

Chapter 1

Exercise Set 1.1

1. (a) 3×3 (b) 3×2 (c) 2×4 (d) 3×1 (e) 3×5 (f) 1×4

2. 1, 4, 9, -1, 3, 8

3. 4, 5, 6, 7, 2, 3

4.
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. (a) $\begin{bmatrix} 1 & 3 \\ 2 & -5 \end{bmatrix}$ and $\begin{bmatrix} 1 & 3 & 7 \\ 2 & -5 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 2 & -4 \\ 1 & 3 & 6 \\ 4 & 6 & -9 \end{bmatrix}$ and $\begin{bmatrix} 5 & 2 & -4 & 8 \\ 1 & 3 & 6 & 4 \\ 4 & 6 & -9 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 3 & -5 \\ 2 & -2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$ and $\begin{bmatrix} -1 & 3 & -5 & -3 \\ 2 & -2 & 4 & 8 \\ 1 & 3 & 0 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 4 \\ 2 & -8 \\ 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 5 & 4 & 9 \\ 2 & -8 & -4 \\ 1 & 2 & 3 \end{bmatrix}$

(e) $\begin{bmatrix} 5 & 2 & -4 \\ 0 & 4 & 3 \\ 1 & 0 & -1 \end{bmatrix}$ and $\begin{bmatrix} 5 & 2 & -4 & 8 \\ 0 & 4 & 3 & 0 \\ 1 & 0 & -1 & 7 \end{bmatrix}$

(f) $\begin{bmatrix} -1 & 3 & -9 \\ 1 & 0 & -4 \\ 1 & 8 & 0 \end{bmatrix}$ and $\begin{bmatrix} -1 & 3 & -9 & -4 \\ 1 & 0 & -4 & 11 \\ 1 & 8 & 0 & 1 \end{bmatrix}$ (g) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 8 \end{bmatrix}$

(h) $\begin{bmatrix} -4 & 2 & -9 & 1 \\ 1 & 6 & -8 & -7 \\ 0 & -1 & 3 & -5 \end{bmatrix}$ and $\begin{bmatrix} -4 & 2 & -9 & 1 & -1 \\ 1 & 6 & -8 & -7 & 15 \\ 0 & -1 & 3 & -5 & 0 \end{bmatrix}$

6. (a) $x_1 + 2x_2 = 3$
 $4x_1 + 5x_2 = 6$ (b) $7x_1 + 9x_2 = 8$
 $6x_1 + 4x_2 = -3$ (c) $x_1 + 9x_2 = -3$
 $5x_1 = 2$

Section 1.1

$$\begin{array}{l} \text{(d)} \quad 8x_1 + 7x_2 + 5x_3 = -1 \\ \quad \quad 4x_1 + 6x_2 + 2x_3 = 4 \\ \quad \quad 9x_1 + 3x_2 + 7x_3 = 6 \end{array} \qquad \begin{array}{l} \text{(e)} \quad 2x_1 - 3x_2 + 6x_3 = 4 \\ \quad \quad 7x_1 - 5x_2 - 2x_3 = 3 \\ \quad \quad 2x_2 + 4x_3 = 0 \end{array}$$

$$\begin{array}{l} \text{(f)} \quad -2x_2 = 4 \\ \quad \quad 5x_1 + 7x_2 = -3 \\ \quad \quad 6x_1 = 8 \end{array} \qquad \begin{array}{l} \text{(g)} \quad x_1 = 3 \\ \quad \quad x_2 = 8 \\ \quad \quad x_3 = 4 \end{array} \qquad \begin{array}{l} \text{(h)} \quad x_1 + 2x_2 - x_3 = 6 \\ \quad \quad x_2 + 4x_3 = 5 \\ \quad \quad x_3 = -2 \end{array}$$

$$7. \quad \begin{array}{l} \text{(a)} \quad \begin{bmatrix} 1 & 3 & -2 & 0 \\ 1 & 2 & -3 & 6 \\ 8 & 3 & 2 & 5 \end{bmatrix} \end{array} \quad \begin{array}{l} \text{(b)} \quad \begin{bmatrix} 2 & 7 & 5 & 1 \\ 0 & -8 & 4 & 3 \\ 3 & -5 & 8 & 9 \end{bmatrix} \end{array} \quad \begin{array}{l} \text{(c)} \quad \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 10 & 0 \\ 0 & -8 & -1 & -1 \end{bmatrix} \end{array}$$

$$\begin{array}{l} \text{(d)} \quad \begin{bmatrix} 1 & 0 & -1 & -6 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 11 & -1 \end{bmatrix} \end{array} \quad \begin{array}{l} \text{(e)} \quad \begin{bmatrix} 1 & 0 & 0 & -23 \\ 0 & 1 & 0 & 17 \\ 0 & 0 & 1 & 5 \end{bmatrix} \end{array} \quad \begin{array}{l} \text{(f)} \quad \begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & 1 & -4 \end{bmatrix} \end{array}$$

8. (a) Create zeros below the leading 1 in the first column.
 x_1 is eliminated from all equations except the first.
- (b) Normalize the (2,2) element, i.e., make the (2,2) element 1. This becomes a leading 1.
 It is now possible to have x_2 in the second equation with coefficient 1.
- (c) Need to have the leading 1 in row 2 to the left of leading 1 in row 3.
 The second equation now contains an x_2 term.
- (d) Create zeros above and below the leading 1 in row 2.
 x_2 is eliminated from all equations except the second.
9. (a) Create zeros above the leading 1 in column 3.
 x_3 is eliminated from all equations except the third.
- (b) Need to have the leading 1 in row 1 to the left of leading 1s in other rows.
 It is now possible to have x_1 in Equation 1 with leading coefficient 1.
- (c) Normalize the (3,3) element, i.e., make the (3,3) element 1. This becomes a leading 1.
 The coefficient of x_3 in the third equation becomes 1.
- (d) Create zeros above the leading 1 in column 3.
 x_3 is eliminated from all equations except the third.

$$10. \text{ (a)} \begin{bmatrix} 1 & -2 & -8 \\ 2 & -3 & -11 \end{bmatrix} \approx \begin{bmatrix} 1 & -2 & -8 \\ 0 & 1 & 5 \end{bmatrix} \begin{matrix} R2+(-2)R1 \\ R1+(2)R2 \end{matrix} \approx \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \end{bmatrix},$$

so the solution is $x_1 = 2$ and $x_2 = 5$.

$$\text{(b)} \begin{bmatrix} 2 & 2 & 4 \\ 3 & 2 & 3 \end{bmatrix} \begin{matrix} (1/2)R1 \\ R1+(-1)R2 \end{matrix} \approx \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 3 \end{bmatrix} \begin{matrix} R2+(-3)R1 \\ R1+(-1)R2 \end{matrix} \approx \begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & -3 \end{bmatrix} \begin{matrix} (-1)R2 \\ R1+(-1)R2 \end{matrix} \approx \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix}, \text{ so the solution is } x_1 = -1, x_2 = 3.$$

$$\text{(c)} \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 2 & -2 & -4 \\ 0 & 1 & -2 & 5 \end{bmatrix} \begin{matrix} (1/2)R2 \\ R3+(-1)R2 \end{matrix} \approx \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 1 & -2 & 5 \end{bmatrix} \begin{matrix} R3+(-1)R2 \\ R1+(-1)R3 \\ R2+R3 \end{matrix} \approx \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & -1 & 7 \end{bmatrix}$$

$$\begin{matrix} (-1)R3 \\ R1+(-1)R3 \\ R2+R3 \end{matrix} \approx \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & -7 \end{bmatrix} \begin{matrix} R1+(-1)R3 \\ R2+R3 \end{matrix} \approx \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -7 \end{bmatrix},$$

so the solution is $x_1 = 10$, $x_2 = -9$, $x_3 = -7$.

$$\text{(d)} \begin{bmatrix} 1 & 1 & 3 & 6 \\ 1 & 2 & 4 & 9 \\ 2 & 1 & 6 & 11 \end{bmatrix} \begin{matrix} R2+(-1)R1 \\ R3+(-2)R1 \end{matrix} \approx \begin{bmatrix} 1 & 1 & 3 & 6 \\ 0 & 1 & 1 & 3 \\ 0 & -1 & 0 & -1 \end{bmatrix} \begin{matrix} R1+(-1)R2 \\ R3+R2 \end{matrix} \approx \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{matrix} R1+(-2)R3 \\ R2+(-1)R3 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}, \text{ so the solution is } x_1 = -1, x_2 = 1, x_3 = 2.$$

$$\text{(e)} \begin{bmatrix} 1 & -1 & 3 & 3 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & -2 & 3 \end{bmatrix} \begin{matrix} R2+(-2)R1 \\ R3+(-3)R1 \end{matrix} \approx \begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & 1 & -4 & -4 \\ 0 & 4 & -11 & -6 \end{bmatrix} \begin{matrix} R1+R2 \\ R3+(-4)R2 \end{matrix} \approx \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & -4 & -4 \\ 0 & 0 & 5 & 10 \end{bmatrix}$$

$$\begin{matrix} (1/5)R3 \\ R1+R3 \\ R2+(4)R3 \end{matrix} \approx \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & -4 & -4 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{matrix} R1+R3 \\ R2+(4)R3 \end{matrix} \approx \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix},$$

so the solution is $x_1 = 1$, $x_2 = 4$, $x_3 = 2$.

Section 1.1

$$\begin{aligned}
 \text{(f)} \quad & \begin{bmatrix} -1 & 1 & -1 & -2 \\ 3 & 1 & 1 & 10 \\ 4 & 2 & 3 & 14 \end{bmatrix} \xrightarrow{(-1)R1} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 3 & 1 & 1 & 10 \\ 4 & 2 & 3 & 14 \end{bmatrix} \xrightarrow{\begin{matrix} R2+(-3)R1 \\ R3+(-4)R1 \end{matrix}} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 4 & -2 & 4 \\ 0 & 6 & -1 & 6 \end{bmatrix} \\
 & \xrightarrow{(1/4)R2} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1/2 & 1 \\ 0 & 6 & -1 & 6 \end{bmatrix} \xrightarrow{\begin{matrix} R1+R2 \\ R3+(-6)R2 \end{matrix}} \begin{bmatrix} 1 & 0 & 1/2 & 3 \\ 0 & 1 & -1/2 & 1 \\ 0 & 0 & 2 & 0 \end{bmatrix} \xrightarrow{(1/2)R3} \begin{bmatrix} 1 & 0 & 1/2 & 3 \\ 0 & 1 & -1/2 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \\
 & \xrightarrow{\begin{matrix} R1+(-1/2)R3 \\ R2+(1/2)R3 \end{matrix}} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \text{ so the solution is } x_1 = 3, x_2 = 1, x_3 = 0.
 \end{aligned}$$

$$\begin{aligned}
 \text{11. (a)} \quad & \begin{bmatrix} 1 & 2 & 3 & 14 \\ 2 & 5 & 8 & 36 \\ 1 & -1 & 0 & -4 \end{bmatrix} \xrightarrow{\begin{matrix} R2+(-2)R1 \\ R3+(-1)R1 \end{matrix}} \begin{bmatrix} 1 & 2 & 3 & 14 \\ 0 & 1 & 2 & 8 \\ 0 & -3 & -3 & -18 \end{bmatrix} \xrightarrow{\begin{matrix} R1+(-2)R2 \\ R3+(3)R2 \end{matrix}} \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 3 & 6 \end{bmatrix} \\
 & \xrightarrow{(1/3)R3} \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{\begin{matrix} R1+R3 \\ R2+(-2)R3 \end{matrix}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix},
 \end{aligned}$$

so the solution is $x_1 = 0, x_2 = 4, x_3 = 2$.

$$\begin{aligned}
 \text{(b)} \quad & \begin{bmatrix} 1 & -1 & -1 & -1 \\ -2 & 6 & 10 & 14 \\ 2 & 1 & 6 & 9 \end{bmatrix} \xrightarrow{\begin{matrix} R2+(2)R1 \\ R3+(-2)R1 \end{matrix}} \begin{bmatrix} 1 & -1 & -1 & -1 \\ 0 & 4 & 8 & 12 \\ 0 & 3 & 8 & 11 \end{bmatrix} \xrightarrow{(1/4)R2} \begin{bmatrix} 1 & -1 & -1 & -1 \\ 0 & 1 & 2 & 3 \\ 0 & 3 & 8 & 11 \end{bmatrix} \\
 & \xrightarrow{\begin{matrix} R1+R2 \\ R3+(-3)R2 \end{matrix}} \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 2 \end{bmatrix} \xrightarrow{(1/2)R3} \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{\begin{matrix} R1+(-1)R3 \\ R2+(-2)R3 \end{matrix}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix},
 \end{aligned}$$

so the solution is $x_1 = 1, x_2 = 1, x_3 = 1$.

$$\text{(c)} \quad \begin{bmatrix} 2 & 2 & -4 & 14 \\ 3 & 1 & 1 & 8 \\ 2 & -1 & 2 & -1 \end{bmatrix} \xrightarrow{(1/2)R1} \begin{bmatrix} 1 & 1 & -2 & 7 \\ 3 & 1 & 1 & 8 \\ 2 & -1 & 2 & -1 \end{bmatrix} \xrightarrow{\begin{matrix} R2+(-3)R1 \\ R3+(-2)R1 \end{matrix}} \begin{bmatrix} 1 & 1 & -2 & 7 \\ 0 & -2 & 7 & -13 \\ 0 & -3 & 6 & -15 \end{bmatrix}$$

Let us swap rows 2 and rows 2 and 3 to avoid awkward fractions at the next step.

Section 1.1

$$\begin{array}{c} \approx \\ R2 \Leftrightarrow R3 \end{array} \begin{bmatrix} 1 & 1 & -2 & 7 \\ 0 & -3 & 6 & -15 \\ 0 & -2 & 7 & -13 \end{bmatrix} \begin{array}{c} \approx \\ (-1/3)R2 \end{array} \begin{bmatrix} 1 & 1 & -2 & 7 \\ 0 & 1 & -2 & 5 \\ 0 & -2 & 7 & -13 \end{bmatrix}$$

$$\begin{array}{c} \approx \\ R1+(-1)R2 \\ R3+(2)R2 \end{array} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 3 & -3 \end{bmatrix} \begin{array}{c} \approx \\ (1/3)R3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{array}{c} \approx \\ R2+(2)R3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix},$$

so the solution is $x_1 = 2, x_2 = 3, x_3 = -1$.

$$(d) \begin{bmatrix} 0 & 2 & 4 & 8 \\ 2 & 2 & 0 & 6 \\ 1 & 1 & 1 & 5 \end{bmatrix} \begin{array}{c} \approx \\ R1 \Leftrightarrow R2 \end{array} \begin{bmatrix} 2 & 2 & 0 & 6 \\ 0 & 2 & 4 & 8 \\ 1 & 1 & 1 & 5 \end{bmatrix} \begin{array}{c} \approx \\ (1/2)R1 \end{array} \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 2 & 4 & 8 \\ 1 & 1 & 1 & 5 \end{bmatrix}$$

$$\begin{array}{c} \approx \\ R3+(-1)R1 \end{array} \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 2 & 4 & 8 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{array}{c} \approx \\ (1/2)R2 \end{array} \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{array}{c} \approx \\ R1+(-1)R2 \end{array} \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{array}{c} \approx \\ R1+(2)R3 \\ R2+(-2)R3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}, \text{ so the solution is } x_1 = 3, x_2 = 0, x_3 = 2.$$

$$(e) \begin{bmatrix} 1 & 0 & -1 & 3 \\ -1 & 0 & 2 & -8 \\ 3 & 1 & -1 & 0 \end{bmatrix} \begin{array}{c} \approx \\ R2+R1 \\ R2+(-3)R1 \end{array} \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 0 & 1 & -5 \\ 0 & 1 & 2 & -9 \end{bmatrix} \begin{array}{c} \approx \\ R2 \Leftrightarrow R3 \end{array} \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & -9 \\ 0 & 0 & 1 & -5 \end{bmatrix}$$

$$\begin{array}{c} \approx \\ R1+R3 \\ R2+(-2)R3 \end{array} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -5 \end{bmatrix}, \text{ so the solution is } x_1 = -2, x_2 = 1, x_3 = -5.$$

$$12 (a) \begin{bmatrix} 3/2 & 0 & 3 & 15 \\ -1 & 7 & -9 & -45 \\ 2 & 0 & 5 & 22 \end{bmatrix} \begin{array}{c} \approx \\ (2/3)R1 \end{array} \begin{bmatrix} 1 & 0 & 2 & 10 \\ -1 & 7 & -9 & -45 \\ 2 & 0 & 5 & 22 \end{bmatrix} \begin{array}{c} \approx \\ R2+R1 \\ R3+(-2)R1 \end{array} \begin{bmatrix} 1 & 0 & 2 & 10 \\ 0 & 7 & -7 & -35 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{array}{c} \approx \\ (1/7)R2 \end{array} \begin{bmatrix} 1 & 0 & 2 & 10 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{array}{c} \approx \\ R1+(-2)R3 \\ R2+R3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix},$$

so the solution is $x_1 = 6$, $x_2 = -3$, $x_3 = 2$.

$$(b) \begin{bmatrix} -3 & -6 & -15 & -3 \\ 2 & 3 & 9 & 1 \\ -4 & -7 & -17 & -4 \end{bmatrix} \xrightarrow{(-1/3)R1} \begin{bmatrix} 1 & 2 & 5 & 1 \\ 2 & 3 & 9 & 1 \\ -4 & -7 & -17 & -4 \end{bmatrix} \xrightarrow{\begin{matrix} R2+(-2)R1 \\ R3+(4)R1 \end{matrix}} \begin{bmatrix} 1 & 2 & 5 & 1 \\ 0 & -1 & -1 & -1 \\ 0 & 1 & 3 & 0 \end{bmatrix}$$

$$\xrightarrow{(-1)R2} \begin{bmatrix} 1 & 2 & 5 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \end{bmatrix} \xrightarrow{\begin{matrix} R1+(-2)R2 \\ R3+(-1)R2 \end{matrix}} \begin{bmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & -1 \end{bmatrix} \xrightarrow{(1/2)R3} \begin{bmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1/2 \end{bmatrix}$$

$$\xrightarrow{\begin{matrix} R1+(-3)R3 \\ R2+(-1)R3 \end{matrix}} \begin{bmatrix} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 3/2 \\ 0 & 0 & 1 & -1/2 \end{bmatrix}, \text{ so the solution is } x_1 = 1/2, x_2 = 3/2, x_3 = -1/2.$$

$$(c) \begin{bmatrix} 3 & 6 & 0 & -3 & 3 \\ 1 & 3 & -1 & -4 & -12 \\ 1 & -1 & 1 & 2 & 8 \\ 2 & 3 & 0 & 0 & 8 \end{bmatrix} \xrightarrow{(1/3)R1} \begin{bmatrix} 1 & 2 & 0 & -1 & 1 \\ 1 & 3 & -1 & -4 & -12 \\ 1 & -1 & 1 & 2 & 8 \\ 2 & 3 & 0 & 0 & 8 \end{bmatrix}$$

$$\xrightarrow{\begin{matrix} R2+(-1)R1 \\ R3+(-1)R1 \\ R4+(-2)R1 \end{matrix}} \begin{bmatrix} 1 & 2 & 0 & -1 & 1 \\ 0 & 1 & -1 & -3 & -13 \\ 0 & -3 & 1 & 3 & 7 \\ 0 & -1 & 0 & 2 & 6 \end{bmatrix} \xrightarrow{\begin{matrix} R1+(-2)R2 \\ R3+(3)R2 \\ R4+R2 \end{matrix}} \begin{bmatrix} 1 & 0 & 2 & 5 & 27 \\ 0 & 1 & -1 & -3 & -13 \\ 0 & 0 & -2 & -6 & -32 \\ 0 & 0 & -1 & -1 & -7 \end{bmatrix}$$

$$\xrightarrow{(-1/2)R3} \begin{bmatrix} 1 & 0 & 2 & 5 & 27 \\ 0 & 1 & -1 & -3 & -13 \\ 0 & 0 & 1 & 3 & 16 \\ 0 & 0 & -1 & -1 & -7 \end{bmatrix} \xrightarrow{\begin{matrix} R1+(-2)R3 \\ R2+R3 \\ R4+R3 \end{matrix}} \begin{bmatrix} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 3 & 16 \\ 0 & 0 & 0 & 2 & 9 \end{bmatrix}$$

$$\xrightarrow{(1/2)R4} \begin{bmatrix} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 3 & 16 \\ 0 & 0 & 0 & 1 & 9/2 \end{bmatrix} \xrightarrow{\begin{matrix} R1+R4 \\ R3+(-3)R4 \end{matrix}} \begin{bmatrix} 1 & 0 & 0 & 0 & -1/2 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 5/2 \\ 0 & 0 & 0 & 1 & 9/2 \end{bmatrix},$$

so the solution is $x_1 = -1/2$, $x_2 = 3$, $x_3 = 5/2$, $x_4 = 9/2$.

Section 1.2

$$\begin{array}{l} \approx \\ R1+R4 \\ R2+(-1)R4 \\ R3+(-3)R4 \end{array} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 & \\ 0 & 1 & 0 & 0 & -2 & \\ 0 & 0 & 1 & 0 & 1 & \\ 0 & 0 & 0 & 1 & 2 & \end{array} \right], \text{ so the solution is } x_1 = -1, x_2 = -2, x_3 = 1, x_4 = 2.$$

13. (a) $\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 4 & 3 & \\ 3 & 5 & 8 & 9 & 7 & \end{array} \right] \xrightarrow{R2+(-3)R1} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 4 & 3 & \\ 0 & -1 & -1 & -3 & -2 & \end{array} \right] \xrightarrow{(-1)R2} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 4 & 3 & \\ 0 & 1 & 1 & 3 & 2 & \end{array} \right] \xrightarrow{R1-2R2}$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & -2 & -1 & \\ 0 & 1 & 1 & 3 & 2 & \end{array} \right], \text{ so } x_1 = 1, x_2 = 1; x_1 = -2, x_2 = 3; \text{ and } x_1 = -1, x_2 = 2.$$

(b) $\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 5 & 1 & \\ 2 & 3 & 1 & 13 & 2 & \end{array} \right] \xrightarrow{R2+(-2)R1} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 5 & 1 & \\ 0 & 1 & 1 & 3 & 0 & \end{array} \right] \xrightarrow{R1+(-1)R2} \left[\begin{array}{ccccc|c} 1 & 0 & -1 & 2 & 1 & \\ 0 & 1 & 1 & 3 & 0 & \end{array} \right],$

so the solutions are in turn $x_1 = -1, x_2 = 1; x_1 = 2, x_2 = 3; \text{ and } x_1 = 1, x_2 =$

0.

(c) $\left[\begin{array}{cccccc|c} 1 & -2 & 3 & 6 & -5 & 4 & \\ 1 & -1 & 2 & 5 & -3 & 3 & \\ 2 & -3 & 6 & 14 & -8 & 9 & \end{array} \right] \xrightarrow{\begin{array}{l} R2+(-1)R1 \\ R3+(-2)R1 \end{array}} \left[\begin{array}{cccccc|c} 1 & -2 & 3 & 6 & -5 & 4 & \\ 0 & 1 & -1 & -1 & 2 & -1 & \\ 0 & 1 & 0 & 2 & 2 & 1 & \end{array} \right]$

$$\xrightarrow{\begin{array}{l} R1+(2)R2 \\ R3+(-1)R2 \end{array}} \left[\begin{array}{cccccc|c} 1 & 0 & 1 & 4 & -1 & 2 & \\ 0 & 1 & -1 & -1 & 2 & -1 & \\ 0 & 0 & 1 & 3 & 0 & 2 & \end{array} \right] \xrightarrow{\begin{array}{l} R1+(-1)R3 \\ R2+R3 \end{array}} \left[\begin{array}{cccccc|c} 1 & 0 & 0 & 1 & -1 & 0 & \\ 0 & 1 & 0 & 2 & 2 & 1 & \\ 0 & 0 & 1 & 3 & 0 & 2 & \end{array} \right],$$

so the solutions are in turn $x_1 = 1, x_2 = 2, x_3 = 3; x_1 = -1, x_2 = 2, x_3 = 0;$
and $x_1 = 0, x_2 = 1, x_3 = 2.$

(d) $\left[\begin{array}{cccccc|c} 1 & 2 & -1 & -1 & 6 & 0 & \\ -1 & -1 & 1 & 1 & -4 & -2 & \\ 3 & 7 & -1 & -1 & 18 & -4 & \end{array} \right] \xrightarrow{\begin{array}{l} R2+R1 \\ R3+(-3)R1 \end{array}} \left[\begin{array}{cccccc|c} 1 & 2 & -1 & -1 & 6 & 0 & \\ 0 & 1 & 0 & 0 & 2 & -2 & \\ 0 & 1 & 2 & 2 & 0 & -4 & \end{array} \right]$

$$\xrightarrow{\begin{array}{l} R1+(-2)R2 \\ R3+(-1)R2 \end{array}} \left[\begin{array}{cccccc|c} 1 & 0 & -1 & -1 & 2 & 4 & \\ 0 & 1 & 0 & 0 & 2 & -2 & \\ 0 & 0 & 2 & 2 & -2 & -2 & \end{array} \right] \xrightarrow{(1/2)R3} \left[\begin{array}{cccccc|c} 1 & 0 & -1 & -1 & 2 & 4 & \\ 0 & 1 & 0 & 0 & 2 & -2 & \\ 0 & 0 & 1 & 1 & -1 & -1 & \end{array} \right]$$

Section 1.2

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 1 & 3 & \\ 0 & 1 & 0 & 0 & 2 & -2 & \\ 0 & 0 & 1 & 1 & -1 & -1 & \end{array} \right]$$

so the solutions are in turn $x_1 = 0$, $x_2 = 0$, $x_3 = 1$; $x_1 = 1$, $x_2 = 2$, $x_3 = -1$;
and $x_1 = 3$, $x_2 = -2$, $x_3 = -1$.

Exercise Set 1.2

1. (a) Yes. (b) Yes.
 (c) No. The second column contains a leading 1, so other elements in that column should be zero.
 (d) No. The second row does not have 1 as the first nonzero number.
 (e) Yes. (f) Yes. (g) Yes.
 (h) No. The second row does not have 1 as the first nonzero number. (i) Yes.

2. (a) No. The leading 1 in row 3 is not to the right of the leading 1 in row 2.
 (b) Yes. (c) Yes.
 (d) No. The fourth and fifth columns contain leading 1s, so the other numbers in those columns should be zeros.
 (e) No. The row containing all zeros should be at the bottom of the matrix.
 (f) Yes.
 (g) No. The leading 1 in row 3 is not to the right of the leading 1 in row 2. Also, since column 3 contains a leading 1, all other numbers in that column should be zero.
 (h) No. The leading 1 in row 3 is not to the right of the leading 1s in rows 1 and 2.
 (i) Yes.

3. (a) $x_1 = 2$, $x_2 = 4$, $x_3 = -3$. (b) $x_1 = 3r + 4$, $x_2 = -2r + 8$, $x_3 = r$.

Section 1.2

(c) $x_1 = -3r + 6$, $x_2 = r$, $x_3 = -2$. (d) There is no solution. The last row gives $0 = 1$.

(e) $x_1 = -5r + 3$, $x_2 = -6r - 2$, $x_3 = -2r - 4$, $x_4 = r$.

(f) $x_1 = -3r + 2$, $x_2 = r$, $x_3 = 4$, $x_4 = 5$.

4. (a) $x_1 = -2r - 4s + 1$, $x_2 = 3r - 5s - 6$, $x_3 = r$, $x_4 = s$.

(b) $x_1 = 3r - 2s + 4$, $x_2 = r$, $x_3 = s$, $x_4 = -7$.

(c) $x_1 = 2r - 3s + 4$, $x_2 = r$, $x_3 = -2s + 9$, $x_4 = s$, $x_5 = 8$.

(d) $x_1 = -2r - 3s + 6$, $x_2 = -5r - 4s + 7$, $x_3 = r$, $x_4 = -9s - 3$, $x_5 = s$.

$$\begin{aligned}
 5. \quad (a) \quad & \left[\begin{array}{cccc} 1 & 4 & 3 & 1 \\ 2 & 8 & 11 & 7 \\ 1 & 6 & 7 & 3 \end{array} \right] \begin{array}{l} R2 + (-2)R1 \\ R3 + (-1)R1 \end{array} \approx \left[\begin{array}{cccc} 1 & 4 & 3 & 1 \\ 0 & 0 & 5 & 5 \\ 0 & 2 & 4 & 2 \end{array} \right] \begin{array}{l} \\ R2 \leftrightarrow R3 \end{array} \approx \left[\begin{array}{cccc} 1 & 4 & 3 & 1 \\ 0 & 2 & 4 & 2 \\ 0 & 0 & 5 & 5 \end{array} \right] \\
 & \approx \begin{array}{l} (1/2)R2 \\ \\ \end{array} \left[\begin{array}{cccc} 1 & 4 & 3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 5 & 5 \end{array} \right] \begin{array}{l} \\ \\ R1 + (-4)R2 \end{array} \approx \left[\begin{array}{cccc} 1 & 0 & -5 & -3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 5 & 5 \end{array} \right] \begin{array}{l} \\ \\ (1/5)R3 \end{array} \approx \left[\begin{array}{cccc} 1 & 0 & -5 & -3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right] \\
 & \begin{array}{l} R1 + (5)R3 \\ R2 + (-2)R3 \end{array} \approx \left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right], \text{ so the solution is } x_1 = 2, x_2 = -1, x_3 = 1.
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & \left[\begin{array}{cccc} 1 & 2 & 4 & 15 \\ 2 & 4 & 9 & 33 \\ 1 & 3 & 5 & 20 \end{array} \right] \begin{array}{l} R2 + (-2)R1 \\ R3 + (-1)R1 \end{array} \approx \left[\begin{array}{cccc} 1 & 2 & 4 & 15 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 1 & 5 \end{array} \right] \begin{array}{l} \\ R2 \leftrightarrow R3 \end{array} \approx \left[\begin{array}{cccc} 1 & 2 & 4 & 15 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{array} \right] \\
 & \begin{array}{l} \\ R1 + (-2)R2 \end{array} \approx \left[\begin{array}{cccc} 1 & 0 & 2 & 5 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{array} \right] \begin{array}{l} \\ \\ R1 + (-2)R3 \\ R2 + (-1)R3 \end{array} \approx \left[\begin{array}{cccc} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right],
 \end{aligned}$$

so the solution is $x_1 = -1$, $x_2 = 2$, $x_3 = 3$.

Section 1.2

$$(c) \left[\begin{array}{cccc} 1 & 1 & 1 & 7 \\ 2 & 3 & 1 & 18 \\ -1 & 1 & -3 & 1 \end{array} \right] \begin{array}{l} \approx \\ R2+(-2)R1 \\ R3+R1 \end{array} \left[\begin{array}{cccc} 1 & 1 & 1 & 7 \\ 0 & 1 & -1 & 4 \\ 0 & 2 & -2 & 8 \end{array} \right] \begin{array}{l} \approx \\ R1+(-1)R2 \\ R3+(-2)R3 \end{array} \left[\begin{array}{cccc} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right],$$

so $x_1 + 2x_3 = 3$ and $x_2 - x_3 = 4$.

Thus the general solution is $x_1 = 3 - 2r$, $x_2 = 4 + r$, $x_3 = r$.

$$(d) \left[\begin{array}{cccc} 1 & 4 & 1 & 2 \\ 1 & 2 & -1 & 0 \\ 2 & 6 & 0 & 3 \end{array} \right] \begin{array}{l} \approx \\ R2+(-1)R1 \\ R3+(-2)R1 \end{array} \left[\begin{array}{cccc} 1 & 4 & 1 & 2 \\ 0 & -2 & -2 & -2 \\ 0 & -2 & -2 & -1 \end{array} \right] \begin{array}{l} \approx \\ (-1/2)R2 \end{array} \left[\begin{array}{cccc} 1 & 4 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & -2 & -1 \end{array} \right]$$

$$\begin{array}{l} \approx \\ R1+(-4)R2 \\ R3+(2)R2 \end{array} \left[\begin{array}{cccc} 1 & 0 & -3 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right], \text{ so there is no solution, since the last row of the}$$

matrix corresponds to the equation $0 = 1$.

$$(e) \left[\begin{array}{cccc} 1 & -1 & 1 & 3 \\ 2 & -1 & 4 & 7 \\ 3 & -5 & -1 & 7 \end{array} \right] \begin{array}{l} \approx \\ R2+(-2)R1 \\ R3+(-3)R1 \end{array} \left[\begin{array}{cccc} 1 & -1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & -2 & -4 & -2 \end{array} \right] \begin{array}{l} \approx \\ R1+R2 \\ R3+(2)R2 \end{array} \left[\begin{array}{cccc} 1 & 0 & 3 & 4 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right],$$

so $x_1 + 3x_3 = 4$ and $x_2 + 2x_3 = 1$.

Thus the general solution is $x_1 = 4 - 3r$, $x_2 = 1 - 2r$, $x_3 = r$.

$$(f) \left[\begin{array}{cccc} 3 & -3 & 9 & 24 \\ 2 & -2 & 7 & 17 \\ -1 & 2 & -4 & -11 \end{array} \right] \begin{array}{l} \approx \\ (1/3)R1 \end{array} \left[\begin{array}{cccc} 1 & -1 & 3 & 8 \\ 2 & -2 & 7 & 17 \\ -1 & 2 & -4 & -11 \end{array} \right] \begin{array}{l} \approx \\ R2+(-2)R1 \\ R2+R1 \end{array} \left[\begin{array}{cccc} 1 & -1 & 3 & 8 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & -3 \end{array} \right]$$

$$\begin{array}{l} \approx \\ R2 \Leftrightarrow R3 \end{array} \left[\begin{array}{cccc} 1 & -1 & 3 & 8 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 1 \end{array} \right] \begin{array}{l} \approx \\ R1+R2 \end{array} \left[\begin{array}{cccc} 1 & 0 & 2 & 5 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 1 \end{array} \right] \begin{array}{l} \approx \\ R1+(-2)R3 \\ R2+R3 \end{array} \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array} \right],$$

so the solution is $x_1 = 3$, $x_2 = -2$, $x_3 = 1$.

$$6. \quad (a) \quad \left[\begin{array}{cccc|c} 3 & 6 & -3 & 6 & \\ -2 & -4 & -3 & -1 & \\ 3 & 6 & -2 & 10 & \end{array} \right] \xrightarrow{(1/3)R1} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 2 & \\ -2 & -4 & -3 & -1 & \\ 3 & 6 & -2 & 10 & \end{array} \right] \xrightarrow{\begin{array}{l} R2+(2)R1 \\ R3+(-3)R1 \end{array}} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 2 & \\ 0 & 0 & -5 & 3 & \\ 0 & 0 & 1 & 4 & \end{array} \right]$$

It is now clear that there is no solution. The last two rows give $-5x_3 = 3$ and $x_3 = 4$.

$$(b) \quad \left[\begin{array}{cccc|c} 1 & 2 & 1 & 7 & \\ 1 & 2 & 2 & 11 & \\ 2 & 4 & 3 & 18 & \end{array} \right] \xrightarrow{\begin{array}{l} R2+(-1)R1 \\ R3+(-2)R1 \end{array}} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 7 & \\ 0 & 0 & 1 & 4 & \\ 0 & 0 & 1 & 4 & \end{array} \right] \xrightarrow{\begin{array}{l} R1+(-1)R2 \\ R3+(-1)R2 \end{array}} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & \\ 0 & 0 & 1 & 4 & \\ 0 & 0 & 0 & 0 & \end{array} \right]$$

so $x_1 + 2x_2 = 3$ and $x_3 = 4$. Thus the general solution is $x_1 = 3 - 2r$, $x_2 = r$, $x_3 = 4$.

$$(c) \quad \left[\begin{array}{cccc|c} 1 & 2 & -1 & 3 & \\ 2 & 4 & -2 & 6 & \\ 3 & 6 & 2 & -1 & \end{array} \right] \xrightarrow{\begin{array}{l} R2+(-2)R1 \\ R3+(-3)R1 \end{array}} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 3 & \\ 0 & 0 & 0 & 0 & \\ 0 & 0 & 5 & -10 & \end{array} \right] \xrightarrow{R2 \leftrightarrow R3} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 3 & \\ 0 & 0 & 5 & -10 & \\ 0 & 0 & 0 & 0 & \end{array} \right]$$

$$\xrightarrow{(1/5)R2} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 3 & \\ 0 & 0 & 1 & -2 & \\ 0 & 0 & 0 & 0 & \end{array} \right] \xrightarrow{R1+R2} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & \\ 0 & 0 & 1 & -2 & \\ 0 & 0 & 0 & 0 & \end{array} \right], \text{ so } x_1 + 2x_2 = 1, \quad x_3 = -2.$$

Thus the general solution is $x_1 = 1 - 2r$, $x_2 = r$, $x_3 = -2$.

$$(d) \quad \left[\begin{array}{cccc|c} 1 & 2 & 3 & 8 & \\ 3 & 7 & 9 & 26 & \\ 2 & 0 & 6 & 11 & \end{array} \right] \xrightarrow{\begin{array}{l} R2+(-3)R1 \\ R3+(-2)R1 \end{array}} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 8 & \\ 0 & 1 & 0 & 2 & \\ 0 & -4 & 0 & -5 & \end{array} \right], \text{ so there is no solution since the}$$

last two rows give $x_2 = 2$ and $-4x_2 = -5$.

$$(e) \quad \left[\begin{array}{cccc|c} 0 & 1 & 2 & 5 & \\ 1 & 2 & 5 & 13 & \\ 1 & 0 & 2 & 4 & \end{array} \right] \xrightarrow{R1 \leftrightarrow R2} \left[\begin{array}{cccc|c} 1 & 2 & 5 & 13 & \\ 0 & 1 & 2 & 5 & \\ 1 & 0 & 2 & 4 & \end{array} \right] \xrightarrow{R3+(-1)R1} \left[\begin{array}{cccc|c} 1 & 2 & 5 & 13 & \\ 0 & 1 & 2 & 5 & \\ 0 & -2 & -3 & -9 & \end{array} \right]$$

Section 1.2

$$\begin{array}{l} \approx \\ R1+(-2)R2 \\ R3+(2)R2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 1 \end{array} \right] \approx \begin{array}{l} \approx \\ R1+(-1)R3 \\ R2+(-2)R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{array} \right],$$

so the solution is $x_1 = 2$, $x_2 = 3$, $x_3 = 1$.

$$(f) \begin{array}{l} \\ \\ \\ R2+(-2)R1 \end{array} \left[\begin{array}{cccc|c} 1 & 2 & 8 & 7 & \\ 2 & 4 & 16 & 14 & \\ 0 & 1 & 3 & 4 & \end{array} \right] \approx \begin{array}{l} \\ \\ \\ R2 \Leftrightarrow R3 \end{array} \left[\begin{array}{cccc|c} 1 & 2 & 8 & 7 & \\ 0 & 0 & 0 & 0 & \\ 0 & 1 & 3 & 4 & \end{array} \right] \approx \begin{array}{l} \\ \\ \\ R1+(-2)R2 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & \\ 0 & 1 & 3 & 4 & \\ 0 & 0 & 0 & 0 & \end{array} \right], \text{ so } x_1 + 2x_3 = -1, x_2 + 3x_3 = 4.$$

Thus the general solution is $x_1 = -1 - 2r$, $x_2 = 4 - 3r$, $x_3 = r$.

$$7. (a) \left[\begin{array}{cccc|c} 1 & 1 & -3 & 10 & \\ -3 & -2 & 4 & -24 & \end{array} \right] \approx \begin{array}{l} \\ \\ \\ R2+(3)R1 \end{array} \left[\begin{array}{cccc|c} 1 & 1 & -3 & 10 & \\ 0 & 1 & -5 & 6 & \end{array} \right] \approx \begin{array}{l} \\ \\ \\ R1+(-1)R2 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 4 & \\ 0 & 1 & -5 & 6 & \end{array} \right],$$

so $x_1 + 2x_3 = 4$ and $x_2 - 5x_3 = 6$. Thus the general solution is $x_1 = 4 - 2r$, $x_2 = 6 + 5r$, $x_3 = r$.

$$(b) \left[\begin{array}{cccc|c} 2 & -6 & -14 & 38 & \\ -3 & 7 & 15 & -37 & \end{array} \right] \approx \begin{array}{l} \\ \\ \\ (1/2)R1 \end{array} \left[\begin{array}{cccc|c} 1 & -3 & -7 & 19 & \\ -3 & 7 & 15 & -37 & \end{array} \right] \approx \begin{array}{l} \\ \\ \\ R2+(3)R1 \end{array} \left[\begin{array}{cccc|c} 1 & -3 & -7 & 19 & \\ 0 & -2 & -6 & 20 & \end{array} \right] \\ \approx \begin{array}{l} \\ \\ \\ (-1/2)R2 \end{array} \left[\begin{array}{cccc|c} 1 & -3 & -7 & 19 & \\ 0 & 1 & 3 & -10 & \end{array} \right] \approx \begin{array}{l} \\ \\ \\ R1+(3)R2 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 2 & -11 & \\ 0 & 1 & 3 & -10 & \end{array} \right],$$

so $x_1 + 2x_3 = -11$ and $x_2 + 3x_3 = -10$. Thus the general solution is $x_1 = -11 - 2r$, $x_2 = -10 - 3r$, $x_3 = r$.

$$(c) \left[\begin{array}{ccccc|c} 1 & 2 & -1 & -1 & 0 & \\ 1 & 2 & 0 & 1 & 4 & \\ -1 & -2 & 2 & 4 & 5 & \end{array} \right] \approx \begin{array}{l} \\ \\ \\ R2+(-1)R1 \\ R3+R1 \end{array} \left[\begin{array}{ccccc|c} 1 & 2 & -1 & -1 & 0 & \\ 0 & 0 & 1 & 2 & 4 & \\ 0 & 0 & 1 & 3 & 5 & \end{array} \right] \approx \begin{array}{l} \\ \\ \\ R1+R2 \\ R3+(-1)R2 \end{array} \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 1 & 4 & \\ 0 & 0 & 1 & 2 & 4 & \\ 0 & 0 & 0 & 1 & 1 & \end{array} \right]$$

$$\approx \begin{array}{l} \\ R1+(-1)R3 \\ R2+(-2)R3 \end{array} \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right], \text{ so } x_1 + 2x_2 = 3 \text{ and } x_3 = 2, \text{ and } x_4 = 1.$$

Thus the general solution is $x_1 = 3 - 2r$, $x_2 = r$, $x_3 = 2$, and $x_4 = 1$.

$$(d) \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 4 & 0 \\ -2 & -4 & 3 & -2 & 0 \end{array} \right] \begin{array}{l} \\ R2+(2)R1 \end{array} \approx \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 4 & 0 \\ 0 & 0 & 3 & 6 & 0 \end{array} \right] \begin{array}{l} \\ (1/3)R2 \end{array} \approx \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 4 & 0 \\ 0 & 0 & 1 & 2 & 0 \end{array} \right],$$

so $x_1 + 2x_2 + 4x_4 = 0$ and $x_3 + 2x_4 = 0$. Thus the general solution is $x_1 = -2r - 4s$, $x_2 = r$, $x_3 = -2s$, $x_4 = s$.

$$(e) \left[\begin{array}{ccccc|c} 0 & 1 & -3 & 1 & 0 \\ 1 & 1 & -1 & 4 & 0 \\ -2 & -2 & 2 & -8 & 0 \end{array} \right] \begin{array}{l} \\ \\ R1 \leftrightarrow R2 \end{array} \approx \left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 0 \\ 0 & 1 & -3 & 1 & 0 \\ -2 & -2 & 2 & -8 & 0 \end{array} \right]$$

$$\approx \begin{array}{l} \\ R3+(2)R1 \end{array} \left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 0 \\ 0 & 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \\ \\ R1+(-1)R2 \end{array} \approx \left[\begin{array}{ccccc|c} 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right],$$

so $x_1 + 2x_3 + 3x_4 = 0$ and $x_2 - 3x_3 + x_4 = 0$. Thus the general solution is $x_1 = -2r - 3s$, $x_2 = 3r - s$, $x_3 = r$, $x_4 = s$.

$$8. (a) \left[\begin{array}{ccccc|c} 1 & 1 & 1 & -1 & -3 \\ 2 & 3 & 1 & -5 & -9 \\ 1 & 3 & -1 & -6 & -7 \\ -1 & -1 & -1 & 0 & 1 \end{array} \right] \begin{array}{l} \\ \\ \\ R4+R1 \end{array} \approx \left[\begin{array}{ccccc|c} 1 & 1 & 1 & -1 & -3 \\ 0 & 1 & -1 & -3 & -3 \\ 0 & 2 & -2 & -5 & -4 \\ 0 & 0 & 0 & -1 & -2 \end{array} \right]$$

$$\approx \begin{array}{l} \\ R1+(-1)R2 \\ R3+(-2)R2 \end{array} \left[\begin{array}{ccccc|c} 1 & 0 & 2 & 2 & 0 \\ 0 & 1 & -1 & -3 & -3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -1 & -2 \end{array} \right] \begin{array}{l} \\ \\ \\ R4+R3 \end{array} \approx \begin{array}{l} \\ R1+(2)R3 \\ R2+(-3)R3 \end{array} \left[\begin{array}{ccccc|c} 1 & 0 & 2 & 0 & -4 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right],$$

so $x_1 + 2x_3 = -4$, $x_2 - x_3 = 3$, $x_4 = 2$.

The general solution is $x_1 = -2r - 4$, $x_2 = r + 3$, $x_3 = r$, $x_4 = 2$.

Section 1.2

$$(b) \left[\begin{array}{cccc|c} 0 & 1 & 2 & 7 & \\ 1 & -2 & -6 & -18 & \\ -1 & -1 & -2 & -5 & \\ 2 & -5 & -15 & -46 & \end{array} \right] \begin{array}{l} \\ R1 \leftrightarrow R2 \\ \\ \end{array} \approx \left[\begin{array}{cccc|c} 1 & -2 & -6 & -18 & \\ 0 & 1 & 2 & 7 & \\ -1 & -1 & -2 & -5 & \\ 2 & -5 & -15 & -46 & \end{array} \right]$$

$$\approx \begin{array}{l} \\ R3+R1 \\ R4+(-2)R1 \end{array} \left[\begin{array}{cccc|c} 1 & -2 & -6 & -18 & \\ 0 & 1 & 2 & 7 & \\ 0 & -3 & -8 & -23 & \\ 0 & -1 & -3 & -10 & \end{array} \right] \begin{array}{l} \\ R1+(2)R2 \\ R3+(3)R2 \\ R4+R2 \end{array} \approx \left[\begin{array}{cccc|c} 1 & 0 & -2 & 4 & \\ 0 & 1 & 2 & 7 & \\ 0 & 0 & -2 & -2 & \\ 0 & 0 & -1 & -3 & \end{array} \right], \text{ and there are no}$$

solutions because the last two rows of the matrix give, respectively, $x_3 = 1$ and $x_3 = 3$.

$$(c) \left[\begin{array}{ccccc|c} 2 & -4 & 16 & -14 & 10 & \\ -1 & 5 & -17 & 19 & -2 & \\ 1 & -3 & 11 & -11 & 4 & \\ 3 & -4 & 18 & -13 & 17 & \end{array} \right] \begin{array}{l} \\ \\ (1/2)R1 \\ \end{array} \approx \left[\begin{array}{ccccc|c} 1 & -2 & 8 & -7 & 5 & \\ -1 & 5 & -17 & 19 & -2 & \\ 1 & -3 & 11 & -11 & 4 & \\ 3 & -4 & 18 & -13 & 17 & \end{array} \right]$$

$$\approx \begin{array}{l} \\ R2+R1 \\ R3+(-1)R1 \\ R4+(-3)R1 \end{array} \left[\begin{array}{ccccc|c} 1 & -2 & 8 & -7 & 5 & \\ 0 & 3 & -9 & 12 & 3 & \\ 0 & -1 & 3 & -4 & -1 & \\ 0 & 2 & -6 & 8 & 2 & \end{array} \right] \begin{array}{l} \\ \\ (1/3)R2 \\ \end{array} \approx \left[\begin{array}{ccccc|c} 1 & -2 & 8 & -7 & 5 & \\ 0 & 1 & -3 & 4 & 1 & \\ 0 & -1 & 3 & -4 & -1 & \\ 0 & 2 & -6 & 8 & 2 & \end{array} \right]$$

$$\approx \begin{array}{l} \\ R1+(2)R2 \\ R3+R2 \\ R4+(-2)R2 \end{array} \left[\begin{array}{ccccc|c} 1 & 0 & 2 & 1 & 7 & \\ 0 & 1 & -3 & 4 & 1 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \end{array} \right], \text{ so } x_1+2x_3+x_4 = 7 \text{ and } x_2-3x_3+4x_4=1.$$

Thus the general solution is $x_1 = 7 - 2r - s$, $x_2 = 1 + 3r - 4s$, $x_3 = r$, $x_4 = s$.

$$(d) \left[\begin{array}{ccccc|c} 1 & -1 & 2 & 0 & 7 & \\ 2 & -2 & 2 & -4 & 12 & \\ -1 & 1 & -1 & 2 & -4 & \\ -3 & 1 & -8 & -10 & -29 & \end{array} \right] \begin{array}{l} \\ R2+(-2)R1 \\ R3+R1 \\ R4+(3)R1 \end{array} \approx \left[\begin{array}{ccccc|c} 1 & -1 & 2 & 0 & 7 & \\ 0 & 0 & -2 & -4 & -2 & \\ 0 & 0 & 1 & 2 & 3 & \\ 0 & -2 & -2 & -10 & -8 & \end{array} \right]$$

Section 1.2

$$\approx \begin{array}{c} \\ R2 \Leftrightarrow R4 \end{array} \left[\begin{array}{ccccc} 1 & -1 & 2 & 0 & 7 \\ 0 & -2 & -2 & -10 & -8 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & -2 & -4 & -2 \end{array} \right] \approx \begin{array}{c} \\ (-1/2)R2 \end{array} \left[\begin{array}{ccccc} 1 & -1 & 2 & 0 & 7 \\ 0 & 1 & 1 & 5 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & -2 & -4 & -2 \end{array} \right]$$

$$\approx \begin{array}{c} \\ R1+R2 \end{array} \left[\begin{array}{ccccc} 1 & 0 & 3 & 5 & 11 \\ 0 & 1 & 1 & 5 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & -2 & -4 & -2 \end{array} \right] \approx \begin{array}{c} \\ R1+(-3)R3 \\ R2+(-1)R3 \\ R4+(2R3) \end{array} \left[\begin{array}{ccccc} 1 & 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{array} \right]$$

The last row gives $0 = 4$, so there is no solution.

$$(e) \left[\begin{array}{ccccc} 1 & 6 & -1 & -4 & 0 \\ -2 & -12 & 5 & 17 & 0 \\ 3 & 18 & -1 & -6 & 0 \end{array} \right] \approx \begin{array}{c} \\ R2+(2)R1 \\ R3+(-3)R1 \end{array} \left[\begin{array}{ccccc} 1 & 6 & -1 & -4 & 0 \\ 0 & 0 & 3 & 9 & 0 \\ 0 & 0 & 2 & 6 & 0 \end{array} \right]$$

$$(1/3)R2 \left[\begin{array}{ccccc} 1 & 6 & -1 & -4 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 2 & 6 & 0 \end{array} \right] \approx \begin{array}{c} \\ R1+R2 \\ R3+(-2)R2 \end{array} \left[\begin{array}{ccccc} 1 & 6 & 0 & -1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right],$$

so $x_1 + 6x_2 - x_4 = 0$ and $x_3 + 3x_4 = 0$.

Thus the general solution is $x_1 = -6r + s$, $x_2 = r$, $x_3 = -3s$, $x_4 = s$.

$$(f) \left[\begin{array}{cccc} 4 & 8 & -12 & 28 \\ -1 & -2 & 3 & -7 \\ 2 & 4 & -8 & 16 \\ -3 & -6 & 9 & -21 \end{array} \right] \approx \begin{array}{c} \\ (1/4)R1 \end{array} \left[\begin{array}{cccc} 1 & 2 & -3 & 7 \\ -1 & -2 & 3 & -7 \\ 2 & 4 & -8 & 16 \\ -3 & -6 & 9 & -21 \end{array} \right]$$

$$\approx \begin{array}{c} \\ R2+R1 \\ R3+(-2)R1 \\ R4+(3)R1 \end{array} \left[\begin{array}{cccc} 1 & 2 & -3 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right] \approx \begin{array}{c} \\ R2 \Leftrightarrow R3 \end{array} \left[\begin{array}{cccc} 1 & 2 & -3 & 7 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\approx \begin{array}{c} \\ (-1/2)R2 \end{array} \left[\begin{array}{cccc} 1 & 2 & -3 & 7 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \approx \begin{array}{c} \\ R1+(3)R2 \end{array} \left[\begin{array}{cccc} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right], \text{ so } x_1 + 2x_2 = 4 \text{ and } x_3 = -1.$$

Thus the general solution is $x_1 = 4 - 2r$, $x_2 = r$, $x_3 = -1$.

$$(g) \left[\begin{array}{ccc|c} 1 & 1 & 2 & \\ 2 & 3 & 3 & R2+(-2)R1 \\ 1 & 3 & 0 & R3+(-1)R1 \\ 1 & 2 & 1 & R4+(-1)R1 \end{array} \right] \approx \left[\begin{array}{ccc|c} 1 & 1 & 2 & \\ 0 & 1 & -1 & R1+(-1)R2 \\ 0 & 2 & -2 & R3+(-2)R2 \\ 0 & 1 & -1 & R4+(-1)R2 \end{array} \right] \approx \left[\begin{array}{ccc|c} 1 & 0 & 3 & \\ 0 & 1 & -1 & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{array} \right],$$

$$\text{so } x_1 = 3, x_2 = -1.$$

9. (a) The system of equations

$$3x_1 + 2x_2 - x_3 + x_4 = 4$$

$$3x_1 + 2x_2 - x_3 + x_4 = 1$$

clearly has no solution, since the equations are inconsistent. To make a system that is less obvious, add another equation to the system and perform an elementary transformation on this new system of three equations. For example, replace the second equation by the sum of the second equation and some multiple (2 in the example below) of the third equation:

$$3x_1 + 2x_2 - x_3 + x_4 = 4$$

$$5x_1 + 4x_2 - x_3 - x_4 = 1$$

$$x_1 + x_2 - x_4 = 0$$

(b) Choose a solution, e.g., $x_1 = 1, x_2 = 2$. Now make up equations thinking of x_1 as 1 and x_2 as 2:

$$x_1 + x_2 = 3$$

$$x_1 + 2x_2 = 5$$

$$x_1 - 2x_2 = -3$$

An easy way to ensure that there are no additional solutions is to include $x_1 = 1$ or $x_2 = 2$ as an equation in the system.

$$10. (a) \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \left[\begin{array}{ccc} 1 & 0 & * \\ 0 & 1 & * \\ 0 & 0 & 0 \end{array} \right] \left[\begin{array}{ccc} 1 & * & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array} \right] \left[\begin{array}{ccc} 1 & * & * \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

no unique no many
solution solution solution solutions

(b)
$$\begin{bmatrix} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \end{bmatrix} \begin{bmatrix} 1 & 0 & * & 0 \\ 0 & 1 & * & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & * & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

unique solution
no solution
many solutions
no solution

$$\begin{bmatrix} 1 & * & 0 & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & * & * & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & * & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

many solutions
no solution
many solutions

11. (a) If $ax_0 + by_0 = 0$ then $k(ax_0 + by_0) = 0$ so that $a(kx_0) + b(ky_0) = 0$. Thus $x = kx_0, y = ky_0$ is a solution. Likewise for the equation $cx + dy = 0$.
- (b) If $ax_0 + by_0 = 0$ and $ax_1 + by_1 = 0$ then $ax_0 + by_0 + ax_1 + by_1 = 0 + 0 = 0$. But $ax_0 + by_0 + ax_1 + by_1 = ax_0 + ax_1 + by_0 + by_1 = a(x_0 + x_1) + b(y_0 + y_1)$ so that $a(x_0 + x_1) + b(y_0 + y_1) = 0$. Thus $x = x_0 + x_1, y = y_0 + y_1$ is a solution. Likewise for the equation $cx + dy = 0$.
12. $a(0) + b(0) = 0$ and $c(0) + d(0) = 0$, so $x = 0, y = 0$ is a solution. Multiply 1st equation by c , 2nd by a to eliminate x . Get $cax + cby = 0$ and $acx + ady = 0$. Subtract, $ady - cby = 0, (ad - bc)y = 0$. Similarly $(ad - bc)x = 0$. If $ad - bc \neq 0, x = 0, y = 0$. If $ad - bc = 0$ the x and y can be anything; thus many solutions. Therefore $x = 0, y = 0$ is the only solution if and only if $ad - bc \neq 0$.
13. (a) and (b), No. If the first system of equations has a unique solution, then the reduced echelon form of the matrix $[A:B_1]$ will be $[I_3 : X]$. The reduced echelon form of $[A:B_2]$ must therefore be $[I_3 : Y]$. So the second system must also have a unique solution.
- (c) If the first system of equations has many solutions, then at least one row of the reduced echelon form of $[A:B_1]$ will consist entirely of zeros. Therefore the corresponding row(s) of the reduced echelon form of $[A:B_2]$ will have zeros in the first three columns. If any such row has a nonzero number in the fourth column, the system will have no solution.

$$14. \quad (a) \quad \left[\begin{array}{ccccc|c} 1 & 1 & 5 & 2 & 3 & \\ 1 & 2 & 8 & 5 & 2 & \\ 2 & 4 & 16 & 10 & 4 & \end{array} \right] \begin{array}{l} \approx \\ R2+(-1)R1 \\ R3+(-2)R1 \end{array} \left[\begin{array}{ccccc|c} 1 & 1 & 5 & 2 & 3 & \\ 0 & 1 & 3 & 3 & -1 & \\ 0 & 2 & 6 & 6 & -2 & \end{array} \right] \begin{array}{l} \approx \\ R1+(-1)R2 \\ R3+(-2)R2 \end{array} \left[\begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 4 & \\ 0 & 1 & 3 & 3 & -1 & \\ 0 & 0 & 0 & 0 & 0 & \end{array} \right],$$

so the general solution to the first system is $x_1 = -1 - 2r$, $x_2 = 3 - 3r$, $x_3 = r$, and the general solution to the second system is $x_1 = 4 - 2r$, $x_2 = -1 - 3r$, $x_3 = r$.

$$(b) \quad \left[\begin{array}{ccccc|c} 1 & 2 & 4 & 8 & 5 & \\ 1 & 1 & 2 & 5 & 3 & \\ 2 & 3 & 6 & 13 & 11 & \end{array} \right] \begin{array}{l} \approx \\ R2+(-1)R1 \\ R3+(-2)R1 \end{array} \left[\begin{array}{ccccc|c} 1 & 2 & 4 & 8 & 5 & \\ 0 & -1 & -2 & -3 & -2 & \\ 0 & -1 & -2 & -3 & 1 & \end{array} \right] \begin{array}{l} \approx \\ (-1)R2 \end{array} \left[\begin{array}{ccccc|c} 1 & 2 & 4 & 8 & 5 & \\ 0 & 1 & 2 & 3 & 2 & \\ 0 & -1 & -2 & -3 & 1 & \end{array} \right],$$

$$\begin{array}{l} \approx \\ R1+(-2)R2 \\ R3+R2 \end{array} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 2 & 1 & \\ 0 & 1 & 2 & 3 & 2 & \\ 0 & 0 & 0 & 0 & 3 & \end{array} \right], \text{ so the general solution to the first system is}$$

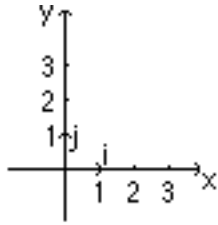
$x_1 = 2$, $x_2 = 3 - 2r$, $x_3 = r$, and the second system has no solution.

15. A 3x3 matrix represents the equations of three lines in a plane. In order for there to be a unique solution, the three lines would have to meet in a point. For there to be many solutions, the three lines would all have to be the same. It is far more likely that the lines will meet in pairs (or that one pair will be parallel), i.e., that there will be no solution, the situation represented by the reduced echelon form I_3 .
16. A 3x4 matrix represents the equations of three planes. In order for there to be many solutions, the three planes must have at least one line in common. For there to be no solutions, either at least two of the three planes must be parallel or the line of intersection of two of the planes must lie in a plane that is parallel to the third plane. It is more likely that the three planes will meet in a single point, i.e., that there will be a unique solution. The reduced echelon form therefore will be $[I_3 : X]$.
17. The difference between no solution and at least one solution is the presence of a nonzero number in the last position of a row that otherwise consists entirely of zeros. Round-off error is more likely to produce a nonzero number when there should be a zero than the reverse. Thus the answer is (b). Thinking geometrically, a small move by one or more of the linear surfaces (round-off error) may destroy a solution if there is one, but probably won't produce a solution if there is none.

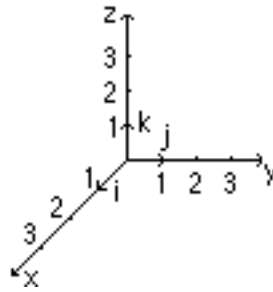
Exercise Set 1.3

Section 1.3

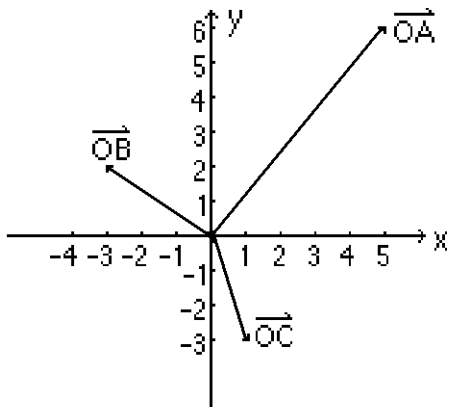
1.



2.



3. (a)



(b)

