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1.1 (a) Unsteady flow during the operation of the gate. Steady flow afterwards. Rapidly varied flow in the immediate vicinity downstream of the gate.

(b) Normally steady , uniform flow

(c) Gradually varied unsteady flow

(d) Rapidly varied unsteady flow

(e) Steady rapidly varied flow

(f) Unsteady flow on both upstream and downstream of the gate

(g) Usually spatially varied steady flow. It can be SVUF also.

1.2 $V = \frac{u_m}{2}$, $\quad \frac{u}{y} = \frac{u_m}{D}$ and hence $u = \frac{y}{D} u_m$

$$\alpha = \frac{\int_0^D u^3 dy}{V^3 D} = \frac{\int_0^D u^3 dy}{\frac{u_m^3}{8} D} = \frac{\frac{u_m^3}{D^3}\int_0^D y^3 dy}{\frac{u_m^3}{8} D}$$

$$= \mathbf{2.0}$$

$$\beta = \frac{\int_0^D u^2 dy}{V^2 D} = \frac{\frac{u_m^2}{D^2}\int_0^D y^2 dy}{\frac{u_m^2}{4} D} = \frac{4}{3} = \mathbf{1.33}$$

1.3 $$\bar{V} = \frac{1}{y_0}\int_0^{y_0} v\, dy = \frac{v_m}{y_0}\frac{1}{y_0^{1/n}}\int_0^{y_0} y^{1/n} dy = \frac{n}{(n+1)} v_m$$

$$\beta = \frac{\int_0^{y_0} v^2 dy}{\bar{V}^2 y_0} = \frac{v_m^2}{\left(\frac{n}{n+1}\right)^2 v_m^2} \cdot \frac{1}{y_0}\int_0^{y_0} \frac{1}{y_0^{2/n}} \cdot y^{2/n}\, dy$$

$$\beta = \frac{(n+1)^2}{n(n+2)}$$

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$$\alpha = \frac{\int_0^{y_0} v^3 dy}{\overline{V}^3 y_0} = \frac{v_m^3}{\left(\frac{n}{n+1}\right)^3 v_m^3} \cdot \frac{1}{y_0} \int_0^{y_0} \frac{1}{y_0^{3/n}} \cdot y^{3/n} \, dy$$

$$\alpha = \frac{(n+1)^3}{n^2(n+3)}$$

1.4 $\beta = \frac{\int u^2 dA}{\overline{u}^2 A} = \frac{\int \left(\overline{u}^2 + (\delta u)^2 + 2\overline{u}\ \delta u\right) dA}{\overline{u}^2 A}$

Since $\int \delta u \cdot dA = 0$, $\beta = 1 + \eta$ where $\eta = \frac{\int (\delta u)^2 dA}{\overline{u}^2 A}$

$$\alpha = \frac{\int u^3 dA}{\overline{u}^3 A} = \frac{\int \left(\overline{u}^3 + (\delta u)^3 + 3\overline{u}\ (\delta u)^2 + 3\overline{u}^2 (\delta u)\right) dA}{\overline{u}^3 A}$$

Since $\int \delta u \cdot dA = 0$ and $\int (\delta u)^3 dA$ is negligibly small ,

$\alpha = 1 + 3\eta$

1.5 $v = C/r \quad r_1 = 4.25 \text{ m} \quad r_2 = 5.75 \text{ m}$

$$Q = \int_{r_1}^{r_2} v \, B \, dr = 6.0$$

$$\int_{r_1}^{r_2} \frac{C}{r} dr = \frac{6.0}{2.0} = 3.0 \qquad C = \frac{3.0}{\ln \frac{r_2}{r_1}} = \frac{3.0}{\ln \frac{5.75}{4.25}} = 9.9245$$

(a) $v = \frac{9.9245}{r}$

$$v_1 = \frac{C}{4.25} = 2.335 \text{ m/s}$$

$$v_2 = \frac{C}{5.75} = 1.726 \text{ m/s}$$

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(b) $$v_{av} = \frac{Q}{B\ y_0} = \frac{6.0}{2.0 \times 1.5} = \mathbf{2.0\ m/s}$$

(c) $$\beta = \frac{\int v^2 dy}{v_{av}^2\ y_0} = \frac{\int_{r_1}^{r_2} \frac{C^2}{r^2} dr}{(2)^2 \times 1.5} = \left[\frac{C^2}{6.0}(-\frac{1}{r})\right]_{4.25}^{5.75}$$

$$=16.415\left[-\frac{1}{5.75} + \frac{1}{4.25}\right] = \mathbf{1.008}$$

$$\alpha = \frac{\int v^3 dy}{v_{av}^3\ y_0} = \frac{\int_{r_1}^{r_2} \frac{C^3}{r^3} dr}{(2)^3 \times 1.5} = \left[\frac{C3}{12.0}(-\frac{1}{2r^2})\right]_{4.25}^{5.75}$$

$$= 40.73\left[-\frac{1}{(5.75)^2} + \frac{1}{(4.25)^2}\right] = \mathbf{1.023}$$

1.6 (a) Refer to Fig. P-1.6 (a)

$$Z_0 + h_1 + \Delta h = Z_0 + (h_1 - y) + k\ y$$

$$\Delta h = (k-1) y$$

$$h_{ep} = Z_0 + h_1 + \frac{1}{h_1}\int_0^{h_1} (k-1) y\ dy$$

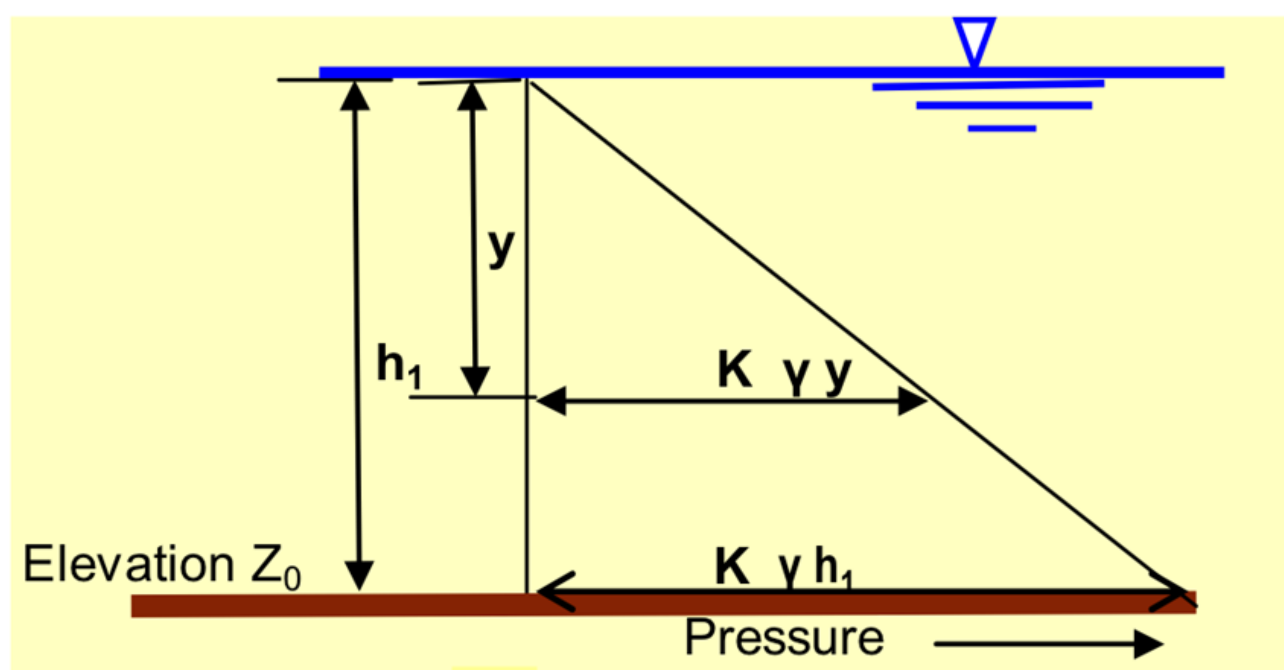

Fig. P -1.6 (a)

$$= Z_0 + h_1 + (k-1)\frac{h_1^2}{2h_1} = Z_0 + h_1 + \frac{(k-1)}{2} h_1$$

$$= Z_0 + \left(\frac{k+1}{2}\right) h_1$$

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(b) Refer to Fig. P-1.6(b)

$$Z_0 + h_1 + \Delta h = Z_0 + (h_1 - y) + k\ h_1$$

$$\Delta h = kh_1 - y$$

$$h_{ep} = Z_0 + h_1 + \frac{1}{h_1}\int_0^{h_1}(kh_1 - y)\ dy$$

$$= Z_0 + h_1 + \frac{1}{h_1}\left[kh_1^2 - \frac{h_1^2}{2}\right]$$

$$= Z_0 + \left(\frac{2k+1}{2}\right)h_1$$

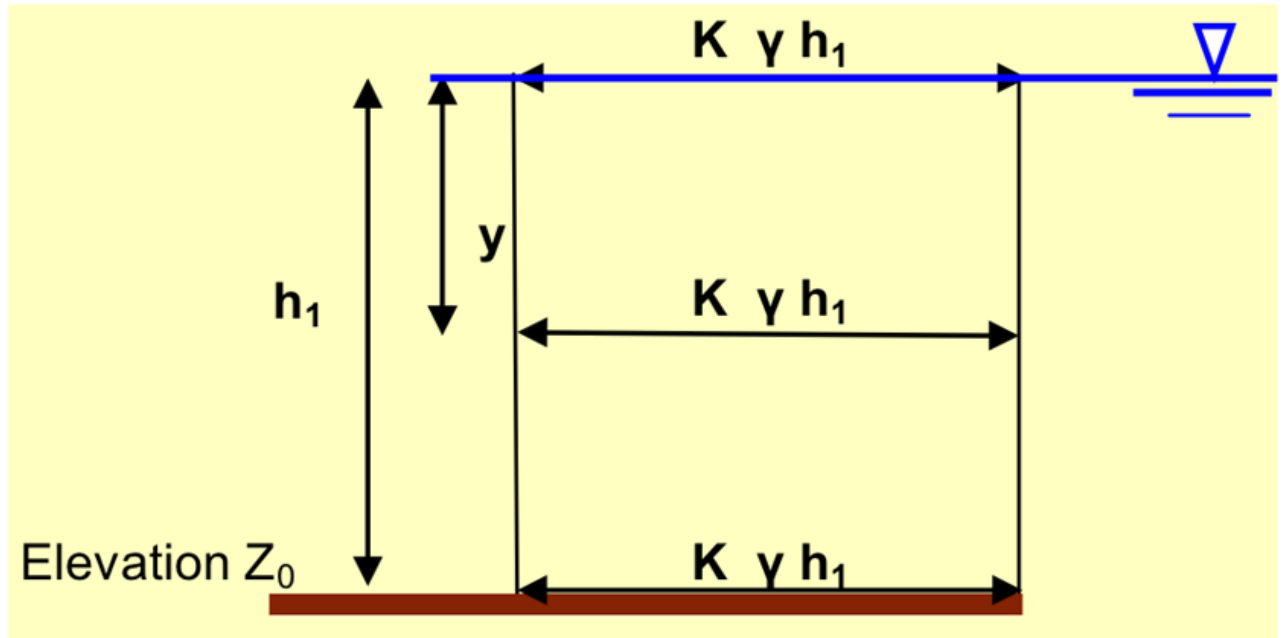

Fig. P -1.6(b)

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1.7 Refer to Fig. P-1.7(a) and (b)

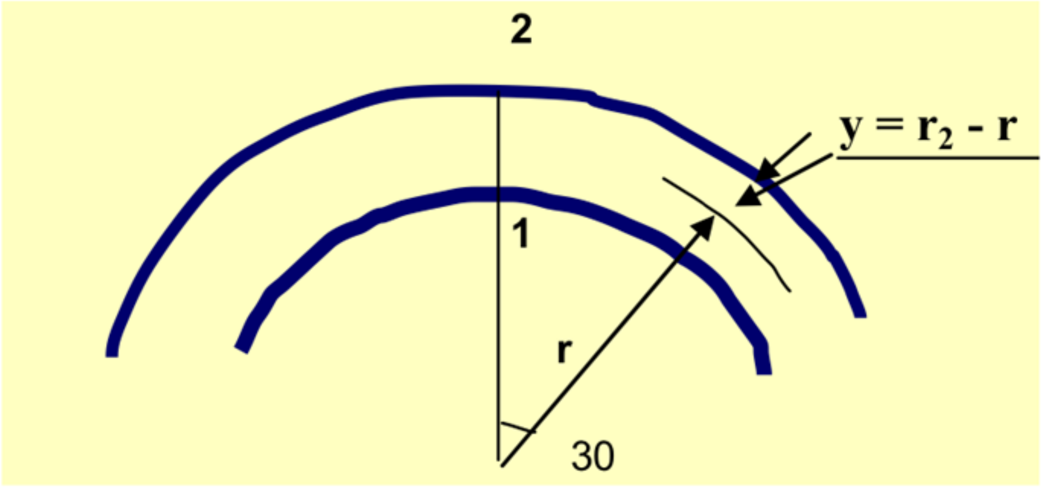

Fig. P 1.7(a)

$$\frac{p}{\gamma} = (r_2 - r)\cos\theta - \frac{a_n}{g}(r_2 - r).$$

= depth below free surface = $(r_2 - r)$.

Hence $\frac{p}{\gamma} = y\cos\theta - \frac{a_n}{g}y$

When $\theta = 30^0$ and $a_n = 0.4\,g$

$$\frac{p}{\gamma} = 0.866y - 0.4y = 0.466y$$

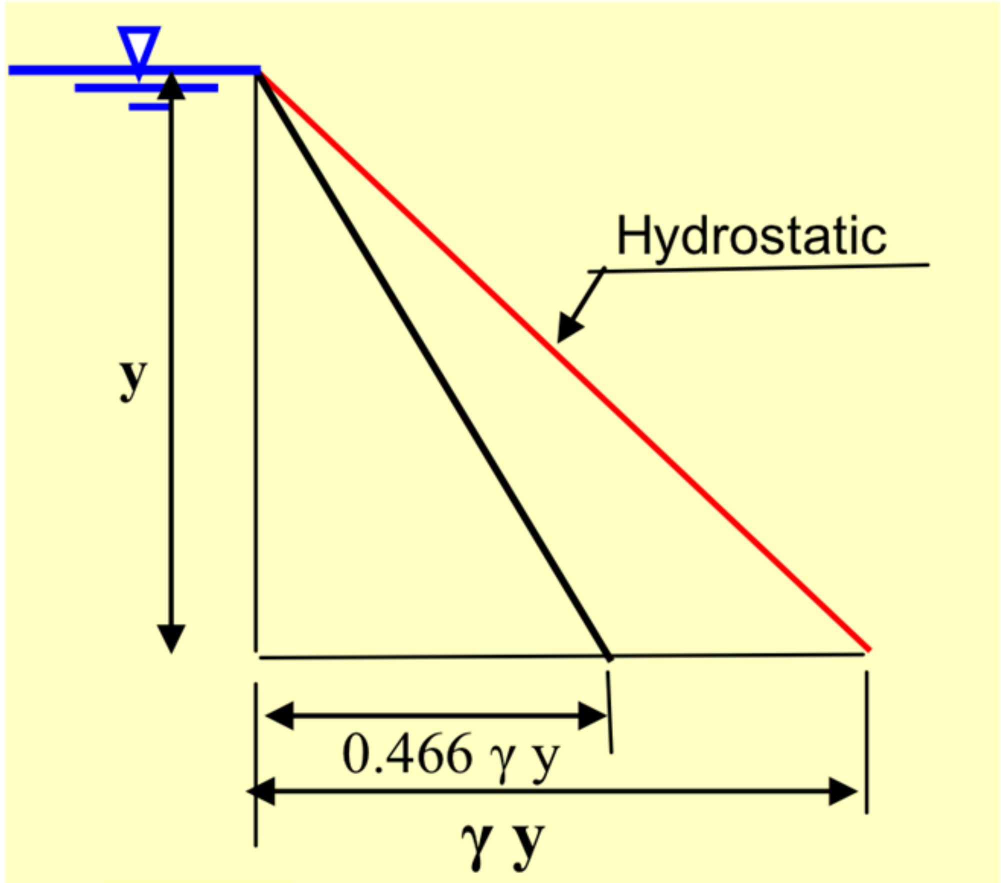

Fig. P 1.7(b)

At the free surface $y = 0$ and hence $\frac{p}{\gamma} = 0$

At the bottom of the channel, $y = 0.75$ m, and $\frac{p}{\gamma} = 0.35$ m.

Hydrostatic pressure distribution gives $\frac{p}{\gamma} = y$.

1.8 Refer to Fig. P-1.8.

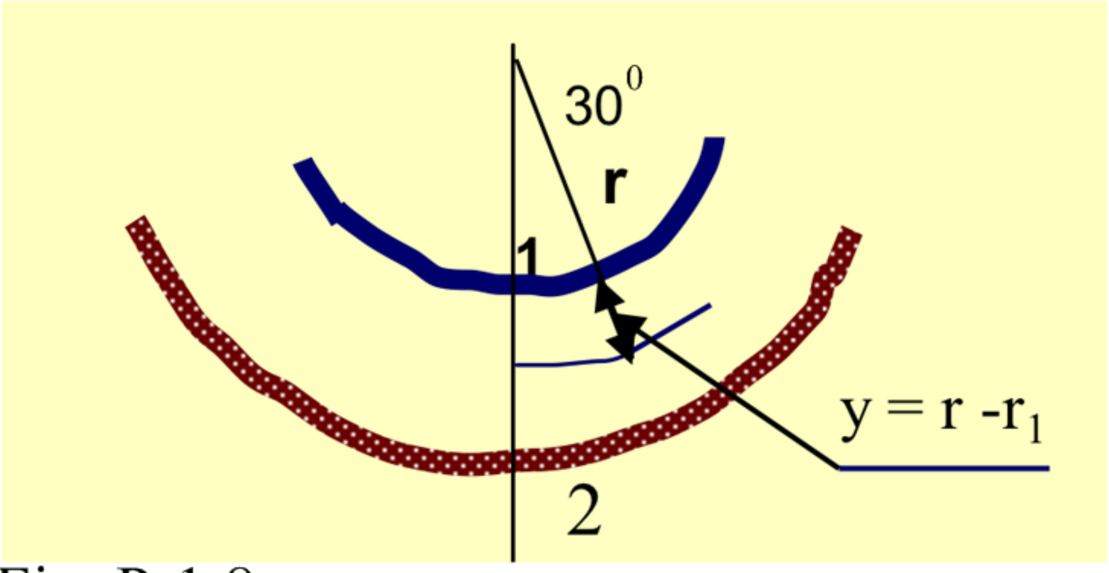

Fig. P-1.8.

$$\frac{p}{\gamma} = y\cos\theta + \frac{a_n}{g}y = 0.866y + 0.4y = 1.266y$$

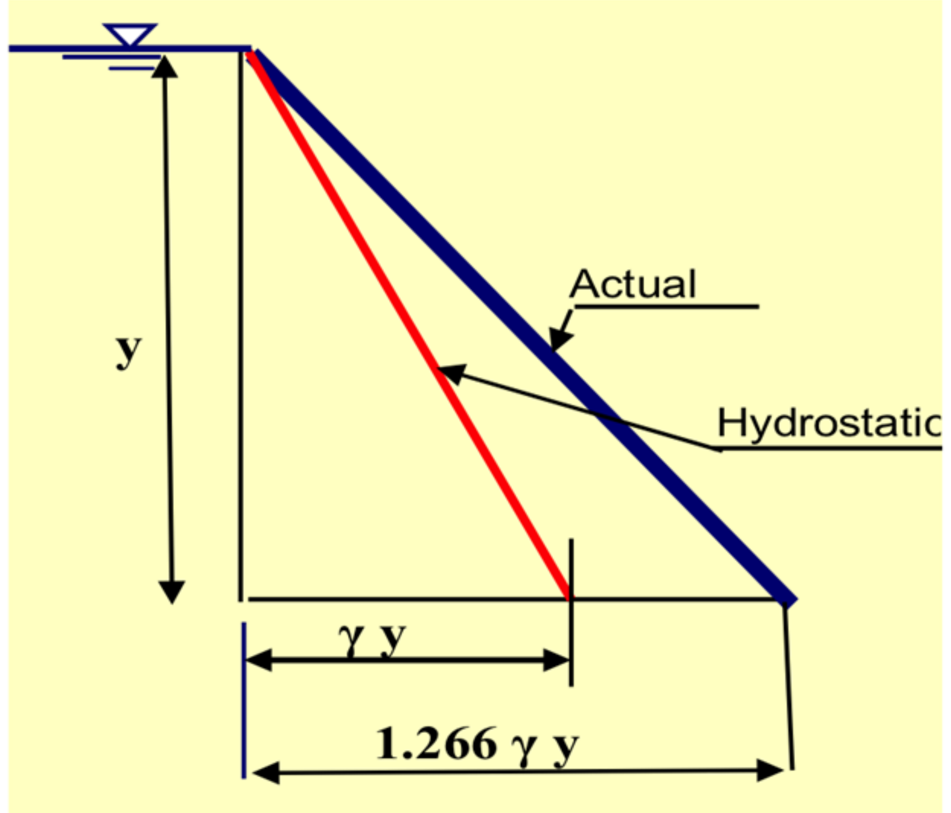

Fig. P - 1.9 (b)

Hydrostatic pressure distribution gives $\frac{p}{\gamma} = y$

Pressure at the bottom of the channel

$$\frac{p}{\gamma} = 1.266 \times 0.75 = \mathbf{0.95\ m}.$$

1.09 Refer to Fig. P- 1.09

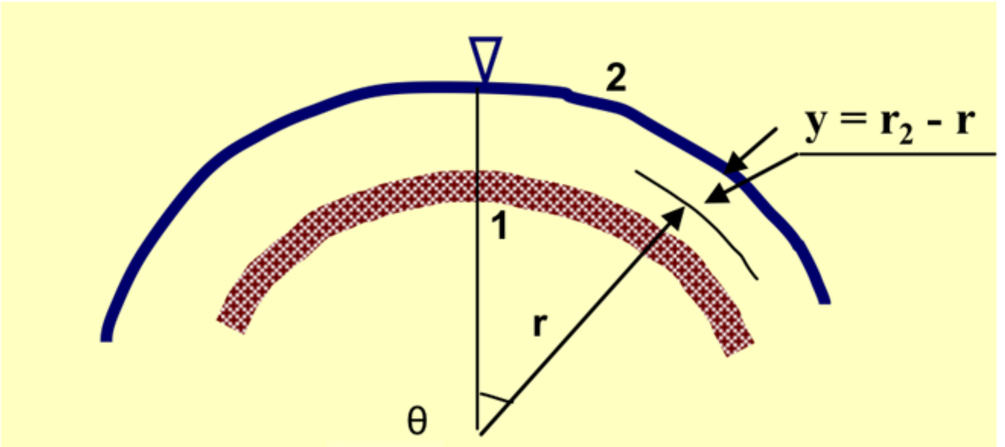

Fig. P-1.09

$$\frac{p}{\gamma} = y\cos\theta - \frac{a_n}{g}y.$$

At point 1 on the crest,

$$\frac{p}{\gamma} = 0 = \text{atmospheric}.$$

Then $\frac{p_1}{\gamma} = y_1\cos\theta - \frac{a_n}{g}y_1 = 0$ or $\boldsymbol{a_n = g\cos\theta}$.

1.10 $$\left(\frac{p}{\gamma} + Z\right) = \int \frac{v^2}{gr} + const = \int \frac{C^2}{gr^3} + const$$

$$= \frac{C^2}{g}\left[-\frac{1}{2r^2}\right] + const$$

At point 1 on the water surface $p_1 = 0$.

$$Z_1 = -\frac{C^2}{2gr_1^2} + \text{const}. \qquad \text{or} \qquad \text{const} = Z_1 + \frac{C^2}{2gr_1^2}$$

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$$\frac{p}{\gamma} = (Z_1 - Z) + \frac{C^2}{g}\left(\frac{1}{r_1^2} - \frac{1}{r^2}\right).$$

At a point h below the free surface, $(Z_1 - Z_2) = h\cos\theta$ and

$$\frac{p}{\gamma} = h\cos\theta + \frac{(v_1^2 - v_2^2)}{2g}$$

1.11 $v = Cr$ and hence $(\frac{p}{\gamma} + Z) = \int \frac{C^2 r}{g} + const$

$$= \frac{C^2 r^2}{2g} + const$$

At point 1 on the free surface $r = r_1, Z = Z_1$ and $\frac{p}{\gamma} = 0$.

$$\therefore \quad \frac{p}{\gamma} = (Z_1 - Z) + \frac{C^2}{2g}(r^2 - r_1^2).$$

At a depth h below the free surface, $(Z_1 - Z_2) = h\cos\theta$ and hence

$$\frac{p}{\gamma} = h\cos\theta + \frac{(v_2^2 - v_1^2)}{2g}$$

1.12 $q = (0.10 \times 1.20 \cos 4^0) + (0.10 \times 1.00 \cos 8^0) + (0.10 \times 0.90 \cos 12^0) + (0.10 \times 0.85 \cos 15^0)$

$=$ **0.3889 m^3/s**

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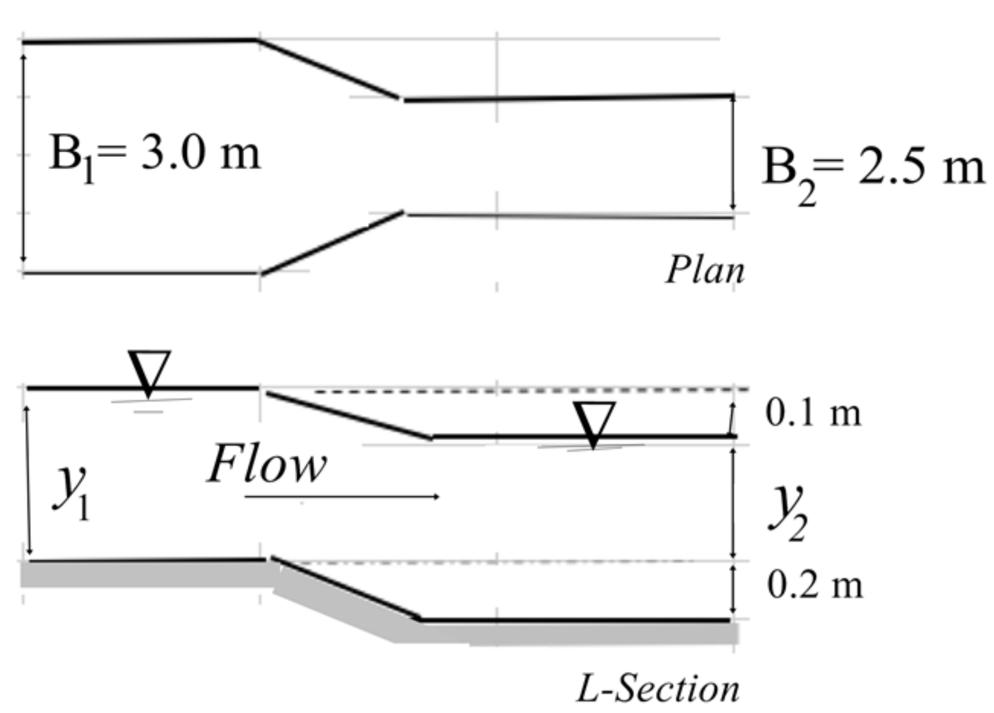
1.13 $B_1 = 3.0$ m, y_1 = depth in 3.0 m channel = 2.0 m

$B_2 = 2.5$ m, y_2 = depth in 2.5 m channel, ΔZ = drop = 0.2 m

Since the drop in water surface = 0.1 m

$y_1 + 0.2 = y_2 + 0.1$

$2.0 + 0.2 = y_2 + 0.1$ Hence $y_2 = 2.1$ m Considering the energy above the bed level of the downstream channel



Schematic sketch of Problem 1.13

$$\Delta Z + y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$0.2 + 2.0 + \frac{V_1^2}{2g} = 2.1 + \frac{V_2^2}{2g}$$

$$\frac{(V_2^2 - V_1^2)}{2g} = 2.2\text{-}2.1 = 0.1 \qquad \text{(i)}$$

Also, $B_1 y_1 V_1 = B_2 y_2 V_2$

$3.0 \times 2.0 \times V_1 = 2.5 \times 2.1 \times V_2$ $\therefore V_1 = 0.875 V_2$

Substituting in Eq. (i), $\frac{V_2^2}{2g}(1 - (0.875)^2) = 0.1$

$V_2^2 = 2 \times 9.81 \times \frac{0.1}{0.2345} = 8.366$

$V_2 = 2.892$ m/s and $V_1 = 2.531$ m/s

Discharge $Q = A_1 V_1 = 3 \times 2 \times 2.531 = \mathbf{15.186\ m^3/s}$

1.14 $Q_1 = 20\ \text{m}^3/\text{s}$, $\frac{\Delta y}{\Delta t} = 0.45$ m/h $= \frac{0.45}{(6060)}$ m/s

T = top width = 25 m, $\Delta x = 1.5$ km = 1500 m

$$\frac{\partial Q}{\partial x} + T\frac{\partial y}{\partial t} = 0$$

$$\frac{(Q_2 - Q_1)}{\Delta x} + \text{T}\ \frac{\Delta y}{\Delta t} = 0$$

$$\frac{(Q_2 - 20)}{1500} + 25\ \frac{0.45}{(60 \times 60)} = 0$$

$$Q_2 = 20 - 25 \times \frac{0.45}{(60 \times 60)} \times 1500 = \mathbf{15.3125\ m^3/s}$$

1.15 Hydraulic jump: $q = \frac{Q}{B} = \frac{44.30}{5.0} = 8.86\ \text{m}^3/\text{s/m}$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 0.5 + \frac{(8.86)^2}{2 \times 9.81 \times (0.5^2)} = 16.50\ \text{m}$$

$$E_2 = y_2 + \frac{V_2^2}{2g} = 0.5 + \frac{(8.86)^2}{2 \times 9.81 \times (5.4)^2} = 5.547\ \text{m}$$

Energy loss $E_L = E_1 - E_2 = 16.50 - 5.547 = \mathbf{10.95\ m}$

1.16 Rectangular channel: $A_1 = 2.0 \times 1.5 = 3.0\ \text{m}^2$

$$V_1 = 10.0/3.0 = 3.33\ \text{m/s}$$

$$y_1 = 1.50\ \text{m, and } \frac{V_1^2}{2g} = 0.566\ \text{m}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2.066\ \text{m}$$

Trapezoidal channel: $A_2 = [3.0 + (1.5 \times 1.0)] = 4.5\ \text{m}^2$

$$V_2 = 10.0/4.5 = 2.222\ \text{m/s}$$

$$y_2 = 1.0\ \text{m, and } \frac{V_2^2}{2g} = 0.252\ \text{m}$$

Energy loss $E_L = 0.2\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) = 0.063\ \text{m}$

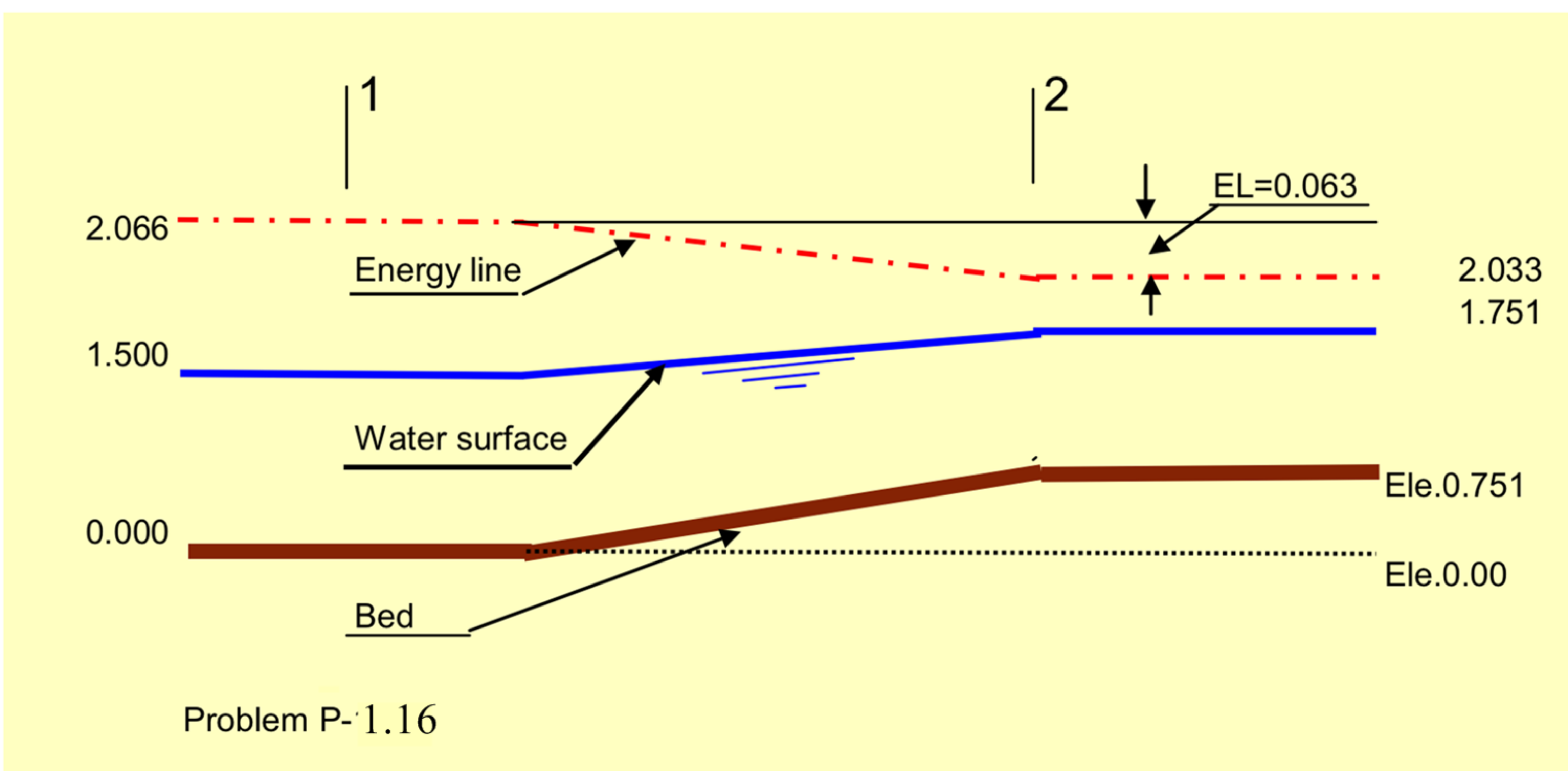


Fig. P-1.16 Schematic Features of the transition

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Refer to Fig. P-1.20. With bed level of section 1 as datum,

Elevation of energy line at 2 = 2.066 – 0.063 = 2.003 m

Elevation of water surface at 2 = 2.003 – 0.252 = 1.751 m

Elevation of bed at 2 = 1.751 – 1.000 = 0.751 m

$\Delta Z =$ **0.751 m** and $\Delta W_s =$ **0.251m**

1.17 Refer to Fig. P-1.17.

$$H_A = Z_A + y_A \cos\theta + \alpha_A \frac{V_A^2}{2g}$$

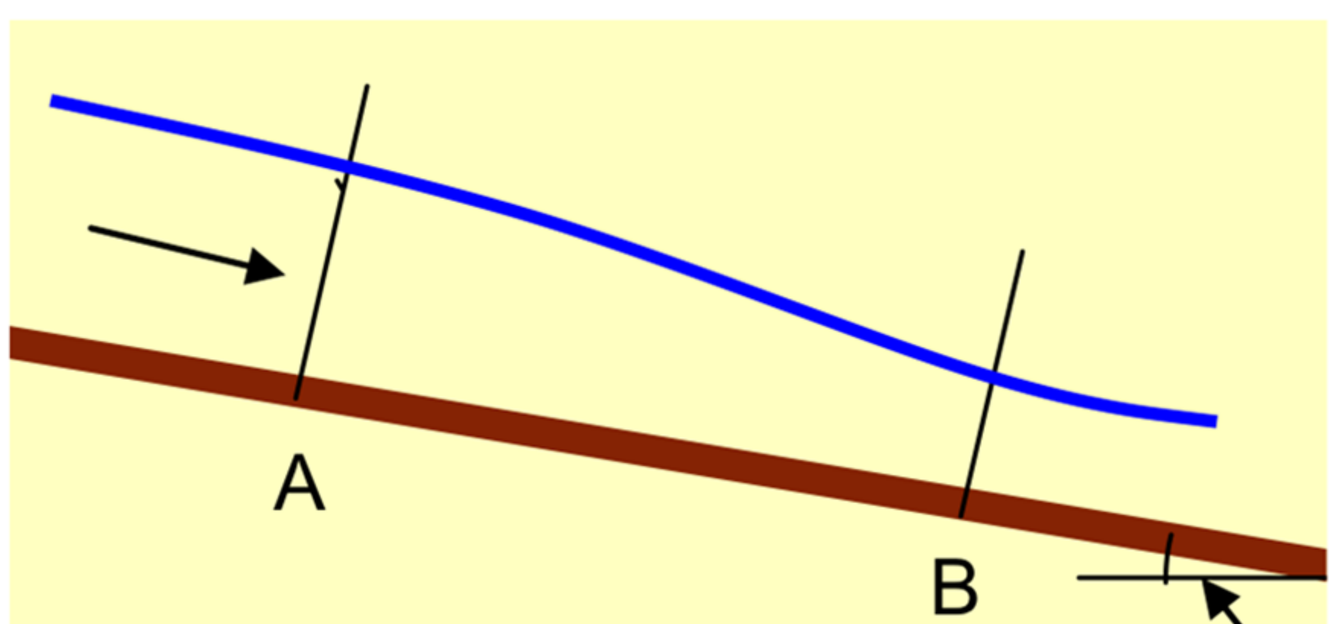

Fig. P-1.17

$$H_A = 15.00 + 1.3 \cos 10^0 + 1.03 \frac{(3.0)^2}{2 \times 9.81} = \mathbf{16.753\ m}$$

h_{pA} = elevation of piezometric head at A

$Z_A + y_A \cos\theta = 15.00 + 1.3 \cos 10^0 =$ **16.280 m**

Similarly $H_B = Z_B + y_B \cos\theta + \alpha_B \frac{V_B^2}{2g}$

$$= 14.60 + 1.2 \cos 10^0 + 1.02 \frac{(V_B)^2}{2 \times 9.81}$$

By continuity $y_A V_A = y_B V_B$,

and hence $V_B = (1.3 \times 3.0)/1.2 = 3.25$ m/s

$$H_B = 14.60 + 1.2\cos 10^0 + 1.03\,\frac{(3.25)^2}{2\times 9.81} = \mathbf{16.331\ m}$$

h_{pB} = elevation of piezometric head at B

$= 14.60 + 1.2\cos 10^0 =$ **15.782 m**

1.18 Refer to Fig. P- 1.18

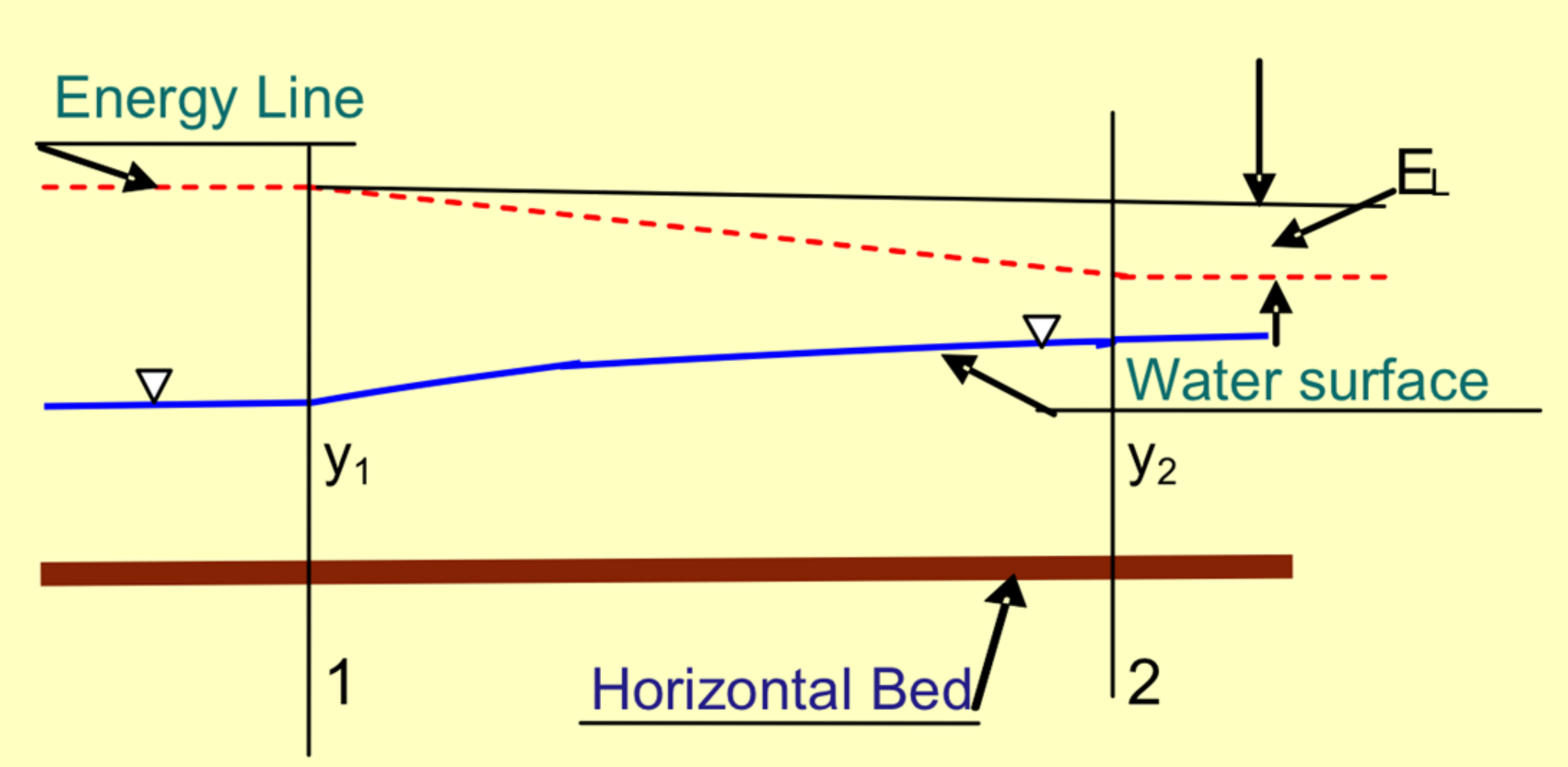

Fig. P- 1.18

$A_1 = 2.0 \times 1.2 = 2.40\ \text{m}^2$

$V_1 = 7.20/2.40 = 3.0$ m/s

$\frac{V_1^2}{2g} = 0.459,\ y_1 = 1.20$ m,

$\alpha_1 = 1.05$

$$H_1 = y_1 + \alpha_1\frac{V_1^2}{2g} = 1.682\ \text{m}$$

For section 2: $A_2 = 3.0 \times 1.4 = 4.20\ \text{m}^2$

$V_2 = 7.20/4.20 = 1.714$ m/s

$\frac{V_2^2}{2g} = 0.150,\ y_2 = 1.40$ m,

$\alpha_2 = 1.15$

$$H_2 = y_2 + \alpha_2\frac{V_2^2}{2g} = 1.573\ \text{m}$$

Energy loss = E_L = 1.682 – 1.573 = **0.109 m**

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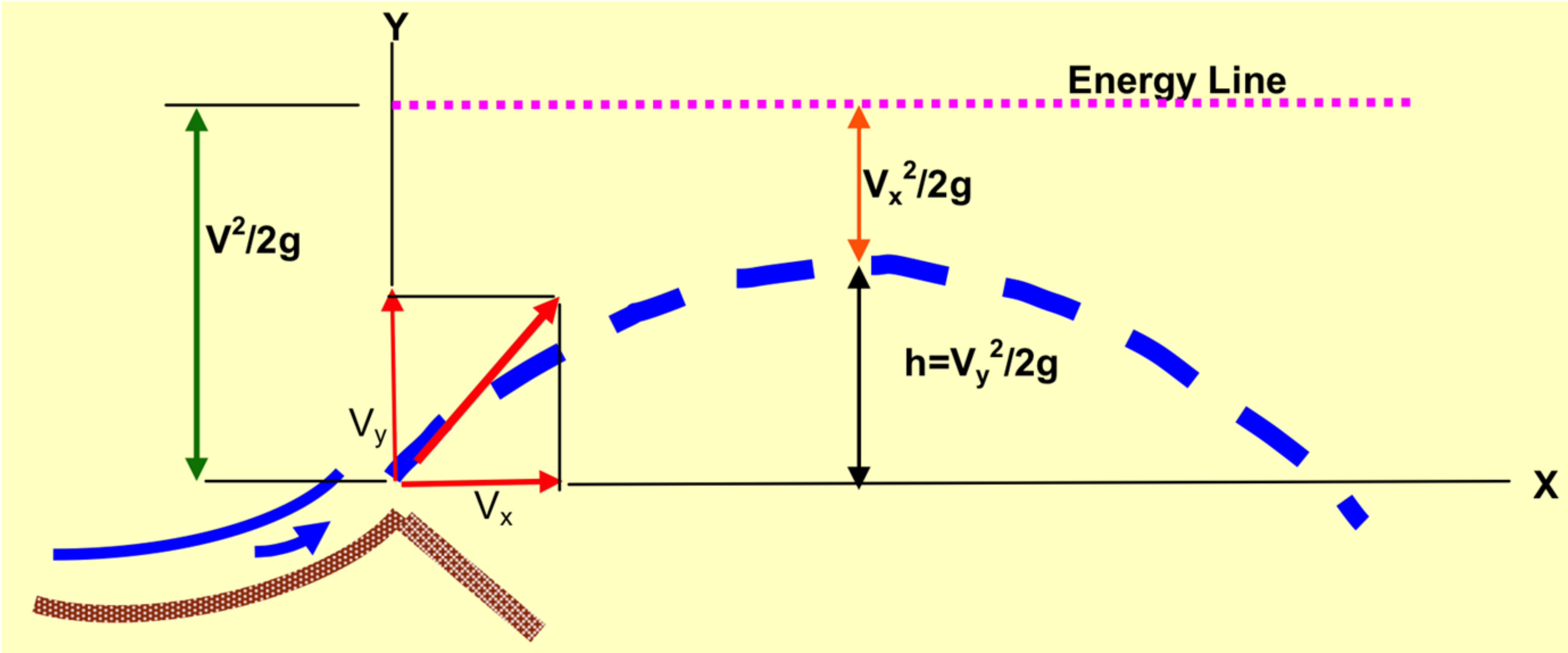
1.19 Refer to Fig. P-1.19. At the maximum point of the trajectory.



See Fig. P–1.19 Trajectory of liquid jet.

From this Fig.
$$h = \frac{V_y^2}{2g} = \frac{(V\sin\theta)^2}{2g} = \frac{(20\times\sin 40^0)^2}{2\times 9.81} = \mathbf{8.42\ m}$$

1.20 (a) $q = 2.5\ \text{m}^3/\text{s/m}$ and $y_2 = 0.30$ m

$$V_2 = \frac{q}{y_2} = \frac{2.5}{0.30} = 8.333\ \text{m/s},\quad V_1 = \frac{2.5}{y_1}$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$y_1 + \frac{(2.5)^2}{2\times 9.81\times y_1^2} = 0.30 + \frac{(8.333)^2}{2\times 9.81} = 3.8395\ \text{m}$$

By trial and error $\boldsymbol{y_1 = 3.818}$ **m**

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1.20 (b) $q = 2.0 \text{ m}^3\text{/s/m} = V_1 y_1 = V_2 y_2$ and $y_1 = 4.0$ m

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$4.00 + \frac{(2.0)^2}{2\times 9.81\times (4.0)^2} = y_2 + \frac{(2.0)^2}{2\times 9.81\times y_2^2}$$

$y_2 + \dfrac{0.2039}{y_2^2} = 4.013$ and by trial and error $\boldsymbol{y_2 = 0.232}$ **m**

1.20 (c) $q = 2.0 \text{ m}^3\text{/s/m} = V_1 y_1 = V_2 y_2$ and $y_1 = 4.0$ m

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + 0.1\frac{V_2^2}{2g} = y_2 + 1.1\frac{V_2^2}{2g}$$

$$4.00 + \frac{(2.0)^2}{2\times 9.81\times (4.0)^2} = y_2 + \frac{1.1\times (2.0)^2}{2\times 9.81\times y_2^2}$$

$y_2 + \dfrac{0.22427}{y_2^2} = 4.013$ and by trial and error $\boldsymbol{y_2 = 0.244}$ **m**

1.20 (d) $$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2}$$

$$(y_1 - y_2) = \frac{q^2}{2g}\left(\frac{1}{y_2^2} - \frac{1}{y_1^2}\right) = \frac{q^2}{2g}\cdot\frac{(y_1 + y_2)}{y_1^2\ y_2^2}$$

$$q = y_1 y_2 \sqrt{\left(\frac{2g}{(y_1 + y_2)}\right)} = (3.0\times 0.25)\sqrt{\frac{(2\times 9.81)}{(3.0 + 0.25)}} = \mathbf{1.843\ m^3.s.m}$$

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Referring to Example 1.21, Force on a sluice gate during flow is evaluated by use of momentum equation as:

$$\frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 - R = \rho q^2\left(\frac{y_1 - y_2}{y_1 y_2}\right)$$

$$R = \frac{1}{2}\varrho g(y_1^2 - y_2^2) - \rho q^2\left(\frac{y_1 - y_2}{y_1 y_2}\right)$$

$$= \rho(y_1 - y_2)\left\{\frac{g}{2}(y_1 + y_2) - \frac{q^2}{(y_1 + y_2)}\right\} \qquad (1)$$

The force on the gate is equal and opposite to *R*. Values of *R* (and hence *F*) are calculated by using Eq.1 for all the four cases of problem 1.20 and are listed below:

y_1 (m)	y_2 (m)	q	*F* (Force per metre width of gate (kN/m)
3.818	0.300	2.500	51.76
4.000	0.232	2.000	61.85
4.000	0.244	2.000	62.67
3.000	0.250	1.843	31.32