Solutions to Selected Exercises

Section 1.1

2. $\{2,4\}$ 3. $\{7,10\}$ 5. $\{1,3,5,7,9,10\}$ 6. $\{2,3,5,6,8,9\}$

8. B 9. $\varnothing$ 11. B 12. $\{1,4\}$ 14. $\{1,7,10\}$

15. $\{2,3,4,5,6,7,8,9,10\}$ 18. $\{n \in \mathbf{Z}^+ \mid n \geq 6\}$ 19. $\{2n-1 \mid n \in \mathbf{Z}^+\}$

21. $\{n \in \mathbf{Z}^+ \mid n \leq 5 \text{ or } n = 2m,\ m \geq 3\}$ 22. $\{2n \mid n \geq 3\}$ 24. $\{1,3,5\}$

25. $\{n \in \mathbf{Z}^+ \mid n \leq 5 \text{ or } n = 2m+1,\ m \geq 3\}$ 27. $\{n \in \mathbf{Z}^+ \mid n \geq 6 \text{ or } n = 2 \text{ or } n = 4\}$

29. 1 30. 3

32. If $x \in A$, then x is one of $1,\ 2,\ 3,\ 4$. Thus $x \in B$. If $x \in B$, then x is one of $4,\ 3,\ 2,\ 1$. Thus $x \in A$. Therefore, $A = B$.s

33. We find that $B = \{2,3\}$. Since A and B have the same elements, they are equal.

34. Let $x \in A$. Then $x = 1,2,3$. If $x = 1$, since $1 \in \mathbf{Z}^+$ and $1^2 < 10$, then $x \in B$. If $x = 2$, since $2 \in \mathbf{Z}^+$ and $2^2 < 10$, then $x \in B$. If $x = 3$, since $3 \in \mathbf{Z}^+$ and $3^2 < 10$, then $x \in B$. Thus if $x \in A$, then $x \in B$.

    Now suppose that $x \in B$. Then $x \in \mathbf{Z}^+$ and $x^2 < 10$. If $x \geq 4$, then $x^2 > 10$ and, for these values of x, $x \notin B$. Therefore $x = 1,2,3$. For each of these values, $x^2 < 10$ and x is indeed in B. Also, for each of the values $x = 1,2,3$, $x \in A$. Thus if $x \in B$, then $x \in A$. Therefore $A = B$.

37. Since $(2)^3 - 2(2)^2 + 4(2) - 8 = 0$, $2 \in B$. Since $2 \notin A$, $A \neq B$.

38. Since $3^2 - 1 > 3$, $3 \notin B$. Since $3 \in A$, $A \neq B$. 41. Equal 42. Equal

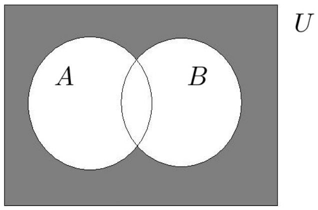
45. Let $x \in A$. Then $x = 1,2$. If $x = 1$,
$$x^3 - 6x^2 + 11x = 1^3 - 6 \cdot 1^2 + 11 \cdot 1 = 6.$$
    Thus $x \in B$. If $x = 2$,
$$x^3 - 6x^2 + 11x = 2^3 - 6 \cdot 2^2 + 11 \cdot 2 = 6.$$
    Again $x \in B$. Therefore $A \subseteq B$.

46. Let $x \in A$. Then $x = (1,1)$ or $x = (1,2)$. In either case, $x \in B$. Therefore $A \subseteq B$.

49. Since $2^3 - 2(2)^2 + 4(2) - 8 = 0$, $2 \in A$. However, $2 \notin B$. Therefore A is not a subset of B.

50. Consider 4, which is in A. If $4 \in B$, then $4 \in A$ and $4 + m = 8$ for some $m \in C$. However, the only value of m for which $4 + m = 8$ is $m = 4$ and $4 \notin C$. Therefore $4 \notin B$. Since $4 \in A$ and $4 \notin B$, A is not a subset of B.

53.



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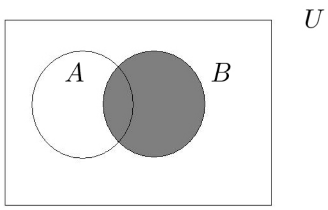

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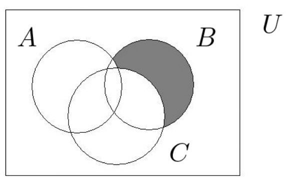

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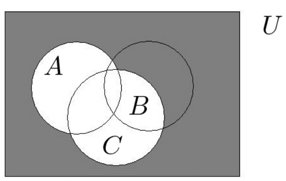

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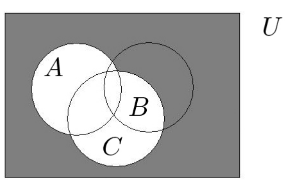
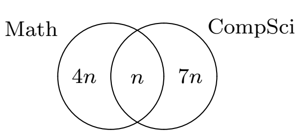


62. 35 63. 105 65. 51

67. Suppose that n students are taking both a mathematics course and a computer science course. Then $4n$ students are taking a mathematics course, but not a computer science course, and $7n$ students are taking a computer science course, but not a mathematics course. The following Venn diagram depicts the situation:



Thus, the total number of students is

$$4n + n + 7n = 12n.$$

The proportion taking a mathematics course is

$$\frac{5n}{12n} = \frac{5}{12},$$

which is greater than one-third.

69. $\{(a,1),(a,2),(b,1),(b,2),(c,1),(c,2)\}$

70. $\{(1,1),(1,2),(2,1),(2,2)\}$

73. $\{(\alpha,a,1),(\alpha,a,2),(\beta,a,1),(\beta,a,2)\}$

74. $\{(1,1,1),(1,2,1),(2,1,1),(2,2,1),(1,1,2),(1,2,2),(2,1,2),(2,2,2)\}$

77. Vertical lines (parallel) spaced one unit apart extending infinitely to the left and right.

79. Consider all points on a horizontal line one unit apart. Now copy these points by moving the horizontal line n units straight up and straight down for all integers $n > 0$. The set of all points obtained in this way is the set $\mathbf{Z} \times \mathbf{Z}$.

80. Ordinary 3-space

82. Take the lines described in the instructions for this set of exercises and copy them by moving n units out and back for all $n > 0$. The set of all points obtained in this way is the set $\mathbf{R} \times \mathbf{Z} \times \mathbf{Z}$.

84. $\{1,2\}$
    $\{1\},\{2\}$

85. $\{a,b,c\}$
    $\{a,b\},\{c\}$
    $\{a,c\},\{b\}$
    $\{b,c\},\{a\}$
    $\{a\},\{b\},\{c\}$

88. False

89. True

91. True

92. False

94. $\varnothing, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}$. All except $\{a,b,c,d\}$ are proper subsets.

95. $2^{12} = 4096$; $2^{12} - 1 = 4095$

98. $B \subseteq A$

99. $A = U$

102. The symmetric difference of two sets consists of the elements in one or the other but not both.

103. $A \triangle A = \varnothing$, $A \triangle \overline{A} = U$, $U \triangle A = \overline{A}$, $\varnothing \triangle A = A$

105. The set of primes

Section 1.2

2. Is a proposition. Negation: $6 + 9 \neq 15$.

3. Not a proposition

4. Is a proposition. Negation: $\pi \neq 3.14$.

6. Is a proposition. Negation: For every positive integer n, $19340 \neq n \cdot 17$.

7. Is a proposition. Negation: Phil Collins was not a member of Genesis.

9. Is a proposition. Negation: The line “Play it again, Sam” does not occur in the movie *Casablanca*.

10. Is a proposition. Some even integer greater than 4 is not the sum of two primes.

12. Not a proposition. The statement is neither true nor false.

13. No heads were obtained. 14. No tails were obtained.

15. No heads or no tails were obtained. 18. True

19. True 21. True 22. False

24.

p	q	$(\neg p \vee \neg q) \vee p$
T	T	T
T	F	T
F	T	T
F	F	T

25.

p	q	$(p \vee q) \wedge \neg p$
T	T	F
T	F	F
F	T	T
F	F	F

27.

p	q	$(p \wedge q) \vee (\neg p \vee \neg q)$
T	T	F
T	F	T
F	T	T
F	F	F

28.

p	q	r	$\neg(p \wedge q) \vee (r \wedge \neg p)$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

30.

p	q	r	$\neg(p \wedge q) \vee (\neg q \vee r)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

32. $\neg(q \wedge r)$. False. 33. $p \vee \neg(q \wedge r)$. True.

35. Lee takes computer science and mathematics.

36. Lee takes computer science or mathematics.

38. Lee takes computer science but not mathematics.

39. Lee takes neither computer science nor mathematics.

41. You do not miss the midterm exam and you pass the course.

42. You play football or you miss the midterm exam or you pass the course.

44. Either you play football and you miss the midterm exam or you do not miss the midterm exam and you pass the course.

46. It is not Monday and either it is raining or it is hot.

47. It is not the case that today is Monday or it is raining, and it is hot.

50. Today is Monday and either it is raining or it is hot, and it is hot or either it is raining or today is Monday.

51. $p \wedge q$ 52. $\neg p \wedge q$ 54. $p \vee q$

55. $(p \vee q) \wedge \neg p$ 56. $\neg p \wedge q$ 57. $p \wedge r \wedge q$

58. $(p \vee r) \wedge q$ 60. $(q \vee \neg p) \wedge \neg r$ 62. $p \wedge \neg r$ 63. $p \wedge q \wedge r$

65. $\neg p \wedge \neg q \wedge r$ 66. $\neg(p \vee q \vee \neg r)$

67.

p	q	p *exor* q
T	T	F
T	F	T
F	T	T
F	F	F

69. Inclusive-or 70. Inclusive-or 72. Exclusive-or 73. Exclusive-or

76. ”concert venue” ”north london” OR ”south london”

77. ”lung disease” -cancer

78. ”european union” country - ”schengen zone”

Section 1.3

2. If Rosa has 160 quarter-hours of credits, then she may graduate.

3. If Fernando buys a computer, then he obtains \$2000.

5. If a person gets that job, then that person knows someone who knows the boss.

6. If you can travel to your dream destination, then you can afford the travel expenses.

8. If a better car is built, then Buick will build it.

9. If the chairperson gives the lecture, then the audience will go to sleep.

11. If the switch is not turned properly, then the light will not be on.

13. Contrapositive of Exercise 2: If Rosa does not graduate, then she does not have 160 quarter-hours of credits.

15. False 16. True 18. False 19. True 21. True 22. True

24. Unknown 25. Unknown 27. True 28. Unknown 30. Unknown

31. Unknown 34. True 35. True 37. True 38. False

40. False 41. True 44. $(q \wedge r) \rightarrow p$ 45. $\neg((r \wedge \neg q) \rightarrow r)$

48. $(\neg p \vee \neg r) \rightarrow \neg q$ 49. $r \rightarrow q$ 51. $q \rightarrow (p \vee r)$ 52. $(p \wedge r) \rightarrow q$

54. If today is Monday, then it is raining and it is not hot.

55. If today is not Monday, then either it is raining or it is hot.

57. If today is Monday and either it is raining or it is hot, then either it is hot, it is raining, or today is Monday.

58. If today is Monday or (it is not Monday and it is not the case that (it is raining or it is hot)), then either today is Monday or it is not the case that (it is hot or it is raining).

60. Let p: $4 > 6$ and q: $9 > 12$. Given statement: $p \rightarrow q$; true. Converse: $q \rightarrow p$; if $9 > 12$, then $4 > 6$; true. Contrapositive: $\neg q \rightarrow \neg p$; if $9 \leq 12$, then $4 \leq 6$; true.

61. Let p: $|1| < 3$ and q: $-3 < 1 < 3$. Given statement: $q \rightarrow p$; true. Converse: $p \rightarrow q$; if $|1| < 3$, then $-3 < 1 < 3$; true. Contrapositive: $\neg p \rightarrow \neg q$; if $|1| \geq 3$, then either $-3 \geq 1$ or $1 \geq 3$; true.

64. $P \not\equiv Q$ 65. $P \equiv Q$ 67. $P \not\equiv Q$ 68. $P \equiv Q$ 70. $P \not\equiv Q$

71. $P \not\equiv Q$

74. Kate will not eat an apple and not drink a glass of water.

75. Either Martin will not eat a banana or not drink a glass of milk.

78. (a) If p and q are both false, $(p \ imp2 \ q) \wedge (q \ imp2 \ p)$ is false, but $p \leftrightarrow q$ is true.
    (b) Making the suggested change does not alter the last line of the *imp2* table.

79.

p	q	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

Section 1.4

2. Invalid

$$\begin{array}{l} p \rightarrow q \\ \neg r \rightarrow \neg q \\ \hline \therefore r \end{array}$$

3. Valid

$$\begin{array}{l} p \leftrightarrow r \\ r \\ \hline \therefore p \end{array}$$

5. Valid

$$\begin{array}{l} p \to (q \lor r) \\ \neg q \land \neg r \\ \hline \therefore \neg p \end{array}$$

7. Valid

$$\begin{array}{l} (p \lor q) \to (r \lor s) \\ p \land \neg r \\ \hline \therefore s \end{array}$$

8. Invalid

$$\begin{array}{l} p \to r \\ q \to s \\ \neg(q \land p) \\ \neg p \\ \hline \therefore s \end{array}$$

11. If 4 megabytes of memory is better than no memory at all, then either we will buy a new computer or we will buy more memory. If we will buy a new computer, then we will not buy more memory. Therefore if 4 megabytes of memory is better than no memory at all, then we will buy a new computer. Invalid.

12. If 4 megabytes of memory is better than no memory at all, then we will buy a new computer. If we will buy a new computer, then we will buy more memory. Therefore, we will buy more memory. Invalid.

14. If 4 megabytes of memory is better than no memory at all, then we will buy a new computer. If we will buy a new computer, then we will buy more memory. 4 megabytes of memory is better than no memory at all. Therefore we will buy more memory. Valid.

16. If the hardware is unreliable or the output is correct, then the while loop is not faulty. If the output is correct, then the while loop is faulty. Either the for loop is faulty or the output is correct. Therefore the hardware is unreliable. Invalid.

17. If, if the for loop is faulty, then the hardware is unreliable, then the while loop is faulty. If, if the while loop is faulty, then the output is correct, then the for loop is faulty. The hardware is unreliable and the output is correct. Either the for loop is faulty or the while loop is faulty. Therefore the for loop is faulty and the while loop is faulty. Invalid.

19. If the for loop is faulty, then the while loop is faulty or the hardware is unreliable. If the while loop is faulty, then the for loop is faulty or the output is correct. Either the for loop is faulty or the while loop is not faulty. The output is not correct. Therefore the for loop is faulty or the hardware is unreliable. Invalid.

21. Valid 22. Valid 24. Valid

25. Suppose that $p_1, p_2, \ldots, p_n$ are all true. Since the argument $p_1, p_2 \ / \therefore p$ is valid, p is true. Since $p, p_3, \ldots, p_n$ are all true and the argument

$$p, p_3, \ldots, p_n \ / \therefore c$$

is valid, c is true. Therefore the argument

$$p_1, p_2, \ldots, p_n \ / \therefore c$$

is valid.

28. Disjunctive syllogism 29. Addition

31. Let p denote the proposition "there is gas in the car," let q denote the proposition "I go to the store," let r denote the proposition "I get a soda," and let s denote the proposition "the car transmission is defective." Then the hypotheses are:

$$p \to q, \ \ q \to r, \ \ \neg r.$$

From $p \to q$ and $q \to r$, we may use the hypothetical syllogism to conclude $p \to r$. From $p \to r$ and $\neg r$, we may use modus tollens to conclude $\neg p$. From $\neg p$, we may use addition to conclude $\neg p \vee s$. Since $\neg p \vee s$ represents the proposition "there is not gas in the car or the car transmission is defective," we conclude that the conclusion does follow from the hypotheses.

32. Let p denote the proposition "Jill can sing," let q denote the proposition "Dweezle can play," let r denote the proposition "I'll buy the compact disk," and let s denote the proposition "I'll buy the compact disk player." Then the hypotheses are:

$$(p \vee q) \to r, \ \ p, \quad s.$$

From p, we may use addition to conclude $p \vee q$. From $p \vee q$ and $(p \vee q) \to r$, we may use modus ponens to conclude r. From r and s, we may use conjunction to conclude $r \wedge s$. Since $r \wedge s$ represents the proposition "I'll buy the compact disk and the compact disk player," we conclude that the conclusion does follow from the hypotheses.

34. The truth table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

shows that whenever p is true, $p \vee q$ is also true. Therefore addition is a valid argument.

35. The truth table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

shows that whenever $p \wedge q$ is true, p is also true. Therefore simplification is a valid argument.

37. The truth table

p	q	r	$p \to q$	$q \to r$	$p \to r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

shows that whenever $p \to q$ and $q \to r$ are true, $p \to r$ is also true. Therefore hypothetical syllogism is a valid argument.

38. The truth table

p	q	$p \vee q$	$\neg p$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	T

shows that whenever $p \vee q$ and $\neg p$ are true, q is also true. Therefore disjunctive syllogism is a valid argument.

Section 1.5

2. The statement is a command, not a propositional function.

3. The statement is not a propositional function since it has no variables.

5. The statement is a command, not a propositional function.

6. The statement is a propositional function. The domain of discourse is the set of real numbers.

8. 1 divides 77. True.

9. 5 divides 77. False.

11. For some n, n divides 77. True.

12. For every n, n does not divide 77. False.

14. It is false that for every n, n divides 77. True.

15. It is false that for some n, n divides 77. False.

17. True

18. False

20. True

21. True

23. False

24. True

26. $\neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$

27. $\neg(P(1) \wedge P(2) \wedge P(3) \wedge P(4))$

29. $\neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$

30. $\neg(P(1) \vee P(2) \vee P(3) \vee P(4))$

33. Some student is taking a math course.

34. Every student is not taking a math course.

36. It is not the case that every student is taking a math course.

37. It is not the case that some student is taking a math course.

40. There is some person such that if the person is a professional athlete, then the person plays soccer. True.

41. Every soccer player is a professional athlete. False.

43. Every person is either a professional athlete or a soccer player. False.

44. Someone is either a professional athlete or a soccer player. True.

46. Someone is a professional athlete and a soccer player. True.

49. $\exists x(P(x) \wedge Q(x))$

50. $\forall x(Q(x) \to P(x))$

54. False 55. False 57. False 58. True

60. No. The suggested replacement returns false if $\neg P(d_1)$ is true, and true if $\neg P(d_1)$ is false.

62. Literal meaning: Every old thing does not covet a twenty-something. Intended meaning: Some old thing does not covet a twenty-something. Let $P(x)$ denote the statement "x is an old thing" and $Q(x)$ denote the statement "x covets a twenty-something." The intended statement is $\exists x(P(x) \wedge \neg Q(x))$.

63. Literal meaning: Every hospital did not attend every lecture. (Domain of discourse: the 63 students.) Intended meaning (most likely): Some students did not attend every lecture. Let $P(x)$ denote the statement "x is a student" and $Q(x)$ denote the statement "x attends every lecture." The intended statement is $\exists x(P(x) \wedge \neg Q(x))$.

65. Literal meaning: Everyone does not have a degree. (Domain of discourse: People in Door County.) Intended meaning: Someone does not have a degree. Let $P(x)$ denote the statement "x has a degree." The intended statement is $\exists x \neg P(x)$.

66. Literal meaning: No lampshade can be cleaned. Intended meaning: Some lampshade cannot be cleaned. Let $P(x)$ denote the statement "x is a lampshade" and $Q(x)$ denote the statement "x can be cleaned." The intended statement is $\exists x(P(x) \wedge \neg Q(x))$.

68. Literal meaning: No person can afford a home. Intended meaning: Some person cannot afford a home. Let $P(x)$ denote the statement "x is a person" and $Q(x)$ denote the statement "x can afford a home." The intended statement is $\exists x(P(x) \wedge \neg Q(x))$.

69. The literal meaning is as Mr. Bush spoke. He probably meant: Someone in this country doesn't agree with the decisions I've made. Let $P(x)$ denote the statement "x agrees with the decisions I've made." Symbolically, the clarified statement is $\exists x \, \neg P(x)$.

71. Literal meaning: Every move does not work out. Intended meaning: Some move does not work out. Let $P(x)$ denote the statement "x is a move" and $Q(x)$ denote the statement "x works out ." The intended statement is $\exists x(P(x) \wedge \neg Q(x))$.

74. Let

$$\begin{aligned} p(x)&: \quad x \text{ is good.} \\ q(x)&: \quad x \text{ is too long.} \\ r(x)&: \quad x \text{ is short enough.} \end{aligned}$$

The domain of discourse is the set of movies. The assertions are

$$\begin{array}{l} \forall x(p(x) \to \neg q(x)) \\ \forall x(\neg p(x) \to \neg r(x)) \\ p(\textit{Love Actually}) \\ q(\textit{Love Actually}). \end{array}$$

By universal instantiation,

$$p(\textit{Love Actually}) \to \neg q(\textit{Love Actually}).$$

Since $p(\textit{Love Actually})$ is true, then $\neg q(\textit{Love Actually})$ is also true. But this contradicts, $q(\textit{Love Actually})$.

77. Let $P(x)$ denote the propositional function "x is a member of the Titans," let $Q(x)$ denote the propositional function "x can hit the ball a long way," and let $R(x)$ denote the propositional function "x can make a lot of money." The hypotheses are