Chapter 1 Circuit Variables and Elements

Solutions to Exercises

E1.1.1 Number of electrons/hr $= \dfrac{-1\,\text{A} \times 1\,\text{s} \times 3600}{-1.6 \times 10^{-19}} = 2.25\times10^{22}$ electrons/hr.

E1.1.2 (a) $\dfrac{5\times10^{18}\times1.6\times10^{-19}}{60} + \dfrac{-2.5\times10^{18}\times(-1.6\times10^{-19})}{60} = 0.02\text{ A} \equiv 20\text{ mA}.$

(b) The sign of the current is reversed to –20 mA

(c) $\dfrac{5\times10^{18}\times1.6\times10^{-19}}{60} + \dfrac{2.5\times10^{18}\times(-1.6\times10^{-19})}{60} = \dfrac{0.4}{60}\text{ A} \equiv \dfrac{20}{3}\text{ mA}.$

E1.2.1 (a) Since electrons move to a more positive voltage, they lose potential energy. Energy loss per electron $= qV = 1.6\times10^{-19}\times10 = 1.6\times10^{-18}$ J.

(b) It is converted to K.E.

(c) $\dfrac{1}{2}mv^2 = 1.6\times10^{-18}$ J; $v = \sqrt{\dfrac{3.2\times10^{-18}}{9.1\times10^{-31}}} = 1.88\times10^{6}$ m/s.

E1.3.1 (a) $6.25\times10^{14}\times1.6\times10^{-18} = 10\times10^{-4}$ J $\equiv$ 1 mJ.

(b) Current in the conventional positive direction is from B to A and of magnitude $6.25\times10^{14}\times1.6\times10^{-19} = 10\times10^{-5}$ A $\equiv$ 100 μA.

(c) The kinetic energy is converted to heat.

(d) $P = V \times I = 10\times10^{-4}$ W $\equiv$ 1 mW.

(e) From (a), K.E. given up per second = 1 mW.

E1.4.1 (a) When cruising, battery delivers $15\times12 = 180$ W.

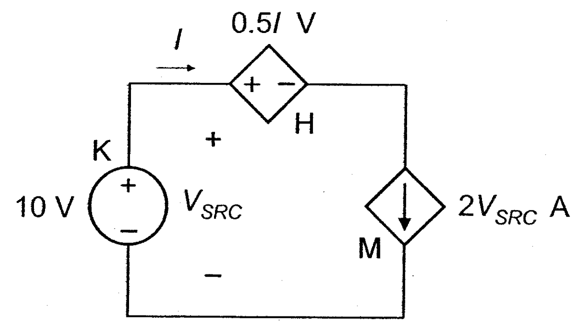
(b) When charging, battery absorbs $20\times12 = 240$ W.

E1.6.1 (a) I is determined by the current source and is equal to $2V_{SRC}$, i.e., 20 A.

(b) $0.5\times I = 10$ V.

(c) K delivers $10\times20 = 200$ W, H absorbs $0.5\,I\times I = 200$ W. Hence, M neither absorbs nor delivers power.

(d) Voltage across M must be zero.



E1.7.1 Each element is rated at 1 kW.

(a) $I = \dfrac{1000}{220} = 4.55$ A, $R = \dfrac{V^2}{P} = \dfrac{(220)^2}{1000} = 48.4\ \Omega$, $G = \dfrac{1}{R} = \dfrac{1}{48.4}$ S $\equiv 20.7$ mS.

(b) $I = \dfrac{2000}{220} = 9.09$ A, $R = \dfrac{(220)^2}{2000} = 24.2\ \Omega$, $G = \dfrac{1}{R} = 41.3$ mS.

E1.8.1 $C = \dfrac{\epsilon_0 \epsilon \times A}{d} = 8.85\times10^{-12}\times5{,}000\times\dfrac{\pi\times(5\times10^{-2})^2}{4}\times\dfrac{1}{10^{-3}} = 8.69\times10^{-8}$ F.

$Q = CV = 50\times8.69\times10^{-8} = 4.34\times10^{-6}$ C.

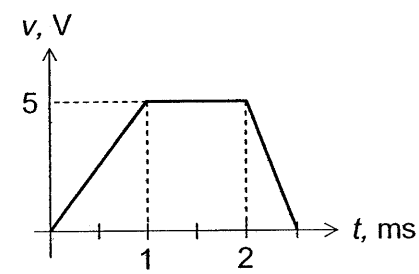
Number of electrons = $\dfrac{4.34\times10^{-6}}{1.6\times10^{-19}} = 2.7\times10^{13}$ electrons.

E1.8.2 (a) $i = C\dfrac{dv}{dt} = 10^{-6}\times\dfrac{5}{10^{-3}} = 5\times10^{-3}$ A $\equiv$

$5\ \text{mA}, 0 < t < 1\ \text{ms};\ i = 0,\ 1 < t < 2\ \text{ms};$

$i = -10^{-6}\times\dfrac{5}{0.5\times10^{-3}} = -10^{-2}\text{A} \equiv -10\ \text{mA},$

$2 < t < 2.5\ \text{ms};\ i = 0,\ t > 2.5\ \text{ms};$



(b) At t = 1.5 ms, q due to the applied voltage equals $Cv = 10^{-6}\times5 \equiv 5\ \mu$C. The total charge is therefore 10 μC.

E1.9.1 (a) $v = L\dfrac{di}{dt} = 10^{-6}\times\dfrac{5}{10^{-3}} = 5\times10^{-3}$ V $\equiv 5$ mV, $0 < t < 1$ ms; $v = 0,\ 1 < t < 2$ ms;

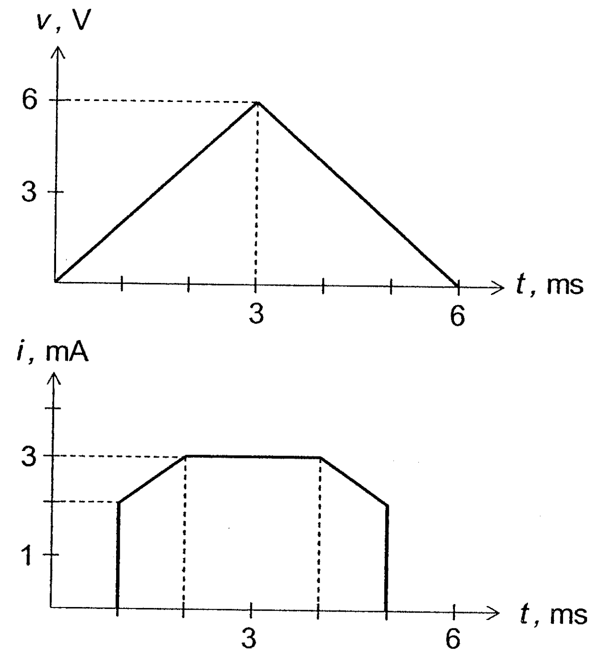
$v = -10^{-6}\times\dfrac{5}{0.5\times10^{-3}} = -10^{-2}$ V $\equiv -10$ mV, $2 < t < 2.5$ ms; $v = 0$, $t > 2$ ms;

(b) At t = 1.5 ms, λ due to the applied voltage equals $\lambda i = 10^{-6}\times5 \equiv 5$ μWb-turns. The total flux linkage is therefore 10 μWb-turns.

Solutions to Problems and Exercises

P1.1.1 $2\times(100\text{ W})\times(6\text{ h})\times(30\text{ days}) \equiv 36\text{ kWh}$.

P1.1.2 (a) $t = 1^-$: $q = \int_0^{1^-} i\,dt = \int_0^{1^-} 0\,dt = 0$.



$t = 2$: $q = \int_1^2 i\,dt = \int_1^2 (1+t)dt = \left[t + \frac{t^2}{2}\right]_1^2 = 2.5\text{ mC}$.

$t = 3$: $q = 2.5 + \int_2^3 i\,dt = 2.5 + \int_2^3 3\,dt = 5.5\text{ mC}$.

$t = 4$: $q = 5.5 + \int_3^4 i\,dt = 5.5 + \int_3^4 3\,dt = 8.5\text{ mC}$.

$t = 5$: $q = 8.5 + \int_4^5 i\,dt = 8.5 + \int_4^5 (-t+7)dt = 8.5 + \left[-\frac{t^2}{2} + 7t\right]_4^5 = 11\text{ mC}$.

$t = 6$: $q = 11 + \int_5^6 i\,dt = \int_5^6 0\,dt = 11\text{ mC}$.

(b) $0 \le t \le 1: p = 0$.

$1 \le t \le 2: p = 2t(t+1) = 2t^2 + 2t\text{ mW}$.

$2 \le t \le 3: p = 2t \times 3 = 6t\text{ mW}$.

$3 \le t \le 4: p = (-2t+12)\times 3 = -6t + 36\text{ mW}$.

$4 \le t \le 5: p = (-2t+12)(-t+7) = 2t^2 - 26t + 84\text{ mW}$.

$5 \le t \le 6: p = 0$.

(c) $\omega(t) = \int_0^6 p\,dt = \int_0^1 0\,dt + \int_1^2 (2t^2+2t)dt + \int_2^3 6t\,dt + \int_3^4 (-6t+36)dt +$

$\int_4^5 (2t^2 - 26t + 84)dt + \int_5^6 0\,dt = \left[\frac{2t^3}{3} + t^2\right]_1^2 + \left[3t^2\right]_2^3 + \left[-3t^2 + 36t\right]_3^4 +$

$\left[\frac{2t^3}{3} - 13t^2 + 84t\right]_4^5$ 136/3 = 45.3 μJ.

P1.1.3 $p = vi = (2t + 1)(4 - 2t) = -4t^2 + 6t + 4$.

(a) $\frac{dp}{dt} = -8t + 6$; $p_{max} \Rightarrow \frac{dp}{dt} = 0 \Rightarrow t = \frac{6}{8} = 0.75s$.

(b) $p = 0 \Rightarrow -4t^2 + 6t + 4 = 0 \Rightarrow t = 2s$; $p = 0$ when either v or i is zero.

(c) At $t = 2$s: $w = \int_0^2 pdt = \int_0^2 (-4t^2 + 6t + 4)dt = \left[-\frac{4}{3}t^3 + 3t^2 + 4t\right]_0^2 = 9.3$ mJ.

At $t = 4$s: $w = \int_0^4 pdt = \int_0^4 (-4t^2 + 6t + 4)dt = \left[-\frac{4}{3}t^3 + 3t^2 + 4t\right]_0^4 = -21.3$ mJ;

power is absorbed by device for $t < 2$ s, and is delivered by device for $2 < t < 4$ s.

P1.1.4 (a) Power is absorbed by device during the first and third quarter cycles, when v and i have the same sign, and is delivered during the second and fourth quarter cycles, when v and i have opposite signs.

(b) $p = vi = \sin 2\pi t \times \cos 2\pi t = 0.5\sin 4\pi t$ W; maximum power absorbed or delivered is 0.5 W.

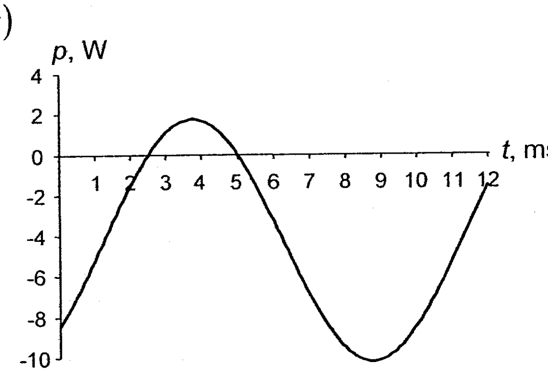
(c) Minimum power absorbed or delivered is 0.5 W.

(d) $P = \int_0^1 0.5\sin 4\pi t = \frac{1}{8\pi}[-\cos 4\pi t]_0^1 = 0$.

P1.1.5 $p = vi = 12\sin(100\pi t - 45^\circ)(\cos 100\pi t)$

$= 6\sin(200\pi t - 45^\circ) + 6\sin(-45^\circ)$

$= 6\sin(200\pi t - 45^\circ) - 3\sqrt{2}$ W.



(a) $P = \frac{1}{T}\int_0^T pdt$

$= \frac{1}{0.01}\int_0^{0.01}\left[6\sin(200\pi t - 45^\circ) - 3\sqrt{2}\right]dt = -3\sqrt{2} = -4.24$ W, since the trigonometric term averages to zero over a complete period.

(b), (c) $\frac{dp}{dt} = 1200\pi\cos(200\pi t - 45^\circ)$ $\frac{dp}{dt} = 0 \Rightarrow \cos(200\pi t - 45^\circ) = 0 \Rightarrow$

$200\pi t - \frac{\pi}{4} = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow t = \frac{3}{800} \equiv 3.75$ ms, or $t = \frac{7}{800} \equiv 8.75$ ms;

substituting in the expression for p gives p_{max} = 1.76 W at t = 3.75 ms, and p_{min} = –10.24 W at t = 8.75 ms.

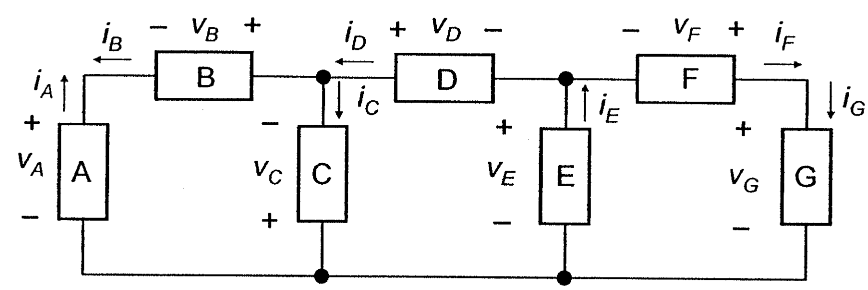
P1.1.6 (a) $0 \le v \le 2$; $p = vi = v(8 - 2v^2) = -2v^3 + 8v$, and $p = 0,\ v \ge 2$.

At v = 1 V; p = 6 W; at v = 2 V, p = 0.

(b) $\dfrac{dp}{dv} = -6v^2 + 8,\ 0 \le v \le 2;\ p_{max} \Rightarrow \dfrac{dp}{dv} = 0 \Rightarrow -6v^2 + 8 = 0 \Rightarrow v = \dfrac{2\sqrt{3}}{3}$ V.

(c) $v(t) = 2e^{-t} \Rightarrow i = 8 - 8e^{-2t};\ q = \int_0^2 i dt = \int_0^2 (8 - 8e^{-2t})dt = [8t + 4e^{-2t}]_0^2 = 12.07$ C.

P1.1.7 According to the assigned positive directions, the direction of



power flow is indicated in the second row of the table below, where D denotes power delivered and A denotes power absorbed. The signed product of the voltage and current for each element is entered in the fifth row if there is a D in the second row, or is entered in the last row if there is an A in the second row. The remaining entries in the fifth and last rows are made so that they have opposite signs in a given column. The sum of the positive quantities in each of the fifth and last rows is 25 and the sum of the negative quantities is also 25. Thus, the total power delivered is 25 W and the total power absorbed is 25 W.

Element	**A**	**B**	**C**	**D**	**E**	**F**	**G**
Power flow	D	A	D	D	D	D	A
Voltage, V	5	–3	–2	5	–3	7	4
Current, A	3	–3	1	–2	–1	1	1
Power delivered, W	15	–9	–2	–10	3	7	–4
Power Absorbed, W	–15	9	2	10	–3	–7	4

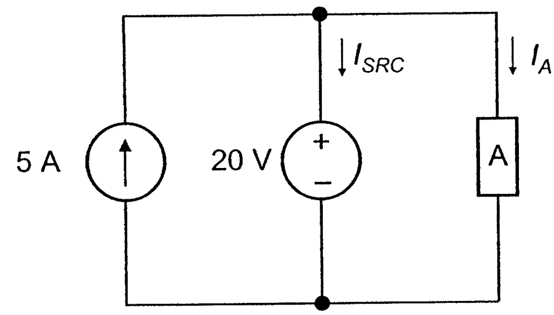
P1.2.1 Voltage across all elements is 20 V. Hence,

$20 \times I_A = 40 \Rightarrow I_A = 2$ A.

Power delivered by 5 A source is $5 \times 20 = 100$ W.

Power absorbed by 20 V source is 100 – 40 = 60 W.

$20 \times I_{SRC} = 60 \Rightarrow I_{SRC} = 3$ A.



P1.2.2 Current through all elements is 6 A.

Power absorbed by A is

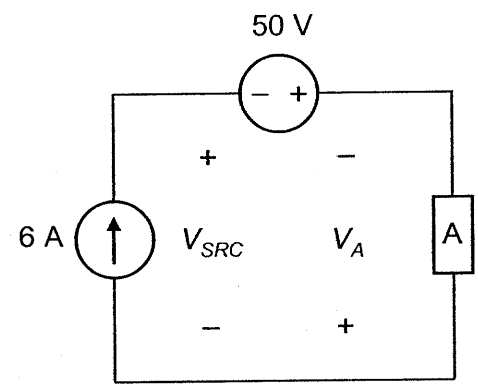
$-V_A \times 6 = 240 \Rightarrow V_A = -40$ V.

Power delivered by 50 V source is $50 \times 6 = 300$ W.

Hence, the 6 A source absorbs 300 – 240 = 60 W.

It follows that

$V_{SRC} \times 6 = -60 \Rightarrow V_{SRC} = -10$ V.

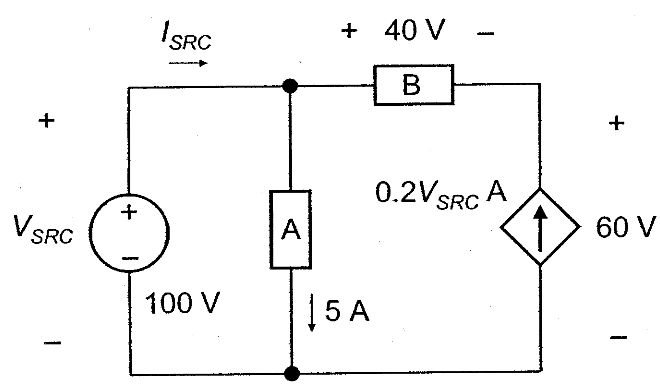


P1.2.3 Power absorbed by A is 500 W.

Power delivered by voltage source is $100I_{SRC}$ W.

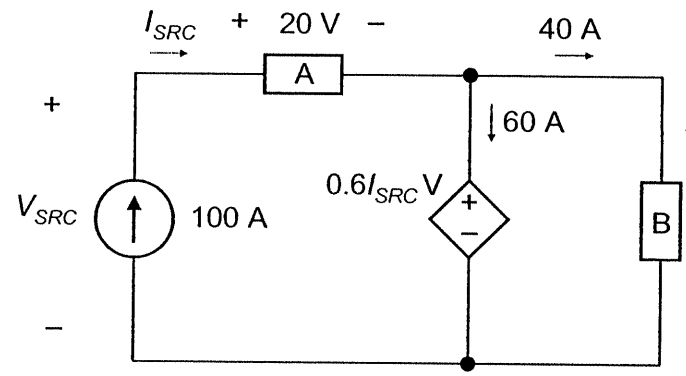
Power delivered by current source is 60×0.2×100 =1200 W.

Power delivered by B is 40×0.2×100 = 800 W.



From conservation of power: 500 = $100I_{SRC}$ + 800 + 1200, so I_{SRC} = -15 A.

P1.2.4 I_{SRC} = 100A, as determined by the current source. Voltage across voltage source and B is 0.6×100 = 60 V. Power absorbed by voltage source is 60×60 = 3600 W.



Power absorbed by B is 60×40 = 2400 W.

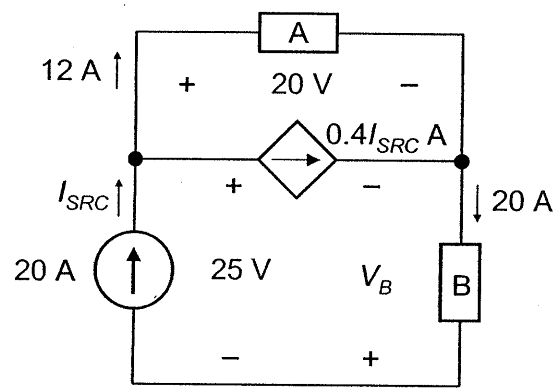
Power absorbed by A is 20×100 = 2000 W. Hence, power delivered by current source is 3600 + 2400 + 2000 = 8000 = V_{SRC}×100, so V_{SRC} = 80 V.

P1.2.5 I_{SRC} = 20 A, as determined by the current source.

Power delivered by independent current source is 25×20 = 500 W.

Power absorbed by A is 20×12 = 240 W.

Power absorbed by dependent current source is 20×0.4×20 = 160 W.



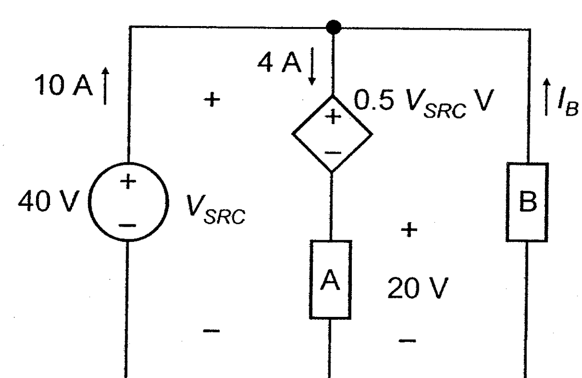
Hence, B must absorb 500 – 240 – 160 = 100 W. It follows that $(-V_B)\times 20 = 100$, so V_B = -5 V.

P1.2.6 Power delivered by independent voltage source is 40×10 = 400 W.

Power absorbed by dependent voltage source is 0.5×40×4 = 80 W.

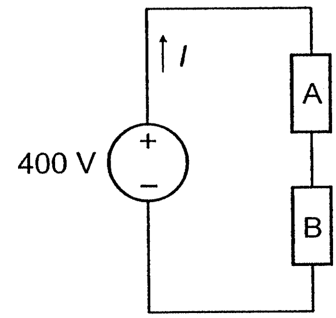
Power absorbed by A is 20×4 = 80 W.

Hence, B must absorb 400 – 80 – 80 = 240 W. It follows that



40×($-I_B$) = 240, so I_B = -6 A. Or, 400 + 40I_B = 160, so I_B = -6 A.

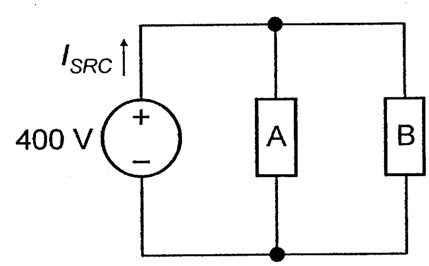
P1.2.7 Power absorbed by each element is $2I^2 \times I = 2I^3$ W.



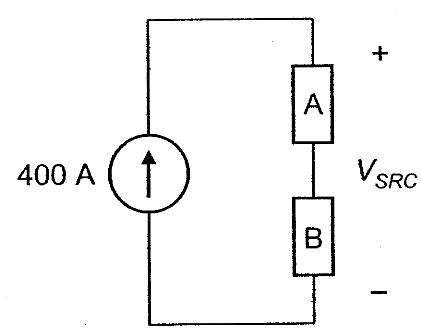
Total power absorbed by A and B is $4I^3$ W.
Power delivered by source is $400 \times I$ W. Hence, $400I = 4I^3$, so $I = 10$ A, since I should be positive for power delivery by the source.
Alternatively, we can say that since A and B are identical, the voltage each of them is 200 V. Hence, $I = \sqrt{\frac{200}{2}} = 10$ A.

The power delivered by the source is, therefore, $400 \times 10 \equiv 4$ kW, whereas each element absorbs $2 \times (10)^3 \equiv 2$ kW, which is also half the power delivered by the source.

P1.2.8 Since the voltage across each element is 400 V, the current through each element is $\sqrt{\frac{400}{2}} = 10\sqrt{2}$ A. The power absorbed by each element is, therefore, $400 \times 10\sqrt{2} \equiv 4\sqrt{2}$ k W. The total power delivered by the source is $8000\sqrt{2} = 400I_{SRC}$ W. It follows that $I_{SRC} = 20\sqrt{2}$ A.



P1.2.9 Voltage across each element is $\sqrt{\frac{400}{2}} = 10\sqrt{2}$ V. Since A and B are identical, $V_{SRC} = 20\sqrt{2}$ V. Power delivered by source is $20\sqrt{2} \times 400 \equiv 8\sqrt{2}$ kW. Power absorbed by each element is $10\sqrt{2} \times 2\left(10\sqrt{2}\right)^2 \equiv 4\sqrt{2}$ kW, which is half the power delivered by the source.



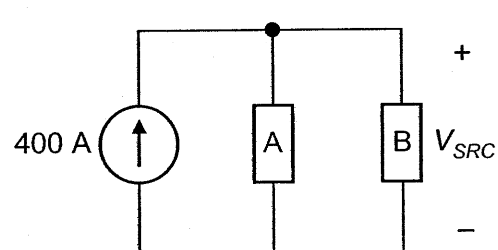
P1.2.10 Power absorbed by each element is

$V_{SRC} \times 2V_{SRC}^2 = 2V_{SRC}^3$ W.

Total power absorbed by A and B is

$4V_{SRC}^3$ W.

Power delivered by source is



$V_{SRC} \times 400$ W. Hence, $400 V_{SRC} = 4V_{SRC}^3$, so $V_{SRC} = 10$ V.

Power absorbed by each element is $2 \times 10^3 \equiv 2$ kW.

Power delivered by source is 4 kW. Since the power absorbed by each element is 2 kW, the current in each element is $\frac{2000}{10} = 200$ A. As expected, the source current divides equally between the identical elements A and B.

P1.3.1 (a) $I = \frac{10{,}000}{220} = 45.5$ A.

(b) $R = \frac{220 \times 220}{10{,}000} = 4.84\ \Omega$.

(c) $G = \frac{10{,}000}{220 \times 220} = \frac{1}{4.84} = 0.21$ S.

P1.3.2 $R_2 = R_1\left[1 + \alpha_m (T_2 - T_1)\right]$.

$70 = 60\left[1 + 0.0039(T_2 - 20)\right] \Rightarrow T_2 = 62.7°\text{C}$.

P1.3.3 $P = \frac{V^2}{R} \Rightarrow V^2 = R \times P = 1.5 \times 10^6 \times 0.5 = 0.75 \times 10^6 \Rightarrow V = 500\sqrt{3} = 866.0\,\text{V}$.

P1.3.4 $1(\text{m}\Omega) \times 10(\text{A}) = 10$ mV; $0.1(\mu\text{S}) \times 100(\text{V}) = 10\ \mu\text{A}$.

P1.3.5 $i = 10^{-9}\left(e^{20v} - 1\right)$.

(a) $v = 0.7V \Rightarrow i = 10^{-9}\left(e^{20 \times 0.7} - 1\right) \cong 10^{-9} \times e^{20 \times 0.7} \equiv 1.20\,\text{mA}$.

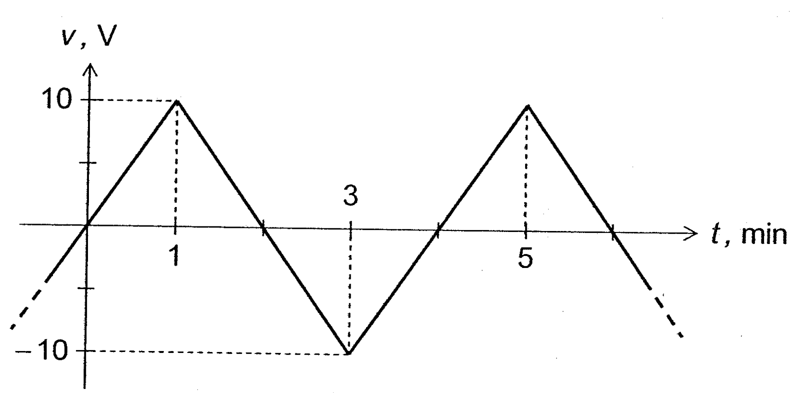
(b) $v = -0.7V \Rightarrow i = 10^{-9}\left(e^{20 \times (-0.7)} - 1\right) \cong -10^{-9} \equiv -1\,\text{nA}$.

P1.3.6 $v = 10t$ V, $0 \le t \le 1$ min;

$= -10t + 20$ V, $1 \le t \le 3$ min;

$= 10t - 40$ V,

$3 \le t \le 4$ min.



(a) $i = \frac{v}{100} = 0.1t$ A,

$0 \le t \le 1$ min;

$= -0.1t + 0.2$ A,

$1 \le t \le 3$ min;

$= 0.1t - 0.4$ A,

$3 \le t \le 4$ min.

(b) $p=\frac{v^2}{R}=t^2$ W, $0\le t\le 1$ min, $p=\frac{(-10t+20)^2}{100}$ W, $1\le t\le 3$ min,

$p=\frac{(10t-40)^2}{100}$ W, $3\le t\le 4$ min.

(c) $P=\frac{1}{T}\int_0^T pdt=\frac{1}{4\times 60}\left[\int_0^1 t^2dt+\int_1^3\left(t^2-4t+4\right)dt+\int_3^4\left(t^2-8t+16\right)dt\right]$

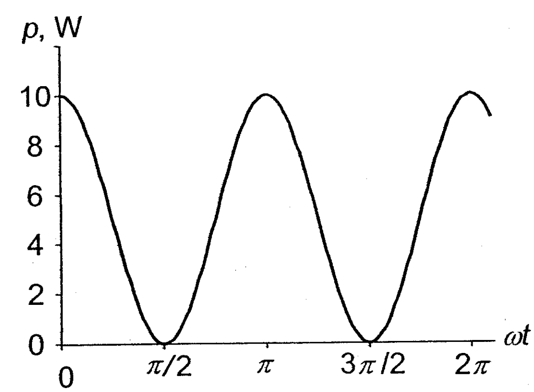
$=\left[\frac{t^3}{3}\right]_0^1+\left[\frac{t^3}{3}-2t^2+4t\right]_1^3+\left[\frac{t^3}{3}-4t^2+16t\right]_3^4=\frac{1}{180}\equiv 5.56\text{ mJ}.$

(d) $V_{avg}=0$, since the waveform is symmetrical about the horizontal axis. This makes $I_{avg}=0$ as well. Thus, $V_{avg}\times I_{avg}=0$, whereas $P\neq 0$. The average power in a resistor is not the product of the average voltage across the resistor and the average current through the resistor, because power, being the product of voltage and current, is a nonlinear quantity.

P1.3.7 $\frac{60\sin 100\pi t}{4\sin 100\pi t}=15\ \Omega$; $P=\frac{1}{2\pi}\int_0^{2\pi}240\sin^2\omega t d(\omega t)=$

$\frac{240}{2\pi}\int_0^{2\pi}\frac{1}{2}(1-\cos 2\omega t)d(\omega t)=\frac{240}{2\pi}\left[\frac{(\omega t)}{2}-\frac{1}{4}\sin 2\omega t\right]_0^{2\pi}=120\text{ W}.$

P1.3.8 $v(t)=10\cos 100\pi t=10\cos\omega t$, where $\omega=100\pi$ rad/s, so that the supply frequency is $\frac{100\pi}{2\pi}=50$ Hz, and the supply period is $\frac{1}{50}\equiv 20$ ms.



(a) $p=\frac{v^2}{R}=\frac{(10\cos 100\pi t)^2}{10}$

$=10\cos^2 100\pi t$ as shown.

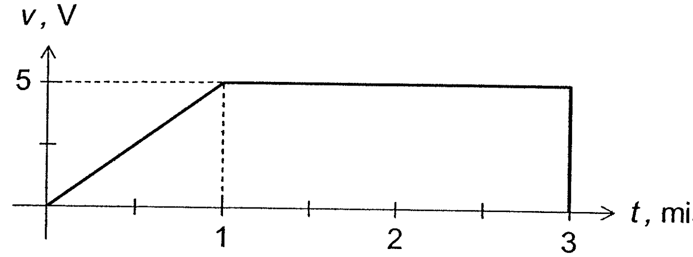
(b) $P=\frac{1}{2\pi}\int_0^{2\pi}10\cos^2\omega t d(\omega t)$

$=\frac{10}{2\pi}\int_0^{2\pi}\frac{1}{2}(1+\cos 2\omega t)d(\omega t)=\frac{10}{2\pi}\times\frac{2\pi}{2}=5\text{ W}$

$w=\int_0^{0.01}pdt=\int_0^{0.01}10\cos^2 100\pi tdt=5\int_0^{0.01}(1+\cos 200\pi t)dt$

$=5\left[t+\frac{\sin 200\pi t}{200\pi}\right]_0^{0.01}=0.05$ J. Since the power dissipated is 5 W, the energy dissipated during one half cycle is 5(W)×0.01(s) = 0.05 J. Note that the average power, i.e., average energy per unit time, is independent of the time scale, but the energy is the integral of instantaneous power with respect to time.

P1.3.9 $v\,(\text{V}) = \begin{cases} t/12 & 0 \le t \le 60\text{ s} \\ 5 & 60 \le t < 180\text{ s} \\ 0 & t > 180\text{ s} \end{cases}$



(a) $0 \le t \le 60$ s; $p = \dfrac{v^2}{R} = \dfrac{1}{5}\left(\dfrac{t}{12}\right)^2 = \dfrac{t^2}{720}$ W, where t is in s.

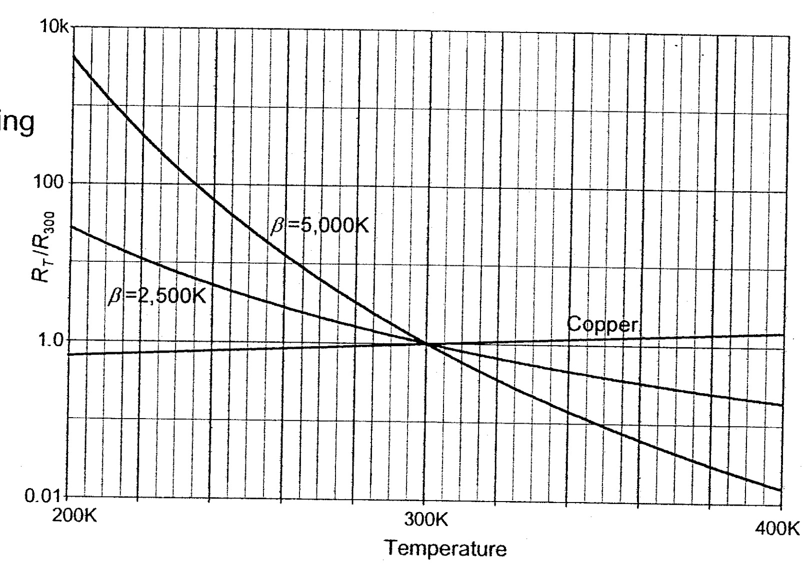
(b) $w = \int_0^{180} p\,dt = \int_0^{60} \dfrac{t^2}{720}\,dt + \int_{60}^{180} \dfrac{25}{5}\,dt = 700$ J.

P1.3.10 $v = 5(\cos\omega t + \cos 2\omega t)^2 = 5(\cos^2\omega t + 2\cos\omega t\cos 2\omega t + \cos^2 2\omega t)$

$= 5[\tfrac{1}{2}(1 + \cos 2\omega t) + \cos\omega t + \cos 3\omega t + \tfrac{1}{2}(1 + \cos 4\omega t)]$ V

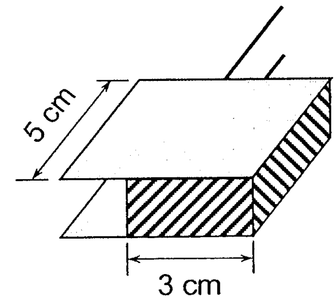
$= 5 + 5\cos\omega t + 2.5\cos 2\omega t + 5\cos 3\omega t + 2.5\cos 4\omega t$ V

P1.3.11 Three plots are shown for thermistors having $\beta = 5{,}000$K and $\beta = 2{,}500$K, as well as copper. The large, negative temperature variation for the thermistors is



evident from the values of R_T/R_{300} on a logarithmic scale. The range of values is: 4,160 to 0.0155 ($\beta = 5{,}000$), 64.5 to 0.125, and 0.61 to 1.39 (copper).

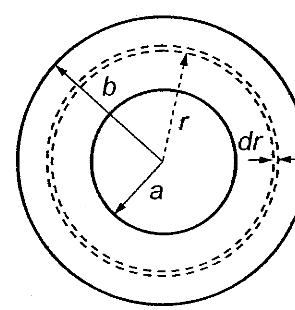
P1.4.1 Let the voltage between the plates be V; $\xi = \dfrac{V}{10^{-3}} =$ 10^3 V/m; D in space filled with air is $10^3\varepsilon_0 V$ C/m^2; charge in this region is $10^3\varepsilon_0 \times 2\times 5\times 10^{-4} V = \varepsilon_0 V$ C; D in space filled with dielectric is $10^4\varepsilon_0 V$ C/m^2; charge in this region is $10^4\varepsilon_0\times 3\times 5\times 10^{-4} V = 15\varepsilon_0 V$ C. Total charge is $16\varepsilon_0 = 16\times 8.85\times 10^{-12} =$ $141.6\times 10^{-12}\times$V C; Capacitance $= \dfrac{141.6\times 10^{-12} V}{V} \equiv$ 0.14 nF.



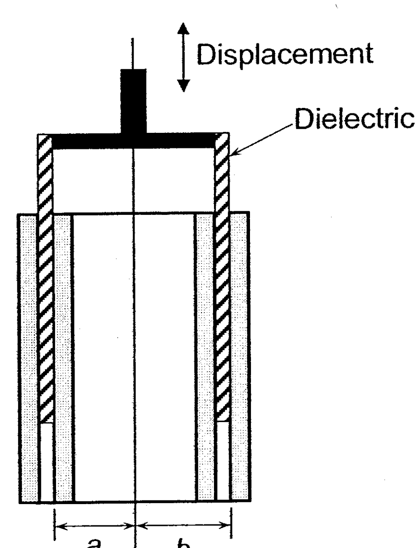
P1.4.2 C/unit area $= \dfrac{\varepsilon}{d}$, or $\varepsilon = 1$ (μF/cm^2)$\times$8 (nm) = $10^{-6}\times 10^4$ (F/m^2)$\times 8\times 10^{-9}$ (m) = 8×10^{-11} F/m. The relative permittivity is $\dfrac{8\times 10^{-11}}{8.85\times 10^{-12}} = 9$.

P1.4.3 Capacitance of a cylindrical shell of radius r, thickness dr, and length l is $\dfrac{\varepsilon\times 2\pi r l}{dr} = dC$. It is convenient to consider the reciprocal of the capacitance. Thus $d\left(\dfrac{1}{C}\right) = \dfrac{1}{2\pi\varepsilon l}\dfrac{dr}{r}$, and

$$\frac{1}{C} = \frac{1}{2\pi\varepsilon l}\int_a^b \frac{dr}{r};\ \frac{1}{C} = \frac{1}{2\pi\varepsilon l}\ln\frac{b}{a},\ \text{or } C = \frac{2\pi\varepsilon l}{\ln\left(\frac{b}{a}\right)}.$$



P1.4.4 Minimum capacitance is when dielectric cylinder is completely out. $C_{min} = \dfrac{2\pi\times 8.85\times 10^{-12}\times 5\times 10^{-2}}{\ln(1.2)} =$ 15.25 pF; $C_{max} = 100C_{min} = 1.525$ nF.



P1.4.5 $i = C\dfrac{dv}{dt} = 2\left[-10Ate^{-10t} + Ae^{-10t} - 10Be^{-10t}\right]$ mA,

$t \geq 0$.

At $t = 0$: $v = 10$ V $\Rightarrow B = 10$ V

$i = 200$ mA $\Rightarrow 2(A - 10B) = 200 \Rightarrow A = 200$ V/s.

P1.4.6 $i = \dfrac{dq}{dt} = \dfrac{d}{dt}(Cv) = \dfrac{d}{dt}\left[C_0\left(1 - e^{-\alpha t}\right)V_0 t^2\right]$

$= C_0V_0\dfrac{d}{dt}\left(t^2 - t^2e^{-\alpha t}\right) = C_0V_0\left(2t - 2te^{-\alpha t} + \alpha t^2e^{-\alpha t}\right)$.

P1.4.7 $q = (5\ \mu\text{F})(20\ \text{V}) = 100\ \mu\text{C}$; each pulse supplies 20 μC; hence $\dfrac{100}{20} = 5$ pulses are required.

P1.4.8 $v = \dfrac{1}{C}\int_0^t idt = \dfrac{1}{2\times10^{-6}}\int_0^t 100\times10^{-6}dt = 50t$ V; $0 \leq t \leq 200$ ms. At 200 ms, $v = 50\times200\times10^{-3} = 10$ V, so $v = 10$ V for $t \geq 200$ ms.

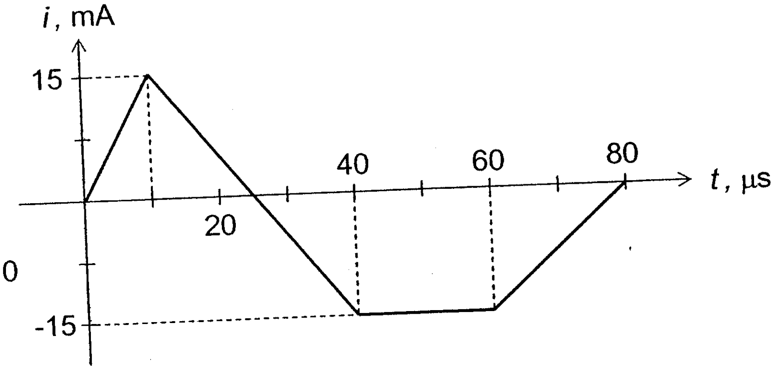
P1.4.9 Differentiating the above, $v = 50$ V for $0 < t < 200$ ms and $v = 0$, $t > 200$ ms. Note that the response at $t = 0$ and at $t = 200$ ms, when the impulses occur, is not defined. Alternatively, the response to the first impulse is $v = \dfrac{1}{C}\int_0^t idt + V(0) = \dfrac{1}{2}\int_{0^-}^{0^+}100\delta(t)dt = 50\text{ V}$, $0 < t < 200$. The response to the second impulse is $= \dfrac{1}{2}\int_{0.2^-}^{0.2^+} -100\delta(t - 0.2)dt + 50 = 0$, $t > 200$ ms. The complete response may be written as: $v = 50u(t) - 50u(t - 0.2)$ V.

P1.4.10 Current pulse: $v = \dfrac{1}{C}\int_0^t idt + v(0) = \dfrac{1}{2\times10^{-6}}\int_0^t 100\times10^{-6}dt - 10 = 50t - 10$ V; $0 \leq t \leq 200$ ms. At 200 ms, $v = 10 - 10 = 0$, so $v = 0$ for $t \geq 200$ ms.

Current impulses: Differentiating the above, $v = 50$ V for $0 < t < 200$ ms and $v = 0$, $t > 200$ ms. Adding the initial voltage of –10 V gives: $v = 40$ V; $0 < t <$ 200 ms and $v = -10$ V; $t > 200$ ms. Alternatively, the response may be obtained as: $v = \dfrac{1}{C}\int_0^t idt + V(0) = \dfrac{1}{2}\left(\int_{0^-}^{0^+}100\delta(t)dt + \int_{0.2^-}^{0.2^+} -\delta(t - 0.2)dt\right) - 10 =$

$= 50u(t) - 50u(t - 0.2) - 10$ V.

P1.4.11 $v = \frac{1}{C}\int_0^t idt + V(t_0)$



(a) $0 \le t \le 10\ \mu s$:

$v = \frac{1}{0.5}\int_0^t 1.5tdt =$ $1.5t^2$ mV, where t is in μs. At t = 10 μs, v = 150 mV.

$10 \le t \le 40\ \mu s$:

$v = \frac{1}{0.5}\int_{10}^t (-t + 25)dt + 150$

$= \left[-t^2 + 50t\right]_{10}^t + 150 = -t^2 + 50t - 250$ mV. At t = 40 μs, v = 150 mV.

$40 \le t \le 60\ \mu s$: $v = \frac{1}{0.5}\int_{40}^t -15dt + 150 = \left[-30t\right]_{40}^t + 150 = -30t + 1350$ mV.

At t = 60 μs, v = –450 mV.

$60 \le t \le 80\ \mu s$: $v = \frac{1}{0.5}\int_{60}^t (0.75t - 60)dt - 450$

$= \left[0.75t^2 - 120t\right]_{60}^t - 450 = 0.75t^2 - 120t + 4050$ mV. At t = 80 μs, v = –750 mV.

$t \ge 80\ \mu s$: v = –750 mV.

Check: Total area = 0.5×15×10 + 0.5×15×15 – 0.5×15×15 – 15×20 – 0.5×15×20 = –375 nC. Hence $v = \frac{-375}{0.5} = -750$ mV.

(b) At $t = 10\ \mu s$, $v = 150$ mV, so $q = CV = 75$ nC.

At $t = 50\ \mu s$, $v = -150$ mV, so $q = CV = -75$ nC.

(c) At $t = 80\ \mu s$, v = –750 mV, so $w = \frac{1}{2}Cv^2 = 0.14\ \mu J$.

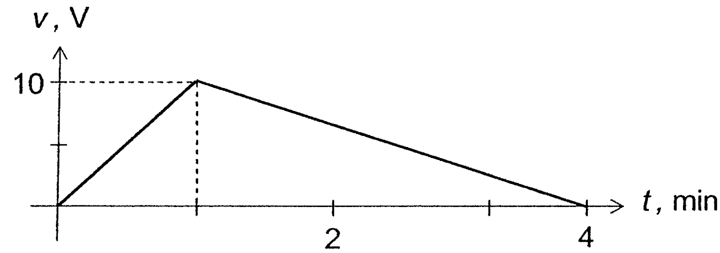
(d) All the expressions derived above for the voltage are increased by 0.5 V.

P1.4.12 $v = \frac{t}{6}$ V, $0 \le t \le 60$ s;

$= -\frac{t}{18} + \frac{40}{3}$ V,

$60 \le t \le 240$ s;

$= 0$, $t \ge 240$ s.



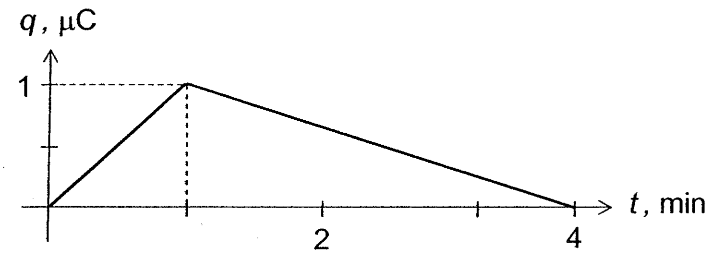
(a) $q = \frac{t}{60}$ μC,

$0 \le t \le 60$ s;

$= -\frac{t}{180} + \frac{4}{3}$ μC,

$60 \le t \le 240$ s;

$= 0$, $t \ge 240$ s.



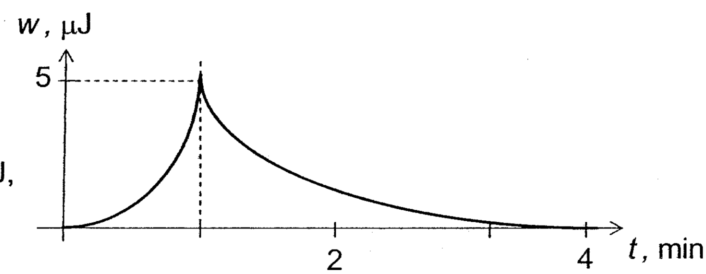
(b) $w = \frac{1}{2}qv = \frac{t^2}{720}$ μJ,

$0 \le t \le 60$ s;

$= \frac{1}{180}\left(\frac{t^2}{36} - \frac{40t}{3} + 1600\right)$ μJ,

$60 \le t \le 240$ s;

$= 0$, $t \ge 240$ s.



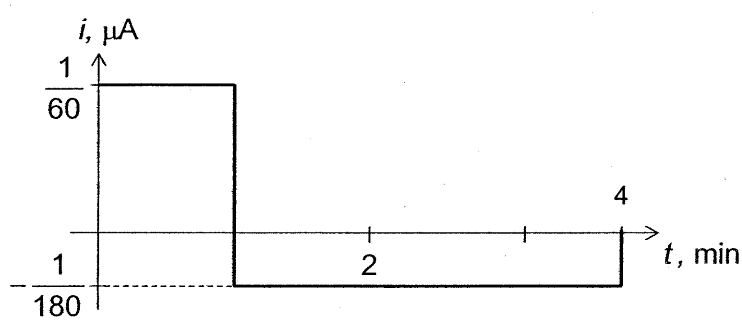
(c) $i = \frac{dq}{dt} = \frac{1}{60}$ μA,

$0 < t < 60$ s;

$= -\frac{1}{180}$ μA,

$60 < t < 240$ s;

$= 0$, $t > 240$ s,



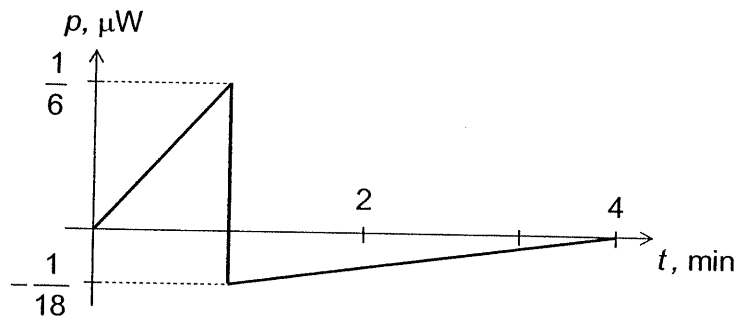
where $\frac{1}{60}$ μA may also be expressed as 1 μC/min.

(d) $p = vi = \dfrac{t}{360}\,\mu\text{W}$, $0 \le t \le 60$ s;

$= \dfrac{1}{180}\left(\dfrac{t}{18} - \dfrac{40}{3}\right)\mu\text{W}$, $60 \le t \le 240$ s;

$= 0$, $t \ge 240$ s, where

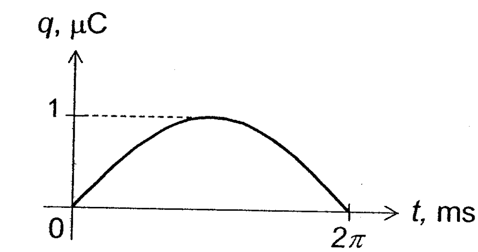
$\dfrac{1}{6}\,\mu\text{W}$ may also be expressed as 10 μJ/min.



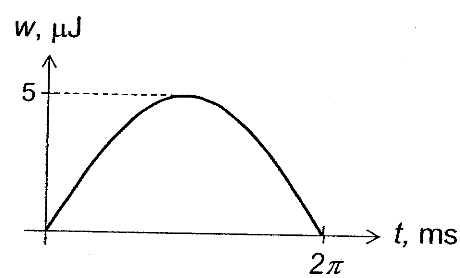
It is seen that $w = \int p\,dt$. Thus $\int_0^t \dfrac{t}{360}dt = \dfrac{t^2}{720}\,\mu\text{J}$, $0 \le t \le 60$ s. At $t = 60$ s, $w = 5\,\mu$J. For $60 \le t \le 240$ s $w = \int_{60}^t \dfrac{1}{180}\left(\dfrac{t}{18} - \dfrac{40}{3}\right)dt + 5$

$= \dfrac{1}{180}\left(\dfrac{t^2}{36} - \dfrac{40t}{3} + 1600\right)\mu\text{J}.$

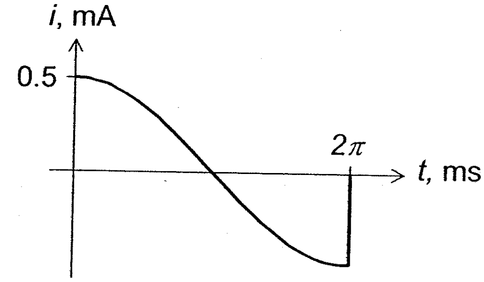
P1.4.13 (a) $q = Cv = 0.1\times10^{-6}\times10\sin(500t) \equiv \sin(500t)\,\mu\text{C}$, $0 \le t \le 2\pi$ ms, and $q = 0$ elsewhere.



(b) $w = \dfrac{1}{2}qv = 5\sin^2(500t)\,\mu\text{J}$, $0 \le t \le 2\pi$ ms, and $w = 0$ elsewhere.



(c) $i = \dfrac{dq}{dt} = 500\cos(500t)\,\mu\text{A}$, $0 \le t \le 2\pi$ ms, and $i = 0$ elsewhere.

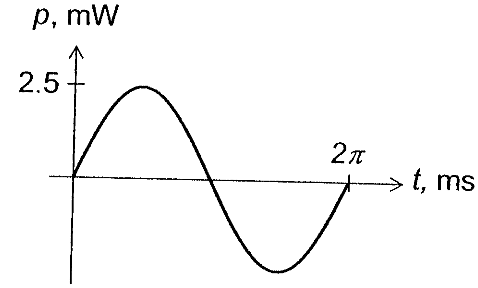


(d) $p = 10\sin(500t) \times 0.5 \times \cos(500t)$

$= 5\sin(500t)\cos(500t) =$

$2.5\sin(1000t)$ mW,

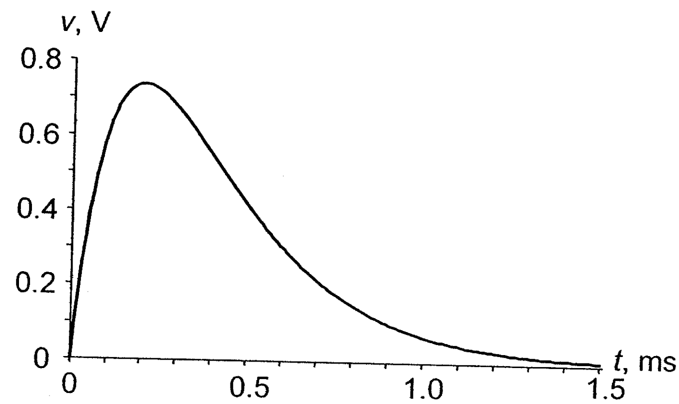
$0 \leq t \leq 2\pi$ ms, and $p = 0$, elsewhere.



$w = \int p dt$. Thus $\int_0^t 2.5\sin(1000t)\, dt =$

$\frac{2.5}{1000}\left[1 - \cos(1{,}000t)\right] \equiv 5\sin^2(500t)$ μJ, as above.

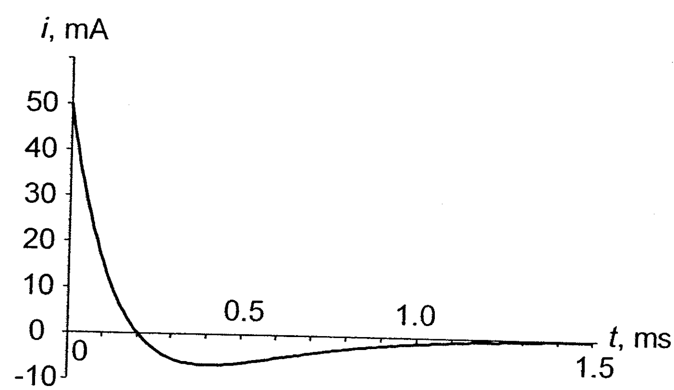
P1.4.14 $v = 10te^{-5t}$ V;



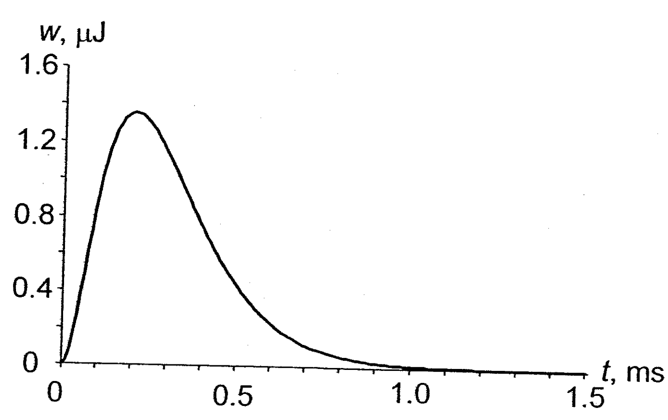
$q = Cv = 50te^{-5t}$ μC;

$i = \frac{dq}{dt} = 50\left(e^{-5t} - 5te^{-5t}\right)$

μC/ms, or mA;



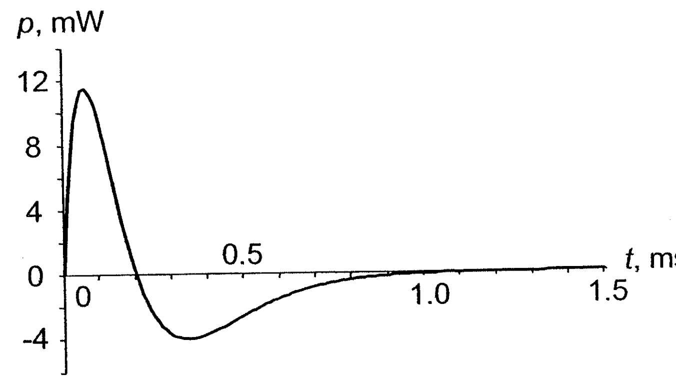
$w = \frac{1}{2}qv = 250t^2e^{-10t}$ μJ;



$p = vi =$

$500\left(te^{-10t} - 5t^2e^{-10t}\right)$m

W.



P1.4.15 (a) $q_C = \int_0^t i_C dt = \int_0^t 10te^{-5t}dt = 10\left[-t\frac{e^{-5t}}{5}\right]_0^t + 2\int_0^t e^{-5t}dt = -2te^{-5t} - \frac{2}{5}e^{-5t} + \frac{2}{5}$

μC. $v_C = \frac{q_C}{C} = -\frac{2}{5}te^{-5t} - \frac{2}{25}e^{-5t} + \frac{2}{25}$V.

(b) i_C is max when $\frac{di_C}{dt} = 0$, or $e^{-5t} - 5te^{-5t} = 0$, which gives t = 0.2 ms. At

this value of t, $v_C = -\frac{0.4}{5}e^{-1} - \frac{2}{25}e^{-1} + \frac{2}{25} \equiv 21.1$ mV.

To verify using MATLAB, enter syms t, then (1/5)*int(10*t*exp(-5*t),0,0.2).

P1.4.16 $q = 0.5v^2 = 0.5(1+t)^2$; $i = \frac{dq}{dt} = (1+t)$; $p = vi = (1+t)^2$; $w = \int_0^t pdt =$

$\int_0^t (1+t)^2 dt = \frac{1}{3}\left[(1+t)^3\right]_0^3 = \frac{63}{3} = 21$ J.

Alternatively, the work done in increasing the charge on the capacitor by dq is vdq, and the total energy stored is $w = \int vdq$. But $dq = vdv$. Hence,

$w = \int_1^4 v^2 dv$, where $v = 1$ when $t = 0$ and $v = 4$ when $t = 3$. This gives

$w = \frac{1}{3}\left[(t+1)^3\right]_1^4 = 21$J.

P1.5.1 $\lambda = 100\times10^{-6} = 10^{-4}$ Wb-turns; $L = \frac{\lambda}{I} = \frac{10^{-4}}{10^{-2}} = 0.01$ H.

P1.5.2 $v = L\frac{di}{dt} = 2(-10Ate^{-10t} + Ae^{-10t} - 10Be^{-10t})$, $t \geq 0$.

At t = 0: i = 10 A $\Rightarrow B = 10$ A

$v = 200$ mV $\Rightarrow 2(A - 10B) = 200 \Rightarrow A = 200$ A/s.

P1.5.3 $v = \frac{d\lambda}{dt} = \frac{d}{dt}(Li) = \frac{d}{dt}\left[L_0\left(1-e^{-\alpha t}\right)I_0 t^2\right]$

$= L_0 I_0 \frac{d}{dt}\left(t^2 - t^2 e^{-\alpha t}\right) = L_0 I_0\left(2t - 2te^{-\alpha t} + \alpha t^2 e^{-\alpha t}\right)$ V.

P1.5.4 $\lambda = (5\ \mu\text{H})(20\ \text{A}) = 100\ \mu$Wb-turns; each pulse supplies 20 μWb-turns; hence $\frac{100}{20} = 5$ pulses are required.

P1.5.5 $i = \frac{1}{L}\int_0^t v dt = \frac{1}{2\times 10^{-6}}\int_0^t 100\times 10^{-6} dt = 50t$ A; $0 \le t \le 200$ ms. At $t = 200$ ms, $i = 50\times 200\times 10^{-3} = 10$ A, so $i = 10$ A for $t \ge 200$ ms.

P1.5.6 $i = \frac{1}{L}\int_0^t v dt + I(0) = \frac{1}{2}\left(\int_{0^-}^{0^+} 100\delta(t)dt + \int_{0.2^-}^{0.2^+} -\delta(t-0.2)dt\right) =$

$= 50u(t) - 50u(t-0.2)$ A. Alternatively, we can differentiate the response of the preceding problem to obtain $i = 50$ A for $0 < t < 200$ ms and $i = 0$, $t > 200$ ms.

P1.5.7 <u>Voltage pulse</u>: $i = \frac{1}{L}\int_0^t v dt + i(0) = \frac{1}{2\times 10^{-6}}\int_0^t 100\times 10^{-6} dt - 10 = 50t - 10$ A; $0 \le t \le 200$ ms. At 200 ms, $i = 10 - 10 = 0$, so $i = 0$ A for $t \ge 200$ ms.

<u>Voltage impulses</u>: Differentiating the above, $i = 50$ A for $0 < t < 200$ ms and $i = 0$, $t > 200$ ms. Adding the initial current of -10 A gives: $i = 40$ A; $0 < t < 200$ ms and $i = -10$ A; $t > 200$ ms. Alternatively, the response may be obtained as:

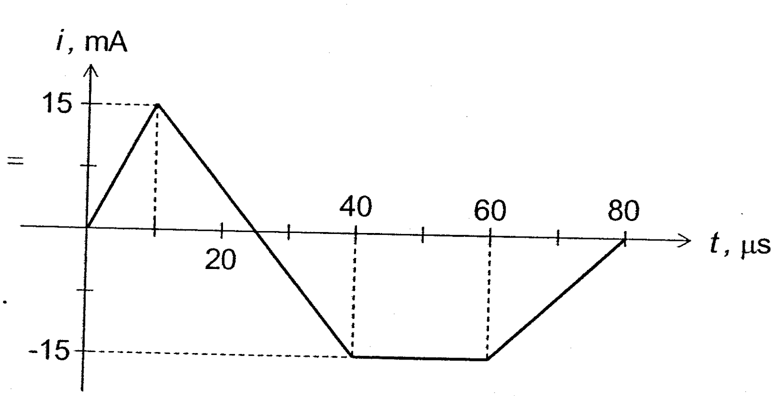
$i = \frac{1}{L}\int_0^t v dt + I(0) = \frac{1}{2}\left(\int_{0^-}^{0^+} 100\delta(t)dt + \int_{0.2^-}^{0.2^+} -\delta(t-0.2)dt\right) - 10 =$

$= 50u(t) - 50u(t-0.2) - 10$ A.

P1.5.8 $i = \frac{1}{L}\int_{t_0}^t v dt + I(t_0)$

(a) $0 \le t \le 10$ μs:

$i = \frac{1}{0.5}\int_0^t 1.5t dt =$ $1.5t^2$ mA, where t is in μs. At $t = 10$ μs, $i = 150$ mA.



$10 \le t \le 40$ μs:

$$i = \frac{1}{0.5}\int_{10}^{t}(-t_1 + 25)dt + 150 \;= \left[-t^2 + 50t\right]_{10}^{t} + 150$$

$= -t^2 + 50t - 250$ mA. At t = 40 μs, v = 150 mA

$40 \le t \le 60$ μs, $i = \frac{1}{0.5}\int_{40}^{t} -15dt + 150 \;= \;= \left[-30t\right]_{40}^{t}$ + 150 = -30t + 1350 mA. At t = 60 μs, i = -450 mA

$60 \le t \le 80$ μs: $i = \frac{1}{0.5}\int_{60}^{t}(0.75t - 60)dt - 450 \;= \left[0.75t^2 - 120t\right]_{60}^{t} - 450 =$ $0.75t^2 - 120t + 4050$ mA. At t = 80 μs, i = -750 mA.

For $t \ge 80$ μs, i = -750 mA

(b) At $t = 10$ μs, $i = 150$ mA, so $\lambda = Li = 0.5$ (μH)×150 mA = 75 nWb-turns.

At $t = 50$ μs, i = -30 × 50 + 1350 = -150 mA, so λ = -75 nWb-turns.

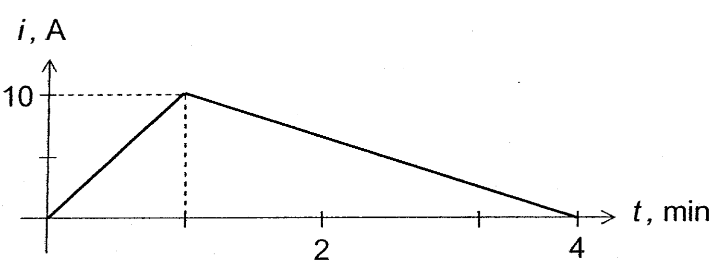
(c) At $t = 80$ μs, i = -750 mA, so $w = \frac{1}{2}Li^2 = 0.14$ μJ.

(d) All the expressions derived above for the current are increased by 0.5 A.

P1.5.9 $i = \frac{t}{6}$ A, $0 \le t \le 60$ s;

$= -\frac{t}{18} + \frac{40}{3}$ A, $60 \le t \le 240$ s;

= 0, $t \ge 240$ s.

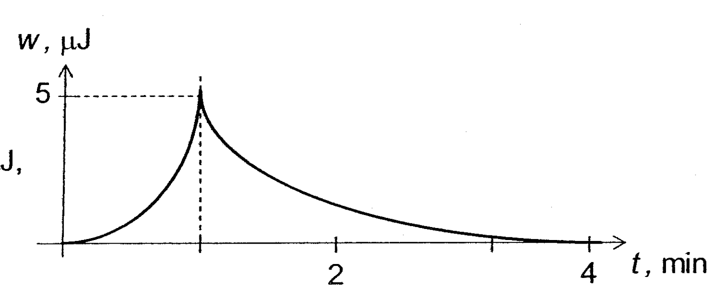


(a) $\lambda = \frac{t}{60}$ μWb-turns, $0 \le t \le 60$ s;

$= -\frac{t}{180} + \frac{4}{3}$ μWb-turns, $60 \le t \le 240$ s = 0, $t \ge 240$ s.

(b) $w = \frac{1}{2}qv = \frac{t^2}{720}$ μJ, $0 \le t \le 60$ s;

$= \frac{1}{180}\left(\frac{t^2}{36} - \frac{40t}{3} + 1600\right)$ μJ, $60 \le t \le 240$ s;



$= 0,\ t \geq 240$ s.

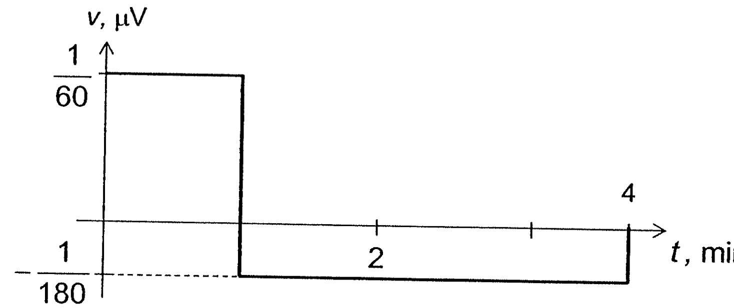
(c) $v = \dfrac{d\lambda}{dt} = \dfrac{1}{60}\ \mu\text{V},$

$0 < t < 60$ s;

$= -\dfrac{1}{180}\ \mu\text{v},$

$60 < t < 240$ s;

$= 0,\ t > 240$ s,



where $\dfrac{1}{60}\ \mu$V may also be expressed as 1 μWb-turns/min.

(d) $p = vi = \dfrac{t}{360}\ \mu\text{W},$

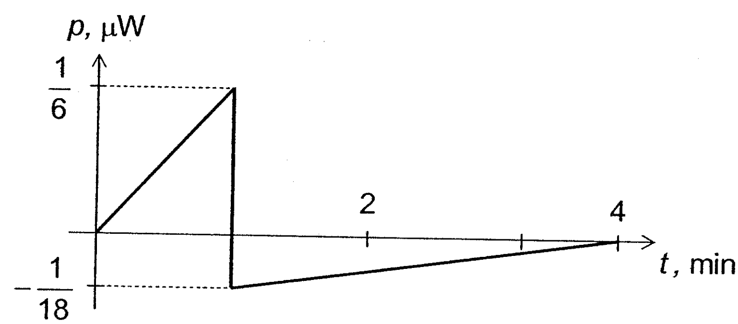
$0 \leq t \leq 60$ s;

$= \dfrac{1}{180}\left(\dfrac{t}{18} - \dfrac{40}{3}\right)\ \mu\text{W},$

$60 \leq t \leq 240$ s;

$= 0,\ t \geq 240$ s, where

$\dfrac{1}{6}\ \mu$W may also be



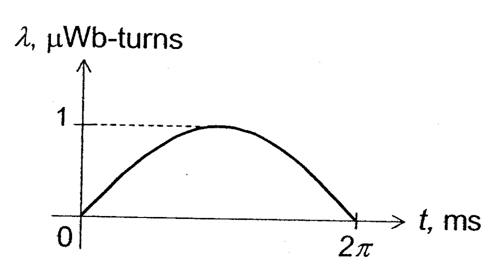
expressed as 10 μJ/min.

t is seen that $w = \int p dt$. Thus $\int_0^t \dfrac{t}{360} dt = \dfrac{t^2}{720}\ \mu\text{J},\ 0 \leq t \leq 60$ s. At $t = 60$ s,

$w = 5$ μJ. For $60 \leq t \leq 240$ s $w = \int_{60}^{t} \dfrac{1}{180}\left(\dfrac{t}{18} - \dfrac{40}{3}\right)dt + 5$

$= \dfrac{1}{180}\left(\dfrac{t^2}{36} - \dfrac{40t}{3} + 1600\right)\ \mu\text{J}.$

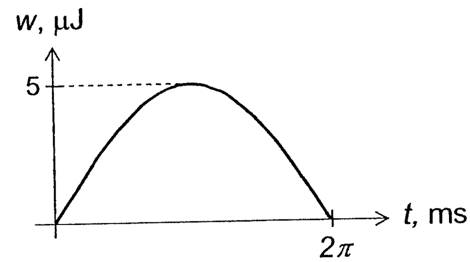
P1.5.10 (a) $\lambda = Li = 0.1\times10^{-6}\times10\sin(500t) \equiv \sin(500t)$ μWb-turns, $0 \leq t \leq 2\pi$ ms, and $\lambda = 0$ elsewhere.



(b) $w = \frac{1}{2}\lambda i = 5\sin^2(500t)$ μJ,

$0 \le t \le 2\pi$ ms, and $w = 0$

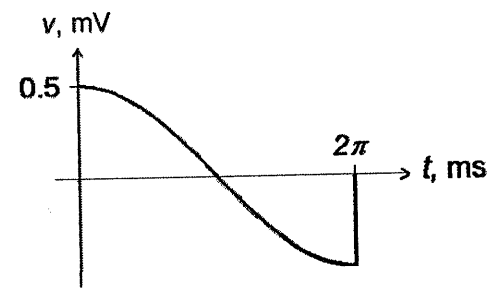
elsewhere.



(c) $v = \frac{d\lambda}{dt} = 500\cos(500t)$ μV,

$0 \le t \le 2\pi$ ms, and $i = 0$

elsewhere.



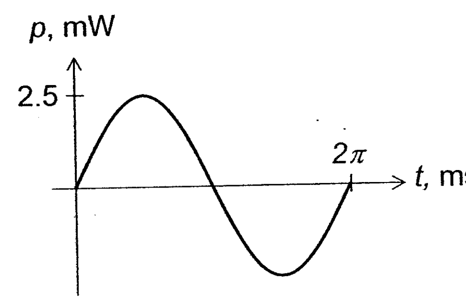
(d) $p = 10\sin(500t) \times 0.5 \times \cos(500t)$

$= 5\sin(500t)\cos(500t) =$

$2.5\sin(1000t)$ mW,

$0 \le t \le 2\pi$ ms, and $p = 0$,

elsewhere.

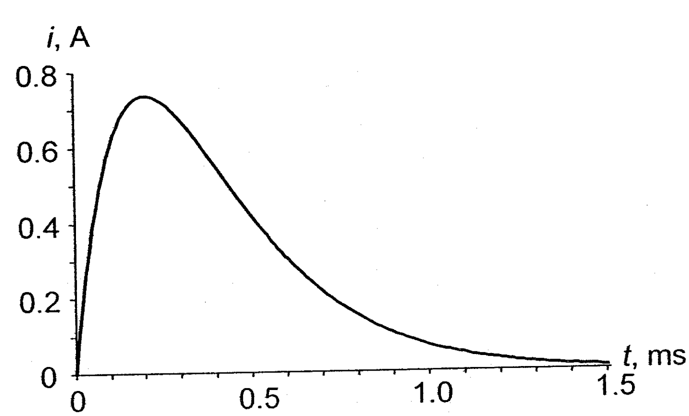


$w = \int p\,dt$. Thus $\int_0^t 2.5\sin(1000t)\,dt =$

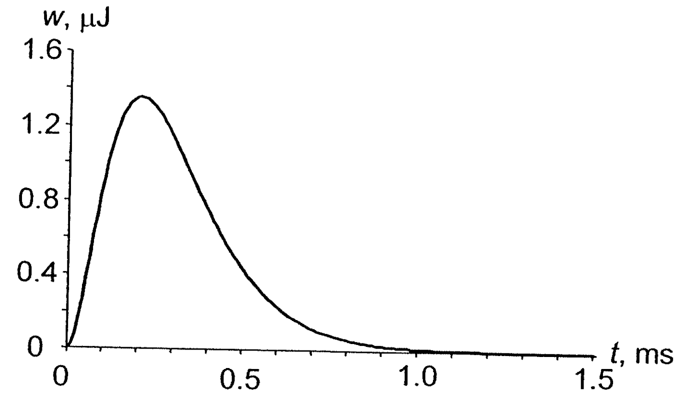
$\frac{2.5}{1000}\left[1 - \cos(1{,}000t)\right] \equiv 5\sin^2(500t)$ μJ, as above.

P1.5.11 $i = 10te^{-5t}$ A

$\lambda = Li = 50te^{-5t}$ μWb-turns;

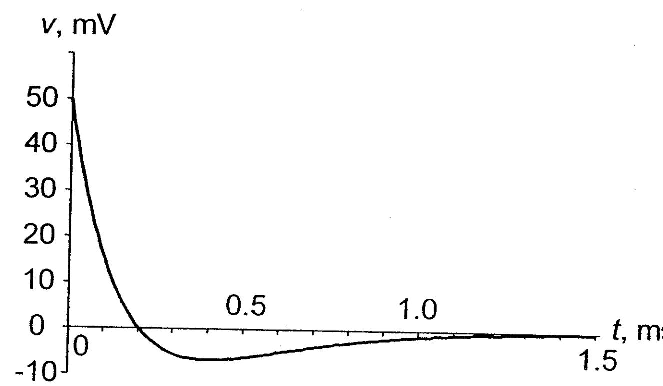


$w = \frac{1}{2}\lambda i = 250t^2 e^{-10t}$ μJ;



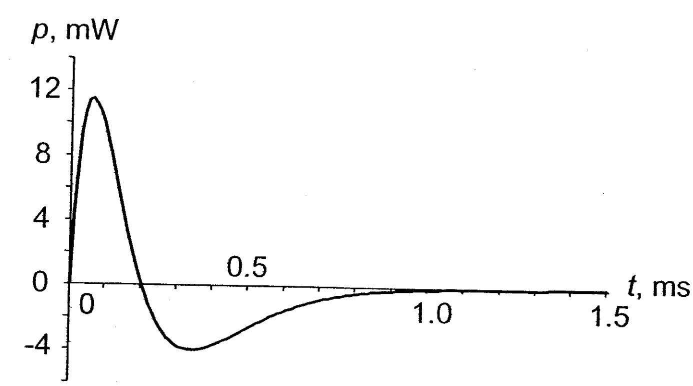
$v = \frac{d\lambda}{dt} = 50\left(e^{-5t} - 5te^{-5t}\right)$

μWb-turns/ms, or mV;



$p = vi =$

$500\left(te^{-10t} - 5t^2 e^{-10t}\right)$ mW.



P1.5.12 (a) $\lambda_L = \int_0^t v_L dt = \int_0^t 10te^{-5t}dt = 10\left[-t\frac{e^{-5t}}{5}\right]_0^t + 2\int_0^t e^{-5t}dt = -2te^{-5t} - \frac{2}{5}e^{-5t} + \frac{2}{5}$

μWb-turns. $i_L = \frac{\lambda_L}{L} = -\frac{2}{5}te^{-5t} - \frac{2}{25}e^{-5t} + \frac{2}{25}$ A.

(b) v_L is max when $\frac{dv_L}{dt} = 0$, or $e^{-5t} - 5te^{-5t} = 0$, which gives t = 0.2 ms. At this value of t, $i_L = -\frac{0.4}{5}e^{-1} - \frac{2}{25}e^{-1} + \frac{2}{25} \equiv 21.1$ mA.

P1.5.13 $\lambda = 0.5i^2 = 0.5(1+t)^2$; $v = \frac{d\lambda}{dt} = (1+t)$; $p = vi = (1+t)^2$; $w = \int_0^t pdt =$

$$\int_0^t (1+t)^2 dt = \frac{1}{3}\left[(1+t)^3\right]_0^3 = \frac{63}{3} = 21 \text{ J.}$$

Alternatively, the work done in increasing the flux linkage in the inductor by $d\lambda$ is $id\lambda$, and the total energy stored is $w = \int id\lambda$. But $d\lambda = idi$. Hence,

$w = \int_1^4 i^2 di$, where $i = 1$ when $t = 0$ and $i = 4$ when $t = 3$. This gives

$$w = \frac{1}{3}\left[(t+1)^3\right]_1^4 = 21\text{J.}$$

P1.6.1 The number of molecules in one gram-molecular weight is given by Avogadro's number (6.025×10^{23}). A 0.1 M solution of $CuCl_2$ contains $0.1(6.025 \times 10^{23})$ Cu^{++} ions per liter and $0.2(6.025 \times 10^{23})$ Cl^- ions per liter.

From Eq. (1.4.1), $i = Aune = 1 \times \left(\frac{0.2}{60}\right) \times \left(\frac{0.1 \times 6.025 \times 10^{23}}{1000}\right) \times \left(2 \times 1.6 \times 10^{-19}\right) +$

$$1 \times \left(-\frac{0.4}{60}\right) \times \left(\frac{0.2 \times 6.025 \times 10^{23}}{1000}\right) \times \left(-1.6 \times 10^{-19}\right) = 0.19 \text{ A.}$$