Chapter 1 Solution

1.3

Given:F_1= $4\hat{i} + 3\hat{j}$ and F_2= $1\hat{i} + 7\hat{j}$

Find: angle θ between F_1 and F_2

Solution: Definition of dot product eqn (1.16 page 10)

$$\mathbf{u} \cdot \mathbf{v} = |u||v|cos\theta = u_1v_1 + u_2v_2 + u_3v_3$$

So

$$F_1 \cdot F_2 = |F_1||F_2|cos\theta = F_{11}F_{21} + F_{12}F_{22} + F_{13}F_{33}$$
$$|F_1| = \sqrt{4^2 + 3^2} = 5$$
$$|F_2| = \sqrt{1^2 + 7^2} = 5\sqrt{2}$$

Therefore

$$5 \times 5\sqrt{2} \times cos\theta = 4 \times 1 + 0 + 3 \times 7$$
$$cos\theta = \frac{4 \times 1 + 0 + 3 \times 7}{5 \times 5\sqrt{2}} = \frac{1}{\sqrt{2}}$$
$$\theta = cos^{-1}(\frac{1}{\sqrt{2}}) = 45^\circ \qquad \text{(ans)}$$

1.4

Given: $F_1 = -5\hat{i} + 3\hat{j}$ and $F_2 = 1\hat{i} - 4\hat{j}$

Find: The cross product of F_1 and F_2

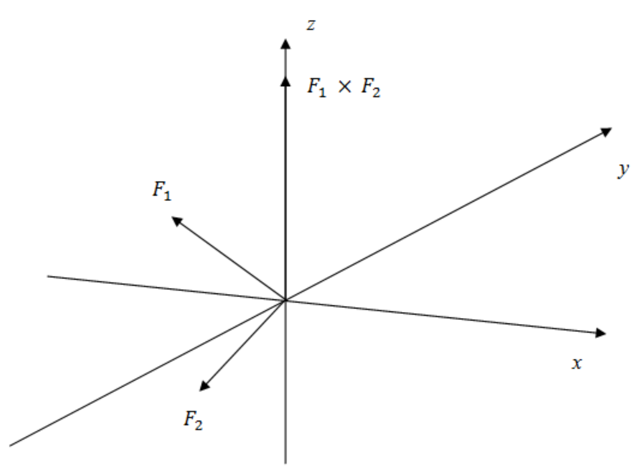
Solution: Definition of cross product (1.22 and 1.23 page 11)

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$
$$= (u_2v_3 - u_3v_2)\hat{i} + (u_3v_1 - v_1u_3)\hat{j} + (u_1v_2 - u_2v_1)\hat{k}$$

So

$$F_1 \times F_2 = (F_{12}F_{23} - F_{13}F_{22})\hat{i} + (F_{13}F_{21} - F_{11}F_{23})\hat{j} + (F_{11}F_{22} - F_{12}F_{21})\hat{k}$$
$$= (3 \times 0 - 0 \times (-4))\hat{i} + (0 \times 1 - (-5) \times 0)\hat{j} + ((-5) \times (-4) - 3 \times 1)\hat{k}$$
$$= 0\hat{i} + 0\hat{j} + 17\hat{k} \qquad \text{(ans)}$$

Sketch:



Result vector as shown

1.5

Given:$r_1 \cdot (r_2 \times r_3)$

Find:volume generated by $r_1 \cdot (r_2 \times r_3)$ and sketch

Solution: Definition of dot product and cross product

$$\mathbf{u} \cdot \mathbf{v} = |u||v|cos\theta$$
$$|\mathbf{u} \times \mathbf{v}| = |u||v|sin\theta$$

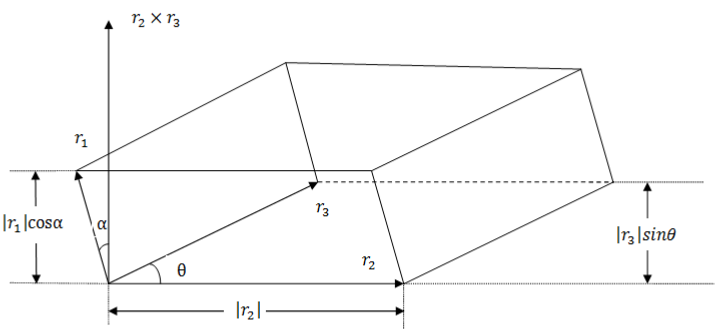
Cross product results in a vector that is perpendicular to both **u** and **v**

So

$$|r_2 \times r_3| = |r_2||r_3|sin\theta$$

θ is the angle between r_2 and r_3

$$r_1 \cdot (r_2 \times r_3) = |r_1| \cdot (|r_2||r_3|sin\theta) \cdot cos\alpha = (|r_1|cos\alpha) \cdot (|r_2||r_3|sin\theta)$$



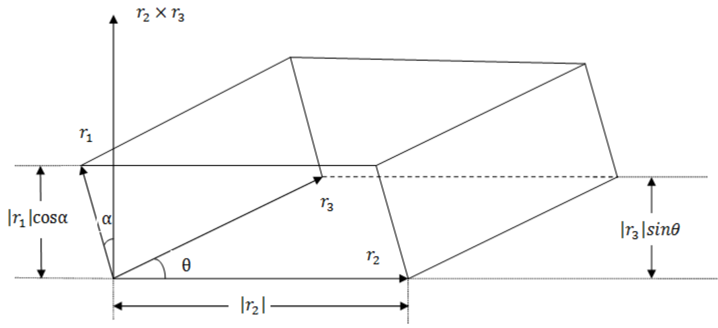
Result volume as shown

1.6

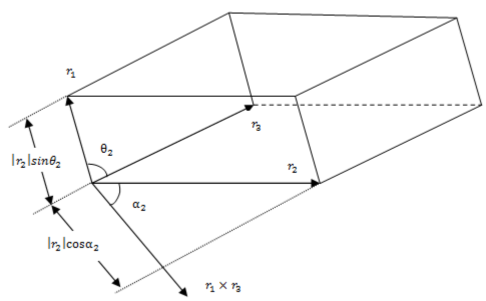
Given: $r_1 \cdot r_2 \times r_3$, $r_2 \cdot r_3 \times r_1$ and $r_3 \cdot r_1 \times r_2$

Find: $r_1 \cdot r_2 \times r_3 = r_2 \cdot r_3 \times r_1 = r_3 \cdot r_1 \times r_2$

Solution: From the result of problem 1.5



1. r_2 and r_3 at the bottom



2. r_2 and r_1 at the bottom

Similarly $r_2 \cdot r_3 \times r_1$ generates a parallelepiped as shown which has the same volume from $r_1 \cdot r_2 \times r_3$. For $r_3 \cdot r_1 \times r_2$, a same parallelepiped will be generated. Volume of one parallelepiped is the same.

Therefore $r_1 \cdot r_2 \times r_3 = r_2 \cdot r_3 \times r_1 = r_3 \cdot r_1 \times r_2$

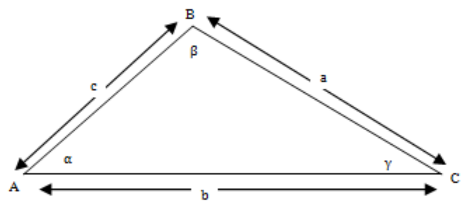
1.7

Given:A triangle as shown

Find:

$$\frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$$

Solution:



$$r_{C/A} = r_{B/A} + r_{C/B}$$
$$r_{C/A} = b\hat{i}$$
$$r_{B/A} = c \cdot cos\alpha\hat{i} + c \cdot sin\alpha\hat{j}$$
$$r_{C/B} = a \cdot cos\gamma\hat{i} - a \cdot sin\gamma\hat{j}$$

So

$$b\hat{i} = (c \cdot cos\alpha\hat{i} + c \cdot sin\alpha\hat{j}) + (a \cdot cos\gamma\hat{i} - a \cdot sin\gamma\hat{j})$$

Compare $\hat{j}$ term on both sides

$$0\hat{j} = c \cdot sin\alpha\hat{j} - a \cdot sin\gamma\hat{j}$$
$$c \cdot sin\alpha = a \cdot sin\gamma$$
$$\frac{c}{sin\gamma} = \frac{a}{sin\alpha}$$

Without loss of generality

$$\frac{a}{sin\alpha} = \frac{b}{sin\beta}$$

Therefore

$$\frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$$

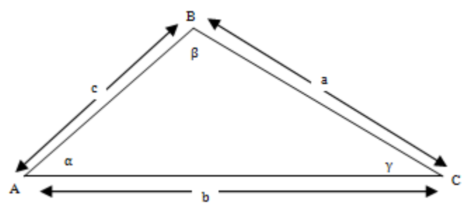
1.8

Given:A triagnle as shown

Find:

$$b^2 = a^2 + c^2 - 2ac \cdot cos\beta$$

Solution:



$$r_{C/A} = r_{B/A} + r_{C/B}$$

$$\begin{aligned} r_{C/A}^2 &= (r_{B/A} + r_{C/B})^2 \\ &= r_{B/A}^2 + r_{C/B}^2 + 2r_{B/A} \cdot r_{C/B} \\ &= |r_{B/A}|^2 + |r_{C/B}|^2 + 2 \cdot |r_{B/A}||r_{C/B}|cos(\pi - \beta) \end{aligned}$$

Since $cos(\pi - \beta) = -\cos\beta$

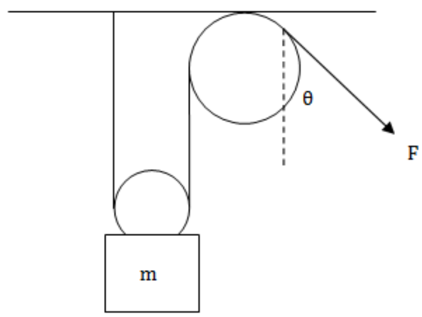
$$b^2 = a^2 + c^2 - 2ac \cdot cos\beta$$

1.11

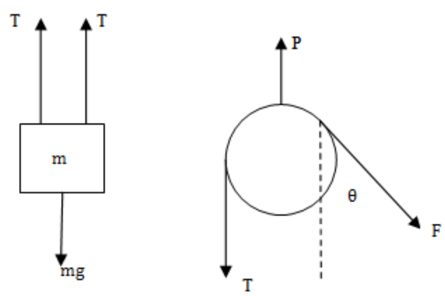
Given:the force F, angle θ, a block with mass m and the system shown

Find:F and θ required to keep the system in static equilibrium

Solution:



1. System given



2. FBD for the system given

$$\sum F_y = 2T - mg = 0$$
$$T = \frac{mg}{2}$$

Since tension in one rope is the same

$$F = T = \frac{mg}{2}$$

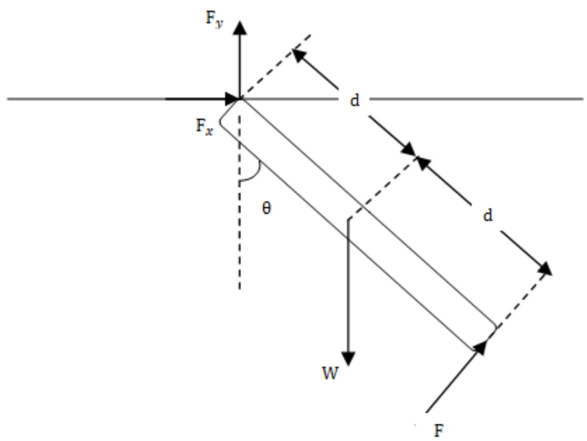
θ can be any value

1.12

Given:$F = 100N$, $d = 1.6m$ and $W = 300N$

Find:reaction force at the pin and angle θ

Solution:



$$\sum M = F \cdot 2d - M \cdot cos\theta \cdot d = 0$$

$$F \cdot 2d = M \cdot sin\theta \cdot d$$

$$2F = M \cdot sin\theta$$

$$sin\theta = \frac{2F}{M} = \frac{200}{300} = \frac{2}{3}$$

$$sin^{-1}\theta(\frac{2}{3}) = 0.730rad = 41.9^\circ \qquad \text{(ans)}$$

$$\sum F_x = F \cdot sin\theta + F_x = 0$$

$$F_x = -F \cdot sin(41.9^\circ) = -100sin(41.9^\circ) = -66.7N \qquad \text{(ans)}$$

$$\sum F_y = F \cdot cos\theta + F_y - W = 0$$

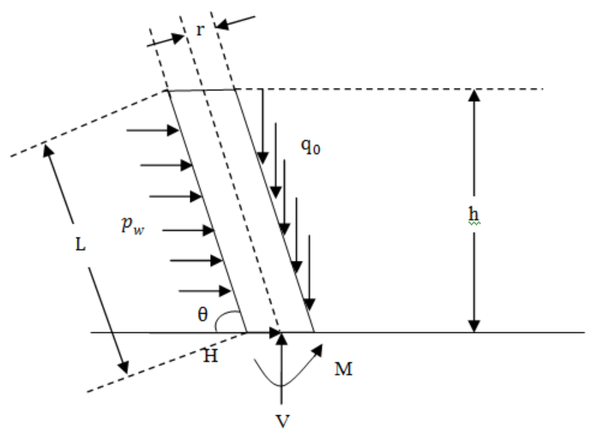
$$F_y = W - F \cdot cos\theta = 200 - 100cos(41.9^\circ) = 125.5N \qquad \text{(ans)}$$

1.13

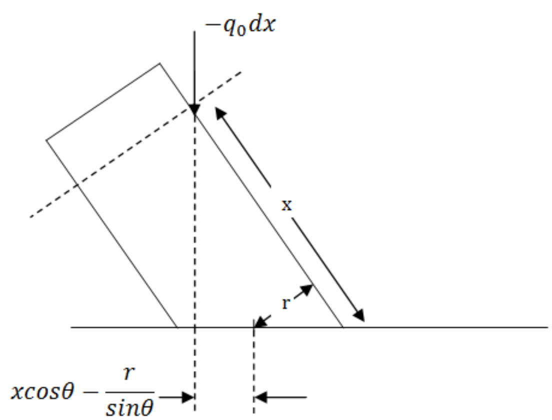
Given:wind pressure p_w, tree trunk radius r

Find:Force and moment at the base of the tree

Solution:



1.FBD for the tree



2.Integral element

$$\sum F_x = p_w L\hat{i} + H\hat{i}$$

$$H = -p_w L \qquad \text{(ans)}$$

$$\sum F_y = V\hat{j} - q_0 L\hat{j}$$

$$V = q_0 L \qquad \text{(ans)}$$

$$\sum M = M\hat{k} + \int_0^L x sin\theta\hat{j} \cdot p_w dx\hat{i} + \int_0^L -(x cos\theta - \frac{r}{sin\theta})\hat{i} \cdot (-q_0) dx\hat{j} = 0$$

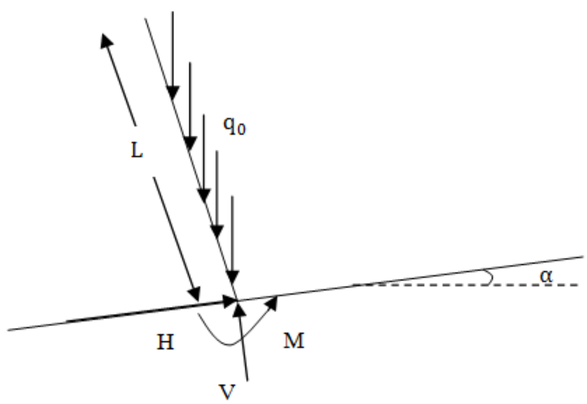
$$M = \frac{1}{2} sin\theta P_W L^2 - \frac{1}{2} cos\theta q_0 L^2 - \frac{r q_0 L}{sin\theta} \qquad \text{(ans)}$$

1.14

Given:tree shown in Fig 1.10(a), the ground has slope with an angle α

Find:Force and Moment at the base

Solution:



1. FBD for the tree

$$\sum F_x = Hcos\alpha\hat{j} - Vsin\alpha\hat{i} = 0$$

$$H = V\frac{sin\alpha}{cos\alpha} \quad (1)$$

$$\sum F_y = Hsin\alpha\hat{j} + Vcos\alpha\hat{j} - q_0 L\hat{j} = 0 \quad (2)$$

Substitute H by (1) in (2)

$$V(\frac{sin\alpha}{cos\alpha} \cdot sin\alpha + cos\alpha) - q_0 L = 0$$

$$V(\frac{sin^2\alpha}{cos\alpha} + \frac{cos^2\alpha}{cos\alpha}) - q_0 L = 0$$

$$V(\frac{1}{cos\alpha}) - q_0 L = 0$$

$$V = q_0 L cos\alpha \quad \text{(ans)}$$

$$H = V\frac{sin\alpha}{cos\alpha} = q_0 L sin\alpha \quad \text{(ans)}$$