1

Basic Facts

1.1 (a) For a thermally perfect gas, by state equation,

$$p = \rho R T$$

Given: $p = 101325$ Pa, $T = 0°\text{C} = 273.15$ K and $\rho = 0.09$ kg/m^3. Therefore,

$$\begin{aligned} R &= \frac{p}{\rho T} \\ &= \frac{101325}{0.09 \times 273.15} \\ &= 4121.7\,\text{J}/(\text{kg}\,\text{K}) \end{aligned}$$

The gas constant of the specific gas under consideration is

$$R = \boxed{4.1217\,\text{kJ}\ (\text{kg}\,\text{K})^{-1}}$$

Also, we know that,

$$R = \frac{R_u}{M}$$

where $R_u = 8314$ J/(kg K) is the universal gas constant and M is the molecular weight of the gas. Therefore,

$$\begin{aligned} M &= \frac{8314}{4121.7} \\ &= \boxed{2.017\,\text{kg}} \end{aligned}$$

(b) By state equation, we have $pv = RT$, where v is the specific volume. Therefore,

$$\begin{aligned} v &= \frac{RT}{p} = \frac{4121.7 \times 343.15}{2.3 \times 101325} \\ &= \boxed{6.07\,\mathrm{m}^3\ \mathrm{kg}^{-1}} \end{aligned}$$

since 1 atm = 101325 Pa.

1.2 By state equation, we have

$$p_1\mathbf{V} = mRT_1 \tag{1}$$

where m is the initial mass and $\mathbf{V}$ is the tank volume. Final mass is $(m+2)$ kg and state equation for the final state is

$$p_2\mathbf{V} = (m+2)RT_2 \tag{2}$$

But $T_1 = T_2$. Therefore, dividing Equation (1) by Equation (2), we get

$$\begin{aligned} \frac{p_1}{p_2} &= \frac{m}{m+2} \\ \frac{120}{240} &= \frac{m}{m+2} \\ m+2 &= 2\,m \\ m &= 2\,\mathrm{kg} \end{aligned}$$

Equation (1) gives

$$\mathbf{V} = \frac{mRT_1}{p_1}$$

For nitrogen $R = 297$ J/(kg K), thus,

$$\begin{aligned} \mathbf{V} &= \frac{2 \times 297 \times 303.15}{120 \times 10^3} \\ &= \boxed{1.5\,\mathrm{m}^3} \end{aligned}$$

1.3 Recorded pressure is

$$\begin{aligned} p &= \rho_{\mathrm{Hg}}\,g\,h \\ &= \left(13.6 \times 10^3\right)(9.81)(0.735) = 98061\,\mathrm{Pa} \\ &= \boxed{98.061\,\mathrm{kPa}} \end{aligned}$$

since $\rho_{\rm Hg} = 13.6 \times 10^3\,{\rm kg/m^3}$.

1.4 Absolute pressure = manometer height + barometer height

$$\begin{aligned} p_{\rm abs} &= 750\ +\ 200\ =\ 950\,{\rm mm\ of\ Hg} \\ &= \left(13.6 \times 10^3\right)(9.81)(0.95)\,{\rm Pa} \\ &= \boxed{126.75\,{\rm kPa}} \end{aligned}$$

1.5 The tank pressure is 250 cm above atmosphere. For standard atmosphere, 1 atm = 760 mm of mercury. Therefore, pressure in the tank is

$$\begin{aligned} p_T &= p_{\rm meas} + p_{\rm atm} = 2500 + 760 \\ &= 3260\,{\rm mm\ of\ mercury} \end{aligned}$$

Also,

$$1\,{\rm atm} = 760\,{\rm mm\ of\ Hg} = 101325\,{\rm Pa}$$

Therefore,

$$\begin{aligned} p_T &= \frac{101325}{760} \times 3260 \\ &= \boxed{434.63\,{\rm kPa}} \end{aligned}$$

1.6 Absolute pressure $= \left(\text{barometric height} - \dfrac{h}{13.6}\right)$ mm of Hg

since h is the water column height, it is converted to mercury column height by dividing by the specific gravity of Hg.

$$\begin{aligned} \text{Absolute pressure} &= 760 - \frac{500}{13.6} = 723.235\,{\rm mm\ of\ Hg} \\ &= \left(13.6 \times 10^3\right)(9.81)\,(0.723235)\ {\rm Pa} \\ &= \boxed{96.49\,{\rm kPa}} \end{aligned}$$

1.7

$$\begin{aligned}\text{Absolute pressure} &= \text{gauge pressure} + \text{atmospheric pressure}\\ \text{Atmospheric pressure} &= (13.6\times10^3)(9.81)\left(\frac{740}{10^3}\right)\\ &= 0.0987\,\text{MPa}\end{aligned}$$

Thus, absolute pressure $= 1.5 + 0.0987 = \boxed{1.5987\,\text{MPa}}$

1.8 The pressure in terms of fluid column is $\rho g h$. Therefore,

$$\begin{aligned}50\times10^3 &= \rho_{\text{Hg}}\, g\, h_{\text{Hg}}\\ h_{\text{Hg}} &= \frac{50\times10^3}{9.81\times13600}\\ &= 0.3747\,\text{m}\end{aligned}$$

In terms of water column, we have

$$\begin{aligned}50\times10^3 &= \rho_w\, g\, h_w\\ h_w &= \frac{50\times10^3}{9.81\times10^3}\\ &= 5.097\,\text{m}\end{aligned}$$

Therefore,

$$\begin{aligned}\triangle h &= h_w - h_{\text{Hg}}\\ &= \boxed{4.7223\,\text{m}}\end{aligned}$$

1.9

$$\begin{aligned}\text{Area of the piston} &= \frac{\pi\, d^2}{4} = \frac{\pi}{4} \times 100^2 = \frac{\pi}{4} \times 10^4\,\text{mm}^2 \\ \text{Pressure} &= \frac{\text{Force}}{\text{Area}} \\ &= \frac{8000}{\pi \times 10^{-2}} \times 4\,\text{Pa} \\ &= \boxed{1018.6\,\text{kPa}}\end{aligned}$$

1.10 By hydrostatic pressure equation, we have

$$\begin{aligned}p &= \rho g h \\ &= (0.8 \times 10^3)(9.81)(2.2) \\ &= \boxed{17.27\,\text{kPa}}\end{aligned}$$

1.11 The speed of sound is given by

$$a = \sqrt{\gamma\, RT}$$

For air, $\gamma = 1.4$, $R = 287\,\text{J/(kg K)}$, thus,

$$\begin{aligned}299 &= \sqrt{1.4 \times 287 \times T} \\ T &= 222.5\,\text{K}\end{aligned}$$

We know that, at standard sea level T = 288 K and the temperature lapse rate is 6.5 degree per kilometer in the troposphere of earth's atmosphere. Therefore,

$$\begin{aligned}T - \lambda h &= 222.5 \;\; (\lambda \text{ is temperature lapse rate}) \\ 288 - 6.5h &= 222.5 \\ h &= \frac{65.5}{6.5}\,\text{km} \\ &= \boxed{10076.9\,\text{m}}\end{aligned}$$

That is, the altitude is 10076.9 m.

1.12 The locations of the aircraft and the observer are as shown in Figure S1.12.

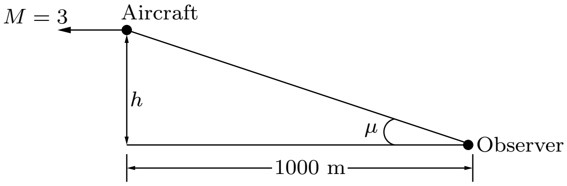

Figure S1.12

where h is the flight altitude. Given: the distance covered $Vt = 1000$ m and the height is at, where, t is the time taken for the sound signal to reach the observer and a is the speed of sound. Therefore,

$$\begin{aligned} Vt &= 1000 \\ t &= \frac{1000}{V} = \frac{1000}{Ma} \\ &= \frac{1000}{3\sqrt{1.4 \times 287 \times 288.15}} \\ &= 0.98\,\mathrm{s} \end{aligned}$$

Thus, the flight altitude is

$$\begin{aligned} h &= at = 340.26 \times 0.98 \\ &= \boxed{333.45\,\mathrm{m}} \end{aligned}$$

1.13 At time $t = 0$, the object is at point 0 and at t = t, it is at point A. Therefore, the distance traveled in time t is Vt and

$$Vt = 10 + 10 + 3 = 23\,\mathrm{m}$$

At the same time duration, the disturbance might have traveled a distance at, where a is the speed of sound. Thus,

$$at = 10 - 3 = 7\,\mathrm{m}$$

Thus, the Mach number of the object is

$$M = \frac{Vt}{at} = \frac{23}{7} = \boxed{3.286}$$

At sea level, $T = 288.15$ K. Therefore, the speed of sound becomes

$$a = \sqrt{\gamma RT} = 340\,\mathrm{m/s}$$

Thus, the velocity of the object is

$$V = M\,a = \boxed{1117.24\,\mathrm{m/s}}$$

The Mach angle μ of the disturbance cone is

$$\mu = \sin^{-1}\left(\frac{1}{M}\right) = \boxed{17.7^\circ}$$

1.14 Assume air to be a perfect gas. The speed of sound is then given by

$$a = \sqrt{\gamma\,RT}$$

(a) $\gamma = 1.4$ and $R = 287\,\mathrm{J/(kg\ K)}$ for air. Therefore,

$$\begin{aligned} a &= \sqrt{1.4 \times 287 \times (25 + 273.15)} \\ &= 346.1\,\mathrm{m/s} \end{aligned}$$

Flight speed $V = 1100/3.6 = 305.6\,\mathrm{m/s}$. Thus, the flight Mach number M is

$$M = \frac{V}{a} = \frac{305.6}{346.1} = \boxed{0.883}$$

(b) At $T = -55^\circ\mathrm{C}$ the sound speed a becomes

$$\begin{aligned} a &= \sqrt{1.4 \times 287 \times (-55 + 273.15)} \\ &= 296.1\,\mathrm{m/s} \end{aligned}$$

Thus,

$$M = \frac{305.6}{296.1} = \boxed{1.03}$$

Note that, for the same flight speed, the plane flies subsonically at sea level but supersonically at high altitude. These aspects must be considered in the design of the plane.

1.15 In the reservoirs, the air is at stagnation state. So, the entropy relation would be

$$s_2 - s_1 = c_p \ln\left(\frac{T_{02}}{T_{01}}\right) - R \ln\left(\frac{p_{02}}{p_{01}}\right)$$

But, $T_{01} = T_{02}$ for adiabatic process. Therefore,

$$\begin{aligned}\Delta s &= R \ln \left(\frac{p_{01}}{p_{02}}\right) \\ &= R \ln \left(\frac{p_{01}}{\frac{1}{2} p_{01}}\right) = R \ln 2 \\ &= \boxed{198.933\,\mathrm{J\ (kg\ K)^{-1}}}\end{aligned}$$

Note: It should be noted that, for entropy only subscripts 2 and 1 are used; since entropy is not defined like static or stagnation entropy.

1.16 Let the initial state be denoted by subscript 1 and expanded state by subscript 2.

(a) Since the cylinder is insulated, preventing any heat transfer what-so-ever, the process is adiabatic. The governing equation for this process is given by

$$p_1 \mathbf{V}_1^{\gamma} = p_2 \mathbf{V}^{\gamma} = \text{constant} \tag{1}$$

Also, from ideal gas state equation

$$\frac{p_1 \mathbf{V}_1}{T_1} = \frac{p_2 \mathbf{V}_2}{T_2} = R \tag{2}$$

From Equations (1) and (2), we get

$$\frac{p_1}{p_2} = \left(\frac{\mathbf{V}_2}{\mathbf{V}_1}\right)^{\gamma} = \left(\frac{T_1}{T_2}\right)^{\gamma/(\gamma-1)}$$

Therefore,

$$\begin{aligned}T_2 &= T_1 \Big/ 10^{(\gamma-1)} = 557.35\,\mathrm{K} \\ \Delta T &= \boxed{-\,842.65\,\mathrm{K}}\end{aligned}$$

(b)

$$\text{Work} = w = \int p dv = \int dh - \int du - \int v dp \tag{3}$$

Also,

$$pv^{\gamma} = \text{constant} \qquad \text{from equation (1)}$$

Differentiating equation (1), we have,

$$p\gamma v^{\gamma-1} dv + v^{\gamma} dp = 0$$

Dividing throughout by $v^{\gamma-1}$ and integrating, we get

$$\int p\gamma dv + \int vdp = 0$$

$$\int vdp = -\gamma \int pdv = -\gamma w \tag{4}$$

Substituting equation (4) into equation (3) and simplifying, we get

$$\begin{aligned}(1-\gamma)\,w &= R\Delta T \\ w &= \frac{R\Delta T}{1-\gamma} = \frac{287 \times (-842.65)}{(-0.4)} \\ &= \boxed{6.04 \times 10^5 \text{ J kg}^{-1}}\end{aligned}$$

Note: Since the process undergone is expansion from a high pressure, the work removed is positive, i.e. work is done by the gas.

(c) Also, from Equation (1)

$$\frac{p_1}{p_2} = \left(\frac{\mathbf{V}_2}{\mathbf{V}_1}\right)^{\gamma} = 10^{1.4} = 25.1189$$

Therefore, pressure ratio = $\boxed{25.1189}$

1.17 $p_1 v_1^{\gamma} = p_2 v_2^{\gamma}$, where v is specific volume, i.e. volume per mbox mass = $\mathbf{V}/m$. Therefore,

$$p_1 \left(\frac{\mathbf{V}_1}{m_1}\right)^{\gamma} = p_2 \left(\frac{\mathbf{V}_2}{m_2}\right)^{\gamma}$$

Also, $\mathbf{V}_1 = \mathbf{V}_2 = \mathbf{V}$ = volume of the tank.

$$\begin{aligned}p_2 &= p_1 \left(\frac{m_2}{m_1}\right)^{\gamma} \\ &= 5 \times 10^5 \times \left(\frac{1}{2}\right)^{1.4} \\ &= \boxed{1.8946 \times 10^5 \text{ Pa}}\end{aligned}$$

From equation of state for a calorically perfect gas,

$$\begin{aligned}\frac{p_1}{p_2} &= \frac{\rho_1}{\rho_2}\frac{T_1}{T_2} \\ T_2 &= \left(\frac{p_2}{p_1}\right)\left(\frac{m_1}{m_2}\right)T_1 \\ &= \left(\frac{1.8946}{5}\right)\times 2\times 500 \\ &= \boxed{378.92\,\mathrm{K}}\end{aligned}$$

1.18 For the isentropic process,

$$\frac{p_1}{p_2} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

(a) Therefore,

$$\begin{aligned}T_2 &= \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}T_1 \\ &= 6^{1/3.5}\times 290 = 483.868\,\mathrm{K}\end{aligned}$$

The change in the temperature is

$$\Delta T = \boxed{193.868\,\mathrm{K}}$$

(b) By first law of thermodynamics, we have

$$du + d(PE) + d(KE) = dq + dw$$

Here, velocity changes are neglected. Therefore,

$$d(KE) = 0$$

Also, assuming

$$d(PE) = 0$$

from thermodynamics first law, we get

$$du = dq + dw$$

Since the process is isentropic, $dq = 0$. Therefore,

$$\begin{aligned}du &= dw = c_v\Delta T = 717.5\times 193.868 \\ &= \boxed{1.39\times 10^5\,\mathrm{J\ kg^{-1}}}\end{aligned}$$

(c) The work done is negative, i.e. work is done on the gas. It has been computed in (b) above.

1.19 Work done by the weight on the piston goes towards increasing the internal energy of the gas. From the first law of thermodynamics

$$E_2 - E_1 = Q + W$$

where, E, Q, and W are respectively the internal energy, heat transferred, and work done. Since no heat is transferred, $Q = 0$. Therefore,

$$E_2 - E_1 = W = \int F.ds$$

where, F is force and ds is distance. At the new equilibrium position, the force acting on the piston face is $F = p_2 A_p$, where A_p is the area of the piston face. The distance traveled by the piston is $ds = (V_1 - V_2)/A_p$, V_1 and V_2 are the initial and final volumes, respectively. Thus, we have,

$$\begin{aligned} E_2 - E_1 &= p_2.A_p(V_1 - V_2)/A_p \\ &= -p_2(V_2 - V_1) \end{aligned}$$

For mbox mass, this reduces to

$$e_2 - e_1 = -p_2(V_2 - V_1)$$

For a calorically perfect gas, $e = c_v T$. Therefore,

$$\begin{aligned} c_v(T_2 - T_1) &= -p_2\left(\frac{RT_2}{p_2} - \frac{RT_1}{p_1}\right) \\ \frac{c_v}{R}\left(\frac{T_2}{T_1} - 1\right) &= -\frac{T_2}{T_1} + \frac{p_2}{p_1} \\ \frac{T_2}{T_1}\left(1 + \frac{c_v}{R}\right) &= \frac{c_v}{R} + \lambda \ \text{(where } \lambda = p_2/p_1\text{)} \end{aligned}$$

But $c_v/R = 1/(\gamma - 1)$. Thus,

$$\begin{aligned} \frac{\gamma}{\gamma - 1}\frac{T_2}{T_1} &= \lambda + \frac{1}{\gamma - 1} \\ \frac{T_2}{T_1} &= \boxed{\frac{1 + (\gamma - 1)\lambda}{\gamma}} \end{aligned}$$

Entropy change for a perfect gas can be written as

$$\begin{aligned}
\Delta s &= c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right) \\
\frac{\Delta s}{R} &= \ln\left[\frac{1}{\lambda}\left\{\frac{1+(\gamma-1)\lambda}{\gamma}\right\}^{\frac{\gamma}{\gamma-1}}\right] \\
s_2 - s_1 &= R\ln\left[\frac{1}{\lambda}\left\{\frac{1+(\gamma-1)\lambda}{\gamma}\right\}^{\frac{\gamma}{\gamma-1}}\right]
\end{aligned}$$

Let $\lambda = 1+\epsilon$, where $\epsilon \ll 1$. Therefore,

$$\begin{aligned}
\frac{\Delta s}{R} &= \frac{\gamma}{\gamma-1}\ln\left[\frac{1+(\gamma-1)(1+\epsilon)}{\gamma}\right] - \ln(1+\epsilon) \\
&= \frac{\gamma}{\gamma-1}\ln\left[\frac{1+\gamma-1+\epsilon(\gamma-1)}{\gamma}\right] - \ln(1+\epsilon) \\
&= \frac{\gamma}{\gamma-1}\ln\left[1+\frac{\gamma-1}{\gamma}\epsilon\right] - \ln(1+\epsilon)
\end{aligned}$$

Expanding the RHS, and retaining only up to second order terms, we get,

$$\begin{aligned}
\frac{\Delta s}{R} &= \frac{\gamma}{\gamma-1}\left[\frac{\gamma-1}{\gamma}\epsilon - \frac{(\gamma-1)^2}{2\gamma^2}\epsilon^2\right] - \left(\epsilon - \frac{\epsilon^2}{2}\right) \\
&= \epsilon - \frac{\gamma-1}{2\gamma}\epsilon^2 - \epsilon + \frac{\epsilon^2}{2} = -\frac{\epsilon^2}{2} + \frac{\epsilon^2}{2\gamma} + \frac{\epsilon^2}{2} \\
&= \boxed{\frac{\epsilon^2}{2\gamma}}
\end{aligned}$$

Note: Work has been done by the weight which is equal to p_2A_p on the gas. The weight has moved by a distance of ds. Therefore, $\Delta E = W.ds = p_2A_p.ds$.

1.20 Since it is an open system,

$$\begin{aligned}
\text{Work done} &= -c_p\,(T_2 - T_1) \\
&= -c_p\left(\frac{T_2}{T_1} - 1\right)T_1 \\
&= -c_p\left(\left\{\frac{p_2}{p_1}\right\}^{\frac{\gamma-1}{\gamma}} - 1\right)T_1 \\
&= -1004.5 \times \left(2^{1/3.5} - 1\right) \times 303 \\
&= \boxed{-\,66.66\,\text{kJ kg}^{-1}}
\end{aligned}$$

1.21 Work done is given by

$$\begin{aligned}
W &= p\,(\mathbf{V}_2 - \mathbf{V}_1) = 101325 \times 6\,(2 - 0.3) \\
&= \boxed{1.0335\,\text{MJ}}
\end{aligned}$$

since 1 atm = 101325 Pa.

1.22 The compression process is given as isentropic. Let subscripts 1 and 2 refer to the initial and final states, respectively. By isentropic process relation, we have

$$\begin{aligned}
\frac{p_1}{\rho_1^\gamma} &= \frac{p_2}{\rho_2^\gamma} \\
\rho_2 &= \left(\frac{p_2}{p_1}\right)^{1/\gamma} \times \rho_1 = \left(\frac{690}{150}\right)^{1/1.3} \times 1.5 \\
&= \boxed{4.85\,\text{kg}\,\text{m}^{-3}}
\end{aligned}$$

1.23 As we know, the relation between temperature and pressure for isentropic change of state may be written as

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

where subscripts 1 and 2 refer to the initial and final states, respectively.

$$\begin{aligned} T_2 &= T_1\left(\frac{p_2}{p_1}\right)^{\frac{0.4}{1.4}} = 298\left(\frac{7}{1}\right)^{0.286} \\ &= \boxed{519.9\,\mathrm{K}} \end{aligned}$$

1.24 By isentropic relation, we have

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

where subscripts 1 and 2 refer to the initial and final states and v is specific volume. For air $\gamma = 1.4$, therefore,

$$\begin{aligned} T_2 &= T_1\left(\frac{v_1}{v_2}\right)^{0.4} \\ &= (30 + 273.15)\,(30)^{0.4} = 1181.7\,\mathrm{K} \\ &= \boxed{908.55^\circ\mathrm{C}} \end{aligned}$$

1.25 (a) We have $c_p = \frac{\gamma}{\gamma-1}R$, therefore, the gas constant is

$$\begin{aligned} R &= \frac{\gamma-1}{\gamma}c_p \\ &= \frac{0.4}{1.4}\times 1000 = 285.7\,\mathrm{J/(kg\ K)} \end{aligned}$$

Also,

$$R = \frac{R_u}{M} = \frac{8314}{M}$$

where M is the molecular weight and R_u is universal gas constant. Thus,

$$M = \frac{8314}{285.7} = \boxed{29.1}$$

(b) By ideal gas state equation, we have

$$p_1\mathbf{V}_1 = mRT_1$$

$$p_2\mathbf{V}_2 = mRT_2$$

where subscripts 1 and 2 refer to initial and final states, respectively. But $p_1 = p_2$, therefore,

$$\begin{aligned}\frac{\mathbf{V}_2}{\mathbf{V}_1} &= \frac{T_2}{T_1} = \frac{50 + 273.15}{200 + 273.15} \\ &= \frac{323.15}{473.15} \\ &= \boxed{0.683}\end{aligned}$$

1.26 For an ideal gas the speed of sound a may be expressed as

$$a = \sqrt{\gamma R T}$$

where γ is the ratio of specific heats and R is the gas constant. For the given gas,

$$R = \frac{R_u}{M} = \frac{8314}{29} = 286.7\,\mathrm{J/(kg\,K)}$$

Therefore,

$$\begin{aligned}400 &= \sqrt{\gamma \times 286.7 \times 373.15} \\ \gamma &= \frac{400^2}{286.7 \times 373.15} = 1.5\end{aligned}$$

The specific heat c_p and c_v can be written as

$$\begin{aligned}c_p &= \frac{\gamma}{\gamma - 1} R \\ c_v &= \frac{R}{\gamma - 1}\end{aligned}$$

Therefore,

$$\begin{aligned}c_p &= (1.5/0.5) \times 286.7 \\ &= \boxed{860.1\,\mathrm{J\,(kg\,K)^{-1}}} \\ c_v &= 286.7/0.5 \\ &= \boxed{573.4\,\mathrm{J\,(kg\,K)^{-1}}}\end{aligned}$$

Note: The ratio of specific heats is $\gamma = c_p/c_v$. For the present case $\gamma = 1.5 = 860.1/573.4$ is correct. This way the answer obtained for c_p and c_v may be checked.

1.27 At the nozzle exit, $V = 390\,\mathrm{m/s}$ and $T = 28 + 273.15 = 301.15\,\mathrm{K}$. The corresponding speed of sound is

$$\begin{aligned} a &= \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 301.15} \\ &= 347.85\,\mathrm{m/s} \end{aligned}$$

Thus,

$$M = \frac{V}{a} = \frac{390}{347.85} = \boxed{1.12}$$

By isentropic relation, we have

$$\begin{aligned} \frac{T_0}{T} &= 1 + \frac{\gamma - 1}{2} M^2 \\ &= 1 + 0.2 \times 1.12^2 = 1.25 \\ T_0 &= 1.25 \times 301.15 = 376.44\,\mathrm{K} \\ &= \boxed{103.29^\circ\mathrm{C}} \end{aligned}$$

For the flow, the stagnation temperature is $T_0 = 376.44\,\mathrm{K}$.

The static temperature is $T = 92.5^\circ\mathrm{C} = 92.5 + 273.15 = 365.65\,\mathrm{K}$. The temperature is

$$\begin{aligned} \frac{T_0}{T} &= \frac{376.44}{365.65} = 1.03 \\ &= 1 + 0.2\,M^2 \end{aligned}$$

Thus,

$$\begin{aligned} M^2 &= \frac{0.03}{0.2} = 0.15 \\ M &= \boxed{0.387} \end{aligned}$$

This is the Mach number at the station where temperature is 92.5°C.

1.28 For hydrogen, the gas constant $R = 8314/2.016 = 4124\,\mathrm{J/(kg\,K)}$.

By isentropic relation, we have

$$\begin{aligned} \frac{T_2}{T_1} &= \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \\ &= \left(\frac{1}{7}\right)^{0.286} = 0.573 \end{aligned}$$

Therefore,

$$\begin{aligned} T_2 &= 0.573 \times T_1 \\ &= 0.573 \times 300 = 171.9\,\text{K} \end{aligned}$$

By energy equation, we have

$$\begin{aligned} h_2 + \frac{V_2^2}{2} &= h_1, \quad \text{since} \quad V_1 = 0 \\ V_2^2 &= 2\,(h_1 - h_2) \end{aligned}$$

But $h = c_p T$, therefore,

$$V_2 = \sqrt{2 c_p \,(T_1 - T_2)}$$

$$\begin{aligned} c_p &= \frac{\gamma}{\gamma - 1} R = \frac{1.4}{0.4} \times 4124 \\ &= 14434\,\text{J/(kg K)} \end{aligned}$$

Thus,

$$\begin{aligned} V_2 &= \sqrt{2 \times 14434\,(300 - 171.9)} \\ &= \boxed{1923\,\text{m s}^{-1}} \end{aligned}$$

The speed of sound is given by

$$\begin{aligned} a_2 &= \sqrt{\gamma R T_2} = \sqrt{1.4 \times 4124 \times 171.9} \\ &= 996.23\,\text{m/s} \end{aligned}$$

Thus,

$$M_2 = \frac{V_2}{a_2} = \frac{1923}{996.23} = \boxed{1.93}$$

The mass flow rate is given by

$$\dot{m} = \rho_2\, A_2\, V_2$$

$$\begin{aligned} \rho_2 &= \frac{p_2}{R T_2}, \text{ by state equation} \\ \rho_2 &= \frac{101325}{4124 \times 171.9}, \quad \text{since 1 atm} = 101325\,\text{Pa} \\ &= 0.143\,\text{kg/m}^3 \end{aligned}$$

Thus,

$$\begin{aligned}\dot{m} &= 0.143 \times 10 \times 10^{-4} \times 1923 \\ &= \boxed{0.275\,\mathrm{kg\ s^{-1}}}\end{aligned}$$

1.29 The given process is a polytropic process with index $n = 1.32$. Since air is given as an ideal gas with constant specific heats, we have from isentropic relations the change of entropy as

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$$

For a polytropic process,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Combining these two equations, we obtain

$$s_2 - s_1 = \left[c_p\left(\frac{n-1}{n}\right) - R\right] \ln\left(\frac{p_2}{p_1}\right)$$

since $c_p = \frac{\gamma}{\gamma-1} R$ for an ideal gas, the above equation may be written as

$$s_2 - s_1 = \frac{(n-\gamma)\,R}{n\,(\gamma-1)} \ln\left(\frac{p_2}{p_1}\right)$$

With the given data,

$$R = \frac{\gamma-1}{\gamma} c_p = \frac{0.4}{1.4} \times 1004 = 287\,\mathrm{J/(kg\,K)}$$

Therefore,

$$\begin{aligned}s_2 - s_1 &= \frac{(1.32-1.4)\,287}{1.32 \times 0.4} \ln\left(\frac{1100}{101}\right) \\ &= \boxed{-103.8\,\mathrm{J\ (kg\,K)^{-1}}}\end{aligned}$$

Note: Since the entropy of the gas decreases for this internally reversible process, heat must be removed from the gas. This is why the cylinder used for such compression process is usually water jacketed. Also, we know that the entropy of the system and surrounding cannot decrease. But in this problem, the entropy decreases. It should be noted that, what decreases is entropy of the system alone and not the combined entropy of the system and surrounding.

1.30 For oxygen, molecular weight M = 32. The gas constant $R = 8314/32 = 259.8$ J/(kg K). Therefore,

$$\begin{aligned} c_p &= \frac{\gamma}{\gamma - 1} R = 909.3\,\mathrm{J/(kg\,K)} \\ c_v &= \frac{c_p}{\gamma} = 649.5\,\mathrm{J/(kg\,K)} \end{aligned}$$

The increase in internal energy is

$$\begin{aligned} \triangle u &= c_v\,(T_2 - T_1) = 649.5\,(125 - 25) \\ &= \boxed{64\,950\,\mathrm{J}\ (\mathrm{kg\,K})^{-1}} \end{aligned}$$

The increase in enthalpy is

$$\begin{aligned} \triangle h &= c_p\,(T_2 - T_1) = 909.3\,(125 - 25) \\ &= \boxed{90\,930\,\mathrm{J}\ (\mathrm{kg\,K})^{-1}} \end{aligned}$$

1.31 Let subscripts 1 and 2 refer to the inlet and exit states, respectively. At state 1, $p_1 = 100$ kPa, $\rho_1 = 1.175$ kg/m^3. Therefore,

$$T_1 = \frac{p_1}{R\rho_1} = \frac{100 \times 10^3}{287 \times 1.175} = 296.5\,\mathrm{K}$$

At state 2, $p_2 = 500$ kPa, $\rho_2 = 5.875\,\mathrm{kg/m^3}$. Therefore,

$$T_2 = \frac{p_2}{R\rho_2} = \frac{500 \times 10^3}{287 \times 5.875} = 296.5\,\mathrm{K}$$

Assuming air to be a perfect gas, the enthalpy difference can be obtained as

$$\begin{aligned} h_2 - h_1 &= c_p\,(T_2 - T_1) \\ &= c_p\,(296.5 - 296.5) \\ &= \boxed{0} \end{aligned}$$

1.32 The entropy change is given by

$$ds = \frac{\delta q}{T}$$

where δq is the reversible heat addition per mbox mass. For an ideal gas

$$\delta q = dh - vdp$$

where dh is the enthalpy change and $dh = c_p dT$. Thus,

$$\delta q = c_p\, dT - vdp$$

Using the above relation, we get the entropy change as

$$ds = c_p \frac{dT}{T} - \frac{v}{T}\, dp$$

By state equation, $pv = RT$, therefore,

$$\frac{v}{T} = \frac{R}{p}$$

Therefore,

$$ds = c_p \frac{dT}{T} - \frac{R}{p}\, dp$$

Taking log of the state equation and differentiating, we get

$$\frac{dT}{T} = \frac{dp}{p} + \frac{dv}{v}$$

Substituting for $\dfrac{dT}{T}$ in ds expression, we obtain

$$ds = c_p \left(\frac{dp}{p} + \frac{dv}{v} \right) - R\, \frac{dp}{p}$$

$$ds = c_p\, \frac{dv}{v} + (c_p - R)\, \frac{dp}{p}$$

But $\quad c_p - c_v = R, \quad c_p - R = c_v$. Thus,

$$\boxed{ds = c_p\, \frac{dv}{v} + c_v\, \frac{dp}{p}}$$

For an isentropic change of state, we have

$$\begin{aligned} 0 &= c_p\, \frac{dv}{v} + c_v\, \frac{dp}{p} \\ c_p\, \frac{dv}{v} &= -c_v\, \frac{dp}{p} \\ \frac{c_p}{c_v}\, \frac{dv}{v} &= -\frac{dp}{p} \end{aligned}$$