = \left(\frac{\hat{a}_x + 2\hat{a}_y}{\sqrt{5}}\right) \sqrt{5}E_1 \frac{e^{-jkz}}{z}$

Linear: 2 components, 0° phase difference

(d) No rotation

$$(e) \quad \hat{\rho}_w = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right), \quad \hat{\rho}_a = \left(\frac{\hat{a}_x + 2\hat{a}_y}{\sqrt{5}} \right)$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left[\left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \cdot \left(\frac{\hat{a}_x + 2\hat{a}_y}{\sqrt{5}} \right) \right]^2 = \frac{|1 + j2|^2}{10} = \frac{5}{10}$$

$$\text{PLF} = \frac{5}{10} = 0.5 = 10 \log_{10}(0.5) = -3 \text{ dB}$$

$$2.39. \quad (a) \quad \underline{E}_w = (4\hat{a}_z + j2\hat{a}_x) E_w \frac{e^{+jky}}{y} = \underbrace{\left(\frac{4\hat{a}_z + j2\hat{a}_x}{\sqrt{20}} \right)}_{\hat{\rho}_w} \sqrt{20} E_w \frac{e^{+jky}}{y}$$

• **Elliptical** (2 components, not of same magnitude, 90° phase difference)

(b) **CW**; x -components leads z -component by 90°; rotate x into z while looking (observing) in the $-y$ direction (from behind the wave).

$$(c) \quad \mathbf{AR} = \frac{4}{2} = 2$$

$$(d) \quad \hat{\rho}_w = \left(\frac{4\hat{a}_z + j2\hat{a}_x}{\sqrt{20}} \right); \quad \hat{\rho}_a = \hat{a}_z$$

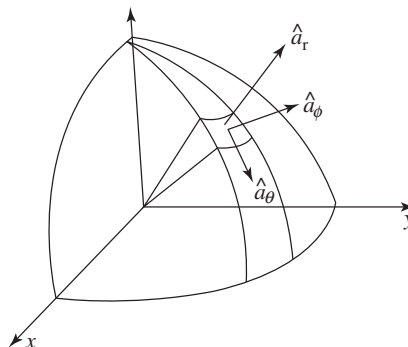
$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \left(\frac{4\hat{a}_z + j2\hat{a}_x}{\sqrt{20}} \right) \cdot \hat{a}_z \right|^2 = \frac{16}{20} = 0.8 = 10 \log_{10}(0.8)$$

$$\text{PLF} = 0.8 = -0.969 \text{ dB}$$

$$2.40. \quad (a) \quad \underline{E}_a = E_0(j\hat{a}_\theta + 2\hat{a}_\phi) f_0(\theta_0, \phi_0) \frac{e^{-jkr}}{r} = E_0 \underbrace{\left(\frac{j\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{5}} \right)}_{\hat{\rho}_a} \sqrt{5} f_0(\theta_0, \phi_0) \frac{e^{-jkr}}{r}$$

$$\hat{\rho}_a = \left(\frac{j\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{5}} \right)$$

Elliptical, CW



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$$\begin{aligned}
 \text{(b) } \underline{E}_w &= E_1(2\hat{a}_\theta + j\hat{a}_\phi)f_1(\theta_0, \phi_0)\frac{e^{+jkr}}{r} \\
 &= E_1 \underbrace{\left(\frac{2\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{5}}\right)}_{\hat{\rho}_w} \sqrt{5}f_1(\theta_0, \phi_0)\frac{e^{+jkr}}{r} \\
 \hat{\rho}_w &= \left(\frac{2\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{5}}\right)
 \end{aligned}$$

Elliptical, CW

$$\text{(c) } \text{PLF} = |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = \left| \left(\frac{j\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{5}}\right) \cdot \left(\frac{2\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{5}}\right) \right|^2 = \left| \frac{2j + j2}{\sqrt{25}} \right|^2 = \left| \frac{4j}{\sqrt{25}} \right|^2$$

$$\boxed{\text{PLF} = \frac{16}{25} = 0.64 = -1.938 \text{ dB}}$$

$$\text{2.41. (a) } \underline{E}_w = E_0(\hat{a}_x \pm j\hat{a}_y)e^{-jkz} \Rightarrow \hat{\rho}_w = \frac{1}{\sqrt{2}}(\hat{a}_x \pm j\hat{a}_y)$$

$$\underline{E}_a \simeq E_1(\hat{a}_\theta - j\hat{a}_\phi)f(r, \theta, \phi) \Rightarrow \hat{\rho}_a = \frac{1}{\sqrt{2}}(\hat{a}_\theta - j\hat{a}_\phi)$$

$$\text{PLF} = \frac{1}{2} |(\hat{a}_x \pm j\hat{a}_y) \cdot (\hat{a}_\theta - j\hat{a}_\phi)|^2 = \frac{1}{2} |(\hat{a}_x \cdot \hat{a}_\theta \pm \hat{a}_y \cdot \hat{a}_\phi) - j(\hat{a}_x \hat{a}_\phi \mp \hat{a}_y \hat{a}_\theta)|^2$$

Converting the spherical unit vectors to rectangular, as it was done in Problem 2.35, leads to

$$\text{PLF} = \frac{1}{2}(\cos \theta \pm 1)^2$$

(b) When

$$\underline{E}_w = E_0(\hat{a}_x \pm j\hat{a}_y)e^{-jkz}$$

$$\underline{E}_a \simeq E_1(\hat{a}_\theta + j\hat{a}_\phi)f(r, \theta, \phi)$$

the PLF is equal to

$$\text{PLF} = \frac{1}{2}(\cos \theta \mp 1)^2$$

$$\text{2.42. } \underline{E}_w = (\hat{a}_\theta \cos \phi - \hat{a}_\phi \sin \phi \cos \theta)f(r, \theta, \phi) \text{ or}$$

$$\underline{E}_w = \left[\frac{\hat{a}_\theta \cos \phi - \hat{a}_\phi \sin \phi \cos \theta}{\sqrt{\cos^2 \phi + \sin^2 \phi \cos^2 \theta}} \right] \sqrt{\cos^2 \phi + \sin^2 \phi \cos^2 \theta} \cdot f(r, \theta, \phi)$$

$$\text{Thus } \hat{\rho}_w = \frac{\hat{a}_\theta \cos \phi - \hat{a}_\phi \sin \phi \cos \theta}{\sqrt{\cos^2 \phi + \sin^2 \phi \cos^2 \theta}}$$

and

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \left(\frac{\hat{a}_\theta \cos \phi - \hat{a}_\phi \sin \phi \cos \theta}{\sqrt{\cos^2 \phi + \sin^2 \phi \cos^2 \theta}} \right) \cdot \hat{a}_x \right|^2$$

Transforming the rectangular unit vector to spherical using

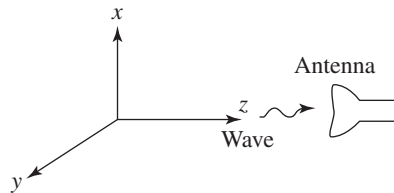
$$\hat{a}_x = \hat{a}_r \sin \theta \cos \phi + \hat{a}_\theta \cos \theta \cos \phi - \hat{a}_\phi \sin \phi$$

the PLF reduces to

$$\text{PLF} = \frac{\cos^2 \theta}{\cos^2 \phi + \sin^2 \phi \cos^2 \theta}$$

The same answer is obtained by transforming the spherical unit vectors to rectangular, as was done in Prob. 2.35.

2.43. $\underline{E}_a \simeq (2\hat{a}_x \pm j\hat{a}_y)f(r, \theta, \phi) = \left(\frac{2\hat{a}_x \pm j\hat{a}_y}{\sqrt{5}} \right) \sqrt{5}f(r, \theta, \phi)$



(a) $\hat{\rho}_w = \left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right) \Rightarrow$ Wave is Right Hand (RH)

$$\hat{\rho}_a = \left(\frac{2\hat{a}_x \pm j\hat{a}_y}{\sqrt{5}} \right)$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

$$= \begin{cases} \frac{9}{10} = -0.4576 \text{ dB using the + sign} & \text{(Antenna is LH in receiving mode and RH in transmitting)} \\ \frac{1}{10} = -10 \text{ dB using the - sign} & \text{(Antenna is RH in receiving mode and LH in transmitting)} \end{cases}$$

(b) $\hat{\rho}_w = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \Rightarrow$ Wave is Left Hand (LH)

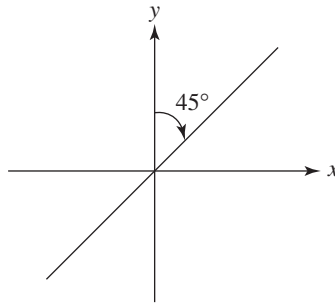
$$\hat{\rho}_a = \left(\frac{2\hat{a}_x \pm j\hat{a}_y}{\sqrt{5}} \right)$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

$$= \begin{cases} \frac{1}{10} = -10 \text{ dB using the + sign} & \text{(Antenna is LH in receiving mode and RH in transmitting)} \\ \frac{9}{10} = -0.4576 \text{ dB using the - sign} & \text{(Antenna is RH in receiving mode and LH in transmitting)} \end{cases}$$

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2.44. For $\hat{\rho}_w$



$$\hat{\rho}_w = \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}; \text{ PLF} = \left| \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}} \cdot \frac{4\hat{a}_x + j\hat{a}_y}{\sqrt{17}} \right|^2$$

$$\text{PLF} = \frac{1}{34} |(\hat{a}_x \cdot 4\hat{a}_x) + (\hat{a}_y \cdot j\hat{a}_y)|^2 = \frac{1}{34} |4 + j|^2 = 0.5 \text{ dimensionless} = -3 \text{ dB}$$

2.45. (a) RHCP; $\hat{\rho}_a = \frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}}$

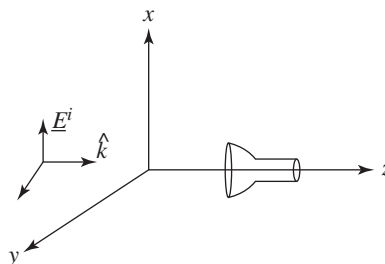
$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \frac{2\hat{a}_x + j\hat{a}_y}{\sqrt{5}} \cdot \frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right|^2 = 0.9 \text{ dimensionless} = -0.46 \text{ dB}$$

(b) LHCP; $\hat{\rho}_a = \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \frac{2\hat{a}_x + j\hat{a}_y}{\sqrt{5}} \cdot \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right|^2 = 0.1 \text{ dimensionless} = -10.0 \text{ dB}$$

2.46. $\underline{E}^i = (\hat{a}_x - j\hat{a}_y)E_0 e^{-jkz} = \left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right) \sqrt{2}E_0 e^{-jkz}$

$$\hat{\rho}_w = \frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \quad \text{CW}$$



$$\begin{aligned} \text{(a)} \quad \underline{E}^a &= (\hat{a}_x + j\hat{a}_y)E_1 e^{+jkz} \\ &= \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \sqrt{2}E_1 e^{+jkz} \\ \hat{\rho}_a &= \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \quad \text{CW} \end{aligned}$$

$$\begin{aligned} \text{PLF} &= |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right) \cdot \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \right|^2 = \left(\frac{1-j^2}{2} \right)^2 = 1 \\ \text{PLF} &= 1 = 0 \text{ dB} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \underline{E}^a &= \left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right) \sqrt{2}E_1 e^{+jkz} \\ \hat{\rho}_a &= \frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \text{PLF} &= |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right) \cdot \left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \right) \right|^2 = \left| \frac{1+j^2}{2} \right|^2 = 0 \\ \text{PLF} &= 0 = -\infty \text{ dB} \end{aligned}$$

$$2.47. \quad \underline{E}_a = (2\hat{a}_\theta + j4\hat{a}_\phi)E_a \frac{e^{-jkr}}{r} = \underbrace{\left(\frac{2\hat{a}_\theta + j4\hat{a}_\phi}{\sqrt{20}} \right)}_{\hat{\rho}_a} 20E_a \frac{e^{-jkr}}{r}$$

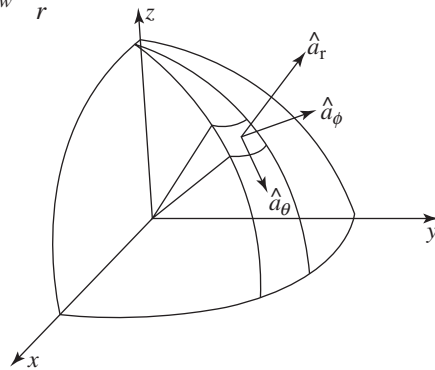
$$\underline{E}_w = (j4\hat{a}_\theta + 2\hat{a}_\phi)E_w \frac{e^{+jkr}}{r} = \underbrace{\left(\frac{j4\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{20}} \right)}_{\hat{\rho}_w} 20E_w \frac{e^{+jkr}}{r}$$

Antenna

- a. Elliptical
- b. CCW
- c. $\mathbf{AR} = \frac{4}{2} = 2$

Wave

- d. Elliptical
- e. CCW
- f. $\mathbf{AR} = \frac{4}{2} = 2$



$$\begin{aligned} \text{g.} \quad \text{PLF} &= |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = \left| \left(\frac{2\hat{a}_\theta + j4\hat{a}_\phi}{\sqrt{20}} \right) \cdot \left(\frac{j4\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{20}} \right) \right|^2 \\ &= \left| \frac{j8 + j8}{20} \right|^2 = \left| \frac{16}{20} \right|^2 = (0.8)^2 = 0.64 \end{aligned}$$

PLF = 0.64 dimensionless = -1.9382 dB

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2.48. $\underline{E}^i = \hat{a}_x E_0 e^{-jkz}, \hat{\rho}_w = \hat{a}_x$

$$\underline{E}^a = (\hat{a}_x + j\hat{a}_y) E_1 e^{+jkz} = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \sqrt{2} E_1 e^{+jkz}$$

$$\hat{\rho}_a = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right)$$

(a) $A_{em} = \frac{\lambda^2}{4\pi} e_o D_0 |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = \frac{\lambda^2}{4\pi} G_0 |\hat{\rho}_a \cdot \hat{\rho}_w|^2$
 $(e_o D_0 = G_0)$

At 10 GHz $\Rightarrow \lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = \frac{3 \times 10^8}{10^{10}} = 3 \times 10^{-2}$

$G_0 = 10 = 10 \log_{10} G_0(\text{dim}) \Rightarrow G_0(\text{dim}) = 10^1 = 10$

$$A_{em} = \frac{\lambda^2}{4\pi} G_0 |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = \frac{(3 \times 10^{-2})^2}{4\pi} (10) \left| \hat{a}_x \cdot \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \right|^2$$

$$= \frac{9 \times 10^{-4}}{4\pi} (10) \left(\frac{1}{2} \right) = \frac{9 \times 10^{-3}}{4\pi} \left(\frac{1}{2} \right) = (0.7162 \times 10^{-3}) \left(\frac{1}{2} \right)$$

$$A_{em} = 0.3581 \times 10^{-3} \text{ m}^2$$

(b) $P_T = A_{em} W^i = (0.3581 \times 10^{-3})(10 \times 10^{-3}) = 3.581 \times 10^{-6}$ Watts
 $P_T = 3.581 \times 10^{-6}$ Watts

2.49. $\underline{E}_w = \hat{a}_z E_w \frac{e^{+jky}}{y}, \hat{\rho}_w = \hat{a}_z, E_a = -\hat{a}_z E_a \frac{e^{-jky}}{y}, \hat{\rho}_a = -\hat{a}_z, W_{\text{inc}} = 100 \times 10^{-3} \frac{W}{\text{cm}^2}$

(a) PLF = $|\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |-\hat{a}_z \cdot \hat{a}_z|^2 = 1 = 0$ dB

(b) For the $\lambda/2$ dipole ($Z_a = 73 + j42.5$) with a loss resistance R_L of 5 ohms:

$$U_n = (E_{\theta n})^2 = (\sin^{1.3} \theta)^2 = \sin^3 \theta \Rightarrow (U_n)_{\text{max}} = 1$$

$$D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}}$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi U \sin \theta d\theta d\phi = \int_0^{2\pi} \left(\int_0^\pi \sin^3 \theta \sin \theta d\theta \right) d\phi = 2\pi \int_0^\pi \sin^4 \theta d\theta$$

$$\int_0^\pi \sin^4 \theta d\theta = -\frac{\sin^3 \theta \cos \theta}{4} \Big|_0^\pi + \frac{4-1}{4} \int_0^\pi \sin^2 \theta d\theta = \frac{3}{4} \int_0^\pi \sin^2 \theta d\theta$$

$$\int_0^\pi \sin^4 \theta d\theta = \frac{3}{4} \int_0^\pi \sin^2 \theta d\theta = \frac{3}{4} \left[\frac{\theta}{2} - \frac{1}{4} \sin(2\theta) \right] = \frac{3\pi}{8}$$

$$P_{\text{rad}} = 2\pi \int_0^{\pi} \sin^4 \theta d\theta = 2\pi \left(\frac{3\pi}{8} \right) = \frac{3\pi^2}{4}$$

$$\therefore D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = D_0 = \frac{4\pi(1)}{3\pi^2/4} = \frac{16}{3\pi} = 1.69765 \text{ (dimensionless)} = 2.298 \text{ dB}$$

Using the equivalent circuit of Figure 1.2 with $R_r = 73$ and $R_L = 5$

$$e_{cd} = \frac{R_r}{R_r + R_L} = \frac{73}{73 + 5} = 0.9359$$

$$\therefore G_0 = e_{cd} D_0 = 0.9359(1.69765) = 1.5888 = 2.011 \text{ dB}$$

$$|\Gamma| = \left| \frac{Z_{in} - Z_c}{Z_{in} + Z_c} \right| = \left| \frac{(Z_a + R_L) - Z_c}{(Z_a + R_L) + Z_c} \right| = \left| \frac{(73 + j42.5 + 5) - 50}{(73 + j42.5 + 5) + 50} \right| = \frac{50.8945}{134.8712} = 0.3774$$

$$|\Gamma|^2 = (0.3774)^2 = 0.1424 \Rightarrow (1 - |\Gamma|^2) = (1 - 0.1424) = 0.8576$$

$$G_{re0} = e_r G_0 = (1 - |\Gamma|^2) G_0 = (0.8576) 1.5888 = 1.3626 \text{ (dim)} = 1.344 \text{ dB}$$

$$\begin{aligned} P_{\text{received}} &= A_{em}(e_{cd})(1 - |\Gamma|^2)\text{PLF} = \left(\frac{\lambda^2}{4\pi} D_0 \right) e_{cd}(1 - |\Gamma|^2)\text{PLF} \\ &= \frac{\pi^2}{4\pi} \underbrace{\left[D_0 e_{cd} (1 - |\Gamma|^2) \right]}_{G_0} (\text{PLF}) W_{inc} \\ &\qquad\qquad\qquad \underbrace{\hspace{10em}}_{G_{re0}} \end{aligned}$$

$$\begin{aligned} P_{\text{received}} &= (W_{inc}) \frac{\lambda^2}{4\pi} G_{re} (\text{PLF}), \quad \lambda = \frac{30 \times 10^9}{10 \times 10^9} = 3 \text{ cm} \\ &= \underbrace{(100 \times 10^{-3})}_{W_{inc}} \underbrace{\left(\frac{(3)^2}{4\pi} \right)}_{3^2/4\pi} \underbrace{(1.3562)}_{G_{re0}} \underbrace{(1)}_{\text{PLF}} = \frac{10^{-1}(9)(1.3562)}{4\pi} \end{aligned}$$

$$P_{\text{received}} = 0.0981 \text{ Watts} = 98.1 \text{ mWatts} = 98.1 \times 10^{-3} \text{ Watts}$$

$$P_{\text{received}} = \underbrace{(100 \times 10^{-3})}_{W_{inc}} \underbrace{\left[\frac{(3)^2}{4\pi} \right]}_{3^2/4\pi} \underbrace{(1.3626)}_{G_{re0}} \underbrace{(1)}_{\text{PLF}} = 97.59 \text{ mW} = 97.59 \times 10^{-3} \text{ W}$$

2.50. $\underline{E}_a = (2\hat{a}_x \pm j\hat{a}_y) E e^{-jkz}$

$$\hat{\rho}_a = \frac{2\hat{a}_x \pm j\hat{a}_y}{\sqrt{5}}$$

(a) $\underline{E}_w = \hat{a}_x E_w \Rightarrow \hat{\rho}_w = \hat{a}_x$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \frac{2}{\sqrt{5}} \right|^2 = \frac{4}{5} = 0.8 \text{ dimensionless} = -0.9691 \text{ dB}$$

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(b) $\underline{E}_w = \hat{a}_y E_w \Rightarrow \hat{\rho}_w = \hat{a}_y$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \frac{1}{\sqrt{5}} \right|^2 = \frac{1}{5} = 0.2 \text{ dimensionless} = -6.9897 \text{ dB}$$

2.51. (a) $E_y = E'_y + E''_y = 3 \cos \omega t + 2 \cos \omega t = 5 \cos \omega t$

$$E_x = E'_x + E''_x = 7 \cos \left(\omega t + \frac{\pi}{2} \right) + 3 \cos \left(\omega t - \frac{\pi}{2} \right)$$

$$= -7 \sin \omega t + 3 \sin \omega t = -4 \sin \omega t$$

$$\text{AR} = \frac{5}{4} = 1.25$$

(b) At $\omega t = 0, \underline{E} = 5\hat{a}_y$
 At $\omega t = \pi/2 \Rightarrow \underline{E} = -4\hat{a}_x \Rightarrow$ Rotation in CCW

2.52. (a) $\text{PLF} = \frac{1}{2}$ independent of $\psi \rightarrow$ must have CP

$\therefore \text{AR} = 1.$

(b) Polarization will be elliptical with major axis aligned with x-axis.

Guess: $\text{AR} = 2$

Verify: $\hat{\rho}_w = (2\hat{a}_x + j\hat{a}_y)/\sqrt{5}$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left| \frac{2 \cos \psi + j \sin \psi}{\sqrt{5}} \right|^2 = \frac{4 \cos^2 \psi + \sin^2 \psi}{5}$$

$\psi = 0 : \text{PLF} = 0.8$

$\psi = 90^\circ : \text{PLF} = 0.2$

(c) $\text{PLF} = 1$ at $\psi = 45^\circ$ and 225°

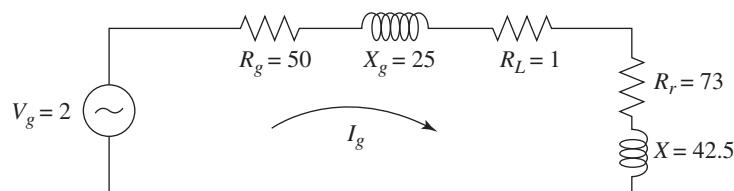
$\text{PLF} = 0$ at $\psi = 135^\circ$ and 315°

Polarization must be linear at an angle of 45°

$\therefore \text{AR} = \infty$

2.53. $I_g = \frac{2}{(50 + 1 + 73) + j(25 + 42.5)} = \frac{2}{124 + j67.5}$

$$= (12.442 - j6.7724) \times 10^{-3} = 14.166 \times 10^{-3} \angle -28.56^\circ$$



(a) $P_s = \frac{1}{2} \text{Re}(V_g \cdot I_g^*) = \text{Re}(12.442 + j6.7724) \times 10^{-3} = 12.442 \times 10^{-3} \text{ W}$

(b) $P_r = \frac{1}{2} |I_g|^2 R_r = 7.325 \times 10^{-3} \text{ W}$

(c) $P_L = \frac{1}{2} |I_g|^2 R_L = 0.1003 \times 10^{-3} \text{ W}$

The remaining supplied power is dissipated as heat in the internal resistor of the generator, or

$$P_g = \frac{1}{2} |I_g|^2 R_g = 5.0169 \times 10^{-3} \text{ W}$$