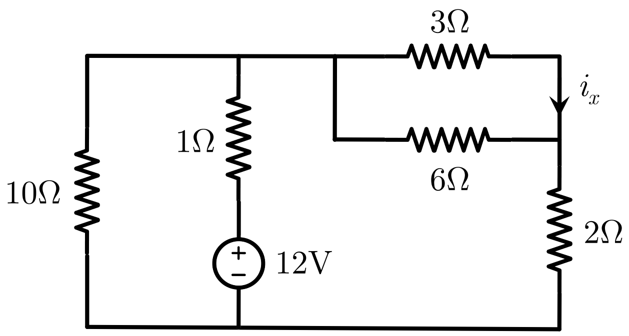
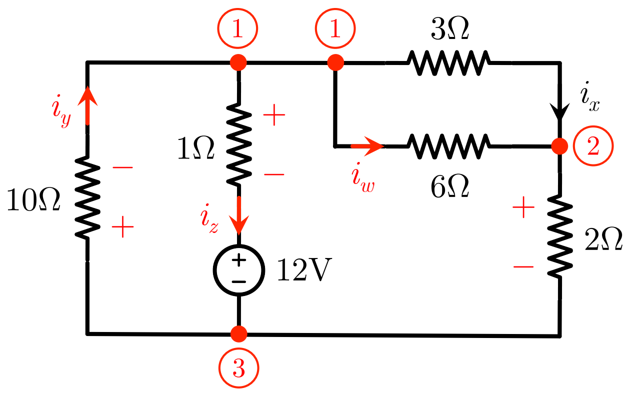
Solution of Exercise 5 (Application of KVL and KCL)

In the following circuit, find the value i_x.



Solution: First we label the nodes from 1 to 3. While a node number can be given to any intersection of multiple wires, we can assign 1 at two intersection points with a direct connection between them.



We also define the current directions (completely arbitrarily) and potentials across the components accordingly using the sign convention. Note that, following the sign convention, the current across a component flows from its positive terminal to negative terminal. Now, we can apply KVL and KCL to solve the problem. First, consider the loop from node 1 to node 2 and back to node 1. We have

KVL($1 \to 2 \to 1$): $3i_x - 6i_w = 0 \longrightarrow i_x = 2i_w$.

This KVL, which contains only two components, is nothing but a current division. In fact, considering the voltage between nodes 1 and 2, i.e., v_{12}, Ohm's law can be used for the 3Ω and 6Ω resistors to derive

$$v_{12} = 3i_x = 6i_w,$$

leading to again $i_x = 2i_w$.

Next, we focus on the loop on the left-hand side, i.e., node 1 to node 3 and back to node 1, leading to

KVL($1 \to 3 \to 1$): $1i_z + 12 + 10i_y = 0 \longrightarrow i_z + 10i_y = -12$.

Note that, we use clockwise direction as a common approach. It is also possible to obtain the same equation by applying KVL in the counterclockwise direction, provided that the signs are used correctly.

Specifically, when going through a component, we use the sign of the first terminal, where *first* is defined depending on the direction (clockwise or counterclockwise). In the clockwise direction, we have $+1i_z$ for the 1Ω resistor, $+12$ for the voltage source, and $+10i_y$ for the 10Ω resistor, leading to the equation above.

At this stage, we have two equations, and four unknowns, i.e., i_x, i_w, i_y, and i_z. Obviously, we need two more equations to arrive at the solution. One option can be a KVL for a sequence of nodes as 1 to 2, 2 to 3, and 3 to 1. From 1 to 2, we can use $6i_w$ or $3i_x$; they are basically the same as indicated by the equation above. In addition, from 3 to 1, we can go through either the 10Ω resistor or the combination of the 12V source and the 1Ω resistor (again it does not matter). We have

KVL($1 \rightarrow 2 \rightarrow 3 \rightarrow 1$): $6i_w + 2(i_x + i_w) - 12 - 1i_z = 0$,

considering that the current through the 2Ω resistor is $i_x + i_w$. Now, using $i_w = i_x/2$, this equation can be rewritten as

$$8i_w + 2i_x - i_z = 12 \longrightarrow 6i_x - i_z = 12.$$

We still need another equation and it appears we have already used KVLs on the available loops. In addition, KCL at node 2 is not useful since we actually used it (by stating that current through the 2Ω resistor is $i_x + i_w$). KCL at node 1 can be used to derive the missing equation as

KCL (1): $i_y - i_z - (i_w + i_x) = 0 \longrightarrow i_y - i_z - (3/2)i_x = 0$.

We note that, when writing KCL, *entering* currents are written as positive and *leaving* as negative. Now, we list all equations as follows.

$$i_z + 10i_y = -12 \quad (1)$$

$$6i_x - i_z = 12 \quad (2)$$

$$i_y - i_z - (3/2)i_x = 0. \quad (3)$$

Using (3) in (2), we have

$$6\left[(2/3)i_y - (2/3)i_z\right] - i_z = 12 \longrightarrow 4i_y - 5i_z = 12. \quad (4)$$

Then, using (1) and (4), we arrive at $i_y = -8/9$A and $i_z = -28/9$A. Therefore, the value of i_x can be obtained as

$$6i_x = 12 + i_z = 12 - 28/9 = 80/9 \longrightarrow i_x = 80/54 = \mathbf{40/27}\text{A}.$$

As a final note, for a given circuit, it may not be obvious which KVL and KCL may lead to useful (or simple) equations that provide trivial solutions. For example, it is common to arrive at true equations, such as $0 = 0$ since some of the KVLs and KCLs can be linearly dependent and they provide the same data. This is the reason for why nodal or mesh analysis (generalization of KCL and KVL) are required for complex problems.