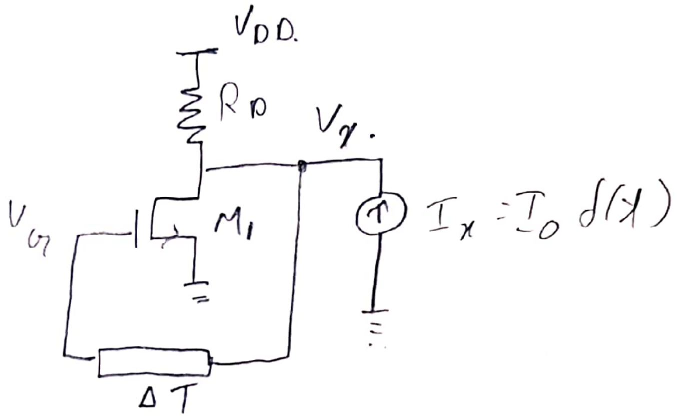
Chapter 1
Oscillator Fundamentals

1.1



By KCL we have.

$$I_x = \frac{V_x}{R_D} + g_m V_{GS}$$

But $V_G = V_x e^{-s\Delta T}$

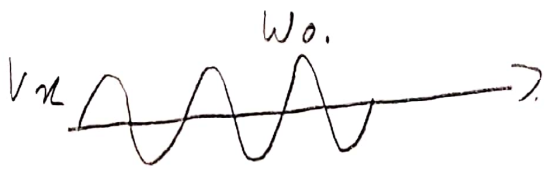
$$\therefore I_x = \frac{V_x}{R_D} + g_m V_x e^{-s\Delta T}$$

$$I_x = V_x \left(\frac{1}{R_D} + g_m e^{-s\Delta T}\right)$$

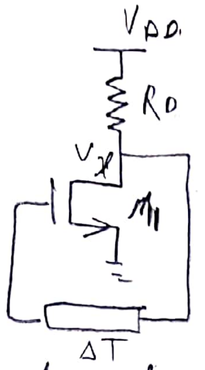
$$V_x(t) = \frac{I_0 \delta(t)}{\frac{1}{R_D} + g_m e^{-s\Delta T}} = \frac{I_0 \delta(t) R_D}{1 + g_m R_D e^{-s\Delta T}}$$

If $g_m R_D = 1$ and $\Delta T \omega = \pi$, then system begins to oscillate.

We know that application of impulse, the circuit oscillates with a constant amplitude. Hence V_x will be

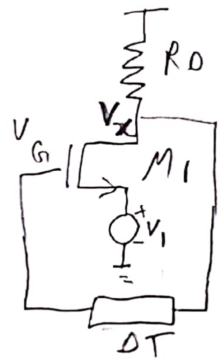


1.2)



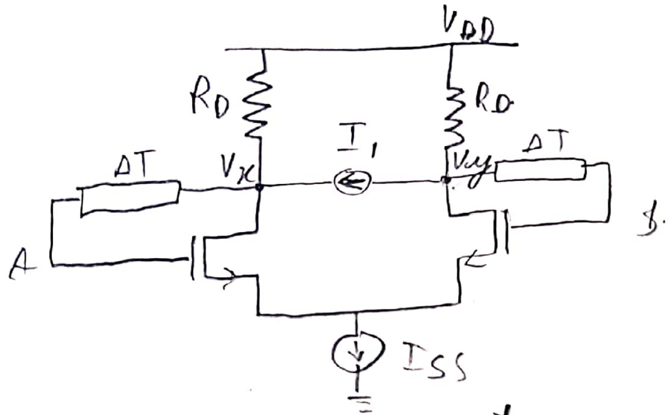
If V_{DD} jumps by ΔV, it acts as an impulse i/p to the loop thus giving rise to oscillation. When V_{DD} increases, V_x increases and V_{DS} increases. If we assume it to be operating in saturation, the current doesn't increase immediately. After a delay of ΔT, V_G increases thus increasing the current through FET and reducing V_x. This process continues and results in oscillations.

1.3)



If we reduce V_1 at source of M_1 from 100 mV to 0, this results in increase in V_{DS}. If we assume that the FET is operating in saturation, then there is no change in current through FET. Since there is no change in voltage or current in the loop, the circuit does not oscillate, assuming there is no noise in the circuit.

1.4



To obtain the impedance between node X and Y, apply a current source between the two nodes.

$V_A = V_x e^{-s\Delta T}$, $V_B = V_y e^{-s\Delta T}$

$$I_1 = \frac{V_x}{R_D} + g_m V_A - \frac{V_y}{R_D} - g_m V_B$$

$$= \frac{V_x}{R_D} + g_m V_x e^{-s\Delta T} - \frac{V_y}{R_D} - g_m V_y e^{-s\Delta T}$$

$$= V_x\left(\frac{1}{R_D} + g_m e^{-s\Delta T}\right) - V_y\left(\frac{1}{R_D} + g_m e^{-s\Delta T}\right)$$

$$I_1 = (V_x - V_y)\left(\frac{1}{R_D} + g_m e^{-s\Delta T}\right)$$

$$Z = \frac{V_x - V_y}{I_1} = \frac{1}{\frac{1}{R_D} + g_m e^{-s\Delta T}} = \frac{R_D}{1 + g_m R_D e^{-s\Delta T}}$$

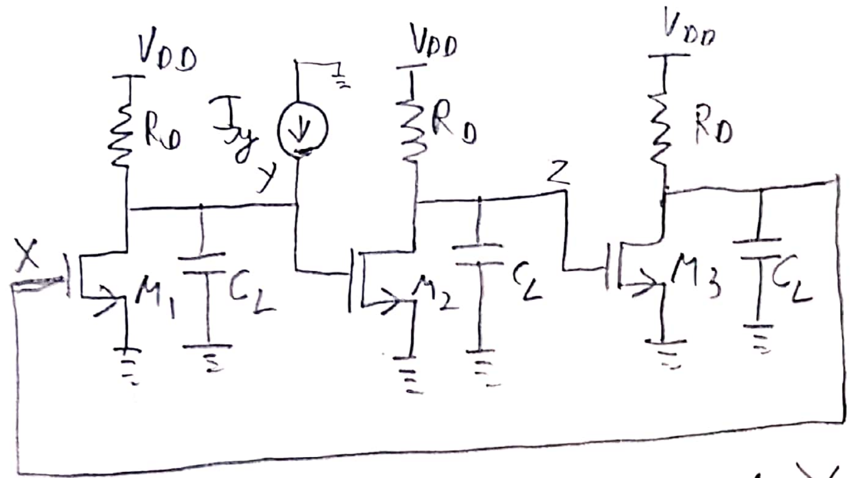
For impedance to go to infinity, denominator should be zero

$$1 + g_m R_D e^{-s\Delta T} = 0 \Rightarrow g_m R_D e^{-s\Delta T} = -1$$

$$\Rightarrow \boxed{g_m R_D = 1} \quad \text{and} \quad \omega_0 \Delta T = \pi$$

$$\Rightarrow \boxed{f_0 = \frac{1}{2\Delta T}}$$

1.5)



Apply current source at node Y.

$$I_y = \frac{V_y}{\left(R_D \parallel \frac{1}{C_L s}\right)} + g_m V_x \quad \text{——} \ (1)$$

But $V_x = -g_m \left(R_D \parallel \frac{1}{C_L s}\right) V_z$

$$= g_m \left(R_D \parallel \frac{1}{C_L s}\right) g_m \left(R_D \parallel \frac{1}{C_L s}\right) V_y \quad \text{—} \ (2)$$

From (1) and (2)

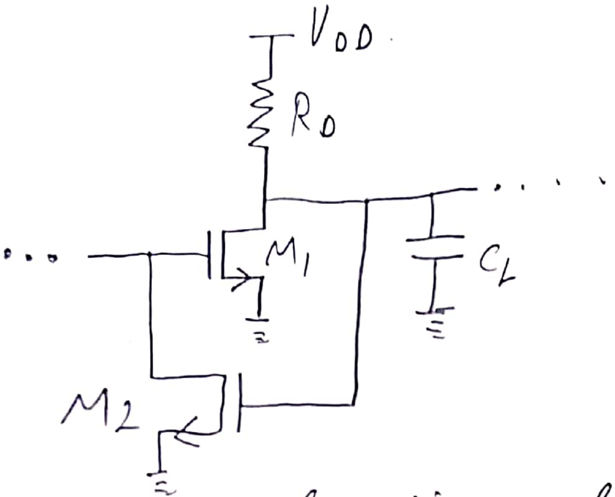
$$I_y = \frac{V_y}{\left(R_D \parallel \frac{1}{C_L s}\right)} + g_m^3 V_y \left(R_D \parallel \frac{1}{C_L s}\right)^2.$$

$$I_y = V_y \left[\frac{1}{\left(R_D \parallel \frac{1}{C_L s}\right)} + g_m^3 \left(R_D \parallel \frac{1}{C_L s}\right)^2\right]$$

$$I_y = V_y \left[\frac{1 + g_m^3 \left(R_D \parallel \frac{1}{C_L s}\right)^3}{R_D \parallel \frac{1}{C_L s}}\right]$$

$$Z_y = \boxed{\frac{V_y}{I_y} = \frac{R_D \parallel \frac{1}{C_L s}}{1 + g_m^3 \left(R_D \parallel \frac{1}{C_L s}\right)^3}}$$

1.6



To find Transfer function of single stage, apply V_x at input of M_1 and check output V_F at drain of M_2.
M_2 is loaded from previous stage.

$$V_F = -g_m V_{out}\left(R_D \parallel \frac{1}{C_L s} \parallel \frac{1}{g_m}\right)$$

$$V_F = g_m\left(R_D \parallel \frac{1}{C_L s} \parallel \frac{1}{g_m}\right) \cdot g_m\left(R_D \parallel \frac{1}{C_L s}\right) V_x .$$

$$H(s) = -\frac{V_F}{V_x} = -g_m^2\left(R_D \parallel \frac{1}{C_L s} \parallel \frac{1}{g_m}\right)\left(R_D \parallel \frac{1}{C_L s}\right)$$

$$\approx -g_m^2\left(R_D \parallel \frac{1}{C_L s}\right)^2$$

$$\therefore \ |H(s)| = 1 \Rightarrow \left|g_m\left(R_D \parallel \frac{1}{C_L s}\right)\right|^2 = 1 \quad \text{—— (1)}$$

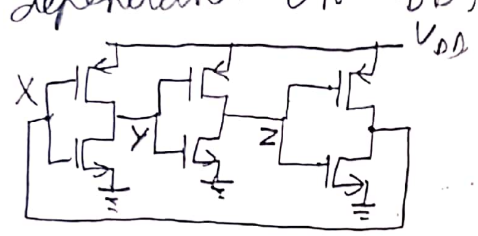
$$H(j\omega) = \frac{g_m^2 R_D^2}{(R_D C_L j\omega + 1)^2} = \frac{g_m^2 R_D^2}{2 R_D C_L j\omega_0 + 1 - R_D^2 C_L^2 \omega_0^2}$$

We need a frequency dependant phase shift of $\frac{-180^\circ}{N}$ per stage

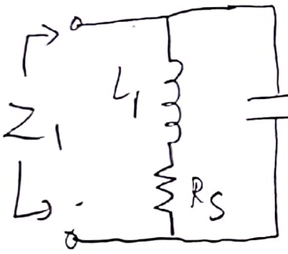
$$\frac{-180^\circ}{N} = -\tan^{-1}\left(\frac{2 R_D C_L \omega_0}{1 - R_D^2 C_L^2 \omega_0^2}\right) \quad \text{—— (2)}$$

Eq^n (1) and (2) give the oscillation condition.

1.7 The frequency of oscillation is defendant on the time delay / profogation delay of the inverter. If the size of each transistor is doubled the capacitance at the node increases. But the resistance offered by the MOS device decreases. The time constant RC of the inverter stays the same, thus keeping the oscillation frequency constant. Moreover the delay is highly defendant on V_{DD}, than on sizing.



1.8



$$Z_1 = (R_S + L_1 S) \parallel \frac{1}{C_1 S}$$

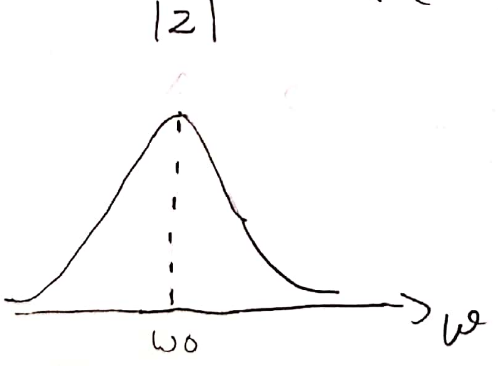
$$= \frac{\frac{R_S + L_1 S}{C_1 S}}{R_S + L_1 S + \frac{1}{C_1 S}}$$

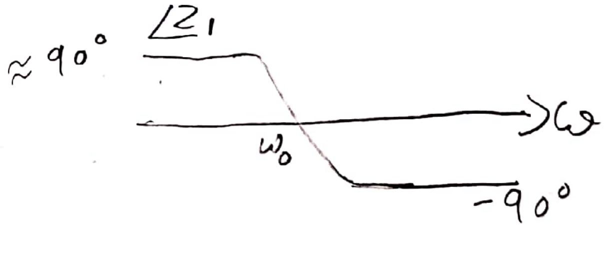
$$Z_1 = \frac{R_S + L_1 S}{R_S C_1 S + L_1 C_1 S^2 + 1}$$

$$Z_1(j\omega) = \frac{R_S + L_1 j\omega}{C_1 R_S j\omega + L_1 C_1 (j\omega)^2 + 1} = \frac{R_S + L_1 j\omega}{j(R_S C_1 \omega) + 1 - L_1 C_1 \omega^2}$$

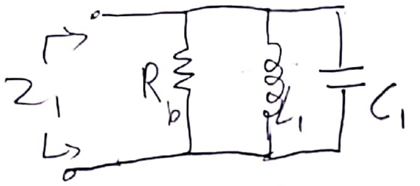
$$|Z_1(s = j\omega)| = \sqrt{\frac{R_S^2 + L_1^2 \omega^2}{(1 - L_1 C_1 \omega^2)^2 + R_S^2 C_1^2 \omega^2}}$$

$$\angle Z_1 = -\tan^{-1}\left[\frac{R_S C_1 \omega}{1 - L_1 C_1 \omega^2}\right]$$





1.9)



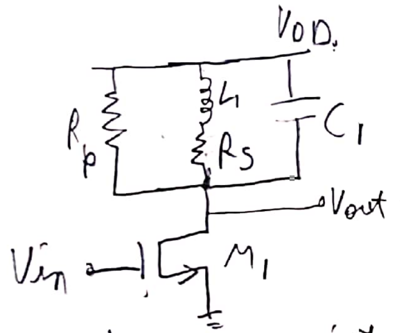
If L_1 doubles and R_S remains constant, then

$$R_p = \frac{L_1^2 \omega^2}{R_S} \text{ increases by 4 times}$$

Since R_p increases by 4 times the slope also increases because $\frac{d}{d\omega}\angle Z_1(j\omega) \approx -2R_p C_1$ and L acts as a better inductor.

If L_1 doubles and R_S also doubles, then R_p increases by only twice and thus increase in slope is less severe.

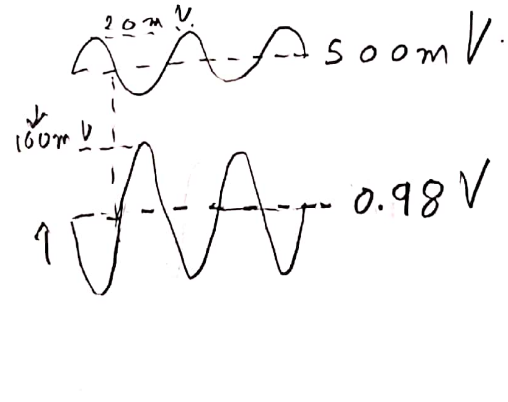
1.10



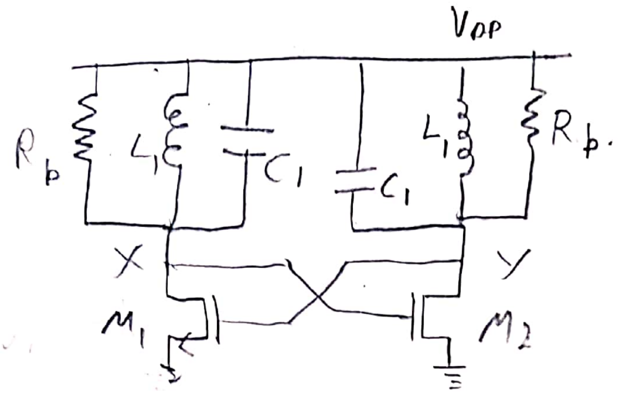
Since there is a resistance in series with L_1, there will be a voltage drop across L from V_{DD} to V_{out} at DC

$$V_{out} = V_{DD} - I_D(R_S \| R_p)$$, But $R_S \| R_p \simeq R_S$

$$V_{out} = V_{DD} - I_D R_S$$
$$= 1V - 0.02$$
$$= 0.98V$$



1.11)



To compute the amplitude of third harmonic we need to find impedance at $3\omega_0$

$$Z_p = R_p \parallel L_1 S \parallel \frac{1}{C_1 S}$$

$$Z_p = R_p \parallel \left(\frac{L_1 S}{L_1 C_1 S^2 + 1}\right) = R_p \parallel \left(\frac{L_1 j\omega}{-L_1 C_1 \omega^2 + 1}\right)$$

$$Z_p\big|_{\omega=3\omega_0} = R_p \parallel \left(\frac{3L_1 j\omega_0}{-L_1 C_1 (3\omega_0)^2 + 1}\right) = R_p \parallel \left(\frac{3L_1 j\omega_0}{-9+1}\right)$$

$$= R_p \parallel \left(\frac{3jL_1 \times \frac{1}{\sqrt{L_1 C_1}}}{-8}\right) = R_p \parallel \left(\frac{3j}{-8}\sqrt{\frac{L_1}{C_1}}\right)$$

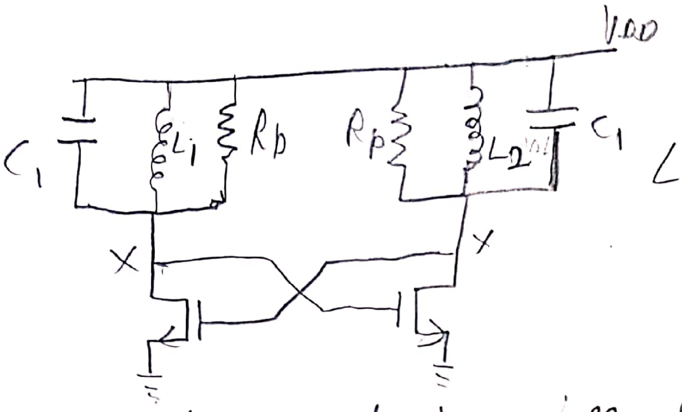
$$Z_p\big|_{\omega=3\omega_0} = \frac{3jR_p\sqrt{L_1}}{-8R_p\sqrt{C_1} + 3j\sqrt{L_1}} \quad \text{or} \quad \frac{3jR_pL_1\omega_0}{-8R_p + 3jL_1\omega_0}$$

We know that $Z_p\big|_{\omega=\omega_0} = R_p$.

$$\therefore \frac{V_{3\omega_0}}{V_{\omega_0}} = \frac{Z\big|_{\omega=3\omega_0}}{Z\big|_{\omega=\omega_0}} = \frac{\frac{3jR_pL_1\omega_0}{-8R_p+3jL_1\omega_0}}{R_p}$$

$$\boxed{\frac{V_{3\omega_0}}{V_{\omega_0}} = \frac{3jL_1\omega_0}{-8R_p+3jL_1\omega_0}}$$

1.12



$L_1 = L_2 + \Delta L$

The phase of each tank is given by

$$\angle Z_1(j\omega) = \frac{\pi}{2} - \tan^{-1}\frac{L_1\omega}{R_p(1-L_1C_1\omega^2)}$$

The sum of phases of both stages should be zero

$$\therefore \frac{\pi}{2} - \tan^{-1}\left(\frac{L_1\omega}{R_p(1-L_1C_1\omega^2)}\right) + \frac{\pi}{2} - \tan^{-1}\left(\frac{L_2\omega}{R_p(1-L_2C_1\omega^2)}\right) = 0$$

But $L_1 = L_2 + \Delta L$ and $\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$$\tan^{-1}\left(\frac{\frac{x_1}{x_2} + \frac{y_1}{y_2}}{1 - \frac{x_1}{x_2}\frac{y_1}{y_2}}\right) = \tan^{-1}\left(\frac{x_1y_2 + y_1x_2}{x_2y_2 - x_1y_1}\right)$$

$$\frac{x_1y_2 + y_1x_2}{x_2y_2 - x_1y_1} = \tan(\pi) = 0$$

$$x_1y_2 + y_1x_2 = 0$$

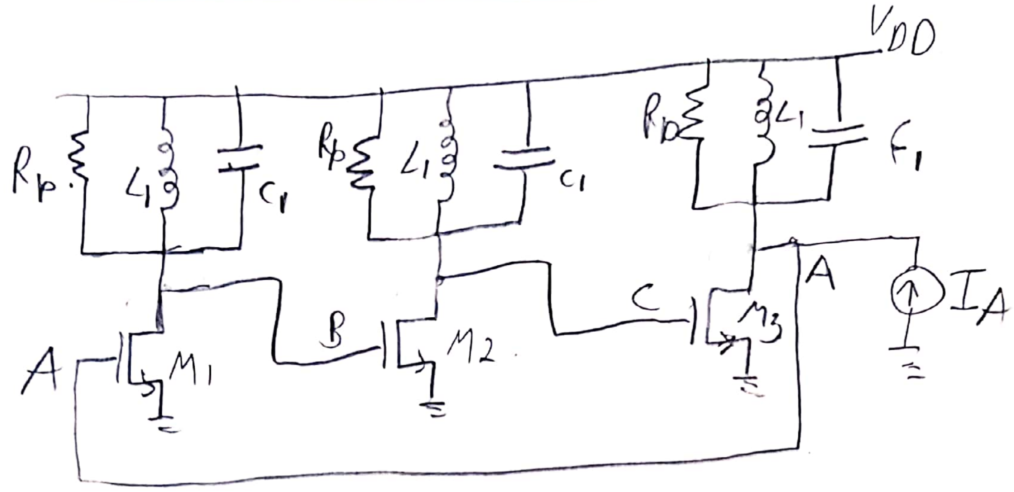
$$R_p[L_2\omega_1 - L_2(L_2+\Delta L)C_1\omega^3 + (L_2+\Delta L)\omega_1 - L_2(L_2+\Delta L)C_1\omega^3 = 0$$

$$2L_2\omega_1 + \Delta L\omega_1 - 2L_2(L_2+\Delta L)C_1\omega^3$$

$$2L_2 + \Delta L = 2L_2(L_2+\Delta L)C_1\omega^2$$

$$\therefore \boxed{\omega = \sqrt{\frac{2L_2+\Delta L}{2L_2(L_2+\Delta L)C_1}}}$$

1.13



$$I_A = \frac{V_A}{R_p \parallel L_1 S \parallel \frac{1}{C_1 S}} + g_m V_C$$

$$= \frac{V_A}{Z} + g_m^3 Z^2 V_A \qquad \text{where } Z = R_p \parallel L_1 S \parallel \frac{1}{C_1 S}$$

$$I_A = V_A \left(\frac{1}{Z} + g_m^3 Z^2 \right)$$

$$\boxed{\frac{V_A}{I_A} = \frac{V_A}{\frac{1}{Z} + g_m^3 Z^2}}$$

For the circuit to oscillate, impedance should go to ∞

i.e. $$\frac{1}{Z} + g_m^3 Z^2 = 0$$

$$1 + g_m^3 Z^3 = 0$$

$$g_m^3 Z^3 = -1$$

$$g_m^3 \left(R_p \parallel L_1 S \parallel \frac{1}{C_1 S} \right)^3 = -1$$

$$g_m^3 \left(\frac{j R_p L_1 \omega_1}{R_p (1 - L_1 C_1 \omega_1^2) + j L_1 \omega_1} \right)^3 = -1$$

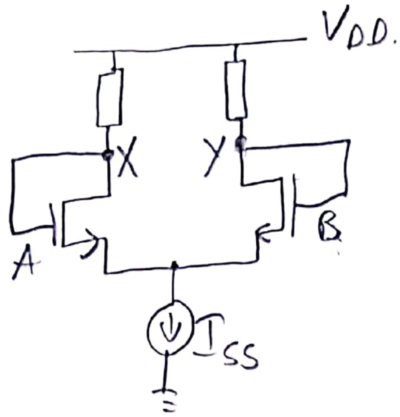
By finding magnitude on both sides and squaring we have

$$\left[\frac{g_m^2 R_p^2 L_1^2 \omega_1^2}{R_p^2 (1 - L_1^2 C_1^2 \omega_1^2)^2 + L_1^2 \omega_1^2} \right]^3 = 1^2, \quad \text{use } \omega_1 = \alpha \omega_0 \text{ in this eq}^n$$

$$\Rightarrow g_m^2 R_p^2 L_1^2 \alpha^2 \omega_0^2 = R_p^2 (1 - \alpha^2)^2 + L_1^2 \alpha^2 \omega_0^2$$

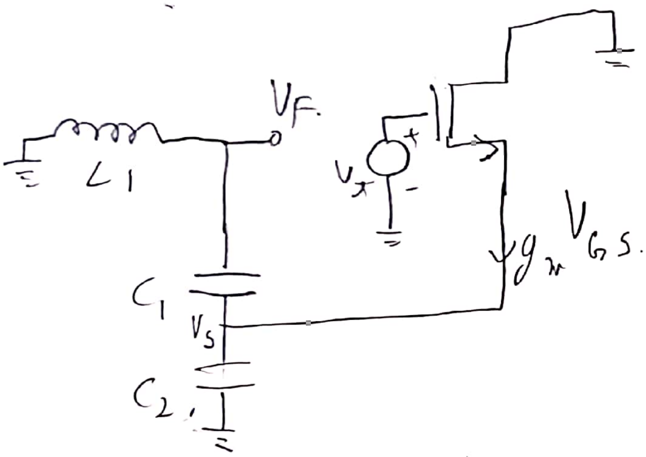
$$\boxed{g_m R_p = \sqrt{1 + \frac{R_p^2}{L_1^2 \omega_0^2} \left(\frac{1}{\alpha} - \alpha \right)^2}} \qquad \text{where } \alpha = \frac{\omega_1}{\omega_0}$$

1.14)



The CS stage provides a 180° phase shift at its output. We would need another 180° phase shift for oscillations to occur. Hence we need a delay line of 180° in series with the gate of the two transistors

1.15



By applying a voltage V_x at input we get

$$\frac{V_S}{V_x} = \frac{g_m Z}{1+g_m Z}$$

, where Z is the impedance seen at the source.

We can see that

$$V_F = V_S \frac{L_1 S}{L_1 S + \frac{1}{C_1 S}}$$

$$V_F = \frac{g_m Z}{1+g_m Z} \left(\frac{L_1 S}{L_1 S + \frac{1}{C_1 S}} \right) V_x.$$