

SOLUTION TO PROBLEM 1.1.

$10\log_{10}(\text{Average } P_b) = Y = 10\log_{10}(\zeta) - G_d G_c \text{ [dB]} - G_d(\bar{\rho} \text{ [dB]})$. Therefore, G_d is the negative of the slope of the curve in the figure. That is, $G_d = -(Y_1 - Y_2)/[10\log_{10}(\bar{\rho}_1) - 10\log_{10}(\bar{\rho}_2)]$. If we choose $10\log_{10}(\bar{\rho}_1) = 20 \text{ dB}$ and $10\log_{10}(\bar{\rho}_2) = 30 \text{ dB}$, then from the curve we find that $Y_1 = -60$ and $Y_2 = -90$. It follows that the diversity gain $G_d = -[-60 - (-90)]/[20 - 30] = -(30)/(-10) = 3$.

The coding gain (in dB) is obtained from the Y-intercept of the curve obtained by plotting $10\log_{10}(P_b)$ versus $10\log_{10}(\bar{\rho})$. Denote the Y-intercept by $B = 10\log_{10}(\zeta) - G_d G_c \text{ [dB]}$. It follows from the figure in the problem that $B \text{ [dB]} = 0 \text{ [dB]} = \zeta \text{ [dB]} - G_d G_c \text{ [dB]}$. If $\zeta = 1$, then $\zeta \text{ [dB]} = 0 \text{ dB}$, so $G_d G_c \text{ [dB]} = 0 \text{ [dB]}$, which implies that $G_c \text{ [dB]} = 0/3 = 0 \text{ [dB]}$. That is, $G_c = 1$.

Since the diversity order is equal to the diversity gain for most MIMO systems without coding (and in particular for those that use QPSK modulation), then $G_d = 3$ implies that $N_d = 3$ for this system, which means that there are 3 independent spatial diversity channels.

The maximum diversity gain for an $N_t \times N_r$ MIMO system is $N_t N_r$. Since the diversity order is equal to the diversity order in this problem, it follows that $N_t N_r = 3$. The only way for this to be true is if $N_t = 3$ and $N_r = 1$ or if $N_t = 1$ and $N_r = 3$. In both cases, the total number of antennas is 4.

SOLUTION TO PROBLEM 1.3

$$\det(A) = ad - bc$$

$$\det(B) = eh - fg$$

$$\det(A)\det(B) = adeh - adfg - bceh + bcfg$$

$$AB = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

$$\det(AB) = (ae + bg)(cf + dh) - (af + bh)(ce + dg)$$

$$= aecf + aedh + bgcf + bgdh - afce - afdg - bhce - bhdg$$

$$= aedh + bgcf - afdg - bhce$$

$$\Rightarrow \det(A)\det(B) = \det(AB)$$

SOLUTION TO PROBLEM 1.4:

We seek to prove that for any square matrix, \mathbf{A} , $\mathbf{A}\mathbf{A}^H$ and $\mathbf{A}^H\mathbf{A}$ are both Hermitian matrices. We will first show that $\mathbf{A}\mathbf{A}^H$ is Hermitian.

To simplify the proof, we will use the fact that for any two matrices, \mathbf{E} and \mathbf{F} , having dimensions of $m \times n$ and $n \times p$, respectively, we can express the (i, j) -th component of $\mathbf{E}\mathbf{F}$ as follows:

$$[\mathbf{E}\mathbf{F}]_{i,j} = \sum_{k=1}^n \mathbf{E}_{i,k} \mathbf{F}_{k,j}. \quad (1)$$

Let $\mathbf{C} \triangleq \mathbf{A}\mathbf{A}^H$ and assume that \mathbf{A} is dimensioned $m \times n$. It follows that

$$\begin{aligned} [\mathbf{C}]_{i,j} &= [\mathbf{A}\mathbf{A}^H]_{i,j} \\ &= \sum_{k=1}^n [\mathbf{A}]_{i,k} [\mathbf{A}^H]_{k,j} \\ &= \sum_{k=1}^n [\mathbf{A}]_{i,k} [\mathbf{A}]_{j,k}^* \end{aligned} \quad (2)$$

It follows that

$$\begin{aligned} [\mathbf{C}]_{j,i}^* &= \sum_{k=1}^n [\mathbf{A}]_{j,k}^* ([\mathbf{A}]_{i,k}^*)^* \\ &= \sum_{k=1}^n [\mathbf{A}]_{j,k}^* [\mathbf{A}]_{i,k} \end{aligned} \quad (3)$$

It follows that $[\mathbf{C}]_{i,j} = [\mathbf{C}]_{j,i}^*$. By the definition in 1.9.1(e), this proves that \mathbf{C} is Hermitian. Similar steps can be used to prove that $\mathbf{A}^H\mathbf{A}$ is also Hermitian.