

Chapter 11

11.1

(a) $\frac{30}{1.6} = 18.76 \text{ kg}$

(b) $W = mg = 18.75(9.8) = 183.75 \text{ N}$

11.2

$$W_{S1} = \rho g V = 7850(9.81)\pi(0.06^2)(0.120) = 104.51 \text{ N}$$

11.3

(a) $100 \text{ kN/m}^2 = \frac{100 \times 10^3 \text{ N}}{\text{m}^2} \times \frac{0.2248 \text{ lb}}{1.0 \text{ N}} \times \frac{1.0 \text{ m}^2}{1550 \text{ in}^2} = 14.50 \text{ lb/in}^2$

(b) $30 \text{ m/s} = \frac{30 \cancel{\text{ m}}}{\text{s}} \times \frac{\text{km}}{1000 \cancel{\text{ m}}} \times \frac{3600 \text{ s}}{1 \text{ h}} = 1080.24 \text{ km/h}$

(c) $800 \text{ slugs} = 800 \text{ slugs} \times \frac{14.593 \text{ kg}}{1.0 \text{ slug}} = 11.67 \times 10^3 \text{ kg} = 11.67 \text{ Mg}$

(d) $\frac{20 \text{ lb}}{\text{ft}^2} \times \frac{4.448 \text{ N}}{1.0 \text{ lb}} \times \frac{1.0 \text{ ft}^2}{0.09290304 \text{ m}^2} = 958 \text{ N/m}^2$

11.4

Second law

$$F = ma$$

Law of gravitation

$$F = \frac{Gm_A m_B}{R^2}$$

$$ma = \frac{Gm_A m_B}{R^2}$$

$$[M] \left[\frac{L}{T^2} \right] = \frac{[G][M][M]}{[L^2]}$$

$$[G] = \left[\frac{L^3}{MT^2} \right]$$

$$= \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$$

11.5

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}mk^2\omega^2$$

Since the dimensions each term must be the same, we have

$$[KE] = [M] \left[\frac{L^2}{T^2} \right] = [M] [k^2] \left[\frac{1}{T^2} \right]$$

Therefore,

$$[k] = [L]$$

In the SI system

$$[KE] = [M] \left[\frac{L^2}{T^2} \right] = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \quad \blacktriangleleft \quad [k] = \text{m} \quad \blacktriangleleft$$

11.6

$$[g][k][x] \left[\frac{1}{W} \right] = \left[\frac{L}{T^2} \right] \left[\frac{F}{L} \right] [L] \left[\frac{1}{F} \right] = \left[\frac{L}{T^2} \right] = [\alpha] \quad \text{Q.E.D.}$$

11.7

$$[F] = [k][x] \quad \left[\frac{ML}{T^2} \right] = [k][L] \quad [k] = \left[\frac{M}{T^2} \right] \quad \blacktriangleleft$$

11.8

$$(a) \quad [mv^2] = \left[\frac{FT^2}{L} \right] \left[\frac{L^2}{T^2} \right] = [FL] \quad \blacktriangleleft$$

$$(b) \quad [mv] = \left[\frac{FT^2}{L} \right] \left[\frac{L}{T} \right] = [FT] \quad \blacktriangleleft$$

$$(c) \quad [ma] = \left[\frac{FT^2}{L} \right] \left[\frac{L}{T^2} \right] = [F] \quad \blacktriangleleft$$

11.9

Rewrite the equation as $y = 1.0 x^2$

$$[y] = [1.0] [x^2] \quad [L] = [1.0] [L^2] \quad [1.0] = \left[\frac{1}{L} \right]$$

$y = x^2$ can be dimensionally correct only if the units of the implied constant 1.0 are mm^{-1} .

11.10

$$(a) [I] = [mR^2] = \left[\frac{FT^2}{L} \right] [L^2] = [FLT^2] \blacktriangleleft$$

$$(b) [I] = [mR^2] = [ML^2] \blacktriangleleft$$

11.11

$$(a) [v^3] = [A][x^2] + [B][v][t^2] \quad \left[\frac{L^3}{T^3} \right] = [A][L^2] + [B] \left[\frac{L}{T} \right] [T^2]$$

$$[A] = \left[\frac{L}{T^3} \right] \blacktriangleleft \quad [B] = \left[\frac{L^2}{T^4} \right] \blacktriangleleft$$

$$(b) [x^2] = [A][t^2] \left[e^{[B][t^2]} \right] \quad [L^2] = [A][T^2][1] \quad [B][T^2] = [1]$$

$$[A] = \left[\frac{L^2}{T^2} \right] \blacktriangleleft \quad [B] = \left[\frac{1}{T^2} \right] \blacktriangleleft$$

11.12

$$m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx = P_0 \sin \omega t$$

$$[m] \left[\frac{d^2 x}{dt^2} \right] = \left[\frac{FT^2}{L} \right] \left[\frac{L}{T^2} \right] = [F]$$

Therefore, the dimension of each term in the expression is $[F]$.

$$+ [c] \left[\frac{dx}{dt} \right] = [c] \left[\frac{L}{T} \right] = [F] \quad [c] = \left[\frac{FT}{L} \right] \blacktriangleleft$$

$$[k][x] = [k][L] = [F] \quad [k] = \left[\frac{F}{L} \right] \blacktriangleleft$$

$$[P_0][\sin \omega t] = [P_0][1] = [F] \quad [P_0] = [F] \blacktriangleleft$$

$$[\omega][t] = [\omega][T] = [1] \quad [\omega] = \left[\frac{1}{T} \right] \blacktriangleleft$$

11.13

$$F = G \frac{m_A m_B}{R^2} \quad G = \frac{FR^2}{m_A m_b} \quad [G] = \frac{[F][L^2]}{[M^2]}$$

$$(a) [G] = \frac{[F][L^2]}{[FT^2/L]^2} = \left[\frac{L^4}{FT^4} \right] \blacktriangleleft$$

$$(b) [G] = \frac{[ML/T^2][L^2]}{[M^2]} = \left[\frac{L^3}{MT^2} \right] \blacktriangleleft$$

11.14

$$(a) [E] = [m][c^2] = \left[\frac{FT^2}{L} \right] \left[\frac{L^2}{T^2} \right] = [FL] \blacktriangleleft$$

$$(b) [E] = [m][c^2] = [M] \left[\frac{L^2}{T^2} \right] = \left[\frac{ML^2}{T^2} \right] \blacktriangleleft$$

11.15

$$F = G \frac{m^2}{R^2} = 6.67 \times 10^{-11} \frac{10^2}{0.5^2} = 2.668 \times 10^{-8} \text{ N}$$

$$W = mg = 10(9.81) = 98.1 \text{ N}$$

$$\frac{F}{W} \times 100\% = \frac{2.668 \times 10^{-8}}{98.1} 100\% = 2.72 \times 10^{-8} \% \blacktriangleleft$$

11.16

$$F = G \frac{m^2}{R^2} = \frac{6.67 \times 10^{-11} \times (3)^2}{(1)^2} = 6 \times 10^{10} \text{ N}$$

11.17

$$m = \frac{WR^2}{GM_e} = \frac{2000(6378 + 1800)^2 \times 10^6}{(6.67 \times 10^{-11})(5.9742 \times 10^{24})} = 336 \text{ kg} \blacktriangleleft$$

11.18

$$g_m = \frac{GM_m}{R_m^2} \quad g_e = \frac{GM_e}{R_e^2}$$

$$\frac{g_m}{g_e} = \frac{M_m}{M_e} \left(\frac{R_e}{R_m} \right)^2 = \frac{0.07348}{5.9742} \left(\frac{6378}{1737} \right)^2 = 0.1658 \approx \frac{1}{6} \text{ Q.E.D}$$

11.19

$$M_e = 5.9742 \times 10^{24} \text{ kg}$$

$$R_e = 6378 \times 10^3 \text{ m}$$

$$W = G \frac{M_e m}{(2R_e)^2} = \frac{6.67 \times 10^{-11} \times (5.9742 \times 10^{24}) \times \frac{150}{9.8}}{(5378 \times 10^3)^2} = 37.5 \text{ N}$$

11.20

$$F = G \frac{M_e m}{R^2} = 6.67 \times 10^{-11} \frac{(1.9891 \times 10^{30})(1.0)}{(149.6 \times 10^9)^2} = 0.00593 \text{ N} \blacktriangleleft$$

11.21

$$G \frac{M_e m}{r^2} = G \frac{M_s m}{(R-r)^2}$$

r = distance from the earth

R = distance between the earth and the sun

$$\frac{M_e}{r^2} = \frac{M_s}{(R-r)^2}$$

$$\frac{M_e}{M_s} = \frac{r^2}{R^2 - 2Rr + r^2}$$

$$\frac{5.9742 \times 10^{24}}{1.9891 \times 10^{30}} = \frac{r^2}{(149.6 \times 10^9)^2 - 2(149.6 \times 10^9)r + r^2}$$

$$0 = 2.238 \times 10^{22} - 2.992 \times 10^{11}r - 3.3294 \times 10^5 r^2$$

$$r = 259 \times 10^6 \text{ m} = 259 \times 10^3 \text{ km} \quad \blacktriangleleft$$

Chapter 12

12.1

$$y = -0.13t^4 + 4.1t^3 + 0.12t^2 \text{ m}$$

$$v = \dot{y} = -0.52t^3 + 12.3t^2 + 0.24t \text{ m/s}$$

$$a = \dot{v} = -1.56t^2 + 24.6t + 0.24 \text{ m/s}^2$$

At maximum velocity:

$$a = 0 \quad -1.56t^2 + 24.6t + 0.24 = 0 \quad t = 15.779 \text{ s}$$

$$\begin{aligned} v_{\max} &= -0.52(15.779)^3 + 12.3(15.779)^2 + 0.24(15.779) \\ &= 1023 \text{ m/s} \end{aligned}$$

$$y = -0.13(15.779)^4 + 4.1(15.779)^3 + 0.12(15.779)^2 = 8080 \text{ m}$$

12.2

(a) $x = -\frac{1}{2}gt^2 + v_0t \quad \therefore v = \dot{x} = -gt + v_0 \quad \therefore a = \ddot{x} = -g$

When $t=0$, then $x=0$ and $v=v_0$. Hence v_0 is the initial velocity.

Since gravity is the only source of acceleration in this problem, g must be the gravitational acceleration.

(b) When $x = x_{\max}$, then $v = 0$. $\therefore -gt + v_0 = 0 \quad \therefore t = \frac{v_0}{g}$

$$\therefore x_{\max} = -\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\left(\frac{v_0}{g}\right) = \frac{v_0^2}{2g}$$

At the end of the flight $x = 0 \quad \therefore -\frac{1}{2}gt^2 + v_0t = 0 \quad \therefore t = \frac{2v_0}{g}$

(c) $v_0 = \frac{(25)^2}{2(9.8)} = 31.89 \text{ m} \quad \therefore t = \frac{2(25)}{9.8} = 5.1 \text{ s}$

12.3

(a) $x = t^3 - 108t$ m
 $v = \dot{x} = 3t^2 - 108$ m/s
 $a = \ddot{x} = 6t$ m/s²

When $t = 0$:

$x = 0$ $v = -108$ m/s $a = 0$

When $t = 10$ s:

$x = (10)^3 - 108(10) = -80.0$ m

$v = 3(10)^2 - 108 = 92.0$ m/s

$a = 6(10) = 60$ m/s²

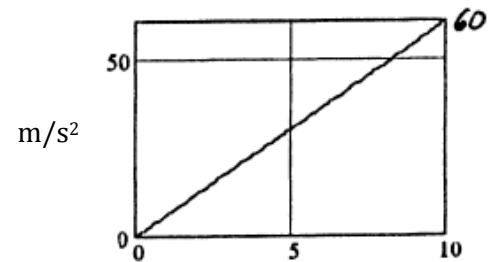
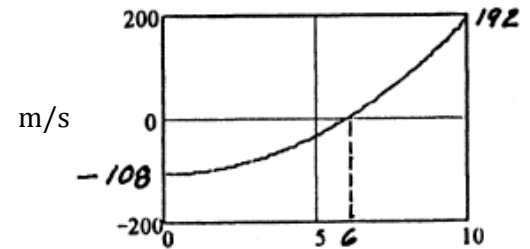
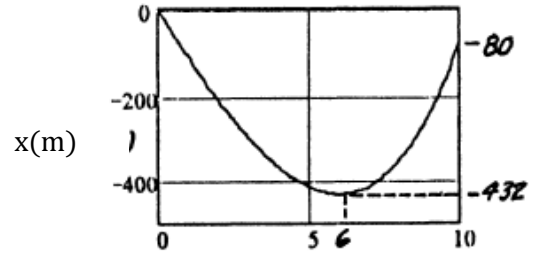
When $v = 0$ ($x = x_{\min}$ or x_{\max}):

$3t^2 - 108 = 0 \quad \therefore t = 6$ s

$\therefore x = (6)^3 - 108(6) = -432.0$ m

(b) Displacement: $\Delta r = 80.0$ m

(c) Distance travelled: $d = 2(432) - 80 = 784$ m



12.4

$x = t^3 - 3t^2 - 45t$ m

$v = \dot{x} = 3t^2 - 6t - 45$ m/s

$a = \dot{v} = 6t - 6$ m/s²

At $t = 8$ s:

$x = 8^3 - 3(8)^2 - 45(8) = -40$ m

$v = 3(8)^2 - 6(8) - 45 = 99$ m/s

$a = 6(8) - 6 = 42$ m/s²

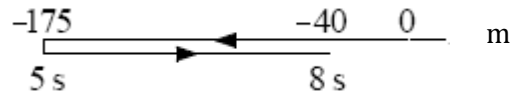
Note that $v(0) < 0$ and $v(8 \text{ s}) > 0$. The reversal of velocity occurs when

$v = 3t^2 - 6t - 45 = 0 \quad t = 5.0$ s

$x = 5^3 - 3(5)^2 - 45 - (5) = -175$ m

At $t=8$ s:

$$s = 175 + (175 - 40) = 310 \text{ m}$$



12.5

(a) $x = t^2 - \frac{t^3}{90} \text{ m}$ $v = \dot{x} = 2t - \frac{t^2}{30} \text{ m/s}$ $v = 0$ when $t = 60$ s

$$x_{\max} = 60^2 - \frac{60^3}{90} = 1200 \text{ m}$$

(b) $a = \dot{v} = 2 - \frac{t}{15} \text{ m/s}^2$ $a = 0$ when $t = 30$ s

$$v_{\max} = 2(30) - \frac{30^2}{30} = 30 \text{ m/s}$$

12.6

(a) $x = v_0(t - t_0 + t_0 e^{-t/t_0})$ $\therefore v = \dot{x} = v_0(1 - e^{-t/t_0})$ \diamond

Since $v \rightarrow v_0$ as $t \rightarrow \infty$, v_0 is the limiting or terminal velocity.

(b) $a = \dot{v} = \frac{v_0}{t_0} e^{-t/t_0}$ \diamond But from part (a): $v_0 - v = v_0 e^{-t/t_0}$ $\therefore a = \frac{v_0 - v}{t_0}$ \diamond

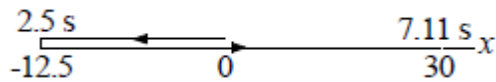
12.7

(a) $x = 2t^2 - 10t \text{ mm}$ $v = \dot{x} = 4t - 10 \text{ mm/s}$

When $x = 30$ mm:

$$2t^2 - 10t = 30 \quad t = 7.110 \text{ s}$$

(b) $v = 0$ when $4t - 10 = 0$ $t = 2.5$ s $x = 2(2.5)^2 - 10(2.5) = -12.5$ mm



$$s = 2(12.5) + 0 = 55.0 \text{ mm}$$

12.8

$$x = 4t^2 - 2 \text{ mm}$$

$$y = \frac{x^2}{12} = \frac{16t^4 - 16t^2 + 4}{12} = \frac{4t^4 - 4t^2 + 1}{3} \text{ mm}$$

When $t = 2$ s:

$$v_x = \dot{x} = 8t = 8(2) = 16 \text{ mm/s}$$

$$v_y = \dot{y} = \frac{16t^3 - 8t}{3} = \frac{16(2)^3 - 8(2)}{3} = 37.33 \text{ mm/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{16^2 + 37.33^2} = 40.6 \text{ mm/s} \blacktriangleleft$$

$$a_x = \dot{v}_x = 8 \text{ mm/s}^2$$

$$a_y = \dot{v}_y = \frac{48t^2 - 8}{3} = \frac{48(2)^2 - 8}{3} = 61.33 \text{ mm/s}^2$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8^2 + 61.33^2} = 61.9 \text{ mm/s}^2 \blacktriangleleft$$

12.9

$$(a) \mathbf{x} = R \left(1 + \frac{1}{2} \cos \omega t \right) \quad \therefore \mathbf{v} = \dot{\mathbf{x}} = -\frac{1}{2} R \omega \sin \omega t \quad \blacklozenge \quad \therefore \mathbf{a} = \dot{\mathbf{v}} = -\frac{1}{2} R \omega^2 \cos \omega t \quad \blacklozenge$$

$$(b) |v|_{\max} = \frac{1}{2} R \omega \quad |a|_{\max} = \frac{1}{2} R \omega^2$$

\therefore Doubling ω would double $|v|_{\max}$ and quadruple $|a|_{\max}$.

12.10

$$x = \sqrt{(v_0 t - b)^2 - b^2}$$

$$\therefore \mathbf{v} = \dot{\mathbf{x}} = \frac{1}{2} [(v_0 t - b)^2 - b^2]^{-1/2} [2v_0(v_0 t - b)] = \frac{v_0(v_0 t - b)}{\sqrt{(v_0 t - b)^2 - b^2}} \quad \blacklozenge$$

$$\therefore \mathbf{a} = \dot{\mathbf{v}} = \frac{v_0^2 [(v_0 t - b)^2 - b^2]^{1/2} - v_0(v_0 t - b) \frac{1}{2} [(v_0 t - b)^2 - b^2]^{-1/2} [2v_0(v_0 t - b)]}{(v_0 t - b)^2 - b^2}$$

$$= -\frac{v_0^2 b^2}{[\sqrt{(v_0 t - b)^2 - b^2}]^3} \quad \blacklozenge$$

12.11

$$(a) v^2 = 2gr_0(r_0/r - 1) + v_0^2$$

Differentiate with respect to time: $2v\dot{v} = 2gr_0(-r_0/r^2)\dot{r}$ or $2va = -2g(r_0/r)^2v$

$$\therefore a = -g(r_0/r)^2 \blacklozenge$$

(b) v_0 is the escape velocity if $v \rightarrow 0$ when $r \rightarrow \infty$.

$$\therefore 0 = \lim_{r \rightarrow \infty} [2gr_0(r_0/r - 1) + v_0^2] \quad \therefore 0 = 2gr_0(0 - 1) + v_0^2 \quad \therefore v_0 = \sqrt{2gr_0} \blacklozenge$$

(c) For earth: $v_0 = \sqrt{2(9.8)(6400 \times 10^3)} = 11200 \text{ m/s}$

12.12

$$\begin{aligned} x &= 15 - 2t^2 \text{ m} & v_x = \dot{x} &= -4t \text{ m/s} & a_x = \dot{v}_x &= -4 \text{ m/s}^2 \\ y &= 15 - 10t + t^2 \text{ m} & v_y = \dot{y} &= -10 + 2t \text{ m/s} & a_y = \dot{v}_y &= 2 \text{ m/s}^2 \end{aligned}$$

(a) At $t = 0$: $\mathbf{v} = -10\mathbf{j} \text{ m/s} \blacktriangleleft$ $\mathbf{a} = -4\mathbf{i} + 2\mathbf{j} \text{ m/s}^2 \blacktriangleleft$

(b) At $t = 5 \text{ s}$: $\mathbf{v} = -20\mathbf{i} \text{ m/s} \blacktriangleleft$ $\mathbf{a} = -4\mathbf{i} + 2\mathbf{j} \text{ m/s}^2 \blacktriangleleft$

12.13

$$\begin{aligned} x &= 66t \text{ m} & v_x = \dot{x} &= 66 \text{ m/s} & a_x = \dot{v}_x &= 0 \\ y &= 86t - 4.91t^2 \text{ m} & v_y = \dot{y} &= 86 - 9.82t \text{ m/s} \\ a_y &= \dot{v}_y &= -9.82 \text{ m/s}^2 \end{aligned}$$

(a) $\mathbf{a} = -9.82\mathbf{j} \text{ m/s}^2 \blacktriangleleft$

(b) When $t = 0$: $\mathbf{v} = 66\mathbf{i} + 86\mathbf{j} \text{ m/s} \blacktriangleleft$

(c) When $y = h$: $v_y = 0$ $86 - 9.82t = 0$ $t = 8.758 \text{ s}$

$$h = 86(8.758) - 4.91(8.758)^2 = 377 \text{ m} \blacktriangleleft$$

(d) When $y = -120 \text{ m}$: $86t - 4.91t^2 = -120$ $t = 18.814 \text{ s}$

$$L = 66(18.814) = 1242 \text{ m} \blacktriangleleft$$

12.14

$$(a) v_x = \dot{x} = v_0 \quad y = b \left(1 - \frac{x^2}{b^2} \right) \quad \therefore v_y = \dot{y} = -\frac{2h}{b^2} x \dot{x} = -\frac{2hv_0}{b^2} x$$

$$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + \left(-\frac{2hv_0}{b^2} x \right)^2} = v_0 \sqrt{1 + \left(\frac{2h}{b^2} x \right)^2} \quad \blacklozenge$$

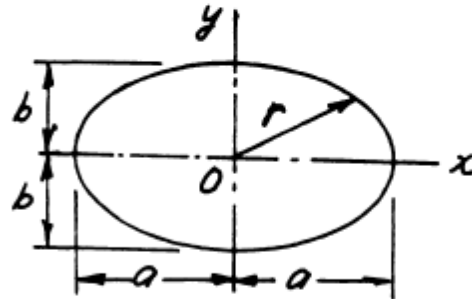
$$(b) a_x = \ddot{x} = 0 \quad a_y = \ddot{y} = -\frac{2hv_0}{b^2} \dot{x} = -\frac{2hv_0^2}{b^2} \quad \therefore a = \sqrt{a_x^2 + a_y^2} = \frac{2hv_0^2}{b^2} \downarrow \quad \blacklozenge$$

12.15

$$(a) x = a \cos \omega t \quad y = b \sin \omega t$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2 \omega t + \sin^2 \omega t = 1$$

which is the equation of ellipse \therefore Q.E.D.



$$(b) v_x = \dot{x} = -a\omega \sin \omega t \quad a_x = \ddot{x} = -a\omega^2 \cos \omega t = -\omega^2 x$$

$$v_y = \dot{y} = b\omega \cos \omega t \quad a_y = \ddot{y} = -b\omega^2 \sin \omega t = -\omega^2 y$$

$$\therefore \mathbf{a} = a_x \mathbf{i} + a_y \mathbf{j} = -\omega^2(x\mathbf{i} + y\mathbf{j}) = -\omega^2 \mathbf{r} \quad \therefore \mathbf{a} \text{ and } \mathbf{r} \text{ are collinear} \quad \therefore \text{Q.E.D.}$$

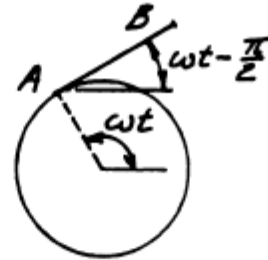
12.16

$$x = R \cos \omega t + R\omega t \sin \omega t \quad y = R \sin \omega t - R\omega t \cos \omega t$$

$$\therefore v_x = \dot{x} = -R\omega \sin \omega t + R\omega \sin \omega t + R\omega^2 t \cos \omega t = R\omega^2 t \cos \omega t$$

$$\therefore v_y = \dot{y} = R\omega \cos \omega t - R\omega \cos \omega t + R\omega^2 t \sin \omega t = R\omega^2 t \sin \omega t$$

$$\therefore \mathbf{v} = v_x \mathbf{i} + v_y \mathbf{j} = R\omega^2 t (\mathbf{i} \cos \omega t + \mathbf{j} \sin \omega t) \quad \therefore \mathbf{v} = R\omega^2 t \diamond$$



$$\overrightarrow{AB} = \overline{AB} [1 \cos(\omega t - \pi/2) + \mathbf{j} \sin(\omega t - \pi/2)]$$

$$= \overline{AB} (\mathbf{i} \sin \omega t - \mathbf{j} \cos \omega t)$$

By inspection: $\mathbf{v} \cdot \overrightarrow{AB} = 0 \quad \therefore \mathbf{v}$ is perpendicular to $AB \quad \therefore$ Q.E.D.

12.17

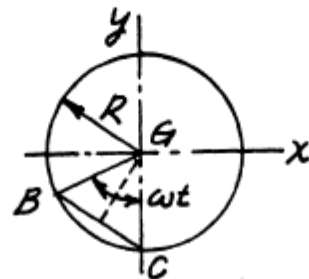
(a) $x = R(\omega t - \sin \omega t) \quad \therefore v_x = \dot{x} = R\omega(1 - \cos \omega t)$

$y = R(1 - \cos \omega t) \quad \therefore v_y = \dot{y} = R\omega \sin \omega t$

$$\therefore v^2 = v_x^2 + v_y^2 = R^2 \omega^2 [(1 - \cos \omega t)^2 + \sin^2 \omega t] = 2R^2 \omega^2 (1 - \cos \omega t) = 4R^2 \omega^2 \sin^2 \frac{\omega t}{2}$$

$$\therefore v = 2R\omega \sin \frac{\omega t}{2}$$

But $\overline{BC} = 2R \sin \frac{\omega t}{2} \quad \therefore v = \omega \overline{BC} \quad \therefore$ Q.E.D.



(b) $a_x = \ddot{x} = R\omega^2 \sin \omega t \quad a_y = \ddot{y} = R\omega^2 \cos \omega t$

$$\therefore \mathbf{a} = a_x \mathbf{i} + a_y \mathbf{j} = (R\omega^2 \sin \omega t) \mathbf{i} + (R\omega^2 \cos \omega t) \mathbf{j}$$

$$\overrightarrow{BG} = (R \sin \omega t) \mathbf{i} + (R \cos \omega t) \mathbf{j} = \frac{1}{\omega^2} \mathbf{a} \quad \therefore \overrightarrow{BG} \text{ and } \mathbf{a} \text{ are parallel} \quad \therefore \text{Q.E.D.}$$

12.18

$$x = R \cos \omega t \quad y = R \sin \omega t \quad z = -\frac{h}{2\pi} \omega t$$

$$\therefore v_x = \dot{x} = -R\omega \sin \omega t \quad v_y = \dot{y} = R\omega \cos \omega t \quad v_z = \dot{z} = -\frac{h\omega}{2\pi}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2 = R^2\omega^2 \sin^2 \omega t + R^2\omega^2 \cos^2 \omega t + \left(\frac{h\omega}{2\pi}\right)^2 = (R\omega)^2 + \left(\frac{h\omega}{2\pi}\right)^2$$

$$\therefore v = R\omega \sqrt{1 + \left[\frac{h}{2\pi R}\right]^2} = \text{constant} \quad \therefore \text{Q.E.D.}$$

$$a_x = \ddot{x} = -R\omega^2 \cos \omega t \quad a_y = \ddot{y} = -R\omega^2 \sin \omega t \quad a_z = \ddot{z} = 0$$

$$a^2 = a_x^2 + a_y^2 + a_z^2 = R^2\omega^4 (\cos^2 \omega t + \sin^2 \omega t) = R^2\omega^4 \quad \therefore a = R\omega^2 = \text{constant} \quad \therefore \text{Q.E.D.}$$

Using the given data:

$$v = (1.2)(4\pi) \sqrt{1 + \left[\frac{0.75}{2\pi(1.2)}\right]^2} = 15.15 \text{ m/s} \quad \blacklozenge \quad a = (1.2)(4\pi)^2 = 189.5 \text{ m/s}^2 \quad \blacklozenge$$

12.19

$$x = 0.8v_0 t \quad y = 0.6v_0 t \quad z = -0.04v_0^2 t^2$$

$$\therefore v_x = \dot{x} = 0.8v_0 \quad v_y = \dot{y} = 0.6v_0 \quad v_z = \dot{z} = -0.08v_0^2 t$$

$$\therefore a_x = \ddot{x} = 0 \quad a_y = \ddot{y} = 0 \quad a_z = \ddot{z} = -0.08v_0^2$$

(a) **At point B:** $x = 4 \text{ cm}$ $\therefore 4 = 0.8v_0 t$ $\therefore t = 5/v_0$ $\therefore v_z = -0.4v_0$

$$\therefore v = \sqrt{v_x^2 + v_y^2 + v_z^2} = v_0 \sqrt{0.8^2 + 0.6^2 + 0.4^2} = 1.0770v_0 \quad \blacklozenge$$

$$a = |a_z| = 0.08v_0^2 \quad \blacklozenge$$

(b) Let θ be the angle between the path and the z-axis at point B.

$$\therefore \cos \theta = \frac{\mathbf{v} \cdot \mathbf{k}}{|\mathbf{v}|} = \frac{-0.4v_0}{1.0770v_0} = -0.3714 \quad \therefore \theta = 111.8^\circ$$

$$\therefore \text{The angle between the path and the x-y plane is: } \theta - 90^\circ = 21.8^\circ \quad \blacklozenge$$

12.20

(a) $\mathbf{r} = (3t^2 + 4t)\mathbf{i} + (-4t^2 + 3t)\mathbf{j} + (-6t + 9)\mathbf{k}$ m

$$\therefore \mathbf{v} = \dot{\mathbf{r}} = (6t + 4)\mathbf{i} + (-8t + 3)\mathbf{j} - 6\mathbf{k} \text{ m/s}$$

$$\therefore \mathbf{a} = \dot{\mathbf{v}} = 6\mathbf{i} - 8\mathbf{j} \text{ m/s}^2$$

(b) The vector normal to the plane formed by \mathbf{v} and \mathbf{a} (the instantaneous plane of motion) is

$$\mathbf{v} \times \mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6t + 4 & -8t + 3 & -6 \\ 6 & -8 & 0 \end{vmatrix} = -48\mathbf{i} - 36\mathbf{j} - 50\mathbf{k}$$

and the corresponding unit vector is

$$\mathbf{n} = \pm \frac{48\mathbf{i} + 36\mathbf{j} + 50\mathbf{k}}{\sqrt{48^2 + 36^2 + 50^2}} = \pm(0.615\mathbf{i} + 0.461\mathbf{j} + 0.640\mathbf{k})$$

Since this vector is independent of t , the orientation of the plane does not vary with the location of the particle. Thus the particle is in plane motion on an inclined plane. Q.E.D.

12.21

$$x = R \cos \omega t \quad y = R \sin \omega t \quad z = \frac{R}{2} \sin 2\omega t$$

$$\therefore v_x = \dot{x} = -R\omega \sin \omega t \quad v_y = \dot{y} = R\omega \cos \omega t \quad v_z = \dot{z} = R\omega \cos 2\omega t$$

$$\therefore v = \sqrt{v_x^2 + v_y^2 + v_z^2} = R\omega \sqrt{\cos^2 \omega t + \sin^2 \omega t + \cos^2 2\omega t} = R\omega \sqrt{1 + \cos^2 2\omega t}$$

$$\therefore v_{\max} = \sqrt{2} R\omega \quad \blacklozenge$$

$$a_x = \ddot{x} = -R\omega^2 \cos \omega t \quad a_y = \ddot{y} = -R\omega^2 \sin \omega t \quad a_z = \ddot{z} = -2R\omega^2 \sin 2\omega t$$

$$\therefore a = \sqrt{a_x^2 + a_y^2 + a_z^2} = R\omega^2 \sqrt{\cos^2 \omega t + \sin^2 \omega t + 4 \sin^2 2\omega t} = R\omega^2 \sqrt{1 + 4 \sin^2 2\omega t}$$

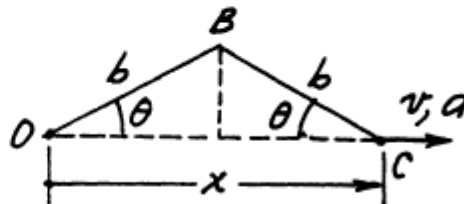
$$\therefore a_{\max} = \sqrt{5} R\omega^2 \quad \blacklozenge$$

12.22

(a) From geometry: $x = 2b \cos \theta$

$$\therefore v = \dot{x} = -2b\dot{\theta} \sin \theta \rightarrow \blacklozenge$$

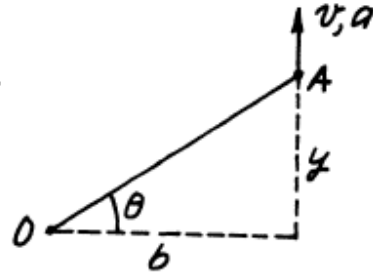
(b) $\therefore a = \dot{v} = -2b(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta) \rightarrow \blacklozenge$



12.23

(a) Geometry: $y = b \tan \theta \quad \therefore v = \dot{y} = b \dot{\theta} \sec^2 \theta \uparrow \blacklozenge$

(b) $\therefore a = \dot{v} = b [\ddot{\theta} \sec^2 \theta + 2 \dot{\theta} \sec \theta (\sec \theta \tan \theta \dot{\theta})]$
 $= b \sec^2 \theta (\ddot{\theta} + 2 \dot{\theta}^2 \tan \theta) \uparrow \blacklozenge$



12.24

$$v = \dot{x} = \frac{dx}{d\theta} \frac{d\theta}{dt} = \frac{dx}{d\theta} \omega = R\omega \left[-\sin \theta + \frac{1}{2} (9 - \sin^2 \theta)^{-1/2} (-2 \sin \theta \cos \theta) \right]$$

$$= -R\omega \sin \theta \left(1 + \frac{\cos \theta}{\sqrt{9 - \sin^2 \theta}} \right) \blacklozenge$$

12.25

$$\dot{\theta} = \frac{1200 \text{ rev}}{1.0 \text{ min}} \times \frac{2\pi \text{ rad}}{1.0 \text{ rev}} \times \frac{1.0 \text{ min}}{60 \text{ s}} = 125.66 \text{ rad/s}$$

$$r = 55 + 10 \cos \theta + 5 \cos 2\theta \text{ mm}$$

$$v = \dot{r} = \frac{dr}{d\theta} \dot{\theta} = (-10 \sin \theta - 10 \sin 2\theta) (125.66) \text{ mm/s}$$

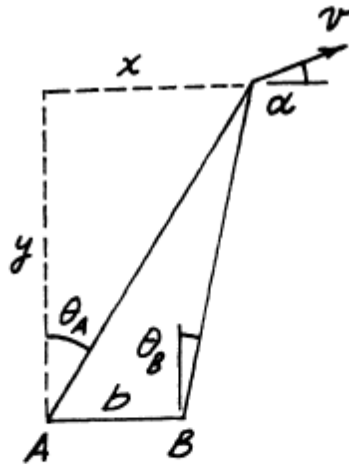
$$a = \dot{v} = \frac{dv}{d\theta} \dot{\theta} = (-10 \cos \theta - 20 \cos 2\theta) (125.66)^2 \text{ mm/s}^2$$

$$|a|_{\max} = 30(125.66)^2 = 474\,000 \text{ mm/s}^2 = 474 \text{ m/s}^2 \text{ (at } t = 0) \blacktriangleleft$$

*12.26

(a) Geometry: $x = y \tan \theta_A = y \tan \theta_B + b \dots\dots\dots$ (a)

$$\therefore y = \frac{b}{\tan \theta_A - \tan \theta_B} = \frac{1000}{\tan 30^\circ - \tan 22^\circ} = 5770 \text{ m} \blacklozenge$$



(b) Differentiate Eq. (a):

$$\dot{x} = y \dot{\theta}_A \sec^2 \theta_A + \dot{y} \tan \theta_A = y \dot{\theta}_B \sec^2 \theta_B + \dot{y} \tan \theta_B \quad (b)$$

$$\therefore \dot{y} = -y \frac{\dot{\theta}_A \sec^2 \theta_A - \dot{\theta}_B \sec^2 \theta_B}{\tan \theta_A - \tan \theta_B}$$

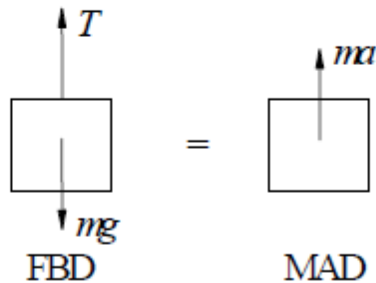
$$= -(5770) \frac{(0.026) \sec^2 30^\circ - (0.032) \sec^2 22^\circ}{\tan 30^\circ - \tan 22^\circ} = 85.11 \text{ m/s}$$

From Eq. (b): $\dot{x} = (5770)(0.026) \sec^2 30^\circ + (85.11) \tan 30^\circ = 249.2 \text{ m/s}$

$$\therefore v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{249.2^2 + 85.11^2} = 263 \text{ m/s} \blacklozenge$$

(c) $\therefore \alpha = \tan^{-1}(\dot{y}/\dot{x}) = \tan^{-1} \frac{85.11}{249.2} = 18.86^\circ \blacklozenge$

12.27

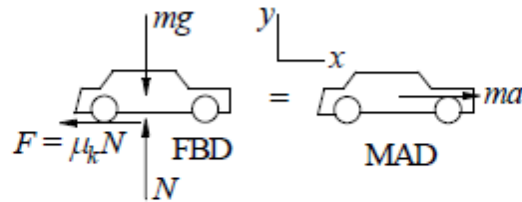


$$v = 4t \text{ m/s} \quad a = \dot{v} = 4 \text{ m/s}^2$$

$$\Sigma F = ma \quad + \uparrow \quad T - mg = ma$$

$$T = m(g + a) = 50(9.81 + 4) = 691 \text{ N} \blacktriangleleft$$

12.28



$$v_0 = 100 \text{ km/h} = 100 \text{ km/h} \left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right) = 27.78 \text{ m/s}$$

$$\Sigma F_y = 0 \quad + \uparrow \quad N - mg = 0 \quad \therefore N = mg$$

$$\Sigma F_x = ma \quad + \rightarrow \quad -\mu_k N = ma \quad \therefore a = -\frac{\mu_k N}{m} = -\mu_k g$$

$$v = \int a \, dt + C_1 = -\mu_k g t + C_1$$

$$x = \int v \, dt + C_2 = -\frac{1}{2} \mu_k g t^2 + C_1 t + C_2$$

When $t = 0$ (initial conditions):

$$x = 0 \quad \therefore C_2 = 0 \quad v = v_0 \quad \therefore C_1 = v_0$$

$$\therefore x = -\frac{1}{2} \mu_k g t^2 + v_0 t \quad v = -\mu_k g t + v_0$$

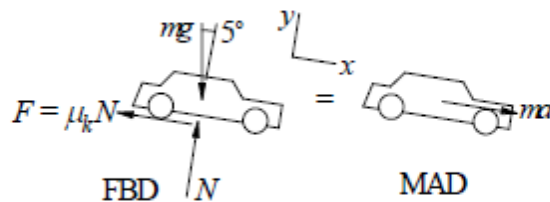
When $v = 0$:

$$-\mu_k g t + v_0 = 0 \quad \therefore t = \frac{v_0}{\mu_k g}$$

$$x = -\frac{1}{2} \mu_k g \left(\frac{v_0}{\mu_k g} \right)^2 + v_0 \left(\frac{v_0}{\mu_k g} \right) = \frac{v_0^2}{2\mu_k g}$$

$$= \frac{27.78^2}{2(0.65)(9.8)} = 60.58 \text{ m}$$

12.29



$$v_0 = 100 \text{ km/h} = 27.78 \text{ m/s}$$

$$\Sigma F_y = 0 + \uparrow N - mg \cos 5^\circ = 0 \quad \therefore N = mg \cos 5^\circ$$

$$\Sigma F_x = ma \overset{+}{\rightarrow} - \mu_k N + mg \sin 5^\circ = ma$$

$$\therefore a = -\frac{\mu_k N}{m} + g \sin 5^\circ = (\sin 5^\circ - \mu_k \cos 5^\circ)g$$

$$= (\sin 5^\circ - 0.65 \cos 5^\circ)9.8 = -5.492 \text{ m/s}^2$$

$$v = \int a \, dt + C_1 = -5.492t + C_1$$

$$x = \int v \, dt + C_2 = -2.746t^2 + C_1t + C_2$$

When $t = 0$ (initial conditions):

$$x = 0 \quad \therefore C_2 = 0 \quad v = v_0 \quad \therefore C_1 = v_0 = 27.78 \text{ m/s}$$

$$x = -2.746t^2 + 27.78t \text{ m}$$

$$v = -5.492t + 27.78 \text{ m/s}$$

When $v = 0$:

$$-5.492t + 27.78 = 0 \quad \therefore t = 5.058 \text{ s}$$

$$x = 2.746(5.058)^2 + 27.78(5.058) = 70.3 \text{ m}$$

12.30

$$a = \frac{F}{m} = \frac{-1.2t}{0.1} = -12t \text{ m/s}^2$$

$$v = \int a_x \, dt + C_1 = -6t^2 + C_1 \text{ m/s}$$

$$x = \int v_x \, dt + C_2 = -2t^3 + C_1t + C_2 \text{ m}$$

When $t = 0$ (initial conditions):

$$x = 0 \quad \therefore C_2 = 0 \quad v = 64 \text{ m/s} \quad \therefore C_1 = 64 \text{ m/s}$$

$$\therefore x = -2t^3 + 64t \text{ m} \quad v = -6t^2 + 64 \text{ m/s}$$

When $t = 4 \text{ s}$:

$$x = -2(4)^3 + 64(4) = 128 \text{ m}$$

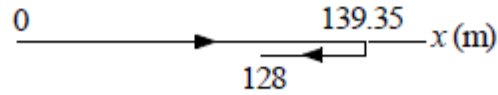
When $v = 0$:

$$-6t^2 + 64 = 0 \quad t = 3.266 \text{ s}$$

$$x = -2(3.266)^3 + 64(3.266) = 139.35 \text{ m}$$

Distance traveled:

$$d = 2(139.35) - 128 = 150.7 \text{ m} \quad \blacktriangleleft$$



12.31

$$a = \frac{F}{m} = \frac{0.04\sqrt{v}}{0.01} = 4\sqrt{v} \text{ m/s}^2$$

$$dt = \frac{dv}{a} \quad t = \int \frac{dv}{a} + C_1 = \int \frac{dv}{4\sqrt{v}} + C_1 = \frac{\sqrt{v}}{2} + C_1$$

When $t = 0.6 \text{ s}$ (initial condition):

$$v = 0.16 \text{ m/s} \quad \therefore 0.6 = \frac{\sqrt{0.16}}{2} + C_1 \quad C_1 = 0.4 \text{ s}$$

$$\therefore t = \frac{\sqrt{v}}{2} + 0.4 \text{ s} \quad \therefore v = (2t - 0.8)^2 = 4t^2 - 3.2t + 0.64 \text{ m/s}$$

$$x = \int v \, dt + C_2 = \frac{4}{3}t^3 - 1.6t^2 + 0.64t + C_2$$

When $t = 0$ (initial condition):

$$x = 0 \quad \therefore C_2 = 0 \quad \therefore x = \frac{4}{3}t^3 - 1.6t^2 + 0.64t \text{ m}$$

When $t = 0.8 \text{ s}$:

$$x = \frac{4}{3}(0.8)^3 - 1.6(0.8)^2 + 0.64(0.8) = 0.1707 \text{ m} \quad \blacktriangleleft$$