

Chapter 1

1.1 (a) $x(t) = A \cos 2\pi f_0 t$: Power signal

$$\begin{aligned} P_x &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t) dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A^2 \cos^2 2\pi f_0 t dt \\ &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \frac{A^2}{2} dt = \frac{A^2}{2} \end{aligned}$$

(b)

$$x(t) = \begin{cases} A \cos 2\pi f_0 t & \text{for } -T_0/2 \leq t \leq T_0/2 \\ 0 & \text{elsewhere} \end{cases}$$

Energy signal

$$E_x = \int_{-\infty}^{\infty} x^2(t) dt = \int_{-T_0/2}^{T_0/2} A^2 \cos^2 2\pi f_0 t dt = \frac{A^2 T_0}{2}$$

(c)

$$x(t) = \begin{cases} A \exp(-at) & \text{for } t > 0, a > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Energy signal

$$\begin{aligned} E_x &= \int_{-\infty}^{\infty} x^2(t) dt = \int_0^{\infty} A^2 \exp(-2at) dt \\ &= \left[\frac{A^2 \exp(-2at)}{-2a} \right]_0^{\infty} = \frac{A^2}{2a} \end{aligned}$$

$$(d) \quad x(t) = \cos t + 5 \cos 2t \quad \text{for } -\infty < t < \infty$$

Power signal

$$P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \cos^2 t + 25 \cos^2 2t \, dt; \quad \begin{array}{l} 2\pi f_0 = 1 \\ T_0 = 2\pi \end{array}$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(\frac{1}{2} + \frac{25}{2} \right) dt = \frac{1}{2\pi} (26\pi) = 13$$

$$\underline{1.2} \quad x(t) = \text{rect}(t/T)$$

$$= \begin{cases} 1 & \text{for } -T/2 \leq t \leq T/2 \\ 0 & \text{elsewhere} \end{cases}$$

$$\text{ESD } \Psi(f) = |X(f)|^2 = T^2 \text{sinc}^2(fT)$$

$$E_x = \int_{-\infty}^{\infty} x^2(t) \, dt = \int_{-T/2}^{T/2} dt = T$$

1.3 Using Equations (1.18) and (1.19)

$$G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0)$$

$$P_x = \int_{-\infty}^{\infty} G_x(f) \, df = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0) \, df$$

$$P_x = \sum_{n=-\infty}^{\infty} |c_n|^2$$

$$\underline{1.4} \quad P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t) dt; \quad 2\pi f_0 = 10$$

$$f_0 = \frac{5}{\pi}$$

$$T_0 = \pi/5$$

$$P_x = \frac{5}{\pi} \int_{-\pi/10}^{\pi/10} 100 \cos^2 10t + 400 \cos^2 20t dt$$

$$= \frac{5}{2\pi} \int_{-\pi/10}^{\pi/10} 100(1 + \cos 20t) + 400(1 + \cos 40t) dt$$

$$= \frac{5}{2\pi} \left[100t + 400t \right]_{-\pi/10}^{\pi/10} = 250 \text{ W}$$

$$\underline{1.5} \quad G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - n f_0)$$

$$c_1 = c_{-1} = \frac{10}{2} = 5; \quad c_2 = c_{-2} = \frac{20}{2} = 10$$

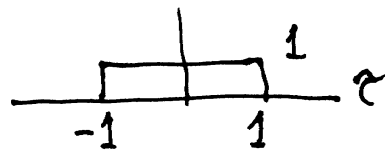
$$c_m = 0 \text{ for } m = 0, \pm 3, \pm 4, \dots$$

$$G_x(f) = (5)^2 \delta\left(f - \frac{5}{\pi}\right) + (5)^2 \delta\left(f + \frac{5}{\pi}\right) \\ + (10)^2 \delta\left(f - \frac{10}{\pi}\right) + (10)^2 \delta\left(f + \frac{10}{\pi}\right)$$

$$P_x = \int_{-\infty}^{\infty} G_x(f) df = 25 + 25 + 100 + 100 \\ = 250 \text{ W}$$

1.6 $\mathcal{F}\{R(\tau)\}$ must be a nonnegative function because $\mathcal{F}\{R(\tau)\} = G(f)$; and, the power spectral density, $G(f)$, must be a nonnegative function.

(a) $x(\tau) = \begin{cases} 1 & \text{for } -1 \leq \tau \leq 1 \\ 0 & \text{otherwise} \end{cases}$



NO $\begin{cases} 1. x(\tau) = x(-\tau) \checkmark \\ 2. x(0) \geq x(\tau) \checkmark \\ 3. \mathcal{F}\{x(\tau)\} \text{ is a positive and negative going function.} \end{cases}$

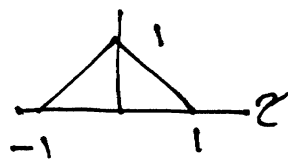
(b) $x(\tau) = \delta(\tau) + \sin 2\pi f_0 \tau$

NO 1. $x(\tau) \neq x(-\tau)$ x

(c) $x(\tau) = \exp(1|\tau|)$

NO $\begin{cases} 1. x(\tau) = x(-\tau) \checkmark \\ 2. x(0) \neq x(\tau) \times \end{cases}$

(d) $x(\tau) = \begin{cases} -\tau + 1 & \text{for } 0 \leq \tau \leq 1 \\ \tau + 1 & \text{for } -1 \leq \tau \leq 0 \end{cases}$



YES $\begin{cases} 1. x(\tau) = x(-\tau) \checkmark \\ 2. x(0) \geq x(\tau) \checkmark \\ 3. \mathcal{F}\{x(\tau)\} = 2 \operatorname{sinc}^2 f \tau \end{cases}$

is a nonnegative function. \checkmark

1.7 (a) $X(f) = \delta(f) + \cos^2 2\pi f$

YES $\left\{ \begin{array}{l} 1. \text{ always real } \checkmark \\ 2. P_x(f) \geq 0 \checkmark \\ 3. P_x(-f) = P_x(f) \checkmark \end{array} \right.$

(b) $X(f) = 10 + \delta(f-10)$

NO $\left\{ \begin{array}{l} 1. \text{ always real } \checkmark \\ 2. P_x(f) \geq 0 \checkmark \\ 3. P_x(-f) \neq P_x(f) \times \end{array} \right.$

(c) $X(f) = \exp(-2\pi|f-10|)$

NO $\left\{ \begin{array}{l} 1. \text{ always real } \checkmark \\ 2. P_x(f) \geq 0 \checkmark \\ 3. P_x(-f) \neq P_x(f) \times \end{array} \right.$

(d) $X(f) = \exp[-2\pi(f^2-10)]$

YES $\left\{ \begin{array}{l} 1. \text{ always real } \checkmark \\ 2. P_x(f) \geq 0 \checkmark \\ 3. P_x(-f) = P_x(f) \checkmark \end{array} \right.$

1.8

$$R_x(t) = \langle A \cos(2\pi f_0 t + \phi) A \cos(2\pi f_0 t + 2\pi f_0 \tau + \phi) \rangle$$

where $\langle \cdot \rangle$ is the time averaging operator $\frac{1}{T_0} \int_{-T_0/2}^{T_0/2} dt$

Upon expanding (see Appendix D),

$R_x(t)$ becomes:

$$R_x(t) = A^2 \left[\cos 2\pi f_0 \tau \langle \cos^2(2\pi f_0 t + \phi) \rangle - \sin 2\pi f_0 \tau \langle \cos(2\pi f_0 t + \phi) \sin(2\pi f_0 t + \phi) \rangle \right]$$

The negative term in the above expression goes to zero, and hence

$$R_x(t) = \frac{A^2}{2} \cos 2\pi f_0 \tau$$

$$P_x = R_x(0) = \frac{A^2}{2}$$

1.9 (a) $R_x(t) = \frac{100}{2} \cos 10t + \frac{400}{2} \cos 20t$

where $2\pi f_0 = 10$

(b) $P_x = R_x(0) = 50 + 200 = 250 \text{ W}$

1.10 (a) average value of $x(t)$

$$\langle x(t) \rangle = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (1 + \cos 2\pi f_0 t) dt = 1$$

(b) the ac power of $x(t)$

$$\langle \sigma^2(t) \rangle = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \cos^2 2\pi f_0 t dt = \frac{1}{2}$$

(c) the rms value of $x(t)$

$$\begin{aligned} \langle x^2(t) \rangle &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (1 + \cos 2\pi f_0 t)^2 dt \\ &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (1 + 2\cos 2\pi f_0 t + \cos^2 2\pi f_0 t) dt = \frac{3}{2} \end{aligned}$$

$$x_{\text{rms}} = \sqrt{\frac{3}{2}}$$

1.11 (a) $\langle X(t) \rangle = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A \cos(2\pi f_0 t + \phi) dt = 0$

$$\langle X^2(t) \rangle = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} [A \cos(2\pi f_0 t + \phi)]^2 dt$$

$$= \frac{A^2}{2}$$

$$1.11 \text{ (b)} \quad E\{X\} = \int_{-\infty}^{\infty} X(\phi) p(\phi) d\phi$$

$p(\phi) = \frac{1}{2\pi}$ since ϕ is uniformly distributed over $(0, 2\pi)$

$$E\{X\} = \int_0^{2\pi} [A \cos(2\pi f_0 t + \phi)] \frac{1}{2\pi} d\phi = 0$$

$$E\{X^2\} = \int_0^{2\pi} [A \cos(2\pi f_0 t + \phi)]^2 \frac{1}{2\pi} d\phi$$

$$= A^2/2$$

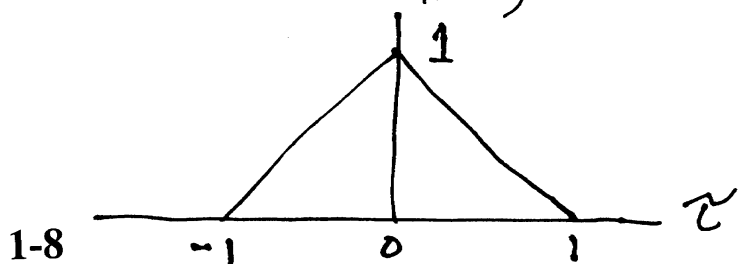
$$\underline{1.12} \quad X(f) = \text{sinc } f$$

$$\Psi_x(f) = |X(f)|^2 = \text{sinc}^2 f$$

$$R_x(\tau) = \mathcal{F}^{-1}\{\Psi_x(f)\}$$

From Table A.1, $R_x(\tau)$ is seen to be the following triangular function:

$$R_x(\tau) = \begin{cases} 1 - |\tau| & \text{for } |\tau| < 1 \\ 0 & \text{elsewhere} \end{cases}$$



$$\underline{1.13} \quad (a) \int_{-\infty}^{\infty} \cos 6t \delta(t-3) dt = \cos 18$$

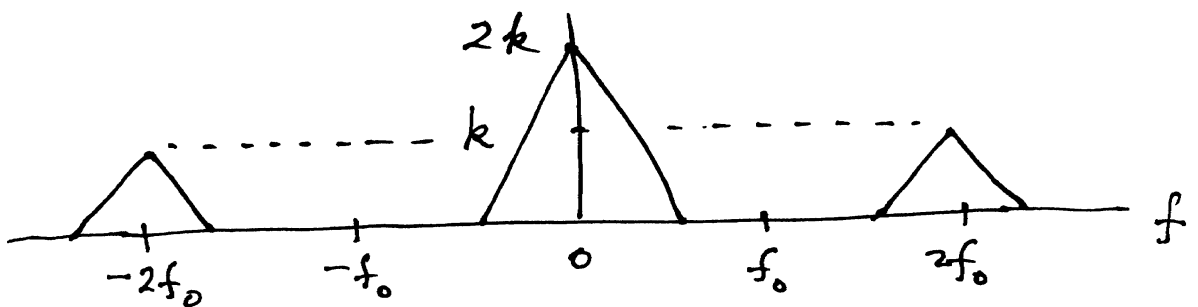
$$(b) \int_{-\infty}^{\infty} 10 \delta(t) (1+t)^{-1} dt = 10$$

$$(c) \int_{-\infty}^{\infty} \delta(t+4) (t^2 + 6t + 1) dt = -7$$

$$(d) \int_{-\infty}^{\infty} \exp(-t^2) \delta(t-2) dt = 0.0183$$

$$\underline{1.14} \quad X_2(f) = k [\delta(f-f_0) + \delta(f+f_0)]$$

$$X_1(f) * X_2(f) = X_1(f) * k [\delta(f-f_0) + \delta(f+f_0)]$$



$$\underline{1.15} \quad (a) P_x = 2 \int_0^{10 \text{ kHz}} G_x df = 2 \int_0^{10 \text{ kHz}} 10^{-6} f^2 df$$

$$= 2 \left[\frac{10^{-6} f^3}{3} \right]_0^{10^4} = 667 \text{ kW}$$

$$(b) P_x = 2 \int_{5 \text{ kHz}}^{10 \text{ kHz}} 10^{-6} f^2 df = 2 \left[\frac{10^{-6} f^3}{3} \right]_{5000}^{10000}$$

$$= 583 \text{ kW}$$

$$\underline{1.16} \quad 10 \log_{10} \left[\frac{100 \times 2 \times \frac{1}{2}}{\frac{1}{2}} \right] = 23 \text{ dB}$$

1.17 (a) Since $|H(f)|$ decreases monotonically with $|f|$, and $|H(0)| = 1$, we can write the following relationship in terms of the -1dB frequency, f_1 .

$$10 \log_{10} |H(f_1)|^2 = -1 \text{ dB}$$

$$\log_{10} \left[\frac{1}{(1 + f_1/f_u)^{2n}} \right] = -\frac{1}{10}$$

$$\left[1 + f_1/f_u \right]^{2n} = 10^{1/10}$$

$$\left[f_1/f_u \right]^{2n} = 10^{1/10} - 1 = 0.2584$$

$$\therefore n \geq \frac{1}{2} \left[\frac{\log 0.2584}{\log (f_1/f_u)} \right]$$

$$\text{For } f_1/f_u = 0.9, \quad n \geq 6.4$$

$$\text{Thus, } n = 7$$

1.17 (b) In the limit, as $n \rightarrow \infty$

$$\left(\frac{f}{f_m}\right)^{2n} \rightarrow 0, \quad |H(f)| \rightarrow 1, \quad \text{for } \left|\frac{f}{f_m}\right| < 1$$

$$\left(\frac{f}{f_m}\right)^{2n} \rightarrow \infty, \quad |H(f)| \rightarrow 0, \quad \text{for } \left|\frac{f}{f_m}\right| > 1$$

Hence, $|H(f)|$ approaches the transfer characteristic of an ideal low-pass filter with a cut-off frequency at f_m , as n approaches infinity.

1.18 $y(t) = \delta(t) * h(t)$

$$Y(f) = 1 * H(f)$$

since $\mathcal{F}\{\delta(t)\} = 1$.

Hence, $y(t) = \mathcal{F}^{-1}\{Y(f)\} = \mathcal{F}^{-1}\{H(f)\}$
 $= h(t)$

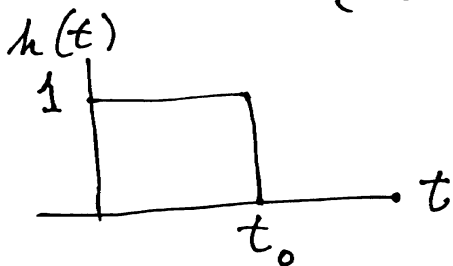
1.19 Let $x(t) = \delta(t)$

$$g(t) = \delta(t) - \delta(t - t_0)$$

$$h(t) = \int_{-\infty}^t [\delta(\tau) - \delta(\tau - t_0)] d\tau$$
$$= u(t) - u(t - t_0)$$

where $u(t)$ is the unit step function defined as follows:

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & \text{elsewhere} \end{cases}$$



1.20 (a) Half-power bandwidth is the bandwidth from half-power point to half-power point. $BW = 2f_0$

where $\frac{1}{2} = \left[\frac{\sin(\pi f_0 10^{-4})}{\pi f_0 10^{-4}} \right]^2$

$$0.707 = \frac{\sin \chi_0}{\chi_0}, \quad \chi_0 = \pi f_0 10^{-4}$$

$$\chi_0 \cong 1.4 \Rightarrow f_0 = 4.46 \text{ kHz} \Rightarrow BW \cong 9 \text{ kHz}$$

1.20 (b) Noise equivalent bandwidth

$$BW = 2 \int_0^{\infty} \left[\frac{\sin(\pi f 10^{-4})}{\pi f 10^{-4}} \right]^2 df$$

$$= \frac{2 \times 10^4}{\pi} \int_0^{\infty} \left[\frac{\sin x}{x} \right]^2 dx$$

$$= 10 \text{ kHz}$$

(c) Null-to-null bandwidth: $BW = 2f_0$
where f_0 is the frequency where

$$\frac{\sin(\pi f_0 10^{-4})}{\pi f_0 10^{-4}} = 0$$

The minimum f_0 corresponding to the first null is found by:

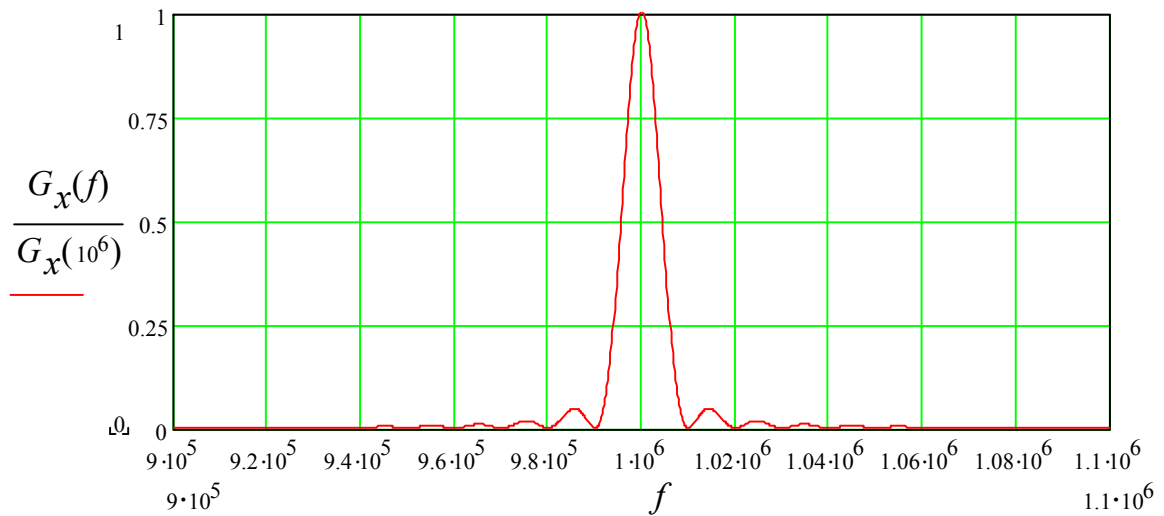
$$\pi f_0 10^{-4} = \pi$$

$$BW = 2f_0 = 20 \text{ kHz}$$

(d) 99% of power bandwidth, $BW = 2f_0$

$$\text{Where } 0.995 = \frac{10^{-4} \int_0^{f_0} \left(\frac{\sin \pi f 10^{-4}}{\pi f 10^{-4}} \right)^2 df}{10^{-4} \int_0^{\infty} \left(\frac{\sin \pi f 10^{-4}}{\pi f 10^{-4}} \right)^2 df}$$

1.20 (d) The normalized spectrum, $G_x(f)/G_x(10^6)$, appears as:



Applying numerical methods with Mathcad ®, the two-sided 99% bandwidth can be found by numerical integration as:

$$\frac{\int_{10^6-f_1}^{10^6+f_1} G_x(f) df}{\int_{-\infty}^{\infty} G_x(f) df} = 0.99$$

where f_1 is found to be equal to 103 kHz. Thus, the two-sided 99% bandwidth is equal to 206 kHz.

Since this bandwidth corresponds to the given spectrum (with signaling rate = 10,000 symbols/s), normalizing it relative to one symbol per second, yields the two-sided 99% bandwidth as $(206 \times 10^3 / 10^4) = 20.6$ Hz for one symbol per second, or in general the 99% bandwidth in terms of the signaling rate, R , is $20.6 \times R$ Hz.

1.20 (e)

35-dB Bandwidth:

$$35\text{-dB attenuation} \Rightarrow 10^{-3.5} = 3.16 \times 10^{-4}$$

Since $\sin^2 x$ is unity for $x = \frac{\pi}{2} (2k+1)$, $k=0, 1, \dots$, the lobe beyond which the attenuation criterion is guaranteed to be met is the minimum k for which

$$10^{-3.5} \geq \frac{1}{\left[\frac{\pi}{2} (2k+1)\right]^2}$$

$$3162.28 \leq \left[\frac{\pi}{2} (2k+1)\right]^2$$

$$56.23 \leq \frac{\pi}{2} (2k+1)$$

$$35.80 \leq 2k+1 \Rightarrow k = 18$$

Thus the 18th sidelobe meets the 35-dB criterion, and the actual 35-dB point will be on the falling edge of the 17th sidelobe. $BW = 2f_0$, where f_0 is the minimum value satisfying:

$$\pi f_0 10^{-4} > \frac{\pi}{2} (35)$$

$$\text{and } \left[\frac{\sin(\pi f_0 10^{-4})}{\pi f_0 10^{-4}} \right]^2 = 10^{-3.5}$$

Solving iteratively, we get: $\pi f_0 10^{-4} = 55.171$
 $BW = 2f_0 = 351.2 \text{ kHz}$

1.20 (f) The absolute bandwidth is infinite, since for any finite test-BW, $10^{-4} \left[\frac{\sin \pi (f-10^6) 10^{-4}}{\pi (f-10^6) 10^{-4}} \right]^2$ will have positive measure beyond it.

Chapter 2

Answers to Problems

The pages in Chapter 2 contain a mix of typeset and handwritten text. The handwritten text is comprised of updates and solutions new to this edition.

Chapter 2

2.1 (a) $\underbrace{00010010}_H, \underbrace{11110011}_O, \underbrace{11101011}_W$

(b) $\underbrace{000}_0, \underbrace{100}_4, \underbrace{101}_5, \underbrace{111}_7, \underbrace{001}_1, \underbrace{111}_7, \underbrace{101}_5, \underbrace{011}_3$

8 symbols

(c) $24 \text{ bits} / 4 \text{ bits per symbol} = 6 \text{ symbols}$

(d) $24 \text{ bits} / 8 \text{ bits per symbol} = 3 \text{ symbols}$

2.2 (a) $800 \text{ char/s} \times 8 \text{ bits/char} = 6400 \text{ bits/s}$

(b) $\frac{6400 \text{ bits/s}}{4 \text{ bits/symbol}} = 1600 \text{ symbols/s}$

2.3 (a) $100 \text{ char/2 s} \times 8 \text{ bits/char} = 400 \text{ bits/s}$

$\frac{400 \text{ bits/s}}{5 \text{ bits/symbol}} = 80 \text{ symbols/s}$

(b) 16-level PCM: $400 \text{ bits/s}, 100 \text{ symbols/s}$

8-level PCM: $400 \text{ bits/s}, 133.3 \text{ symbols/s}$

4-level PCM: $400 \text{ bits/s}, 200 \text{ symbols/s}$

2-level PCM: $400 \text{ bits/s}, 400 \text{ symbols/s}$