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## **Introduction to Econometrics (4th Edition)**

by

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### **Solutions to End-of-Chapter Exercises: Chapter 2\***

\*Limited distribution: **For Instructors Only**. Answers to all odd-numbered questions are provided to students on the textbook website.

2.1. (a) Probability distribution function for  $Y$

Outcome (number of heads)	$Y = 0$	$Y = 1$	$Y = 2$
Probability	0.25	0.50	0.25

(b) Cumulative probability distribution function for  $Y$

Outcome (number of heads)	$Y < 0$	$0 \leq Y < 1$	$1 \leq Y < 2$	$Y \geq 2$
Probability	0	0.25	0.75	1.0

(c)  $\mu_Y = E(Y) = (0 \times 0.25) + (1 \times 0.50) + (2 \times 0.25) = 1.00$

Using Key Concept 2.3:  $\text{var}(Y) = E(Y^2) - [E(Y)]^2$ ,

and

$$E(Y^2) = (0^2 \times 0.25) + (1^2 \times 0.50) + (2^2 \times 0.25) = 1.50$$

so that

$$\text{var}(Y) = E(Y^2) - [E(Y)]^2 = 1.50 - (1.00)^2 = 0.50.$$

2.2. We know from Table 2.2 that  $\Pr(Y = 0) = 0.22$ ,  $\Pr(Y = 1) = 0.78$ ,  $\Pr(X = 0) = 0.30$ ,  $\Pr(X = 1) = 0.70$ . So

(a)

$$\begin{aligned}\mu_Y &= E(Y) = 0 \times \Pr(Y = 0) + 1 \times \Pr(Y = 1) \\ &= 0 \times 0.22 + 1 \times 0.78 = 0.78,\end{aligned}$$

$$\begin{aligned}\mu_X &= E(X) = 0 \times \Pr(X = 0) + 1 \times \Pr(X = 1) \\ &= 0 \times 0.30 + 1 \times 0.70 = 0.70.\end{aligned}$$

(b)

$$\begin{aligned}\sigma_X^2 &= E[(X - \mu_X)^2] \\ &= (0 - 0.70)^2 \times \Pr(X = 0) + (1 - 0.70)^2 \times \Pr(X = 1) \\ &= (-0.70)^2 \times 0.30 + 0.30^2 \times 0.70 = 0.21,\end{aligned}$$

$$\begin{aligned}\sigma_Y^2 &= E[(Y - \mu_Y)^2] \\ &= (0 - 0.78)^2 \times \Pr(Y = 0) + (1 - 0.78)^2 \times \Pr(Y = 1) \\ &= (-0.78)^2 \times 0.22 + 0.22^2 \times 0.78 = 0.1716.\end{aligned}$$

(c)

$$\begin{aligned}\sigma_{XY} &= \text{cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)] \\ &= (0 - 0.70)(0 - 0.78)\Pr(X = 0, Y = 0) + (0 - 0.70)(1 - 0.78)\Pr(X = 0, Y = 1) \\ &\quad + (1 - 0.70)(0 - 0.78)\Pr(X = 1, Y = 0) + (1 - 0.70)(1 - 0.78)\Pr(X = 1, Y = 1) \\ &= (-0.70) \times (-0.78) \times 0.15 + (-0.70) \times 0.22 \times 0.15 + 0.30 \times (-0.78) \times 0.07 + 0.30 \times 0.22 \times 0.63 \\ &= 0.084,\end{aligned}$$

$$\text{corr}(X, Y) = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = \frac{0.084}{\sqrt{0.21 \times 0.1716}} = 0.4425.$$

2.3. For the two new random variables  $W = 3 + 6X$  and  $V = 20 - 7Y$ , we have:

(a)

$$\begin{aligned}E(V) &= E(20 - 7Y) = 20 - 7E(Y) = 20 - 7 \times 0.78 = 14.54, \\E(W) &= E(3 + 6X) = 3 + 6E(X) = 3 + 6 \times 0.70 = 7.2.\end{aligned}$$

(b)

$$\begin{aligned}\sigma_W^2 &= \text{var}(3 + 6X) = 6^2 \sigma_X^2 = 36 \times 0.21 = 7.56, \\ \sigma_V^2 &= \text{var}(20 - 7Y) = (-7)^2 \times \sigma_Y^2 = 49 \times 0.1716 = 8.4084.\end{aligned}$$

(c)

$$\sigma_{WV} = \text{cov}(3 + 6X, 20 - 7Y) = 6(-7)\text{cov}(X, Y) = -42 \times 0.084 = -3.52$$

$$\text{corr}(W, V) = \frac{\sigma_{WV}}{\sigma_W \sigma_V} = \frac{-3.528}{\sqrt{7.56 \times 8.4084}} = -0.4425.$$

2.4. (a)  $E(X^3) = 0^3 \times (1-p) + 1^3 \times p = p$

(b)  $E(X^k) = 0^k \times (1-p) + 1^k \times p = p$

(c)  $E(X) = 0.3$

$$\text{var}(X) = E(X^2) - [E(X)]^2 = 0.3 - 0.09 = 0.21$$

Thus,  $\sigma = \sqrt{0.21} = 0.46$ .

To compute the skewness, use the formula from exercise 2.21:

$$\begin{aligned} E(X - \mu)^3 &= E(X^3) - 3[E(X^2)][E(X)] + 2[E(X)]^3 \\ &= 0.3 - 3 \times 0.3^2 + 2 \times 0.3^3 = 0.084 \end{aligned}$$

Alternatively,  $E(X - \mu)^3 = [(1-0.3)^3 \times 0.3] + [(0-0.3)^3 \times 0.7] = 0.084$

Thus, skewness  $= E(X - \mu)^3 / \sigma^3 = .084 / 0.46^3 = 0.87$ .

To compute the kurtosis, use the formula from exercise 2.21:

$$\begin{aligned} E(X - \mu)^4 &= E(X^4) - 4[E(X)][E(X^3)] + 6[E(X)]^2[E(X^2)] - 3[E(X)]^4 \\ &= 0.3 - 4 \times 0.3^2 + 6 \times 0.3^3 - 3 \times 0.3^4 = 0.0777 \end{aligned}$$

Alternatively,  $E(X - \mu)^4 = [(1-0.3)^4 \times 0.3] + [(0-0.3)^4 \times 0.7] = 0.0777$

Thus, kurtosis is  $E(X - \mu)^4 / \sigma^4 = .0777 / 0.46^4 = 1.76$

- 2.5. Let  $X$  denote temperature in °F and  $Y$  denote temperature in °C. Recall that  $Y = 0$  when  $X = 32$  and  $Y = 100$  when  $X = 212$ .

This implies  $Y = (100/180) \times (X - 32)$  or  $Y = -17.78 + (5/9) \times X$ .

Using Key Concept 2.3,  $\mu_X = 70^\circ\text{F}$  implies that  $\mu_Y = -17.78 + (5/9) \times 70 = 21.11^\circ\text{C}$ , and  $\sigma_X = 7^\circ\text{F}$  implies  $\sigma_Y = (5/9) \times 7 = 3.89^\circ\text{C}$ .

2.6. The table shows that  $\Pr(X=0, Y=0) = 0.026$ ,  $\Pr(X=0, Y=1) = 0.576$ ,  
 $\Pr(X=1, Y=0) = 0.009$ ,  $\Pr(X=1, Y=1) = 0.389$ ,  $\Pr(X=0) = 0.602$ ,  
 $\Pr(X=1) = 0.398$ ,  $\Pr(Y=0) = 0.035$ ,  $\Pr(Y=1) = 0.965$ .

(a)

$$\begin{aligned} E(Y) &= \mu_y = 0 \times \Pr(Y=0) + 1 \times \Pr(Y=1) \\ &= 0 \times 0.035 + 1 \times 0.965 = 0.965. \end{aligned}$$

(b)

$$\begin{aligned} \text{Unemployment Rate} &= \frac{\#(\text{unemployed})}{\#(\text{labor force})} \\ &= \Pr(Y=0) = 1 - \Pr(Y=1) = 1 - E(Y) = 1 - 0.965 = 0.035. \end{aligned}$$

(c) Calculate the conditional probabilities first:

$$\Pr(Y=0|X=0) = \frac{\Pr(X=0, Y=0)}{\Pr(X=0)} = \frac{0.026}{0.602} = 0.043,$$

$$\Pr(Y=1|X=0) = \frac{\Pr(X=0, Y=1)}{\Pr(X=0)} = \frac{0.576}{0.602} = 0.957,$$

$$\Pr(Y=0|X=1) = \frac{\Pr(X=1, Y=0)}{\Pr(X=1)} = \frac{0.009}{0.398} = 0.023,$$

$$\Pr(Y=1|X=1) = \frac{\Pr(X=1, Y=1)}{\Pr(X=1)} = \frac{0.389}{0.398} = 0.978.$$

The conditional expectations are

$$\begin{aligned} E(Y|X=1) &= 0 \times \Pr(Y=0|X=1) + 1 \times \Pr(Y=1|X=1) \\ &= 0 \times 0.023 + 1 \times 0.978 = 0.978, \end{aligned}$$

$$\begin{aligned} E(Y|X=0) &= 0 \times \Pr(Y=0|X=0) + 1 \times \Pr(Y=1|X=0) \\ &= 0 \times 0.043 + 1 \times 0.957 = 0.957. \end{aligned}$$

(d) Use the solution to part (b),

$$\text{Unemployment rate for college graduates} = 1 - E(Y|X=1) = 1 - 0.978 = 0.023.$$

$$\text{Unemployment rate for non-college graduates} = 1 - E(Y|X=0) = 1 - 0.957 = 0.043$$

(e) The probability that a randomly selected worker who is reported being unemployed is a college graduate is

$$\Pr(X = 1|Y = 0) = \frac{\Pr(X = 1, Y = 0)}{\Pr(Y = 0)} = \frac{0.009}{0.035} = 0.257.$$

The probability that this worker is a non-college graduate is

$$\Pr(X = 0|Y = 0) = 1 - \Pr(X = 1|Y = 0) = 1 - 0.257 = 0.743.$$

(f) Educational achievement and employment status are not independent because they do not satisfy that, for all values of  $x$  and  $y$ ,

$$\Pr(X = x|Y = y) = \Pr(X = x).$$

For example, from part (e)  $\Pr(X = 0|Y = 0) = 0.743$ , while from the table  $\Pr(X = 0) = 0.602$ .

2.7. Using obvious notation,  $C = M + F$ ; thus  $\mu_C = \mu_M + \mu_F$  and  $\sigma_C^2 = \sigma_M^2 + \sigma_F^2 + 2\text{cov}(M, F)$ . This implies

(a)  $\mu_C = 40 + 45 = \$85,000$  per year.

(b)  $\text{corr}(M, F) = \frac{\text{cov}(M, F)}{\sigma_M \sigma_F}$ , so that  $\text{cov}(M, F) = \sigma_M \sigma_F \text{corr}(M, F)$ . Thus  $\text{cov}(M, F) = 12 \times 18 \times 0.80 = 172.80$ , where the units are squared thousands of dollars per year.

(c)  $\sigma_C^2 = \sigma_M^2 + \sigma_F^2 + 2\text{cov}(M, F)$ , so that  $\sigma_C^2 = 12^2 + 18^2 + 2 \times 172.80 = 813.60$ , and  $\sigma_C = \sqrt{813.60} = 28.524$  thousand dollars per year.

(d) First you need to look up the current Euro/dollar exchange rate in the Wall Street Journal, the Federal Reserve web page, or other financial data outlet. Suppose that this exchange rate is  $e$  (say  $e = 0.85$  Euros per Dollar or  $1/e = 1.18$  Dollars per Euro); each 1 Dollar is therefore worth  $e$  Euros. The mean is therefore  $e \times \mu_C$  (in units of thousands of euros per year), and the standard deviation is  $e \times \sigma_C$  (in units of thousands of euros per year). The correlation is unit-free, and is unchanged.

2.8.  $\mu_Y = E(Y) = 1$ ,  $\sigma_Y^2 = \text{var}(Y) = 4$ . With  $Z = \frac{1}{2}(Y - 1)$ ,

$$\mu_Z = E\left(\frac{1}{2}(Y - 1)\right) = \frac{1}{2}(\mu_Y - 1) = \frac{1}{2}(1 - 1) = 0,$$

$$\sigma_Z^2 = \text{var}\left(\frac{1}{2}(Y - 1)\right) = \frac{1}{4}\sigma_Y^2 = \frac{1}{4} \times 4 = 1.$$

2.9.

		Value of Y					Probability Distribution of X
		14	22	30	40	65	
Value of X	1	0.02	0.05	0.10	0.03	0.01	0.21
	5	0.17	0.15	0.05	0.02	0.01	0.40
	8	0.02	0.03	0.15	0.10	0.09	0.39
Probability distribution of Y		0.21	0.23	0.30	0.15	0.11	1.00

(a) The probability distribution is given in the table above.

$$E(Y) = 14 \times 0.21 + 22 \times 0.23 + 30 \times 0.30 + 40 \times 0.15 + 65 \times 0.11 = 30.15$$

$$E(Y^2) = 14^2 \times 0.21 + 22^2 \times 0.23 + 30^2 \times 0.30 + 40^2 \times 0.15 + 65^2 \times 0.11 = 1127.23$$

$$\text{var}(Y) = E(Y^2) - [E(Y)]^2 = 218.21$$

$$\sigma_Y = 14.77$$

(b) The conditional probability of  $Y|X = 8$  is given in the table below

Value of Y				
14	22	30	40	65
0.02/0.39	0.03/0.39	0.15/0.39	0.10/0.39	0.09/0.39

$$E(Y|X = 8) = 14 \times (0.02/0.39) + 22 \times (0.03/0.39) + 30 \times (0.15/0.39) + 40 \times (0.10/0.39) + 65 \times (0.09/0.39) = 39.21$$

$$E(Y^2|X = 8) = 14^2 \times (0.02/0.39) + 22^2 \times (0.03/0.39) + 30^2 \times (0.15/0.39) + 40^2 \times (0.10/0.39) + 65^2 \times (0.09/0.39) = 1778.7$$

$$\text{var}(Y) = 1778.7 - 39.21^2 = 241.65$$

$$\sigma_{Y|X=8} = 15.54$$

(c)

$$E(XY) = (1 \times 14 \times 0.02) + (1 \times 22 \times 0.05) + \dots + (8 \times 65 \times 0.09) = 171.7$$

$$\text{cov}(X, Y) = E(XY) - E(X)E(Y) = 171.7 - 5.33 \times 30.15 = 11.0$$

$$\text{corr}(X, Y) = \text{cov}(X, Y) / (\sigma_X \sigma_Y) = 11.0 / (2.60 \times 14.77) = 0.286$$

2.10. Using the fact that if  $Y \sim N(\mu_Y, \sigma_Y^2)$  then  $\frac{Y - \mu_Y}{\sigma_Y} \sim N(0, 1)$  and Appendix Table 1,

we have

$$(a) \Pr(Y \leq 3) = \Pr\left(\frac{Y-1}{2} \leq \frac{3-1}{2}\right) = \Phi(1) = 0.8413.$$

(b)

$$\begin{aligned} \Pr(Y > 0) &= 1 - \Pr(Y \leq 0) \\ &= 1 - \Pr\left(\frac{Y-3}{3} \leq \frac{0-3}{3}\right) = 1 - \Phi(-1) = \Phi(1) = 0.8413. \end{aligned}$$

(c)

$$\begin{aligned} \Pr(40 \leq Y \leq 52) &= \Pr\left(\frac{40-50}{5} \leq \frac{Y-50}{5} \leq \frac{52-50}{5}\right) \\ &= \Phi(0.4) - \Phi(-2) = \Phi(0.4) - [1 - \Phi(2)] \\ &= 0.6554 - 1 + 0.9772 = 0.6326. \end{aligned}$$

(d)

$$\begin{aligned} \Pr(6 \leq Y \leq 8) &= \Pr\left(\frac{6-5}{\sqrt{2}} \leq \frac{Y-5}{\sqrt{2}} \leq \frac{8-5}{\sqrt{2}}\right) \\ &= \Phi(2.1213) - \Phi(0.7071) \\ &= 0.9831 - 0.7602 = 0.2229. \end{aligned}$$

2.11. (a) 0.90

(b) 0.05

(c) 0.05

(d) When  $Y \sim \chi_{10}^2$ , then  $Y/10 \sim F_{10, \infty}$ .

(e)  $Y = Z^2$ , where  $Z \sim N(0,1)$ , thus  $\Pr(Y \leq 1) = \Pr(-1 \leq Z \leq 1) = 0.32$ .

2.12. (a) 0.05

(b) 0.950

(c) 0.953

(d) The  $t_{df}$  distribution and  $N(0, 1)$  are approximately the same when  $df$  is large.

(e) 0.10

(f) 0.01

2.13. (a)  $E(Y^2) = \text{Var}(Y) + \mu_Y^2 = 1 + 0 = 1$ ;  $E(W^2) = \text{Var}(W) + \mu_W^2 = 100 + 0 = 100$ .

(b)  $Y$  and  $W$  are symmetric around 0, thus skewness is equal to 0; because their mean is zero, this means that the third moment is zero.

(c) The kurtosis of the normal is 3, so  $3 = \frac{E(Y - \mu_Y)^4}{\sigma_Y^4}$ ; solving yields  $E(Y^4) = 3$ ; a similar calculation yields the results for  $W$ .

(d) First, condition on  $X = 0$ , so that  $S = W$ :

$$E(S|X = 0) = 0; E(S^2|X = 0) = 100, E(S^3|X = 0) = 0, E(S^4|X = 0) = 3 \times 100^2$$

Similarly,

$$E(S|X = 1) = 0; E(S^2|X = 1) = 1, E(S^3|X = 1) = 0, E(S^4|X = 1) = 3.$$

From the large of iterated expectations

$$E(S) = E(S|X = 0) \times \Pr(X = 0) + E(S|X = 1) \times \Pr(X = 1) = 0$$

$$E(S^2) = E(S^2|X = 0) \times \Pr(X = 0) + E(S^2|X = 1) \times \Pr(X = 1) = 100 \times 0.01 + 1 \times 0.99 = 1.99$$

$$E(S^3) = E(S^3|X = 0) \times \Pr(X = 0) + E(S^3|X = 1) \times \Pr(X = 1) = 0$$

$$E(S^4) = E(S^4|X = 0) \times \Pr(X = 0) + E(S^4|X = 1) \times \Pr(X = 1) \\ = 3 \times 100^2 \times 0.01 + 3 \times 1 \times 0.99 = 302.97$$

(e)  $\mu_S = E(S) = 0$ , thus  $E(S - \mu_S)^3 = E(S^3) = 0$  from part (d). Thus skewness = 0.

Similarly,  $\sigma_S^2 = E(S - \mu_S)^2 = E(S^2) = 1.99$ , and  $E(S - \mu_S)^4 = E(S^4) = 302.97$ .

Thus, kurtosis =  $302.97 / (1.99^2) = 76.5$