

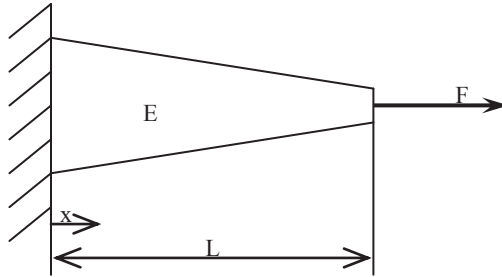
SOLUTIONS MANUAL for
 Finite Element Analysis of Composite Materials Using ANSYS® 2nd Edition

Chapter 1

Solution to Problems in Chapter 1 of FEACM Using ANSYS – 2nd Edition

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Problem 1.1



The PVW says that:

$$\int_V (\sigma_{ij} \delta \epsilon_{ij} dV) - \int_S (t_i \delta u_i dS) - \int_V (f_i \delta u_i dV) = 0$$

Assuming \$u(x)\$ as: \$u(x) = C_0 + C_1 x + C_2 x^2\$

Using the boundary condition \$u(x=0) = 0 \xrightarrow{THEN} C_0 = 0\$

Because of it's a case of uniaxial load without body forces (\$f_i = 0\$), the PVW can be rewritten as:

$$\int_0^L (\sigma_x \delta \epsilon_x A_x dx) - F \delta u(x=L) = 0$$

So, we can calculate:

$$\begin{aligned} \delta u(x) &= x \delta C_1 + x^2 \delta C_2 \\ \epsilon_x &= \frac{\partial u}{\partial x} = C_1 + 2C_2 x \\ \delta \epsilon_x &= \delta C_1 + 2x \delta C_2 \\ \sigma_x &= E \epsilon_x = E(C_1 + 2C_2 x) \end{aligned}$$

And

$$\begin{aligned} A_x &= A_1 \frac{(L-x)}{L} + A_2 \frac{x}{L} \\ A_x &= A_1 - \frac{(A_1 - A_2)}{L} x \end{aligned}$$

Now,

$$E \int_0^L (C_1 + 2C_2x)(\delta C_1 + 2x\delta C_2) \left(A_1 - \frac{(A_1 - A_2)}{L}x \right) dx - F(L\delta C_1 + L^2\delta C_2) = 0$$

$$E \int_0^L (C_1\delta C_1 + 2C_1x\delta C_2 + 2C_2x\delta C_1 + 4C_2x^2\delta C_2) \left(A_1 - \frac{(A_1 - A_2)}{L}x \right) dx - F(L\delta C_1 + L^2\delta C_2) = 0$$

$$E \int_0^L (4C_2x^2\delta C_2 + 2(C_1\delta C_2 + C_2\delta C_1)x + C_1\delta C_1) \left(A_1 - \frac{(A_1 - A_2)}{L}x \right) dx - F(L\delta C_1 + L^2\delta C_2) = 0$$

$$EA_1 \int_0^L (4C_2x^2\delta C_2 + 2(C_1\delta C_2 + C_2\delta C_1)x + C_1\delta C_1) dx - E \frac{(A_1 - A_2)}{L} \int_0^L (4C_2x^3\delta C_2 + 2(C_1\delta C_2 + C_2\delta C_1)x^2 + C_1\delta C_1x) dx - F(L\delta C_1 + L^2\delta C_2) = 0$$

$$EA_1 \left(\frac{4}{3}L^3C_2\delta C_2 + L^2(C_1\delta C_2 + C_2\delta C_1) + LC_1\delta C_1 \right) - E \frac{(A_1 - A_2)}{L} \left(L^4C_2\delta C_2 + \frac{2}{3}L^3(C_1\delta C_2 + C_2\delta C_1) + \frac{1}{2}L^2C_1\delta C_1 \right) - F(L\delta C_1 + L^2\delta C_2) = 0$$

Reordering the equation to express it like a linear combination of the variations of C_i 's

$$\delta C_1 \left(EA_1L - \frac{1}{2}E(A_1 - A_2)L \right) C_1 + \left(EA_1L^2 - \frac{2}{3}E(A_1 - A_2)L^2 \right) C_2 - FL \\ + \delta C_2 \left(\frac{4}{3}EA_1L^3C_2 + EA_1L^2C_1 - E(A_1 - A_2)L^3C_2 - \frac{2}{3}E(A_1 - A_2)L^2C_1 - FL^2 \right) = 0$$

Since δC_i 's are independent variations of C_i 's values there are two equations involved in the above one. The solution of those equations (2x2 system) will give the values of coefficients C_i 's.

$$\left(EA_1L - \frac{1}{2}E(A_1 - A_2)L \right) C_1 + \left(EA_1L^2 - \frac{2}{3}E(A_1 - A_2)L^2 \right) C_2 = FL$$

$$\left(EA_1L^2 - \frac{2}{3}E(A_1 - A_2)L^2 \right) C_1 + \left(\frac{4}{3}EA_1L^3 - E(A_1 - A_2)L^3 \right) C_2 = FL^2$$

Calling

$$\alpha = EA_1 - \frac{1}{2}E(A_1 - A_2)$$

$$\beta = EA_1 - \frac{2}{3}E(A_1 - A_2)$$

$$\chi = \frac{4}{3}EA_1 - E(A_1 - A_2)$$

Then

$$\alpha C_1 + \beta LC_2 = F$$

$$\beta LC_1 + \chi LC_2 = F$$

The solution for this system is:

$$C_2 = \frac{(\beta - \alpha)F}{(\beta^2 - \alpha\chi)L}$$

$$C_1 = \frac{(\chi - \beta)F}{(\alpha\chi - \beta^2)}$$

Finally, the displacement solution is:

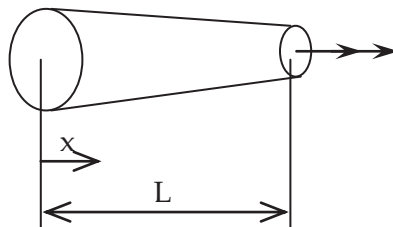
$$u(x) = \frac{(\chi - \beta)F}{(\alpha\chi - \beta^2)}x + \frac{(\beta - \alpha)F}{(\beta^2 - \alpha\chi)L}x^2$$

In terms of the known data

$$u(x) = \frac{6A_2}{E(A_1^2 + 4A_1A_2 + A_2^2)}Fx + \frac{3(A_1 - A_2)}{E(A_1^2 + 4A_1A_2 + A_2^2)}\frac{Fx}{L}$$

When $A_1 = A_2 = A$ the well known result $u(x) = \frac{Fx}{EA}$ is gotten.

Problem 1.2



The PVW says that:

$$\int_V (\sigma_{ij} \delta \epsilon_{ij} dV) - \int_S (T_i \delta \theta_i dS) - \int_V (f_i \delta \theta_i dV) = 0$$

Assuming $\theta(x)$ as: $\theta(x) = C_0 + C_1 x + C_2 x^2$

Using the boundary condition $\theta(x=0) = 0 \xrightarrow{\text{THEN}} C_0 = 0$

Because of it's a case of pure torsion without body forces ($f_i = 0$), the *PVW* can be rewritten as:

$$\int_0^L (\tau_{xy} \delta (\frac{1}{2} \gamma_{xy}) A_x dx) - T \delta \theta(x=L) = 0$$

So, we can calculate:

$$\delta \theta(x) = x \delta C_1 + x^2 \delta C_2$$

$$\gamma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = r_x (C_1 + 2C_2 x)$$

$$u = 0 \xrightarrow{\frac{\partial}{\partial y}} \frac{\partial u}{\partial y} = 0$$

$$v = r_x \theta(x) \xrightarrow{\frac{\partial}{\partial x}} \frac{\partial v}{\partial x} = r_x (C_1 + 2C_2 x)$$

$$\delta (\frac{1}{2} \gamma_{xy}) = \frac{r_x}{2} (\delta C_1 + 2x \delta C_2)$$

$$\tau_{xy} = G \gamma_{xy} = G r_x (C_1 + 2C_2 x)$$

And

$$A_x = A_1 \frac{(L-x)}{L} + A_2 \frac{x}{L}$$

$$A_x = A_1 - \frac{(A_1 - A_2)}{L} x = \pi r_x^2$$

$$r_x^2 = r_1^2 - \frac{(r_1^2 - r_2^2)}{L} x$$

Now,

$$\int_0^L G r_x (C_1 + 2C_2 x) \frac{1}{2} r_x (\delta C_1 + 2x \delta C_2) \pi r_x^2 dx - T (L \delta C_1 + L^2 \delta C_2) = 0$$

Solving the integral and organizing the result

$$\begin{aligned} & \delta C_1 \left(\frac{1}{12} \pi G \right) \left(2C_1 (r_1^4 + r_1^2 r_2^2 + r_2^4) + LC_2 (r_1^4 + 2r_1^2 r_2^2 + 3r_2^4) \right) \\ & + \delta C_2 \left(\frac{1}{60} \pi G \right) \left(5LC_1 (r_1^4 + 2r_1^2 r_2^2 + 3r_2^4) + 4L^2 C_2 (r_1^4 + 3r_1^2 r_2^2 + 6r_2^4) \right) \\ & = T\delta C_1 + LT\delta C_2 \end{aligned}$$

The coefficients of the terms δC_i 's must be equal on RHS and LHS, therefore a 2x2 system can be written.

$$\begin{aligned} 2\alpha C_1 + \beta LC_2 &= \delta \\ 5\beta C_1 + 4\chi LC_2 &= 5\delta \\ \text{with} \\ \alpha &= r_1^4 + r_1^2 r_2^2 + r_2^4 \\ \beta &= r_1^4 + 2r_1^2 r_2^2 + 3r_2^4 \\ \chi &= r_1^4 + 3r_1^2 r_2^2 + 6r_2^4 \\ \delta &= \frac{12T}{\pi G} \end{aligned}$$

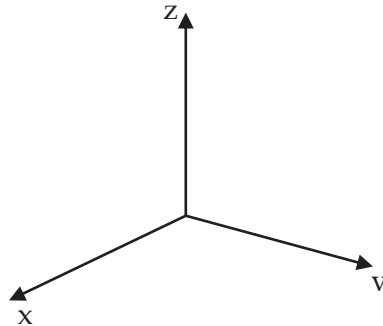
Solving the system

$$\begin{aligned} C_1 &= \left(\frac{5\beta - 4\chi}{5\beta^2 - 8\alpha\chi} \right) \delta \\ C_2 &= \left(\frac{2\alpha - \beta}{5\beta^2 - 8\alpha\chi} \right) \frac{5\delta}{L} \end{aligned}$$

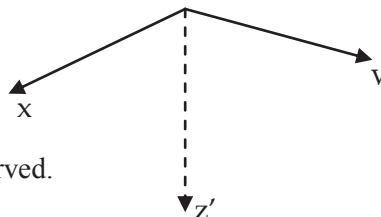
When $r_1 = r_2 = r$ the well known result $\theta = \frac{2Tx}{G\pi r^4}$ is gotten.

Problem 1.3

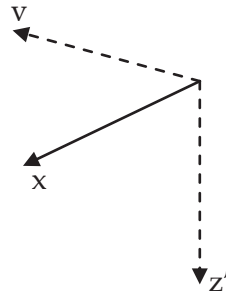
Initial system



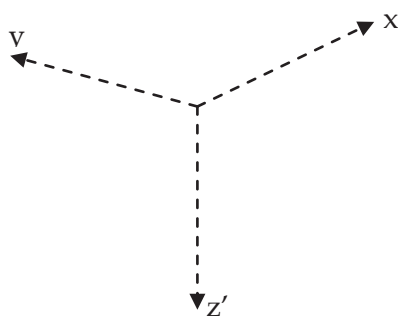
- (a) The reflection of z-axis about the plane x-y can be reached by one of the following four rotations: $\theta = (+\pi)$ or $(-\pi)$ about the x-axis or y-axis.



- (b) The reflection of y-axis about the plane x-z can be reached by one of the following four rotations: $\theta = (+\pi)$ or $(-\pi)$ about the x-axis or z-axis.



- (c) x' -axis represents the reflection of x-axis about the plane y-z. It can be reached by one of the following four rotations: $\theta = (+\pi)$ or $(-\pi)$ about the y-axis or z-axis.



Notice that the new system doesn't accomplish the right hand rule.

$$\text{Rotation matrix: } a_{ij} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Problem 1.4

Rotation about the x-axis

$$a_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix} \xrightarrow{\theta=\pi} a_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Rotation about the y-axis

$$a_{ij} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix} \xrightarrow{\theta=\pi} a_{ij} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Rotation about the x-axis

$$a_{ij} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\theta=\pi} a_{ij} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 1.5

Scilab™ is a free, open source software with a syntax virtually identical to MATLAB®
Scilab can be downloaded from <http://scilab.org>

The verification requested in this problem can be done and understood best with a short Scilab program. Note there is a function defined to construct the T matrix (1.34). MATLAB code to produce (1.34) in symbolic form is provided in the textbook immediately below (1.34) but requires you to have the symbolic package, which like everything else cost money, so I have switched to Scilab for simple numerical code, and to Maxima™ (also free) for symbolic manipulation.

```
// Program P1p5.sce http://scilab.org
clc; funcprot(0);
mode(0); //displays every line unless suppressed by ; (MATLAB default)
```

```
sigma = [[10,2,1];[2,5,1];[1,1,3]]
a = [[sqrt(3)/2,1/2,0];[-1/2,sqrt(3)/2,0];[0,0,1]]//Ex.1.2
```

```
// (1.26)
sigmap = a * sigma * a'
```

```
// (1.29)
sigmaVoigt = [sigma(1,1);sigma(2,2);sigma(3,3);...
             sigma(2,3);sigma(1,3);sigma(1,2)]
T = Transform(a)
sigmapp = T * sigmaVoigt
//end program
```

```
function [T,Tbar] = Transform(a)
// T(6,6), Tbar(6,6) : stress and strain transformation matrices
// a(3,3) vector transformation matrix
// tensor indexes i,j,p,q = 1..3
// Voigt contracted notation indexes alpha,bbeta = 1..6
// transformation matrix [T]
T(1:6,1:6) = 0;
for i=1:1:3
for j=1:1:3
if i==j; alpha = j; else alpha = 9-i-j; end
for p=1:1:3
for q=1:1:3
if p==q bbeta = p; else bbeta = 9-p-q; end
T(alpha,bbeta) = 0;
if alpha<=3 & bbeta<= 3; T(alpha,bbeta)=a(i,p)*a(i,p); end
```

```

if alpha> 3 & bbeta<= 3; T(alpha,bbeta)=a(i,p)*a(j,p); end
if alpha<=3 & bbeta>3; T(alpha,bbeta)=a(i,q)*a(i,p)+a(i,p)*a(i,q);end
if alpha>3 & bbeta>3; T(alpha,bbeta)=a(i,p)*a(j,q)+a(i,q)*a(j,p);end
end
end
end
end
end
R = eye(6,6); R(4,4)=2; R(5,5)=2; R(6,6)=2;// Reuter matrix
Tbar = R*T*R^(-1)
endfunction

```

Problem 1.6

The transversely isotropic material can be considered as a simplification of the orthotropic material, when it has the same properties in the transverse-to-the-fibers plane (*2-3 plane*). For this case the set of elastic properties describing the material reduces from 13 to 5, namely $E_1, E_2, G_{12}, \nu_{12}, \nu_{23}$ (in engineering terms). The simplification can be shown using the compliance matrix for two materials.

$$[S]_{ORTHO} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0 \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{bmatrix}$$

$$[S]_{TRANS} = \begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{12} & S_{23} & S_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix}; \text{ But } S_{44}=2(S_{22}-S_{23}).$$

In terms of the Engineering Constants

$$[S]_{TRANS} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{21}/E_2 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & -\nu_{23}/E_2 & 1/E_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{bmatrix}; \text{ And } G_{23} = \frac{E_2}{2(1 + \nu_{23})}$$

A MATLAB® routine, named *P1p6.m* (code attached), was built to construct both stiffness and compliance matrices for a transversely isotropic single layer laminate, provided that its five properties are given in a file

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named *props.dat* as a column vector in the following order: E_1 , E_2 , G_{12} , ν_{12} , ν_{23} . The routine calculate the compliance matrix from the elastic properties and invert it to get the stiffness matrix.

The routine was verified using the following data set:

Material: E-Glass / Isophthalic Polyester (took from [4])

$E_1 = 37.9$ GPa; $E_2 = 11.3$ GPa; $G_{12} = 3.3$ GPa; $\nu_{12} = 0.3$; $\nu_{23} = 0.38$ (from the matrix)

The results were:

1	Compliance matrix [S]					
2	0.026	-0.008	-0.008	0.000	0.000	0.000
3	-0.008	0.088	-0.034	0.000	0.000	0.000
4	-0.008	-0.034	0.088	0.000	0.000	0.000
5	0.000	0.000	0.000	0.244	0.000	0.000
6	0.000	0.000	0.000	0.000	0.303	0.000
7	0.000	0.000	0.000	0.000	0.000	0.303
8						
9	Stiffness matrix [C]					
10	41.492	5.986	5.986	0.000	0.000	0.000
11	5.986	14.071	5.882	0.000	0.000	0.000
12	5.986	5.882	14.071	0.000	0.000	0.000
13	0.000	0.000	0.000	4.094	0.000	0.000
14	0.000	0.000	0.000	0.000	3.300	0.000
15	0.000	0.000	0.000	0.000	0.000	3.300

CODE PROBLEM 1.6 (P1p6.m)

```
% Routine that solve the Problem 1.6. It builds up the Stiffness and
% Compliance matrix for a transversely isotropic single layer composite
% provided its five elastic properties in the file props.dat
clear; clc;
% Read data
load props.dat
% Engineering properties
E1 = props(1); E2 = props(2); G12 = props(3); nu12 = props(4); nu23 = props(5);
% Compliance matrix [S]
S(1:3,1:3) = [1/E1 -nu12/E1 -nu12/E1;
             -nu12/E1 1/E2 -nu23/E2;
             -nu12/E1 -nu23/E2 1/E2];
S(4,4) = 2*(1+nu23)/E2;
S(5,5) = 1/G12; S(6,6) = S(5,5);
% Stiffness matrix [C]
C = inv(S);
% Write data
out1 = fopen('matrices.dat','w');
fprintf(out1,'Compliance_matrix_[S]\n');
fprintf(out1,'%8.3f %8.3f %8.3f %8.3f %8.3f\n',S);
fprintf(out1,'\n');
fprintf(out1,'Stiffness_matrix_[C]\n');
```

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```
fprintf(out1,'%8.3f %8.3f %8.3f %8.3f %8.3f %8.3f\n',C);
fclose(out1);
```

Problem 1.7

A rotation of an angle θ around the x_3 -axis is defined by:

$$a_{ij} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From a_{ij} the values m_i , l_i and n_i (director cosines) can be obtained:

$$\begin{aligned} l_1 &= \cos \theta & m_1 &= \sin \theta & n_1 &= 0 \\ l_2 &= -\sin \theta & m_2 &= \cos \theta & n_2 &= 0 \\ l_3 &= 0 & m_3 &= 0 & n_3 &= 1 \end{aligned}$$

Now, the transformations matrices $[T]$ and $[\bar{T}]$ can be obtained using:

$$[T] = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & 2m_1n_1 & 2l_1n_1 & 2l_1m_1 \\ l_2^2 & m_2^2 & n_2^2 & 2m_2n_2 & 2l_2n_2 & 2l_2m_2 \\ l_3^2 & m_3^2 & n_3^2 & 2m_3n_3 & 2l_3n_3 & 2l_3m_3 \\ l_2l_3 & m_2m_3 & n_2n_3 & m_2n_3 + n_2m_3 & l_2n_3 + n_2l_3 & l_2m_3 + m_2l_3 \\ l_1l_3 & m_1m_3 & n_1n_3 & m_1n_3 + n_1m_3 & l_1n_3 + n_1l_3 & l_1m_3 + m_1l_3 \\ l_1l_2 & m_1m_2 & n_1n_2 & m_1n_2 + n_1m_2 & l_1n_2 + n_1l_2 & l_1m_2 + m_1l_2 \end{bmatrix}$$

$$[\bar{T}] = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & m_1n_1 & l_1n_1 & l_1m_1 \\ l_2^2 & m_2^2 & n_2^2 & m_2n_2 & l_2n_2 & l_2m_2 \\ l_3^2 & m_3^2 & n_3^2 & m_3n_3 & l_3n_3 & l_3m_3 \\ 2l_2l_3 & 2m_2m_3 & 2n_2n_3 & m_2n_3 + n_2m_3 & l_2n_3 + n_2l_3 & l_2m_3 + m_2l_3 \\ 2l_1l_3 & 2m_1m_3 & 2n_1n_3 & m_1n_3 + n_1m_3 & l_1n_3 + n_1l_3 & l_1m_3 + m_1l_3 \\ 2l_1l_2 & 2m_1m_2 & 2n_1n_2 & m_1n_2 + n_1m_2 & l_1n_2 + n_1l_2 & l_1m_2 + m_1l_2 \end{bmatrix}$$

Due to the material is transversely isotropic both the compliance and stiffness matrix are defined like in the problem 1.6. Their transformations are given by:

$$\begin{aligned} [C] &= [T]^T * [C'] * [\bar{T}] \\ [S] &= [T]^T * [S'] * [\bar{T}] \end{aligned}$$

A MATLAB®'s routine was built (code attached) which reads the information about $[C']$, $[S']$ and θ from a file named *matrices.dat*, written with the following format:

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Text: *Compliance_matrix_[S]*
Data: *Formatted array 6x6*
Text: *<<<Blank>>>*
Text: *Stiffness_matrix_[S]*
Data: *Formatted array 6x6*
Text: *<<<Blank>>>*
Text: *Rotation_angle_theta*
Data: *Single value of theta in Degrees*

Using the matrices gotten from the previous problem and making a rotation of 45 degrees over z -axis it's obtained:

```

1 Compliance_matrix_[S]
2 0.176 -0.127 -0.021 0.000 0.000 -0.016
3 -0.127 0.176 -0.021 0.000 0.000 -0.015
4 -0.021 -0.021 0.088 0.000 0.000 0.013
5 0.000 0.000 0.000 0.273 0.030 0.000
6 0.000 0.000 0.000 0.030 0.273 0.000
7 -0.031 -0.031 0.026 0.000 0.000 0.065
8
9 Stiffness_matrix_[C]
10 18.534 15.234 5.934 0.000 0.000 13.711
11 15.234 18.534 5.934 0.000 0.000 13.710
12 5.934 5.934 14.071 0.000 0.000 0.104
13 0.000 0.000 0.000 3.697 -0.397 0.000
14 0.000 0.000 0.000 -0.397 3.697 0.000
15 6.855 6.855 0.052 0.000 0.000 21.796

```

CODE PROBLEM 1.7 (P1p7.m)

```

% Routine that solves the problem 1.7. It transforms the Stiffness and
% Compliance matrix for a transversely isotropic single layer composite
% from its material coordinates to another system of coordinates
clear; clc;
% Read data
inp1 = fopen('matrices.dat','r');
dummy = fscanf(inp1,'%s',1); % first text line
% Compliance matrix
S = fscanf(inp1,'%f %f %f %f %f %f',[6,6]);
dummy = fscanf(inp1,'%s',1); % second text line
% Stiffness matrix
C = fscanf(inp1,'%f %f %f %f %f %f',[6,6]);
dummy = fscanf(inp1,'%s',1); % third text line
% Rotation angle (theta)
the = fscanf(inp1,'%f',1);
fclose(inp1);
% Director cosines
l1 = cos(the*pi/180); m1 = sin(the*pi/180); n1 = 0;
l2 = -m1; m2 = l1; n2 = n1;
l3 = 0; m3 = 0; n3 = 1;

```

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```
% Transformation matrices
T = [l1^2 m1^2 n1^2 2*m1*n1 2*l1*n1 2*l1*m1;
     l2^2 m2^2 n2^2 2*m2*n2 2*l2*n2 2*l2*m2;
     l3^2 m3^2 n3^2 2*m3*n3 2*l3*n3 2*l3*m3;
     l2*l3 m2*m3 n2*n3 m2*n3+n2*m3 l2*n3+n2*l3 l2*m3+m2*l3;
     l1*l3 m1*m3 n1*n3 m1*n3+n1*m3 l1*n3+n1*l3 l1*m3+m1*l3;
     l1*l2 m1*m2 n1*n2 m1*n2+n1*m2 l1*n2+n1*l2 l1*m2+m1*l2];
Tbar = [l1^2 m1^2 n1^2 m1*n1 l1*n1 l1*m1;
        l2^2 m2^2 n2^2 m2*n2 l2*n2 l2*m2;
        l3^2 m3^2 n3^2 m3*n3 l3*n3 l3*m3;
        2*l2*l3 2*m2*m3 2*n2*n3 m2*n3+n2*m3 l2*n3+n2*l3 l2*m3+m2*l3;
        2*l1*l3 2*m1*m3 2*n1*n3 m1*n3+n1*m3 l1*n3+n1*l3 l1*m3+m1*l3;
        2*l1*l2 2*m1*m2 2*n1*n2 m1*n2+n1*m2 l1*n2+n1*l2 l1*m2+m1*l2];
% transformations
Cg = T.*C.*Tbar; Sg = Tbar.*S.*T;
% Write data
out1 = fopen('mglobal.dat','w');
fprintf(out1,'Compliance_matrix_[S]\n');
fprintf(out1,'%8.3f %8.3f %8.3f %8.3f %8.3f %8.3f\n',Sg);
fprintf(out1,'\n');
fprintf(out1,'Stiffness_matrix_[C]\n');
fprintf(out1,'%8.3f %8.3f %8.3f %8.3f %8.3f %8.3f\n',Cg);
fclose(out1);
```

Problem 1.8

For the same material used in the problem 1.5 (E-Glass / Isophthalic Polyester), we have the same results that were previously gotten. Now the equation (1.92) for transversely isotropic material is given by:

$$C_{11} = \frac{1 - \nu_{23}^2}{E_2 \Delta}$$

$$C_{12} = \frac{\nu_{12} + \nu_{12} \nu_{23}}{E_1 E_2 \Delta}$$

$$C_{13} = C_{12}$$