Solutions

Chapter 1

1.1 If $\Omega_{\mathrm{M}} = \Omega_{\mathrm{T}} = 1$, then $\Omega_{\mathrm{M}}(a) = \Omega_{\mathrm{T}}(a) = 1$ for all $a(t)$. Structure formation never ceases as larger and larger regions of negative Newtonian energy detach from the expansion.

The values of the Ω's for the $\Omega_{\mathrm{M}} = \Omega_{\mathrm{T}} = 0.3$ and the $\Omega_{\mathrm{M}} = 0.3$, $\Omega_{\Lambda} = 0.7$ models are shown in Fig. 1.

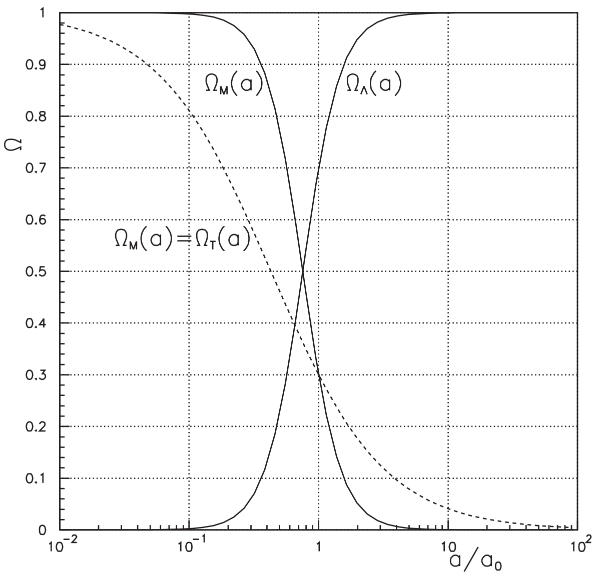

Fig. 1 The solid lines show $\Omega_{\mathrm{M}}(a)$ and $\Omega_{\Lambda}(a)$ for ($\Omega_{\mathrm{M}} = 0.3$, $\Omega_{\Lambda} = 0.7$). The dashed line shows $\Omega_{\mathrm{M}}(a)$ for ($\Omega_{\mathrm{M}} = \Omega_{\mathrm{T}} = 0.3$). The universe remains matter dominated for a longer period in the first case

1.2 The time that has passed since the universe became vacuum dominated is (including only the vacuum energy density)

$$\frac{t_0 - t_{m=v}}{H_0^{-1}} \sim \int \frac{da}{a\sqrt{\Omega_\Lambda}} = \frac{1}{3\sqrt{\Omega_\Lambda}} \ln(\Omega_\Lambda/\Omega_{\rm M}) = 0.39\ . \tag{1}$$

Numerical integration including both matter and vacuum gives 0.32.

The duration of the matter-dominated epoch is (including only the matter density)

$$\frac{t_{m=v} - t_{r=m}}{H_0^{-1}} \sim a_0^{-3/2} \int \frac{da}{a\sqrt{\Omega_{\rm M} a^{-3}}} \sim (2/3)\frac{1}{\Omega_\Lambda^{1/2}} = 0.78\ . \tag{2}$$

Numerical integration including matter and vacuum gives 0.69.

The duration of the radiation-dominated epoch is (including only the radiation density)

$$\frac{t_{r=m} - t_{\rm inf}}{H_0^{-1}} \sim a_0^{-2} \int \frac{da}{a\sqrt{\Omega_{\rm R} a^{-4}}} \sim (1/2)\frac{\Omega_{\rm R}^{3/2}}{\Omega_{\rm M}^2} = 5.4 \times 10^{-6} \tag{3}$$

for $\Omega_{\rm R} = 1.68\Omega_\gamma \sim 8.5 \times 10^{-5}$ (three massless neutrino species). Numerical integration including both radiation and matter gives 4.2×10^{-6}. The time would not change by much if you had taken $a_{\rm inf} = 0$.

The time when the first nuclei formed:

$$\frac{t_{\rm nuc} - t_{\rm inf}}{H_0^{-1}} \sim a_0^{-2} \int \frac{da}{a\sqrt{\Omega_{\rm R} a^{-4}}} \sim \frac{(3 \times 10^{-9})^2}{2\sqrt{\Omega_\Lambda}} \sim 4.9 \times 10^{-16}, \tag{4}$$

i.e. 3.4 min.

1.3 The universe is expanding today because it was expanding yesterday (see (1.58)). It was expanding yesterday because.....

It will be difficult to get an ultimate explanation since it will require knowledge of the physics that was in charge of things before the expansion began.

Chapter 2

2.1 The flux from a typical galaxy of redshift $z \ll 1$ is

$$\phi \sim \frac{2 \times 10^{10} L_\odot/(2\text{eV/photon})}{4\pi(zd_H)^2} \sim 100\,\text{m}^{-2}\text{s}^{-1}/z^2\ . \tag{5}$$

The ratio of the flux of nearby galaxies to that of nearby stars is

$$\frac{2\times 10^{10}L_\odot/(1\,\mathrm{Mpc})^2}{L_\odot/(1\,\mathrm{pc})^2} \sim 2\times 10^{-2}\,. \qquad (6)$$

2.2 The total number of stellar photons can be roughly estimated as follows:

$$\begin{aligned} n_{\mathrm{starlight}} &\sim J_0 H_0^{-1}/(2\,\mathrm{eV/photon}) \sim 10^8 L_\odot \mathrm{Mpc}^{-3} H_0^{-1}/2 \\ &\sim 2\times 10^3 \mathrm{m}^{-3}\,, \end{aligned} \qquad (7)$$

which is much less than the number of CMB photons.

The number of hydrogen nuclei transformed in order to produce these photons is

$$n_{p\to{}^4\mathrm{He}} \sim 2000\,\mathrm{m}^{-3}\frac{2\,\mathrm{eV}}{6\,\mathrm{MeV}} \sim 0.6\times 10^{-3}\mathrm{m}^{-3} \qquad (8)$$

or about 0.3×10^{-2} of the available hydrogen. Only a small amount of hydrogen has been transformed since most of it is still in intergalactic space.

2.3 Compton scattering dominates with a mean free path of order

$$(n_e(t_0)\sigma_\mathrm{T})^{-1} \sim 600 d_\mathrm{H}\,, \qquad (9)$$

where we have assumed that all matter is ionized (as suggested by the Gunn–Peterson effect).

2.4 It is possible to count the number of galaxies with a redshift less than z. The volume of the corresponding space is $V = (4\pi/3)z^3 d_\mathrm{H}^3 \propto h_{70}^{-3}$ so the measured number density is $\propto h_{70}^3$.

Luminosities are determined by multiplying a measured flux by $(z d_\mathrm{H})^2$ and are, therefore, proportional to h_{70}^{-2}. The luminosity density $\sim n_\mathrm{gal} L_\mathrm{gal}$ is, therefore, proportional to h_{70}.

Galactic masses are determined from the rotation curve, $M \sim v^2 r/G$. The radial distance r is proportional to the measured angular size and the redshift-determined distance so the mass is proportional to h_{70}^{-1}. Multiplying by n_gal gives a mass density associated with galaxies proportional to h_{70}^2. Dividing by the critical density gives an Ω independent of h_{70}.

2.5 For NGC1365 Cepheids (Fig. 2.28) we have

$$V(10\,\mathrm{days}) \sim 27.5\,, \qquad (10)$$

while for LMC Cepheids (Fig. 2.5) we have

$$V(10\,\mathrm{days}) \sim 14.3\,, \qquad (11)$$