Chapter 1

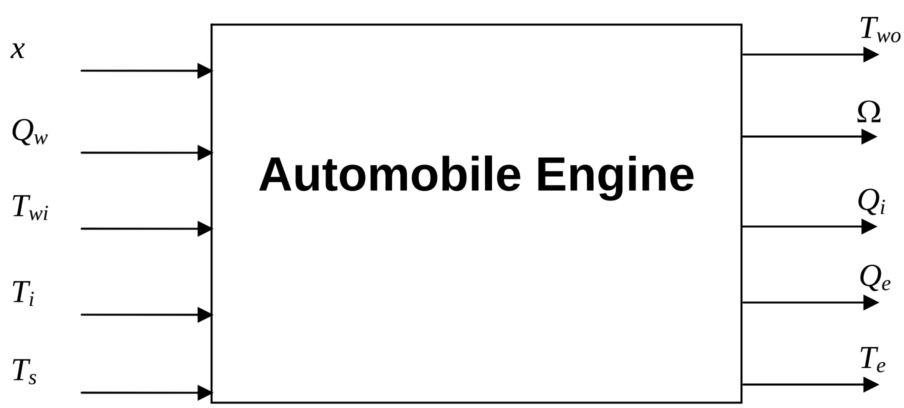
Author's notes:

The systems shown in the end-of-chapter problems in Chapter one are meant to be representative of real-world engineering systems. As such, the definition of input and output might be open to some interpretation and other instructors, indeed other students, may have different and defensible interpretations than those laid out here. Please consider these to be guides at best.

In addition, we arranged the problems such that the first four problems are well-defined, with all interesting variables called out and labeled. The latter problems, on the other hand, are less structured, requiring more judgment and, hopefully, discussion among the readers.

JFG

Problem 1.1

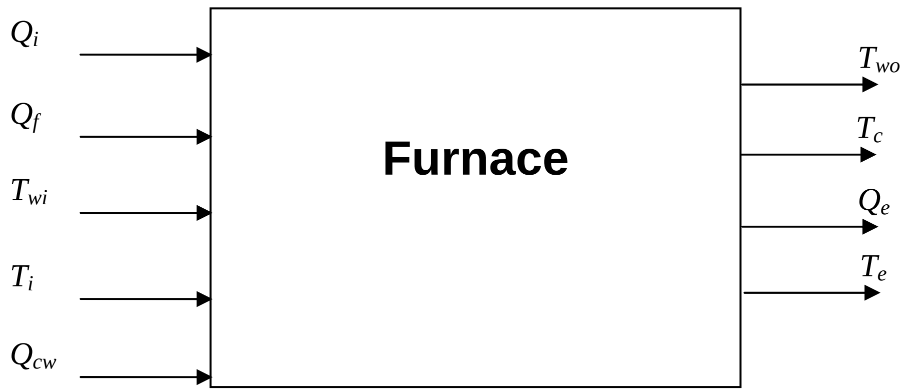


Note: Depending on the model of the system connected to the output shaft (i.e. the clutch and/or transmission), the torque on the output shaft might be better considered as an output.

Problem 1.2

Note: Again, the true causality of this system cannot be determined without more information about the rest of the system. For example, a case can be made that the shaft torque is an input, being determined from the best model while the generated current is an output. However, we show this causality because it reflects the more common case in which the current is determined by the electrical load and the shaft torque will be whatever it takes to produce that current.

Problem 1.3



Problem 1.4

There are three independent energy storing elements in this system. The crown gear, storing kinetic energy due to its inertia, the beaters (not independent) which also store kinetic energy and the beater shafts, which store energy in their compliance.

Their energy equations are:

$$E_k = \frac{1}{2} J_c \Omega_c^2$$

$$E_k = 2\left(\frac{1}{2} J_b \Omega_b^2\right)$$

where J_b is the inertia of one beater.

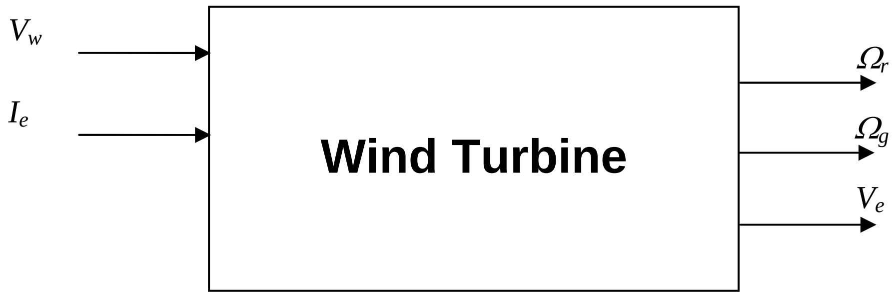
$$E_p = 2\left(\frac{1}{2} K_t (\theta_c - \theta_b)^2\right)$$

and Kt is the torsional stiffness of one beater shaft

It would be reasonable to assume Ti as the input and the beater speed as an output. The work being done on the eggs would be modeled as an algebraic equation relating the torque ‘felt’ by the beaters to the speed of the beaters.

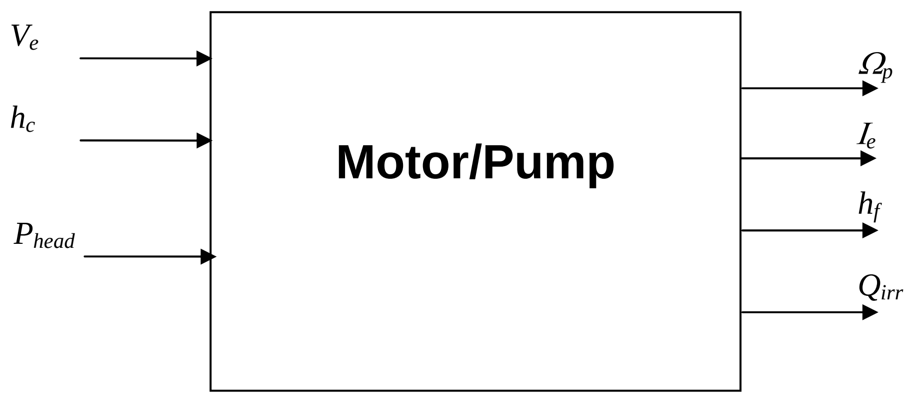
Problem 1.5

Part (a)



The obvious energy storing element in this sub-model is the inertia of the rotor itself. Those more familiar with the operations of a modern wind turbine and/or mechanical transmissions, would recognize that there is some (not much) inherent compliance in the shafts and gears that make up a drivetrain. The model of a multi-gear drivetrain can be as complicated or as simple as we desire. For the time being, we'll leave it at this.

Part (b)



Note that I include the canyon height (or depth, depending on you perspective) as an input to the system. Strictly speaking, this is a parameter (like mass, or spring rate) but I include it to make it clear that the elements that determine the load against which you pump are often determined external to the problem. In this respect, the pressure head and canyon height are closely related.

Part (c)

When you combine the two subsystems, you have a different perspective and many of the variables become internal to the model and neither inputs nor outputs.

Finally, here's a table of what I believe to be the most important energy storing elements. As I said at the opening of this chapter, your mileage may vary.

Element	Type	Parameter	Comments
Rotor Inertia	A-Type	Rotary inertia of element	Dominant inertia in the turbine (by far)
Pump/motor Inertia	A-Type	Combined inertia of pump and motor	
Motor Windings	T-Type	Inductance of motor windings	Motors are highly inductive loads
Storage tank	A-Type	Fluid capacitance of storage tank	Capacitive element, potential energy of fluid

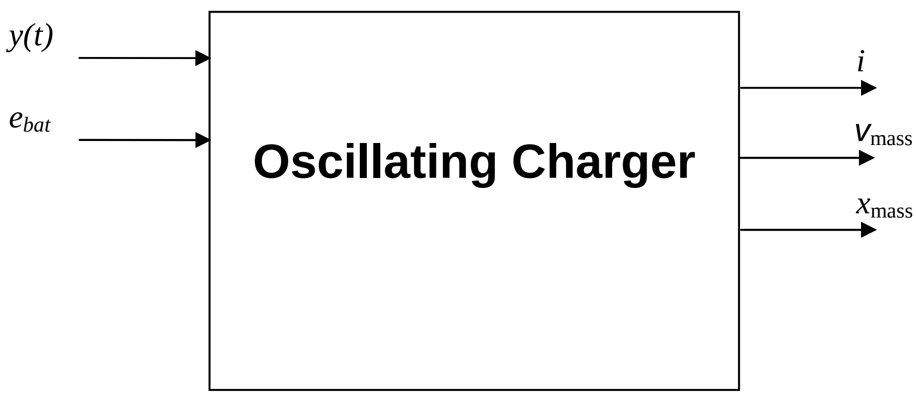
Problem 1.6

The obvious energy storing element in the artificial heart is the inertia of the motor rotor and two pusherplates, all of which are constrained to move together. Not so clear are the two fluid compliance elements embodied in the two blood sacs. A more thorough model of the heart might include the inductance of the DC motor windings, but the dynamics associated with the change in current are likely to be much faster than the mechanical and fluid dynamics.

Problem 1.7

The energy storing elements are:

- mass, *m*
- spring, *k*
- coil inductance, *L*
- and, of course, the battery



Problem 1.8

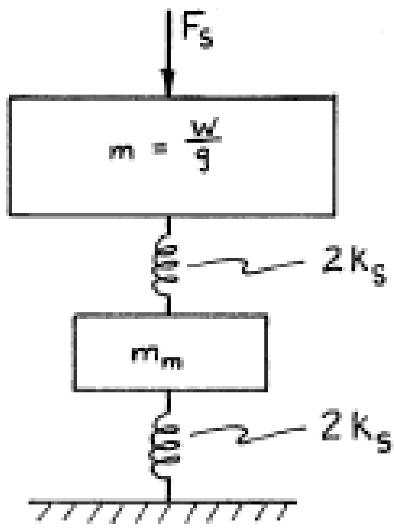
The point of this problem is to focus on the concept of independent energy storing elements. In the case where the wheelchair is moving in a straight line, all the inertial elements described move together through kinematic constraints. Therefore, only one variable is needed to describe the energy. On the other hand, as the wheelchair is turning, then the two wheels (and their driving components, are not turning at the same speed and two independent elements are seen. More subtly, when the wheel chair is turning, the mass of the chair has energy from two different motions, the linear velocity and the rotational velocity about a vertical axis.

Again, the focus here should be on the thought process of identifying energy storing elements, and the variables that go with them.

CHAPTER 2

PROBLEM 2.1

a) Since the attached mass m is only five times the self-mass of the spring and the frequency of excitation may result in self-vibration of the spring, a massless spring model for the heavy spring is not suitable. Since the excitation frequency is less than the lowest natural frequency of the distributed-parameter model of the spring, the three-ideal-element model shown below should approximate its behavior in the 'clamped-clamped' mode (i.e. the mode in which it is constrained displacement-wise at both of its ends).



Here m_m is the mass which results in a natural frequency within the lumped-parameter model of the heavy spring which is equal to the lowest natural frequency of self-vibration of the distributed-parameter model of the spring.

This model will behave at very low frequencies like a pure spring have net stiffness k_s (since $1/k_s = 1/(2k_s) + 1/(2k_s)$), and at increasing frequencies of excitation it will begin to vibrate within itself in a manner very similar to that of the distributed-parameter model until the excitation frequency approaches the lowest natural frequency in the 'clamped-clamped' mode.

From Marks' Handbook, p. 5-75, the natural frequencies of the distributed-parameter model in the 'clamped-clamped' mode are given by:

$$f_n = c_n \left[\frac{gAE}{wl^2} \right]^{.5} , \quad c_n = \frac{1}{2n} , \quad n = 1, 2, 3, \ldots$$

For the lowest frequency mode, $n = 1$, so that

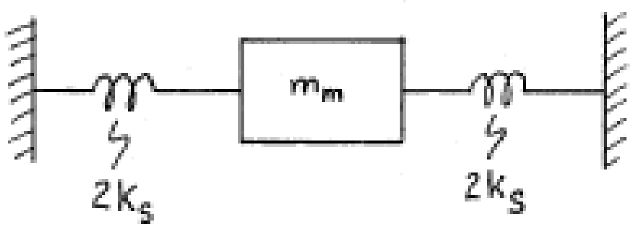
$$(f_n)_{distrib} = .5(k_s m_s)^{.5}$$

where $k_s = AE/l$ = self-stiffness of spring

$m_s = wl/g$ = self-mass of spring

A = spring rod area
E = modulus of elasticity
l = spring rod length
w = weight per unit length of rod
g = acceleration of gravity

From vibration theory the natural frequency of the three-lumped-element model:



is given by:

$$(f_n)_{lumped} = \omega_n/2\pi = (1/2\pi)(4k_s/m_m)^{.5}$$

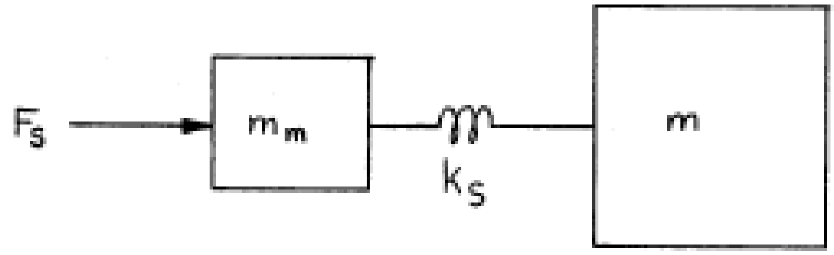
Equating $(f_n)_{distrib}$ and $(f_n)_{lumped}$ yields:

$$(1/2)(k_s/m_s)^{.5} = (1/\pi)(k_s/m_m)^{.5}$$

or

$$m_m = (4/\pi^2)m_s \approx 0.4\ m_s$$

b) Here the 'free-clamped' mode (left end constrained force-wise and right end constrained displacement-wise) of self-vibration for the distributed model is indicated, approximated by the following lumped parameter model:



This model will behave as a pure spring having stiffness k_s at very low excitation frequencies because the portion of the transmitted force F_s required to accelerate the mass m_m will be negligible, and at increasing excitation frequencies, it will exhibit a self-vibration very similar to that of the distributed-parameter model until the excitation frequency approaches the lowest natural frequency in the 'free-clamped' mode of vibration for the distributed parameter model.

From <u>Marks' Handbook</u> again,

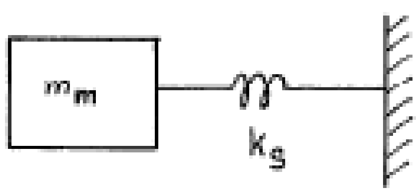
$$f_n = C_n \left[\frac{gAE}{wl^2} \right]^{.5}, \qquad C_n = \frac{2n - 1}{4}$$

so that the lowest frequency mode, $n = 1$, and

$$(f_n)_{distrib} = (1/4)(k_s/m_s)^{.5}$$

where k_s = self-stiffness of the spring
m_s = self-mass of the spring

From vibration theory the natural frequency for the two-lumped-element model:



is given by:

$$(f_n)_{lumped} = \omega_n/2\pi = (1/2\pi)(k_s/m_m)^{.5}$$

Equating $(f_n)_{distrib}$ with $(f_n)_{lumped}$

$$(1/4)(k_s/m_s)^{.5} = (1/2\pi)(k_s/m_m)^{.5}$$

which is then solved for m_m giving,

$$m_m = (2/\pi)^2 m_s = 0.4m_s$$