

CHAPTER 1

P. E. 1.1

(a) $\mathbf{A} + \mathbf{B} = (1,0,3) + (5,2,-6) = (6,2,-3)$

$$|\mathbf{A} + \mathbf{B}| = \sqrt{36 + 4 + 9} = \underline{\underline{7}}$$

(b) $5\mathbf{A} - \mathbf{B} = (5,0,15) - (5,2,-6) = \underline{\underline{(0,-2,21)}}$

(c) The component of \mathbf{A} along \mathbf{a}_y is $A_y = \underline{\underline{0}}$

(d) $3\mathbf{A} + \mathbf{B} = (3,0,9) + (5,2,-6) = (8,2,3)$

A unit vector parallel to this vector is

$$\begin{aligned} \mathbf{a}_{11} &= \frac{(8,2,3)}{\sqrt{64 + 4 + 9}} \\ &= \underline{\underline{\pm(0.9117\mathbf{a}_x + 0.2279\mathbf{a}_y + 0.3419\mathbf{a}_z)}} \end{aligned}$$

P. E. 1.2 (a) $\mathbf{r}_p = \underline{\underline{\mathbf{a}_x - 3\mathbf{a}_y + 5\mathbf{a}_z}}$

$$\mathbf{r}_R = \underline{\underline{3\mathbf{a}_y + 8\mathbf{a}_z}}$$

(b) The distance vector is

$$\mathbf{r}_{QR} = \mathbf{r}_R - \mathbf{r}_Q = (0,3,8) - (2,4,6) = \underline{\underline{-2\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z}}$$

(c) The distance between Q and R is

$$|\mathbf{r}_{QR}| = \sqrt{4 + 1 + 4} = \underline{\underline{3}}$$

P. E. 1.3 Consider the figure shown on the next page:

$$\mathbf{u}_z = \mathbf{u}_p + \mathbf{u}_w = -350\mathbf{a}_x + \frac{40}{\sqrt{2}}(-\mathbf{a}_x + \mathbf{a}_y)$$

$$= -378.28\mathbf{a}_x + 28.28\mathbf{a}_y \text{ km/hr}$$

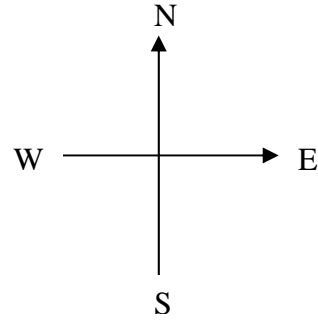
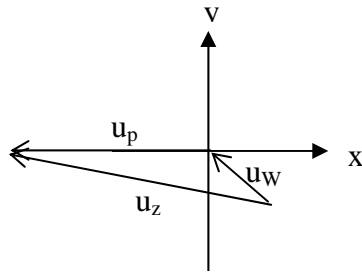
or

$$\mathbf{u}_z = 379.3 \angle 175.72^\circ \text{ km/hr}$$

Where \mathbf{u}_p = velocity of the airplane in the absence of wind

\mathbf{u}_w = wind velocity

\mathbf{u}_z = observed velocity

**P. E. 1.4**

Using the dot product,

$$\cos\theta_{AB} = \frac{\mathbf{A} \cdot \mathbf{B}}{AB} = \frac{-13}{\sqrt{10}\sqrt{65}} = -\sqrt{\frac{13}{50}}$$

$$\underline{\underline{\theta_{AB} = 120.66^\circ}}$$

P. E. 1.5

$$\begin{aligned} \text{(a) } \mathbf{E}_F &= (\mathbf{E} \cdot \mathbf{a}_F) \mathbf{a}_F = \frac{(\mathbf{E} \cdot \mathbf{F})\mathbf{F}}{|\mathbf{F}|^2} = \frac{-10(4, -10, 5)}{141} \\ &= \underline{\underline{-0.2837\mathbf{a}_x + 0.7092\mathbf{a}_y - 0.3546\mathbf{a}_z}} \end{aligned}$$

$$\begin{aligned} \text{(b) } \mathbf{E} \times \mathbf{F} &= \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 0 & 3 & 4 \\ 4 & -10 & 5 \end{vmatrix} = (55, 16, -12) \\ \mathbf{a}_{E \times F} &= \underline{\underline{\pm (0.9398, 0.2734, -0.205)}} \end{aligned}$$

P. E. 1.6 $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$ showing that \mathbf{a} , \mathbf{b} , and \mathbf{c} form the sides of a triangle.

$$\mathbf{a} \cdot \mathbf{b} = 0,$$

hence it is a right angle triangle.

$$\begin{aligned} \text{Area} &= \frac{1}{2} |\mathbf{a} \times \mathbf{b}| = \frac{1}{2} |\mathbf{b} \times \mathbf{c}| = \frac{1}{2} |\mathbf{c} \times \mathbf{a}| \\ \frac{1}{2} |\mathbf{a} \times \mathbf{b}| &= \frac{1}{2} \begin{vmatrix} 4 & 0 & -1 \\ 1 & 3 & 4 \end{vmatrix} = \frac{1}{2} |(3, -17, 12)| \\ \text{Area} &= \frac{1}{2} \sqrt{9 + 289 + 144} = \underline{\underline{10.51}} \end{aligned}$$

P. E. 1.7

$$(a) P_1P_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{25 + 4 + 64} = \underline{\underline{9.644}}$$

$$(b) \mathbf{r}_P = \mathbf{r}_{P_1} + \lambda(\mathbf{r}_{P_2} - \mathbf{r}_{P_1})$$

$$= (1, 2, -3) + \lambda(-5, -2, 8)$$

$$= \underline{\underline{(1 - 5\lambda, 2 - 2\lambda, -3 + 8\lambda)}}$$

(c) The shortest distance is

$$d = P_1P_3 \sin \theta = |\mathbf{P}_1\mathbf{P}_3 \times \mathbf{a}_{P_1P_2}|$$

$$= \frac{1}{\sqrt{93}} \begin{vmatrix} 6 & -3 & 5 \\ -5 & -2 & 8 \end{vmatrix}$$

$$= \frac{1}{\sqrt{93}} |(-14, -73, -27)| = \underline{\underline{8.2}}$$

Prob.1.1

$$\mathbf{r}_{OP} = 4\mathbf{a}_x - 5\mathbf{a}_y + \mathbf{a}_z$$

$$\mathbf{a}_{r_{OP}} = \frac{\mathbf{r}_{OP}}{|\mathbf{r}_{OP}|} = \frac{(4, -5, 1)}{\sqrt{(16 + 25 + 1)}} = \underline{\underline{0.6172\mathbf{a}_x - 0.7715\mathbf{a}_y + 0.1543\mathbf{a}_z}}$$

Prob. 1.2Method 1:

$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A, \quad \mathbf{r}_{BC} = \mathbf{r}_C - \mathbf{r}_B, \quad \mathbf{r}_{CA} = \mathbf{r}_A - \mathbf{r}_C$$

$$\mathbf{r}_{AB} + \mathbf{r}_{BC} + \mathbf{r}_{CA} = \mathbf{r}_B - \mathbf{r}_A + \mathbf{r}_C - \mathbf{r}_B + \mathbf{r}_A - \mathbf{r}_C = \mathbf{0}$$

Method 2

$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A = (-2, 0, 3) - (4, -6, 2) = (-6, 6, 1)$$

$$\mathbf{r}_{BC} = \mathbf{r}_C - \mathbf{r}_B = (10, 1, -7) - (-2, 0, 3) = (12, 1, -10)$$

$$\mathbf{r}_{CA} = \mathbf{r}_A - \mathbf{r}_C = (4, -6, 2) - (10, 1, -7) = (-6, -7, 9)$$

$$\mathbf{r}_{AB} + \mathbf{r}_{BC} + \mathbf{r}_{CA} = (0, 0, 0) = \mathbf{0}$$

Prob. 1.3

(a)

$$\begin{aligned} \mathbf{A} - 3\mathbf{B} &= (4, -2, 6) - 3(12, 18, -8) = (4, -2, 6) - (36, 54, -24) \\ &= \underline{\underline{(-32, -56, -30)}} \end{aligned}$$

(b)

$$2\mathbf{A} + 5\mathbf{B} = 2(4, -2, 6) + 5(12, 18, -8) = (68, 86, -28)$$

$$|\mathbf{B}| = \sqrt{12^2 + 18^2 + 8^2} = \sqrt{532} = 23.065$$

$$(2\mathbf{A} + 5\mathbf{B}) / |\mathbf{B}| = (68, 86, -28) / 23.065 = 2.948\mathbf{a}_x + 3.728\mathbf{a}_y - 1.214\mathbf{a}_z$$

(c)

$$\mathbf{a}_x \times \mathbf{A} = \begin{vmatrix} 1 & 0 & 0 \\ 4 & -2 & 6 \end{vmatrix} = \underline{\underline{-6\mathbf{a}_y - 2\mathbf{a}_z}}$$

(d)

$$\mathbf{B} \times \mathbf{a}_x = \begin{vmatrix} 12 & 18 & -8 \\ 1 & 0 & 0 \end{vmatrix} = -8\mathbf{a}_y - 18\mathbf{a}_z$$

$$(\mathbf{B} \times \mathbf{a}_x) \cdot \mathbf{a}_y = -8$$

Prob. 1.4

$$(a) \mathbf{A} \square \mathbf{B} = (10, -6, 8) \square (1, 0, 2) = 10 + 16 = \underline{\underline{26}}$$

$$(b) \mathbf{A} \times \mathbf{B} = \begin{vmatrix} 10 & -6 & 8 \\ 1 & 0 & 2 \end{vmatrix} = (-12 - 0)\mathbf{a}_x + (8 - 20)\mathbf{a}_y + (0 + 6)\mathbf{a}_z \\ = \underline{\underline{-12\mathbf{a}_x - 12\mathbf{a}_y + 6\mathbf{a}_z}}$$

$$(c) 2\mathbf{A} - 3\mathbf{B} = (20, -12, 16) - (3, 0, 6) = \underline{\underline{17\mathbf{a}_x - 12\mathbf{a}_y + 10\mathbf{a}_z}}$$

Prob. 1.5

$$(a) \mathbf{A} - \mathbf{B} + \mathbf{C} = (-2, 5, 1) + (-1, 0, -3) + (4, -6, 10) = \underline{\underline{(1, -1, 8)}}$$

$$(b) \mathbf{B} \times \mathbf{C} = \begin{vmatrix} 1 & 0 & 3 \\ 4 & -6 & 10 \end{vmatrix} = (18, 2, -6)$$

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (-2, 5, 1) \cdot (18, 2, -6) = -36 + 10 - 6 = \underline{\underline{-32}}$$

$$(c) \cos \theta_{AB} = \frac{\mathbf{A} \cdot \mathbf{B}}{AB} = \frac{-2 + 0 + 3}{\sqrt{4 + 25 + 1}\sqrt{1 + 9}} = 0.05773 \rightarrow \theta_{AB} = \underline{\underline{86.69^\circ}}$$

Prob. 1.6

$$(a) \mathbf{B} \times \mathbf{C} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix} = \mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z$$

$$\mathbf{A} \square (\mathbf{B} \times \mathbf{C}) = (1, 0, -1) \square (1, -2, 1) = 1 + 0 - 1 = \underline{\underline{0}}$$

$$(b) \quad \mathbf{A} \times \mathbf{B} = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z$$

$$(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = (1, -2, 1) \cdot (0, 1, 2) = 0 - 2 + 2 = \underline{\underline{0}}$$

$$(c) \quad \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \begin{vmatrix} 1 & 0 & -1 \\ 1 & -2 & 1 \end{vmatrix} = \underline{\underline{-2\mathbf{a}_x - 2\mathbf{a}_y - 2\mathbf{a}_z}}$$

$$(d) \quad (\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = \begin{vmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = \underline{\underline{-5\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z}}$$

Prob.1.7

$$(a) \quad \mathbf{T} = \underline{(4, 6, -1)} \text{ and } \mathbf{S} = \underline{(10, 12, 8)}$$

$$(b) \quad r_{TS} = r_S - r_T = (10, 12, 8) - (4, 6, -1) = \underline{\underline{6\mathbf{a}_x + 6\mathbf{a}_y + 9\mathbf{a}_z}}$$

$$(c) \quad TS = |r_{TS}| = \sqrt{36 + 36 + 81} = \underline{\underline{12.37}}$$

Prob. 1.8

(a) If \mathbf{A} and \mathbf{B} are parallel, $\mathbf{B} = k\mathbf{A}$, where k is a constant.

$$B_x = kA_x, \quad B_y = kA_y, \quad B_z = kA_z$$

$$\text{For } B_z, \quad 3 = k(-1) \quad \rightarrow \quad k = -3$$

$$B_x = \alpha = kA_x = (-3)(4) = -12$$

$$B_y = \beta = kA_y = (-3)(2) = -6$$

$$\text{Hence,} \quad \underline{\underline{\alpha = -12, \beta = -6}}$$

(b) If \mathbf{A} and \mathbf{B} are perpendicular to each other,

$$\mathbf{A} \cdot \mathbf{B} = 0 \quad \longrightarrow \quad \underline{\underline{4\alpha + 2\beta - 3 = 0}}$$

Prob. 1.9

$$(a) \quad \mathbf{A} \times \mathbf{a}_y = \begin{vmatrix} 10 & 5 & -2 \\ 0 & 1 & 0 \end{vmatrix} = \underline{\underline{2\mathbf{a}_x + 10\mathbf{a}_z}}$$

$$(b) \quad \mathbf{A} \cdot \mathbf{a}_z = \underline{\underline{-2}}$$

$$(c) \quad \cos \theta_z = \frac{\mathbf{A} \cdot \mathbf{a}_z}{\sqrt{100 + 25 + 4}} = \frac{-2}{11.358} \quad \rightarrow \quad \theta_z = \underline{\underline{100.14^\circ}}$$

Prob. 1.10

(a) $A \cdot B = AB \cos \theta_{AB}$

$A \times B = AB \sin \theta_{AB} \mathbf{a}_n$

$(A \cdot B)^2 + |A \times B|^2 = (AB)^2 (\cos^2 \theta_{AB} + \sin^2 \theta_{AB}) = (AB)^2$

(b) $\mathbf{a}_x \cdot (\mathbf{a}_y \times \mathbf{a}_z) = \mathbf{a}_x \cdot \mathbf{a}_x = 1$. Hence,

$$\frac{\mathbf{a}_y \times \mathbf{a}_z}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z} = \frac{\mathbf{a}_x}{1} = \mathbf{a}_x$$

$$\frac{\mathbf{a}_z \times \mathbf{a}_x}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z} = \frac{\mathbf{a}_y}{1} = \mathbf{a}_y$$

$$\frac{\mathbf{a}_x \times \mathbf{a}_y}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z} = \frac{\mathbf{a}_z}{1} = \mathbf{a}_z$$

Prob. 1.11

(a) $\mathbf{P} + \mathbf{Q} = (6, 2, 0)$, $\mathbf{P} + \mathbf{Q} - \mathbf{R} = (7, 1, -2)$

$|\mathbf{P} + \mathbf{Q} - \mathbf{R}| = \sqrt{49 + 1 + 4} = \sqrt{54} = \underline{\underline{7.3485}}$

(b) $\mathbf{P} \cdot \mathbf{Q} \times \mathbf{R} = \begin{vmatrix} 2 & -1 & -2 \\ 4 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} = 2(6-2) + (8+2) - 2(4+3) = 8+10-14 = \underline{\underline{4}}$

$\mathbf{Q} \times \mathbf{R} = \begin{vmatrix} 4 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} = (4, -10, 7)$

$\mathbf{P} \cdot \mathbf{Q} \times \mathbf{R} = (2, -1, -2) \cdot (4, -10, 7) = 8+10-14 = \underline{\underline{4}}$

(c) $\mathbf{Q} \times \mathbf{P} = \begin{vmatrix} 4 & 3 & 2 \\ 2 & -1 & -2 \end{vmatrix} = (-4, 12, -10)$

$\mathbf{Q} \times \mathbf{P} \cdot \mathbf{R} = (-4, 12, -10) \cdot (-1, 1, 2) = 4+12-20 = \underline{\underline{-4}}$

or $\mathbf{Q} \times \mathbf{P} \cdot \mathbf{R} = \mathbf{R} \cdot \mathbf{Q} \times \mathbf{P} = \begin{vmatrix} -1 & 1 & 2 \\ 4 & 3 & 2 \\ 2 & -1 & -2 \end{vmatrix} = -(-6+2) - (-8-4) + 2(-4-6) = \underline{\underline{-4}}$

(d) $(\mathbf{P} \times \mathbf{Q}) \cdot (\mathbf{Q} \times \mathbf{R}) = (4, -12, 10) \cdot (4, -10, 7) = 16+120+70 = \underline{\underline{206}}$

(e) $(\mathbf{P} \times \mathbf{Q}) \times (\mathbf{Q} \times \mathbf{R}) = \begin{vmatrix} 4 & -12 & 10 \\ 4 & -10 & 7 \end{vmatrix} = \underline{\underline{16\mathbf{a}_x + 12\mathbf{a}_y + 8\mathbf{a}_z}}$

$$(f) \cos \theta_{PR} = \frac{\mathbf{P} \cdot \mathbf{R}}{|\mathbf{P}| |\mathbf{R}|} = \frac{(-2-1-4)}{\sqrt{4+1+4} \sqrt{1+1+4}} = \frac{-7}{3\sqrt{6}} = -0.9526$$

$$\theta_{PR} = \underline{\underline{162.3^\circ}}$$

$$(g) \sin \theta_{PQ} = \frac{|\mathbf{P} \times \mathbf{Q}|}{|\mathbf{P}| |\mathbf{Q}|} = \frac{\sqrt{16+144+100}}{3\sqrt{16+9+4}} = \frac{\sqrt{260}}{3\sqrt{29}} = 0.998$$

$$\theta_{PQ} = \underline{\underline{86.45^\circ}}$$

Prob. 1.12

$$\mathbf{A} \cdot \mathbf{B} = (4, -6, 1) \cdot (2, 0, 5) = 8 - 0 + 5 = 13$$

$$(a) |\mathbf{B}|^2 = 2^2 + 5^2 = 29$$

$$\mathbf{A} \cdot \mathbf{B} + 2|\mathbf{B}|^2 = 13 + 2 \times 29 = \underline{\underline{71}}$$

(b)

$$\mathbf{a}_\perp = \pm \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$$

$$\text{Let } \mathbf{C} = \mathbf{A} \times \mathbf{B} = \begin{vmatrix} 4 & -6 & 1 \\ 2 & 0 & 5 \end{vmatrix} = (-30, -18, 12)$$

$$\mathbf{a}_\perp = \pm \frac{\mathbf{C}}{|\mathbf{C}|} = \pm \frac{(-30, -18, 12)}{\sqrt{30^2 + 18^2 + 12^2}} = \pm \underline{\underline{(-0.8111\mathbf{a}_x - 0.4867\mathbf{a}_y + 0.3244\mathbf{a}_z)}}$$

Prob. 1.13

$$\mathbf{P} \cdot \mathbf{Q} = (2, -6, 5) \cdot (0, 3, 1) = 0 - 18 + 5 = \underline{\underline{-13}}$$

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} 2 & -6 & 5 \\ 0 & 3 & 1 \end{vmatrix} = \underline{\underline{-21\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z}}$$

$$\cos \theta_{PQ} = \frac{\mathbf{P} \cdot \mathbf{Q}}{PQ} = \frac{-13}{\sqrt{10}\sqrt{65}} = -0.51 \quad \longrightarrow \quad \theta_{PQ} = \underline{\underline{120.66^\circ}}$$

Prob. 1.14

\mathbf{P} and \mathbf{Q} are orthogonal if the angle between them is 90° . Hence

$$\mathbf{P} \cdot \mathbf{Q} = PQ \cos \theta = 0$$

$$\mathbf{P} \cdot \mathbf{Q} = (2, 4, -6) \cdot (5, 2, 3) = 10 + 8 - 18 = 0$$

showing that they are perpendicular or orthogonal.

Prob. 1.15

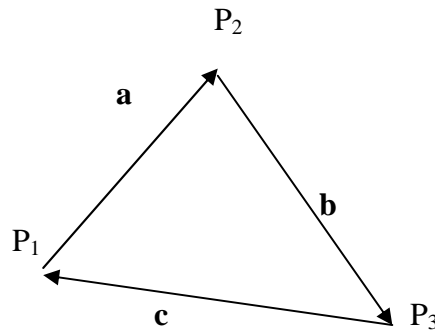
(a) Using the fact that

$$(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{B} \cdot \mathbf{C})\mathbf{A},$$

we get

$$\mathbf{A} \times (\mathbf{A} \times \mathbf{B}) = -(\mathbf{A} \times \mathbf{B}) \times \mathbf{A} = \underline{\underline{(\mathbf{B} \cdot \mathbf{A})\mathbf{A} - (\mathbf{A} \cdot \mathbf{A})\mathbf{B}}}$$

$$\begin{aligned} \text{(b) } \mathbf{A} \times (\mathbf{A} \times (\mathbf{A} \times \mathbf{B})) &= \mathbf{A} \times [(\mathbf{A} \cdot \mathbf{B})\mathbf{A} - (\mathbf{A} \cdot \mathbf{A})\mathbf{B}] \\ &= (\mathbf{A} \cdot \mathbf{B}) (\mathbf{A} \times \mathbf{A}) - (\mathbf{A} \cdot \mathbf{A}) (\mathbf{A} \times \mathbf{B}) \\ &= \underline{\underline{-\mathbf{A}^2 (\mathbf{A} \times \mathbf{B})}} \end{aligned}$$

since $\mathbf{A} \times \mathbf{A} = \mathbf{0}$ **Prob. 1.16**

$$\mathbf{a} = \mathbf{r}_{p_2} - \mathbf{r}_{p_1} = (1, -2, 4) - (5, -3, 1) = (-4, 1, 3)$$

$$\text{(a) } \mathbf{b} = \mathbf{r}_{p_3} - \mathbf{r}_{p_2} = (3, 3, 5) - (1, -2, 4) = (2, 5, 1)$$

$$\mathbf{c} = \mathbf{r}_{p_1} - \mathbf{r}_{p_3} = (5, -3, 1) - (3, 3, 5) = (2, -6, -4)$$

Note that $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$

$$\mathbf{a} \cdot \mathbf{b} = -8 + 5 + 3 = 0 \quad \longrightarrow \quad \text{perpendicular}$$

$$\mathbf{b} \cdot \mathbf{c} = 4 - 30 - 4 \neq 0$$

$$\mathbf{c} \cdot \mathbf{a} = -8 - 6 - 12 \neq 0$$

Hence $\underline{\underline{P_2}}$ is a right angle.

$$\begin{aligned} \text{(b) Area} &= \frac{1}{2} |\mathbf{a} \times \mathbf{b}| = \frac{1}{2} \begin{vmatrix} -4 & 1 & 3 \\ 2 & 5 & 1 \end{vmatrix} = \frac{1}{2} |(1-15)\mathbf{a}_x + (6+4)\mathbf{a}_y + (-20-2)\mathbf{a}_z| \\ &= \frac{1}{2} |(-14, 10, -22)| = \frac{1}{2} \sqrt{196 + 100 + 484} = \underline{\underline{13.96}} \end{aligned}$$

Prob. 1.17

Given $r_P = (-1, 4, 8)$, $r_Q = (2, -1, 3)$, $r_R = (-1, 2, 3)$

(a) $|PQ| = \sqrt{9 + 25 + 25} = \underline{7.6811}$

(b) $\underline{PR} = -2a_y - 5a_z$

(c)

$$QP = (-1, 4, 8) - (2, -1, 3) = (-3, 5, 5)$$

$$QR = (-1, 2, 3) - (2, -1, 3) = (-3, 3, 0)$$

$$\frac{QP \cdot QR}{|QP| |QR|} = \frac{9 + 15 + 0}{\sqrt{59} \sqrt{18}} = 0.7365$$

$$\angle PQR = \cos^{-1}(0.7365) = \underline{42.64^\circ}$$

(d) Area = $\frac{1}{2} |QP \times QR| = \frac{1}{2} \begin{vmatrix} -3 & 5 & 5 \\ -3 & 3 & 0 \end{vmatrix} = 0.5 |(-15, -15, 8)| = 0.5 \sqrt{225 + 225 + 36} = \underline{10.677}$

(e) Perimeter = $PQ + QR + RP = \sqrt{59} + \sqrt{18} + \sqrt{29} = \underline{17.31}$

Prob. 1.18

Let R be the midpoint of PQ.

$$r_R = \frac{1}{2} \{(2, 4, -1) + (12, 16, 9)\} = (7, 10, 4)$$

$$OR = \sqrt{49 + 100 + 16} = \sqrt{165} = 12.845$$

$$t = \frac{OR}{v} = \frac{12.845}{300} = \underline{42.82 \text{ ms}}$$

Prob. 1.19

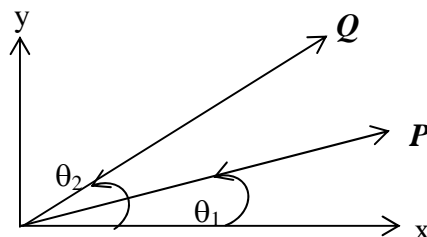
Area = Twice the area of a triangle

$$= |D \times E| = \begin{vmatrix} 4 & 1 & 5 \\ -1 & 2 & 3 \end{vmatrix} = (3 - 10)a_x + (-5 - 12)a_y + (8 + 1)a_z$$

$$= |(-7, -19, 9)| = \sqrt{49 + 361 + 81} = \underline{22.16}$$

Prob. 1.20

(a) Let **P** and **Q** be as shown below:



$$|\mathbf{P}| = \cos^2 \theta_1 + \sin^2 \theta_1 = 1, |\mathbf{Q}| = \cos^2 \theta_2 + \sin^2 \theta_2 = 1,$$

Hence \mathbf{P} and \mathbf{Q} are unit vectors.

$$(b) \mathbf{P} \cdot \mathbf{Q} = (1)(1)\cos(\theta_2 - \theta_1)$$

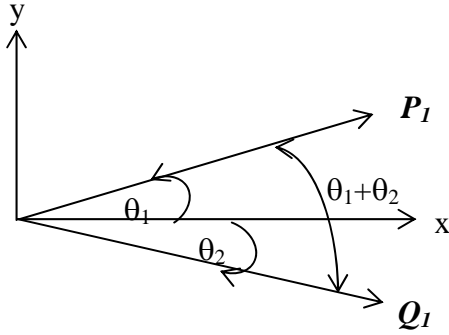
But $\mathbf{P} \cdot \mathbf{Q} = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2$. Thus,

$$\underline{\underline{\cos(\theta_2 - \theta_1) = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2}}$$

Let $\mathbf{P}_1 = \mathbf{P} = \cos \theta_1 \mathbf{a}_x + \sin \theta_1 \mathbf{a}_y$ and

$$\mathbf{Q}_1 = \cos \theta_2 \mathbf{a}_x - \sin \theta_2 \mathbf{a}_y.$$

\mathbf{P}_1 and \mathbf{Q}_1 are unit vectors as shown below:



$$\mathbf{P}_1 \cdot \mathbf{Q}_1 = (1)(1)\cos(\theta_1 + \theta_2)$$

But $\mathbf{P}_1 \cdot \mathbf{Q}_1 = \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2$,

$$\underline{\underline{\cos(\theta_2 + \theta_1) = \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2}}$$

Alternatively, we can obtain this formula from the previous one by replacing θ_2 by $-\theta_2$ in \mathbf{Q} .

(c)

$$\frac{1}{2} |\mathbf{P} - \mathbf{Q}| = \frac{1}{2} |(\cos \theta_1 - \cos \theta_2) \mathbf{a}_x + (\sin \theta_1 - \sin \theta_2) \mathbf{a}_y|$$

$$= \frac{1}{2} \sqrt{\cos^2 \theta_1 + \sin^2 \theta_1 + \cos^2 \theta_2 + \sin^2 \theta_2 - 2 \cos \theta_1 \cos \theta_2 - 2 \sin \theta_1 \sin \theta_2}$$

$$= \frac{1}{2} \sqrt{2 - 2(\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2)} = \frac{1}{2} \sqrt{2 - 2 \cos(\theta_2 - \theta_1)}$$

Let $\theta_2 - \theta_1 = \theta$, the angle between \mathbf{P} and \mathbf{Q} .

$$\frac{1}{2} |\mathbf{P} - \mathbf{Q}| = \frac{1}{2} \sqrt{2 - 2 \cos \theta}$$

But $\cos 2A = 1 - 2 \sin^2 A$.

$$\frac{1}{2} |\mathbf{P} - \mathbf{Q}| = \frac{1}{2} \sqrt{2 - 2 + 4 \sin^2 \theta / 2} = \sin \theta / 2$$

Thus,

$$\frac{1}{2} |\mathbf{P} - \mathbf{Q}| = \left| \sin \frac{\theta_2 - \theta_1}{2} \right|$$

Prob. 1.21

$$\boldsymbol{\omega} = \frac{\boldsymbol{\omega}(1, -2, 2)}{3} = (1, -2, 2), \quad \mathbf{r} = \mathbf{r}_p - \mathbf{r}_o = (1, 3, 4) - (2, -3, 1) = (-1, 6, 3)$$

$$\mathbf{u} = \boldsymbol{\omega} \times \mathbf{r} = \begin{vmatrix} 1 & -2 & 2 \\ -1 & 6 & 3 \end{vmatrix} = (-18, -5, 4)$$

$$\underline{\underline{\mathbf{u} = -18\mathbf{a}_x - 5\mathbf{a}_y + 4\mathbf{a}_z}}$$

Prob. 1.22

$$\mathbf{r}_1 = (1, 1, 1), \quad \mathbf{r}_2 = (1, 0, 1) - (0, 1, 0) = (1, -1, 1)$$

$$\cos \theta = \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{r_1 r_2} = \frac{(1-1+1)}{\sqrt{3}\sqrt{3}} = \frac{1}{3} \quad \longrightarrow \quad \underline{\underline{\theta = 70.53^\circ}}$$

Prob. 1.23

$$(a) T_s = \mathbf{T} \cdot \mathbf{a}_s = \frac{\mathbf{T} \cdot \mathbf{S}}{|\mathbf{S}|} = \frac{(2, -6, 3) \cdot (1, 2, 1)}{\sqrt{6}} = \frac{-7}{\sqrt{6}} = \underline{\underline{-2.8577}}$$

$$(b) \mathbf{S}_T = (\mathbf{S} \cdot \mathbf{a}_T) \mathbf{a}_T = \frac{(\mathbf{S} \cdot \mathbf{T}) \mathbf{T}}{T^2} = \frac{-7(2, -6, 3)}{7^2}$$

$$\underline{\underline{= -0.2857\mathbf{a}_x + 0.8571\mathbf{a}_y - 0.4286\mathbf{a}_z}}$$

$$(c) \sin \theta_{TS} = \frac{|\mathbf{T} \times \mathbf{S}|}{|\mathbf{T}| |\mathbf{S}|} = \frac{\begin{vmatrix} 2 & -6 & 3 \\ 1 & 2 & 1 \end{vmatrix}}{7\sqrt{6}} = \frac{|(-12, 1, 10)|}{7\sqrt{6}} = \frac{\sqrt{245}}{7\sqrt{6}} = 0.9129$$

$$\Rightarrow \theta_{TS} = \underline{\underline{65.91^\circ}}$$

Prob. 1.24

$$\text{Let } \mathbf{A} = \mathbf{A}_{B\parallel} + \mathbf{A}_{B\perp}$$

$$\mathbf{A}_{B\parallel} = (\mathbf{A} \cdot \mathbf{a}_B) \mathbf{a}_B = \frac{\mathbf{A} \cdot \mathbf{B}}{\mathbf{B} \cdot \mathbf{B}} \mathbf{B}$$

Hence,

$$\mathbf{A}_{B\perp} = \mathbf{A} - \mathbf{A}_{B\parallel} = \mathbf{A} - \frac{\mathbf{A} \cdot \mathbf{B}}{\mathbf{B} \cdot \mathbf{B}} \mathbf{B}$$

Prob. 1.25

$$(a) \quad \mathbf{A} \cdot \mathbf{B} = 20 + 0 - 10 = \underline{\underline{10}}$$

$$(b) \quad \mathbf{A} \times \mathbf{B} = \begin{vmatrix} 20 & 15 & -10 \\ 1 & 0 & 1 \end{vmatrix} = \underline{\underline{15\mathbf{a}_x - 30\mathbf{a}_y - 15\mathbf{a}_z}}$$

$$(c) \quad \mathbf{A}_B = (\mathbf{A} \cdot \mathbf{a}_B) \mathbf{a}_B = \frac{(\mathbf{A} \cdot \mathbf{B}) \mathbf{B}}{B^2} = \frac{10(\mathbf{a}_x + \mathbf{a}_z)}{2} = \underline{\underline{5\mathbf{a}_x + 5\mathbf{a}_z}}$$

Prob. 1.26

$$\mathbf{A} \cdot \mathbf{a}_x = A_x = A \cos \alpha \quad \rightarrow \quad \cos \alpha = \frac{A_x}{A} = \frac{2}{\sqrt{4+16+36}} = 0.2673 \quad \rightarrow \quad \underline{\underline{\alpha = 74.5^\circ}}$$

$$\cos \beta = \frac{A_y}{A} = \frac{-4}{\sqrt{56}} = -0.5345 \quad \rightarrow \quad \underline{\underline{\beta = 122.31^\circ}}$$

$$\cos \gamma = \frac{A_z}{A} = \frac{6}{\sqrt{56}} = 0.8018 \quad \rightarrow \quad \underline{\underline{\gamma = 36.7^\circ}}$$

Prob.1.27

$$(a) \quad \mathbf{H}(1, 3, -2) = 6\mathbf{a}_x + \mathbf{a}_y + 4\mathbf{a}_z$$

$$\mathbf{a}_H = \frac{(6, 1, 4)}{\sqrt{36+1+16}} = \underline{\underline{0.8242\mathbf{a}_x + 0.1374\mathbf{a}_y + 0.5494\mathbf{a}_z}}$$

$$(b) \quad |\mathbf{H}| = 10 = \sqrt{4x^2y^2 + (x+z)^2 + z^4}$$

or

$$\underline{\underline{100 = 4x^2y^2 + x^2 + 2xz + z^2 + z^4}}$$

Prob. 1.28

$$\mathbf{R} = R\mathbf{a}_R, \quad R = 4$$

$$\mathbf{a}_R = \frac{\mathbf{P} \times \mathbf{Q}}{|\mathbf{P} \times \mathbf{Q}|}$$

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} 2 & -4 & 1 \\ 1 & 2 & 0 \end{vmatrix} = -2\mathbf{a}_x + \mathbf{a}_y + 8\mathbf{a}_z$$

$$\mathbf{a}_R = \frac{-2\mathbf{a}_x + \mathbf{a}_y + 8\mathbf{a}_z}{\sqrt{4+1+64}} = -0.2408\mathbf{a}_x + 0.1204\mathbf{a}_y + 0.9631\mathbf{a}_z$$

$$\mathbf{R} = R\mathbf{a}_R = 4(-0.2408\mathbf{a}_x + 0.1204\mathbf{a}_y + 0.9631\mathbf{a}_z) = \underline{\underline{-0.9631\mathbf{a}_x + 0.4815\mathbf{a}_y + 3.852\mathbf{a}_z}}$$

An alternate choice of \mathbf{R} is $-0.9631\mathbf{a}_x + 0.4815\mathbf{a}_y + 3.852\mathbf{a}_z$

Prob. 1.29

(a) At (1, -2, 3), $x = 1, y = -2, z = 3$.

$$\mathbf{G} = \mathbf{a}_x + 2\mathbf{a}_y + 6\mathbf{a}_z, \quad \mathbf{H} = -6\mathbf{a}_x + 3\mathbf{a}_y - 3\mathbf{a}_z$$

$$G = \sqrt{1+4+36} = \underline{\underline{6.403}}$$

$$H = \sqrt{36+9+9} = \underline{\underline{7.348}}$$

$$(b) \mathbf{G} \cdot \mathbf{H} = -6 + 6 - 18 = \underline{\underline{-18}}$$

$$(c) \cos \theta_{GH} = \frac{\mathbf{G} \cdot \mathbf{H}}{GH} = \frac{-18}{6.403 \times 7.348} = -0.3826$$

$$\underline{\underline{\theta_{GH} = 112.5^\circ}}$$

Prob. 1.30

$$(a) \mathbf{H} = 10(2)(16)\mathbf{a}_x - 8(-8)\mathbf{a}_y + 12(4)\mathbf{a}_z = 320\mathbf{a}_x + 64\mathbf{a}_y + 48\mathbf{a}_z$$

$$\text{Let } \mathbf{F} = \mathbf{a}_x - \mathbf{a}_y$$

$$(b) \mathbf{H}_F = (\mathbf{H} \cdot \mathbf{a}_F)\mathbf{a}_F = \frac{(\mathbf{H} \cdot \mathbf{F})\mathbf{F}}{F^2} = \frac{(320 - 64)(1, -1, 0)}{1+1} = 128\mathbf{a}_x - 128\mathbf{a}_y$$

Prob. 1.31

(a) At (1,2,3), $\mathbf{E} = (2,1,6)$

$$|\mathbf{E}| = \sqrt{4+1+36} = \sqrt{41} = \underline{\underline{6.403}}$$

(b) At (1,2,3), $\mathbf{F} = (2,-4,6)$

$$\begin{aligned} \mathbf{E}_F &= (\mathbf{E} \cdot \mathbf{a}_F) \mathbf{a}_F = \frac{(\mathbf{E} \cdot \mathbf{F}) \mathbf{F}}{|\mathbf{F}|^2} = \frac{36}{56} (2, -4, 6) \\ &= \underline{\underline{1.286 \mathbf{a}_x - 2.571 \mathbf{a}_y + 3.857 \mathbf{a}_z}} \end{aligned}$$

(c) At (0,1,-3), $\mathbf{E} = (0,1,-3)$, $\mathbf{F} = (0,-1,0)$

$$\mathbf{E} \times \mathbf{F} = \begin{vmatrix} 0 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix} = (-3, 0, 0)$$

$$\mathbf{a}_{\mathbf{E} \times \mathbf{F}} = \pm \frac{\mathbf{E} \times \mathbf{F}}{|\mathbf{E} \times \mathbf{F}|} = \underline{\underline{\pm \mathbf{a}_x}}$$

Prob. 1.32

(a) At P, $x = -1$, $y = 2$, $z = 4$

$$\mathbf{D} = 8\mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z, \quad \mathbf{E} = -10\mathbf{a}_x + 24\mathbf{a}_y + 128\mathbf{a}_z$$

$$\mathbf{C} = \mathbf{D} + \mathbf{E} = -2\mathbf{a}_x + 20\mathbf{a}_y + 126\mathbf{a}_z$$

(b) $\mathbf{C} \cdot \mathbf{a}_x = C \cos \theta_x \longrightarrow \cos \theta_x = \frac{\mathbf{C} \cdot \mathbf{a}_x}{C} = \frac{-2}{\sqrt{2^2 + 20^2 + 126^2}} = -0.01575$

$$\underline{\underline{\theta_x = 90.9^\circ}}$$