CHAPTER 1 MAGNETIC AND MAGNETICALLY COUPLED CIRCUITS

Short Problems

SP 1.2-1

$$Z = \frac{\tilde{V}}{\tilde{I}} = 1\angle 180^{\circ} = -1$$
$$S = \tilde{V}\tilde{I} = -1\,\text{W}$$

Negative indicates generator operation.

SP 1.2-2

$$V(t) = \sqrt{2}\cos(377t)$$
$$I(t) = \sqrt{2}\cos(377t + \pi)$$
$$P(t) = V(t)I(t) = -2\cos^2(377t) = -1 - \cos(754t) = -1 + \cos(754t + \pi)$$

SP1.2-3

$$A + B = \sqrt{2}(1 + j) = \sqrt{2}\sqrt{2}(1\angle 45^{\circ}) = 2\angle 45^{\circ}$$
$$AB = \sqrt{2}\sqrt{2}\,j = 2j = 2\angle 90^{\circ}$$

SP 1.2-4 For simplicity, and without lack of generality, neglect resistance and assume the voltage is along the real axis. In this case the relationship between voltage and current is given by:

$$V\angle 0 = j(X_L - X_C)\tilde{I} + \tilde{E}$$

One can observe that the magnitude and phase of $\tilde{E}$ controls the magnitude and direction of $\tilde{I}$. For example, if $\tilde{E} = 2\angle 0^{\circ}$ and $X_L > X_c$, $\tilde{I}$ will lead $\tilde{V}$. In contrast, if $\tilde{E} = 0.5\angle 0^{\circ}$ and $X_L > X_c$, $\tilde{I}$ will lag $\tilde{V}$.

SP1.3-1. $\mathrm{N}i = \mathscr{R}_{ab}(\Phi_1 + \Phi_2) + \mathscr{R}_{bcda}\Phi_1$

$$\Phi_1 = \frac{1}{\mathscr{R}_{bcda}}\left[\mathrm{N}i - \mathscr{R}_{ab}(\Phi_1 + \Phi_2)\right]$$
$$\Phi_1 = \frac{1}{358{,}099}\left[-(109{,}419)(2.547 \times 10^{-3})\right] = 2.014 \times 10^{-3}\ \text{Wb}$$

SP1.3-2. $\tilde{\Phi}_1 + \tilde{\Phi}_2 = \dfrac{\mathrm{N}\tilde{I}}{\mathscr{R}_{ab} + \mathscr{R}_{eq}}$

$$= \frac{(100)\ 10\angle{-30^{\circ}}}{109{,}419 + 283{,}148} = 2.547 \times 10^{-3}\ \angle{-30^{\circ}}\ \text{Wb, rms}$$

SP1.3-3. With windings as shown in Fig. 1B-1 and with the center leg removed, the total mmf is

$$\text{mmf}_t = \text{mmf}_1 + \text{mmf}_2 = N_1 I_1 + N_2 I_2$$
$$= (150)(9) + (90)(-15) = 0$$

SP1.3-4

$$\tilde{\phi}_2 = \sqrt{1.065^2 + 0.427^2} \text{ x } 10^{-3} < \tan^{-1}\left(\frac{0.427}{1.065}\right)$$
$$\tilde{\phi}_2 = \sqrt{2}\ 1.147 \text{ x } 10^{-3} \cos\left(\omega_e t + 21.8^o\right)$$

SP1.4-1. From Fig. 1.3-1, $H_i = 200$ A/m for $B_i = 1$ Wb/m^2. Also,

$$H_g = \frac{B_g}{\mu_0} = \frac{1}{A\pi \text{ x } 10^{-7}} = 7.958 \text{ x } 10^5 \text{ A/m.}$$

From (1.3-3),

$$H_i l_i + H_g l_g = NI$$
$$(200)(1) + (7.958 \times 10^5)(0.001) = (500)I$$

Thus, I = 1.99 A

SP1.5-1. $\mathcal{R}_m = 2\mathcal{R}_y = (2)(37{,}600)\ H^{-1}$

$$L_{11} = \frac{N_1^2}{\mathcal{R}_m} = \frac{(150)^2}{(2)(37{,}600)} = 299.2 \text{ mH}$$
$$L_{22} = \frac{N_2^2}{\mathcal{R}_m} = \frac{(90)^2}{(2)(37{,}600)} = 107.7 \text{ mH}$$

$$L_{12} = \frac{N_1 N_2}{\mathcal{R}_m} = \frac{(150)(90)}{(2)(37{,}600)} = 179.5 \text{ mH}$$

SP1.5-2. During steady-state conditions, the time rate-of-change of i_1 is zero; therefore, a voltage is not induced in the 2-winding. Hence, for the 2-winding open or short circuited $I_1 = \frac{V}{r_1} = \frac{12}{6} = 2$ A and $I_2 = 0$.

SP1.5-3. $Z = r_1 + j\,\omega_e\,(L_{l1} + L_{m1}) = 6 + j\,(100)(13.5 + 263.9) \times 10^{-3}$

$$= 6 + j\,27.74 = 28.38\ \underline{/77.8^\circ}$$
$$\tilde{I}_1 = \frac{\tilde{V}_1}{Z} = \frac{10/0^\circ}{28.38/77.8^\circ} = 0.352\ \underline{/-77.8^\circ}\text{ A}$$

SP1.6-1. The peak-to-peak value of λ as the system approaches steady-state operation is $(0.95)(\sqrt{2})(0.29) = 0.389$ V·s. Now, $\lambda = L_{ml}(i_1 + i_2')$ or $i_1 + i_2' = \frac{\lambda}{L_{m1}}$. The amplitude of $i_1 + i_2' = \frac{0.389}{(2)(263.9 \times 10^{-3})}$. Hence,

$$|\tilde{I}_1 + \tilde{I}_2'| = \frac{0.389}{(\sqrt{2})(2)(263.9 \times 10^{-3})} \cong 0.5 \text{ A}$$

SP1.6-2. $Z = (r_1 + r_2') + j\,\omega_e(L_{l1} + L_{l2}')$

$$\phi = \tan^{-1}\frac{\omega_e(L_{l1} + L_{l2}')}{r_1 + r_2'} = \tan^{-1}\frac{(377)(13.5+13.5)\times 10^{-3}}{6+5} = 42.8^\circ$$

SP1.6-3. $Z = (r_1 + r_2' + R_L') + j\left[\omega_e(L_{l1} + L_{l2}) + X_L'\right]$

$$= (6+5+21) + j\left[(1000)(13.5+13.5)\times 10^{-3} + 5\right] = 32 + j\,32\ \Omega$$

$$\tilde{I}_2' \cong -\tilde{I}_1 = -\frac{\tilde{V}_1}{Z} \cong \frac{-110\,\underline{/0^\circ}}{32 + j\,32} \cong -2.4\,\underline{/-45^\circ}$$

SP1.7-1. $$\frac{d[L_m(x)\,i]}{dt} = \frac{\partial[L_m(x)\,i]}{\partial i}\frac{di}{dt} + \frac{\partial[L_m(x)\,i]}{\partial x}\frac{dx}{dt}$$

$$= L_m(x)\frac{di}{dt} + i\frac{\partial L_m(x)}{\partial x}\frac{dx}{dt}$$

$$= \frac{k}{x}\frac{di}{dt} + i\left(-\frac{k}{x^2}\right)\frac{dx}{dt}$$

Now, $x = t$ and $i = t$; thus,

$$\frac{d[L_m(x)\,i]}{dt} = \frac{k}{t}\frac{dt}{dt} - t\frac{k}{t^2}\frac{dt}{dt} = \frac{k}{t} - \frac{k}{t} = 0$$

SP1.7-2. For $\theta_r(0)$, $\mathscr{R}$ is minimum thus $L(\theta_r)$ is maximum. Hence,

$$L_m(0) = L_A + L_B$$

$$L_m\left(\frac{\pi}{2}\right) = L_A - L_B; \text{ thus,}$$

$$L(\theta_r) = L_l + L_A + L_B\cos 2\theta_r$$

SP1.7-3. The self-inductances, L_{11} and L_{22}, are independent of current if saturation is neglected; therefore, they are unchanged. The mutual inductance is positive if the mmfs aid and negative when they oppose. Here, the mmfs oppose when $\theta_r = 0$ and aid when $\theta_r = \pi$. Thus,

$$L_{12} = -L_{sr}\cos\theta_r$$

SP1.7-4. $L_{12} = L_{sr} \cos \theta_r$

$$v_2 = \frac{d[L_{12}\ i_1]}{dt} = \frac{\partial [L_{12}\ i_1]}{\partial i_1} \frac{di_1}{dt} + \frac{\partial [L_{12}\ i_1]}{\partial \theta_r} \frac{d\theta_r}{dt}$$

However, $\frac{di_1}{dt} = 0$ since i_1 is constant; thus,

$$v_2 = \frac{\partial [(0.1 \cos \theta_r)(1)]}{\partial \theta_r}(100) = -(0.1)(100) \sin \theta_r = -10 \sin 100t \text{ V}$$

Long Problems

1.
$$\mathcal{R}_g = \frac{l_g}{\mu_g A_g} = \frac{0.004}{(4\pi \times 10^{-7})(0.04)^2} = 1{,}989{,}440 \text{ H}^{-1}$$

$$\mathcal{R}_i = \frac{l_i}{\mu_i A_i} = \frac{(200)(0.004)}{(1500)(4\pi \times 10^{-7})(0.04)^2} = \frac{200}{1500} \mathcal{R}_g = 265{,}258 \text{ H}^{-1}$$

$$\Phi = \frac{N i}{\mathcal{R}_g + \mathcal{R}_i} = \frac{(100)(2)}{1{,}989{,}440 + 265{,}258} = 8.87 \times 10^{-5} \text{ Wb}$$

2. The parallel branches now have equal reluctance. Hence, from Example 1A

$$\mathcal{R}_{eq} = \frac{1}{2}(\mathcal{R}_{bef} + \mathcal{R}_{fg} + \mathcal{R}_{gha}) = \frac{1}{2}(1{,}352{,}816) = 676{,}408 \text{ H}^{-1}$$

$$\Phi_1 + \Phi_2 = \frac{N i}{\mathcal{R}_{ab} + \mathcal{R}_{eq}} = \frac{(100)(10)}{109{,}419 + 676{,}408} = 1.2725 \times 10^{-3} \text{ Wb}$$

3. The reluctance of the iron is

$$\mathcal{R}_m = \frac{l}{\mu_i A} = \frac{(4)(0.25)}{(4000)(4\pi \times 10^{-7})(0.05)^2} = 79{,}577 \text{ H}^{-1}$$

$$L_{12} = \frac{N_1 N_2}{\mathcal{R}_m} = \frac{(50)(100)}{79{,}577} = 0.0628 \text{ H}$$

$$L_{m1} = \frac{N_1^2}{\mathcal{R}_m} = \frac{50^2}{79{,}577} = 0.0314 \text{ H}$$

$$L_{m2} = \frac{N_2^2}{\mathcal{R}_m} = \frac{100^2}{79{,}577} = 0.1257 \text{ H}$$

4. $$\mathscr{R}_m = \frac{l}{\mu A} = \frac{(2\pi)(1)}{(4\pi\times 10^{-7})(\frac{1}{4\pi}\times 10^{4})\,\pi\,(0.05)^2} = 8\times 10^5\ \mathrm{H}^{-1}$$

$$L_{12} = -\frac{N_1 N_2}{\mathscr{R}_m} = -\frac{(100)(200)}{8\times 10^5} = -25\ \mathrm{mH}$$

5. The flux through N_2 due to current in N_1 may be written

$$\Phi_{21} = \frac{\mathscr{R}_x}{\mathscr{R}_x + \mathscr{R}_y}\,\frac{N_1 i_1}{\mathscr{R}_m}$$

where $\mathscr{R}_m = 10\,\mathscr{R}_y + \dfrac{\mathscr{R}_x \mathscr{R}_y}{\mathscr{R}_x + \mathscr{R}_y}$; thus,

$$\Phi_{21} = \frac{N_1 i_1}{11\,\mathscr{R}_y + 10\,\dfrac{\mathscr{R}_y^2}{\mathscr{R}_x}}$$

$$L_{21} = \frac{N_1 N_2}{11\,\mathscr{R}_y + 10\,\dfrac{\mathscr{R}_y^2}{\mathscr{R}_x}}$$

By a similar method, it can be shown that $L_{12} = L_{21}$.

6. (a) $L_{m1} = L_{11} - L_{l1} = 90\ \mathrm{mH}$

$$L'_{l2} = (\frac{N_1}{N_2})^2 L_{l2} = (\frac{100}{50})^2 (2.5\times 10^{-3}) = 10\ \mathrm{mH}$$

$$r'_2 = (\frac{N_1}{N_2})^2 r_2 = (\frac{100}{50})^2 (2.5) = 10\ \Omega$$

(b) $L_{m2} = L_{22} - L_{l2} = 22.5\ \mathrm{mH}$

$$L'_{l1} = (\frac{N_2}{N_1})^2 L_{l1} = (\frac{50}{100})^2 (10\times 10^{-3}) = 2.5\ \mathrm{mH}$$

$$r'_1 = (\frac{N_2}{N_1})^2 r_1 = (\frac{50}{100})^2 (10) = 2.5\ \Omega$$

7. (a) $L_{12} = -\dfrac{N_1 N_2}{\mathscr{R}_m}$

(b) $\lambda_1 = L_{11}\, i_1 + L_{12}\, i_2$

$\lambda_2 = L_{21}\, i_1 - L_{22}\, i_2$

(c) $\lambda_1 = L_{l1}\, i_1 + L_{m1}\,(i_1 - i_2')$

$\lambda_2' = -L_{l2}'\, i_2' + L_{m1}\,(i_1 - i_2')$

(d) v_1 is unchanged

$v_2' = -r_2'\, i_2' + \dfrac{d\lambda_2'}{dt}$

8. $\tilde{V}_1 = \dfrac{10}{2\sqrt{2}}\ \underline{/0^\circ} = 3.54\ \underline{/0^\circ}\ V$

$Z = r_1 + r_2' + j\,\omega_e\,(L_{l1} + L_{l2}') = 10 + 10 + j\,(2\pi)(30)(30 + 30) \times 10^{-3}$
$= 20 + j\,11.31\ \Omega$

$\tilde{I}_1 = \dfrac{\tilde{V}_1}{Z} = \dfrac{3.54\ /0^\circ}{20 + j\,11.31} = 0.154\ \underline{/-29.5^\circ}\ A$

9. Since $\omega_e = 400$, $X_{m1} = 400\ \Omega$. Neglecting the magnetizing current $i_1 = -i_2'$.

(a) $\tilde{V}_1 = \dfrac{2}{\sqrt{2}}\ \underline{/0^\circ} = \sqrt{2}\ \underline{/0^\circ}$

$\tilde{I}_1 = \dfrac{\tilde{V}_1}{(r_1 + r_2 + R_L) + j\,\omega_e\,(L_{l1} + L_{l2})} = \dfrac{\sqrt{2}\ /0^\circ}{4 + j\,(400)(0.02)} = 0.158\ \underline{/-63.4^\circ}\ A$

(b) $I_1 = \sqrt{2}\ 0.158 \cos(400t - 63.4^\circ)$

10. $Z = r_1 + (\frac{N_1}{N_2})^2 (r_2 + R_L) + j\,\omega_e \left[L_{l1} + (\frac{N_1}{N_2})^2 L_{l2}\right]$

$= 1 + (\frac{1}{2})^2 (2 + 4) + j\,(400)\left[+ (\frac{1}{2})^2 (0.04)\right] = 2.5 + j\,8\ \Omega$

$\tilde{I}_1 = \frac{\tilde{V}_1}{Z} = \frac{2\ \angle 0^\circ}{2.5 + j\,8} = 0.239\ \angle -72.6^\circ\ A$

$\tilde{I}_2^{'} = -\tilde{I}_1 = -0.239\ \angle -72.6^\circ\ A$

$\tilde{V}_2^{'} = -R_L^{'}\,\tilde{I}_2^{'} = -(\frac{1}{2})^2 (4)(-0.239\ \angle -72.6^\circ) = 0.239\ \angle -72.6^\circ\ V$

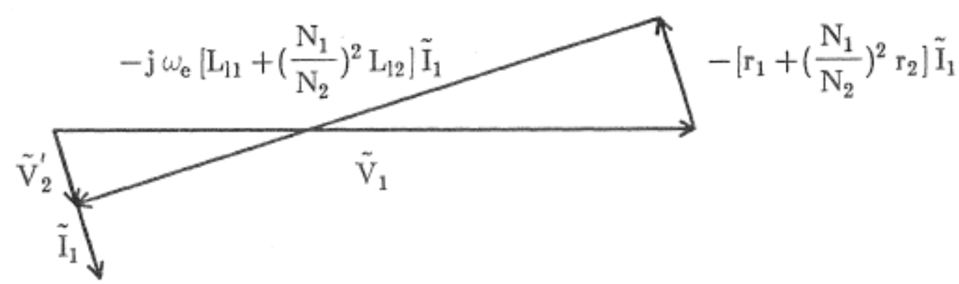
Another method

$\tilde{V}_2^{'} = \tilde{V}_1 - \tilde{I}_1 \left\{ r_1 + (\frac{N_1}{N_2})^2 r_2 + j\,\omega_e \left[L_{l1} + (\frac{N_1}{N_2})^2 L_{l2}\right]\right\}$

$= 2\ \angle 0^\circ - 0.239\ \angle -72.6^\circ (1.5 + j\,8)$

$= 2\ \angle 0^\circ - (1.5)(0.239\ \angle -72.6^\circ) - j\,(8)(0.239\ \angle -72.6^\circ)$

$= 2\angle 0^\circ - 0.359\ \angle -72.6^\circ - j\,1.912\ \angle -72.6^\circ = 0.239\ \angle -72.6^\circ\ V$



Phasor Diagram.

11. $L_m(x) = \dfrac{k}{k_o + x}$

$$k = \frac{N^2 \mu_0 A_i}{2} = \frac{(500)^2(4\pi \times 10^{-7})(4 \times 10^{-4})}{2} = 2\pi \times 10^{-5}$$

$$k_0 = \frac{l_i}{2\mu_{ri}} = \frac{20 \times 10^{-2}}{(2)(1000)} = 10^{-4}$$

$$L_m(x) = \frac{2\pi \times 10^{-5}}{10^{-4} + x} \text{ H}$$

The approximation for $x > 0$ is

$$L_m(x) = \frac{2\pi \times 10^{-5}}{x} \text{ H}$$

Now to find minimum value of x

$$\frac{2\pi \times 10^{-5}}{x} = 1.1\ \frac{2\pi \times 10^{-5}}{10^{-4} + x}$$

Solving for x yields x = 1 mm. Thus, the approximate expression is 10% in error at x = 1 mm and less than this for x > 1 mm.

12. $L_m(x) = \dfrac{k}{k_0 + x}$

$$\frac{\partial L_m(x)}{\partial x} = \frac{-k}{(k_0 + x)^2}$$

$$v = ri + (L_l + \frac{k}{k_0 + x})\frac{di}{dt} - i\frac{k}{(k_0 + x)^2}\frac{dx}{dt}$$

$$v = r\sqrt{2}\, I_s \cos\omega_e t - (L_l + \frac{k}{k_0 + t})\,\omega_e \sqrt{2}\, I_s \sin\omega_e t - \sqrt{2}\, I_s \cos\omega_e t\left[\frac{k}{(k_0 + t)^2}\right]$$

Gathering terms

$$v = \left[r - \frac{k}{(k_0 + t)^2}\right]\sqrt{2}\, I_s \cos\omega_e t - (L_l + \frac{k}{k_0 + t})\,\omega_e \sqrt{2}\, I_s \sin\omega_e t$$

Taking the limit as $t \rightarrow \infty$

$$v = r\sqrt{2}\, I_s \cos\omega_e t - L_l\, \omega_e \sqrt{2}\, I_s \sin\omega_e t$$

which is the voltage equation for a linear r-L circuit. In phasor form,

$$\tilde{V} = (r + j\,\omega_e L_l)\tilde{I}$$

13. $$v = r\,i + \left[L_l + L_m(\theta_r)\right]\frac{di}{dt} + i\,\frac{\partial L_m(\theta_r)}{\partial \theta_r}\,\frac{d\theta_r}{dt}$$

$$v = r\,i + \left[L_l + L_A - L_B \cos 2\theta_r\right]\frac{di}{dt} + 2\,\omega_r\, i\, L_B \sin 2\theta_r$$

14. $$v_1 = r_1\, i_1 + \frac{d\lambda_1}{dt}$$

$$v_2 = r_2\, i_2 + \frac{d\lambda_2}{dt}$$

$$\lambda_1 = L_{11}\, i_1 + (L_{sr} \cos \theta_r)\, i_2$$

$$\lambda_2 = L_{22}\, i_2 + (L_{sr} \cos \theta_r)\, i_1$$

$$\frac{d\lambda_1}{dt} = \frac{\partial \lambda_1}{\partial i_1}\frac{di_1}{dt} + \frac{\partial \lambda_1}{\partial i_2}\frac{di_2}{dt} + \frac{\partial \lambda_1}{\partial \theta_r}\frac{d\theta_r}{dt}$$

$$= L_{11}\frac{di_1}{dt} + L_{sr}\cos\theta_r \frac{di_2}{dt} - i_2\, \omega_r\, L_{sr} \sin\theta_r$$

$$\frac{d\lambda_2}{dt} = L_{22}\frac{di_2}{dt} + L_{sr}\cos\theta_r \frac{di_1}{dt} - i_1\, \omega_r\, L_{sr} \sin\theta_r;\ \text{thus,}$$

$$v_1 = r_1\, i_1 + L_{11}\frac{di_1}{dt} + L_{sr}\cos\theta_r \frac{di_2}{dt} - i_2\, \omega_r\, L_{sr}\sin\theta_r$$

$$v_2 = r_2\, i_2 + L_{22}\frac{di_2}{dt} + L_{sr}\cos\theta_r \frac{di_1}{dt} - i_1\, \omega_r\, L_{sr}\sin\theta_r$$