Chapter One

Problem 1. The first law of thermodynamics for the control volume shown in Fig. 1.5 can be written as (ref. Table 1.1):

$$\frac{\partial}{\partial t}\int_{C.V}\left(u+\frac{V^2}{2}+\phi\right)\rho\, dv + \int_{C.S}\left(i+\frac{1}{2}V^2+\phi\right)\rho\vec{V}\cdot\vec{n}\, dA = q - \dot{W} \quad \text{--- (1)}$$

since the flow is steady, $\dot{m} = \rho_1 V_1 A_1 = \rho_2 V_2 A_2$ --- (2)

and

$$\int_{C.S}\left(i+\frac{1}{2}V^2+\phi\right)\rho\vec{V}\cdot\vec{n}\, dA = \left(i_2+\frac{1}{2}V_2^2+gz_2\right)\rho_2 V_2 A_2 - \left(i_1+\frac{1}{2}V_1^2+gz_1\right)\rho_1 V_1 A_1$$

$$= \dot{m}\left[\left(i_2+\frac{1}{2}V_2^2+gz_2\right)-\left(i_1+\frac{1}{2}V_1^2+gz_1\right)\right] \quad \text{--- (3)}$$

Therefore (1) becomes,

$$q + \dot{m}\left(i_1+\frac{1}{2}V_1^2+gz_1\right) = \dot{m}\left(i_2+\frac{V_2^2}{2}+gz_2\right) + \frac{d}{dt}\left\{M\left(u+\frac{V^2}{2}+gz\right)\right\}_{C.V} + W \quad \text{--- (4)}$$

where M, u and V are the mass, internal energy and the velocity of the control volume respectively.

<u>Problem 2.</u> Taking a control volume just outside the pipe-bend, we can write the momentum equation as:

$$\vec{F} = \frac{\partial}{\partial t}\int_{C.V.} \rho \vec{V} d\forall + \int_{C.S.} \vec{V} \rho \vec{V}\cdot\vec{n}\, dA \qquad (1)$$

The first integral on the right-hand side is zero because of the steady-state conditions, and the second integral reduces to integration over section 1 and 2.

Assuming that the fluid is incompressible,

$$\int_{C.S.} \vec{V}\rho \vec{V}\cdot\vec{n}\, dA = -\rho V_1^2 A_1 \vec{i} + \rho V_2^2 A_2 (\cos\alpha\, \vec{i} - \sin\alpha\, \vec{j}) \quad -(2)$$

where $\vec{V} = u\vec{i} + v\vec{j}$

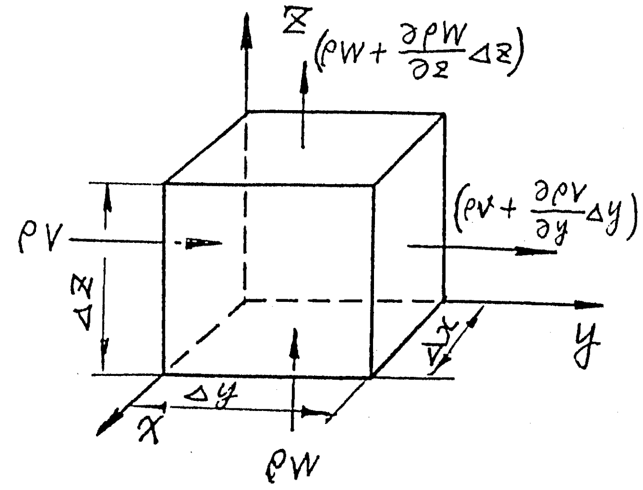
The net force acting on the C.V. is

$$\vec{F} = -R_x \vec{i} - R_y \vec{j} + P_1 A_1 \vec{i} - P_2 A_2 (\cos\alpha\, \vec{i} - \sin\alpha\, \vec{j}) \quad -(3)$$

sub. (2) and (3) into (1) and equating the terms in x- and y-direction respectively, we get:

$$\begin{cases} R_x = (P_1 + \rho V_1^2) A_1 - (P_2 + \rho V_2^2) A_2 \cos\alpha \\ R_y = (P_2 + \rho V_2^2) A_2 \sin\alpha \end{cases}$$

Problem 3. Take the volume element $\Delta x \Delta y \Delta z$ as the control volume and assume the velocity to be uniform on each of the six surfaces.



The figure shows the mass fluxes in the y- and z-directions. The net mass flow rate into the control volume is,

$$-\left(\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right]\Delta x \Delta y \Delta z$$

The rate of increase of mass within the control volume is,

$$\frac{\partial \rho}{\partial t}\Delta x \Delta y \Delta z$$

From the law of conservation of mass,

$$\frac{\partial \rho}{\partial t}\Delta x \Delta y \Delta z = -\left(\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right]\Delta x \Delta y \Delta z$$

$$\therefore \quad \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

or. $$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho \vec{V}) = 0$$

where $\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$

Problem 4. The heat transfer coefficient is defined as,

$$h = \frac{-k_f\left(\frac{\partial T_f}{\partial n}\right)_s}{T_w - T_\infty} \quad \text{—— (1)}$$

$$T_f(y) = 20 + 80e^{-800y}$$

$$\left(\frac{\partial T_f}{\partial n}\right)_s = \left(\frac{dT_f}{dy}\right)_{y=0} = -800 \times 80 e^{-800y}\Big|_{y=0} = -64\times10^3 \ ^\circ C/m \quad -(2)$$

$$T_w = (T_f)_{y=0} = 20 + 80e^{-800y}\Big|_{y=0} = 100\ ^\circ C \quad \text{—— (3)}$$

$$T_\infty = \lim_{y\to\infty}(20 + 80e^{-800y}) = 20\ ^\circ C \quad \text{—— (4)}$$

sub. (2), (3) and (4) into (1), we obtain,

$$h = \frac{(-0.62\ W/mK)(-64\times10^3\ ^\circ C/m)}{100\ ^\circ C - 20\ ^\circ C} = 496\ W/m^2K$$

Problem 5. Assume that the surface area of the cylinder is A, then the solar energy absorbed by the cylinder is,

$$Q_{abs} = \alpha_s \cdot 1500\,A \quad (W) \quad \text{—— (1)}$$

and the energy emitted by the cylinder is

$$Q_{em} = \epsilon A \sigma T_{cy}^4 \quad (W) \quad \text{—— (2)}$$

since $\alpha = \epsilon$, at equilibrium we have,

$$\sigma T_{cy}^4 = 1500$$

$$\therefore \quad T_{cy} = \left(\frac{1500\ W/m^2}{5.6697\times10^{-8}\ W/m^2K^4}\right)^{1/4} = 403.3\ K$$

Problem 6. From vector identity:

$$\int_S \vec{Y}(\vec{X}\cdot d\vec{s}) = \int_V \{\vec{Y}\,\nabla\cdot\vec{X} + (\vec{X}\cdot\nabla)\vec{Y}\}\,dv$$

we have,

$$\int_{c.s} \vec{V}\rho\vec{V}\cdot\vec{n}\,dA = \int_{c.v} \{\vec{V}\,\nabla\cdot(\rho\vec{V}) + (\rho\vec{V}\cdot\nabla)\vec{V}\}\,dv$$

so,
$$\vec{F} = \frac{\partial}{\partial t}\int_{c.v}\rho\vec{V}\,dv + \int_{c.s}\vec{V}\rho\vec{V}\cdot\vec{n}\,dA$$

$$= \int_{c.v}\{\vec{V}\frac{\partial\rho}{\partial t} + \rho\frac{\partial\vec{V}}{\partial t}\}\,dv + \int_{c.v}\{\vec{V}\,\nabla\cdot(\rho\vec{V}) + (\rho\vec{V}\cdot\nabla)\vec{V}\}\,dv$$

$$= \int_{c.v}\left\{\vec{V}\underbrace{\left\{\cancel{\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{V})}\right\}}_{0} + \rho\left(\frac{\partial\vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V}\right)\right\}\,dv$$

$$= \int_{c.v}\rho\left(\frac{\partial\vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V}\right)\,dv$$

$$= \int_{c.v}\rho\frac{D\vec{V}}{Dt}\,dv$$

Problem 7. By the use of Gauss' Theorem the L.H.S of Eq. (1.32) can be written as,

$$\frac{\partial}{\partial t}\int_{c.v} e\rho\,dv + \int_{c.v}\nabla\cdot(e\rho\vec{V})\,dv$$

$$= \int_{c.v}\left(e\frac{\partial\rho}{\partial t} + \rho\frac{\partial e}{\partial t} + e\,\nabla\cdot(\rho\vec{V}) + \rho(\vec{V}\cdot\nabla)e\right)\,dv$$

$$= \int_{c.v}\left\{e\underbrace{\left(\cancel{\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{V})}\right)}_{0} + \rho\left(\frac{\partial e}{\partial t} + (\vec{V}\cdot\nabla)e\right)\right\}\,dv$$

so,
$$\int_{c.v}\rho\frac{De}{Dt}\,dv = \dot{q}_{c.s} - \dot{W}$$

Chapter Two

Problem 2.1 Under the conditions given in the problem, the Navier-Stokes equations reduce to:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{1}{\mu}\frac{dP}{dz} \qquad \text{—— (1)}$$

where $\frac{dP}{dz}$ is a constant, since the R.H.S. is a function of z only and the L.H.S. is a function of r only.

The boundary conditions are:

$$\begin{cases} w=0 & \text{at } r=r_0 \qquad \text{—— (2)} \\ \frac{dw}{dr}=0 & \text{at } r=0 \qquad \text{—— (3)} \end{cases}$$

Integrating (1) twice:

$$w=\frac{1}{4\mu}\frac{dP}{dz}r^2+C_1\ln r+C_2 \qquad \text{—— (4)}$$

From B.C.s (2) and (3),

$$C_1=0, \quad C_2=-\frac{1}{4\mu}\frac{dP}{dz}r_0^2$$

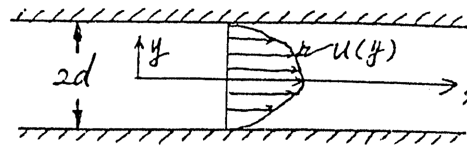
so, $$w=\frac{r_0^2}{4\mu}\frac{dP}{dz}\left[\left(\frac{r}{r_0}\right)^2-1\right] \qquad \text{—— (5)}$$

The average velocity:

$$w_m=\frac{1}{\pi r_0^2}\int_0^{r_0}2\pi r\,w\,dr=\frac{r_0^2}{8\mu}\frac{dp}{dz} \qquad \text{—— (6)}$$

so, $$\frac{w}{w_m}=2\left[1-\left(\frac{r}{r_0}\right)^2\right] \qquad \text{—— (7)}$$

Problem 2.2 From the statements of the problem,



$V=0$, $W=0$, $\frac{\partial u}{\partial x}=0$, $\frac{\partial u}{\partial z}=0$.

The N-S equations in y- and z-direction reduce to: $\frac{\partial P}{\partial y}=0$ and $\frac{\partial P}{\partial z}=0$ respectively.

The one in the x-direction reduces to,

$$\frac{dP}{dx}=\mu\frac{d^2u}{dy^2} \qquad \text{—— (1)}$$

We can deduce that $\frac{dP}{dx}=$ const., since the R.H.S is a function of only and the L.H.S. is a function of x only.

The boundary conditions are:

$$\begin{cases} u=0 & \text{at } y=d \qquad \text{—— (2)} \\ \frac{du}{dy}=0 & \text{at } y=0 \qquad \text{—— (3)} \end{cases}$$

Integrating (1) twice,

$$u=\frac{1}{2\mu}\frac{dP}{dx}y^2+Ay+B \qquad \text{—— (4)}$$

From B.C.s (2) and (3),

$$A=0, \quad \text{and} \quad B=-\frac{1}{2\mu}\frac{dP}{dx}d^2$$

So. $$u=\frac{1}{2\mu}\frac{dP}{dx}(y^2-d^2) \qquad \text{—— (5)}$$

Problem 2.2 (Continued)

At $y=0$, $u=u_{max}$,

$$u_{max} = -\frac{1}{2\mu}\frac{dP}{dx}d^2, \qquad \text{——(6)}$$

so, the velocity distribution in terms of u_{max} is,

$$\frac{u}{u_{max}} = 1-\left(\frac{y}{d}\right)^2 \qquad \text{——(7)}$$

The average velocity is,

$$u_m = \frac{1}{d}\int_0^d u\,dy = -\frac{1}{3\mu}\frac{dP}{dx}d^2 \qquad \text{——(8)}$$

so, the velocity distribution in terms of u_m is,

$$\frac{u}{u_m} = \frac{3}{2}\left\{1-\left(\frac{y}{d}\right)^2\right] \qquad \text{——(9)}$$

Also,

$$u_{max} = \frac{3}{2}u_m \qquad \text{——(10)}$$

Problem 2.3 Assume the downward direction to be the possitive y-direction. The N-S equation is,

$$\rho \frac{DV}{Dt} = \rho g - \frac{\partial P}{\partial y} + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial V}{\partial x} + \frac{\partial u}{\partial y}\right)\right] + \frac{\partial}{\partial y}\left[\mu\left(2\frac{\partial V}{\partial y} - \frac{2}{3}\nabla\cdot\vec{V}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial y} + \frac{\partial V}{\partial z}\right)\right] -$$

Introducing the dimensionless variables:

$$\bar{x} = \frac{x}{L}, \quad \bar{y} = \frac{y}{L}, \quad \bar{z} = \frac{z}{L}, \quad \bar{u} = \frac{u}{U_0}, \quad \bar{w} = \frac{w}{U_0}$$

$$\bar{P} = \frac{P - P_\infty}{\rho U_0^2}, \qquad \bar{t} = \frac{t U_0}{L}$$

where L is the characteristic length of the problem, U_0 is a reference velocity, and P_∞ is the reference pressure.

Sub. all the dimesionless variables into ( ), we get,

$$\frac{D\bar{V}}{D\bar{t}} = \bar{g} - \frac{\partial \bar{P}}{\partial \bar{y}} + \frac{\partial}{\partial \bar{x}}\left[\frac{1}{Re}\left(\frac{\partial \bar{V}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}}\right)\right] + \frac{\partial}{\partial \bar{y}}\left[\frac{1}{Re}\left(2\frac{\partial \bar{V}}{\partial \bar{y}} - \frac{2}{3}\nabla\cdot\vec{\bar{V}}\right)\right] +$$
$$+ \frac{\partial}{\partial \bar{z}}\left[\frac{1}{Re}\left(\frac{\partial \bar{W}}{\partial \bar{y}} + \frac{\partial \bar{V}}{\partial \bar{z}}\right)\right]$$

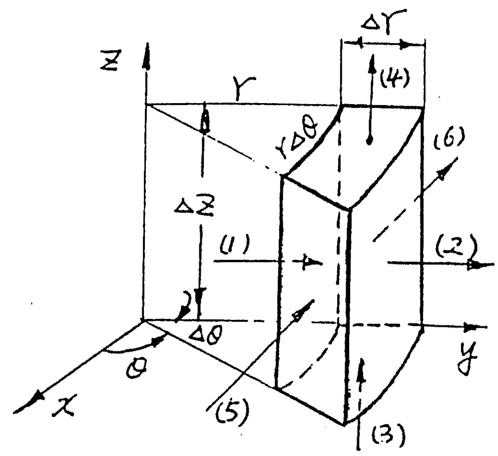
where $\bar{g} = \frac{U_0^2}{gL} = Fr$, called Froude Number,

and $Re = \frac{\rho U_0 L}{\mu}$, called Reynolds Number.

So, the conditions for dynamical similarity are

$$Fr_1 = Fr_2 \quad \text{and} \quad Re_1 = Re_2$$

Problem 2.4 The control valume is as shown in the Fig.



The mass flow rate into the C.V. through surface (1) is $(\rho V_r)\cdot r\Delta\theta\cdot\Delta z$.

- Taking linear approximation, the mass out-flow through surface (2) is, $(\rho V_r) r\Delta\theta\Delta z + \frac{\partial}{\partial r}(\rho V_r r\Delta\theta\Delta z)\Delta r$.

So, the net mass in-flow is, $-\frac{\partial}{\partial r}(\rho r V_r)\Delta r\Delta\theta\Delta z$

The mass in-flow through surface (3) is. $\rho V_z \cdot \Delta r \cdot r\Delta\theta$, and the mass out-flow through surface (4) is,

$\rho V_z r\Delta r\Delta\theta + \frac{\partial}{\partial z}(\rho V_z r\Delta r\Delta\theta)\Delta z$, so the net mass in-flow is $-\frac{\partial}{\partial z}(\rho V_z) r\Delta r\Delta\theta\Delta z$.

Similarly, the net mass in-flow rate in the tangential direction is

$$-\rho V_\theta \Delta r\Delta z - \frac{\partial}{\partial\theta}(\rho V_\theta \Delta r\Delta z)\Delta\theta + \rho V_\theta \Delta r\Delta z = -\frac{\partial}{\partial\theta}(\rho V_\theta)\Delta r\Delta\theta\Delta z$$

The increase rate of mass in the C.V. is

$$\frac{\partial}{\partial t}(\rho\Delta r\Delta z r\Delta\theta)$$

From the law of mass conservation:

$$\frac{\partial\rho}{\partial t}\Delta r\Delta z\Delta\theta = -\frac{\partial}{\partial r}(\rho r V_r)\Delta r\Delta\theta\Delta z - \frac{\partial}{\partial z}(\rho V_z) r\Delta r\Delta\theta\Delta z - \frac{\partial}{\partial\theta}(\rho V_\theta)\Delta r\Delta\theta\Delta z$$

or. $$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r V_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(\rho V_\theta) + \frac{\partial}{\partial z}(\rho V_z) = 0$$