

## Chapter 1

# The Structure of Metals

### Qualitative Problems

- 1.25. Explain your understanding of why the study of the crystal structure of metals is important.

The study of crystal structure is important for a number of reasons. Basically, the crystal structure influences a material's performance from both a design and manufacturing standpoint. For example, the number of slip systems in a crystal has a direct bearing on the ability of a metal to undergo plastic deformation without fracture. Similarly, the crystal structure has a bearing on strength, ductility and corrosion resistance. Metals with face-centered cubic structure, for example, tend to be ductile whereas hexagonal close-packed metals tend to be brittle. The crystal structure and size of atom determines the largest interstitial sites, which has a bearing on the ability of that material to form alloys, and with which materials, as interstitials or substitutionals.

- 1.26. What is the significance of the fact that some metals undergo allotropism?

Allotropism (also called polymorphism) means that a metal can change from one crystal structure to another. Since properties vary with crystal structures, allotropism is useful and essential in heat treating of metals to achieve desired properties (Chapter 4). A major application is hardening of steel, which involves the change in iron from the fcc structure to the bcc structure (see Fig. 1.3). By heating the steel to the fcc structure and quenching, it develops into martensite, which is a very hard, hence strong, structure.

- 1.27. Is it possible for two pieces of the same metal to have different recrystallization temperatures? Is it possible for recrystallization to take place in some regions of a part before it does in other regions of the same part? Explain.

Two pieces of the same metal can have different recrystallization temperatures if the pieces have been cold worked to different amounts. The piece that was cold worked to a greater extent (higher strains), will have more internal energy (stored energy) to drive the recrystallization process, hence its recrystallization temperature will be lower. Recrystallization may also occur in some regions of the part before others if it has been unevenly strained (since varying amounts of cold work have different recrystallization temperatures), or if the part has different thicknesses in various sections. The thinner sections will heat up to the recrystallization temperature faster.

- 1.28. Describe your understanding of why different crystal structures exhibit different strengths and ductilities.

Different crystal structures have different slip systems, which consist of a slip plane (the closest packed plane) and a slip direction (the close-packed direction). The fcc structure has 12 slip systems, bcc has 48, and hcp has 3. The ductility of a metal depends on how many of the slip systems can be operative. In general, fcc and bcc structures possess higher ductility than hcp structures, because they have more slip systems. The shear strength of a metal decreases for decreasing  $b/a$  ratio ( $b$  is inversely proportional to atomic density in the slip plane and  $a$  is the plane spacing), and the  $b/a$  ratio depends on the slip system of the chemical structure (see Section 1.4).

- 1.29. A cold-worked piece of metal has been recrystallized. When tested, it is found to be anisotropic. Explain the probable reason.

The anisotropy of the workpiece is likely due to preferred orientation remaining from the recrystallization process. Copper is an example of a metal that has a very strong preferred orientation after annealing. Also, it has been shown that below a critical amount of plastic deformation, typically 5%, no recrystallization occurs.

- 1.30. What materials and structures can you think of (other than metals) that exhibit anisotropic behavior?

This is an open-ended problem and the students should be encouraged to develop their own answers. However, some examples of anisotropic materials are wood, polymers that have been cold worked, bone, any woven material (such as cloth) and composite materials.

- 1.31. Two parts have been made of the same material, but one was formed by cold working and the other by hot working. Explain the differences you might observe between the two.

There are a large number of differences that will be seen between the two materials, including:

1. The cold worked material will have a higher strength than the hot worked material, and this will be more pronounced for materials with high strain hardening exponents.
2. Since hardness (see Section 2.6.2) is related to strength, the cold worked material will also have a higher hardness.
3. The cold worked material will have smaller grains and the grains will be elongated.
4. The hot worked material will probably have fewer dislocations, and they will be more evenly distributed.
5. The cold worked material can have a superior surface finish when in an as-formed condition. Also, it can have better tolerances.
6. A cold worked material will have a lower recrystallization temperature than a hot worked material.

- 1.32. Explain the importance of homologous temperature.

The homologous temperature is defined as the ratio of a metal's current temperature to its melting temperature on an absolute scale (Kelvin or Rankine, not Celsius or Fahrenheit). This is important for determining whether or not the metal will encounter recrystallization or grain growth. The homologous temperature is more important than actual temperature, because recrystallization occurs at very different temperatures for different metals, but the homologous temperature effects are fairly consistent. The homologous temperature therefore allows one to distinguish between cold, warm, and hot working, as discussed in Table 1.2.

- 1.33. Do you think it might be important to know whether a raw material to be used in a manufacturing process has anisotropic properties? What about anisotropy in the finished product? Explain.

Anisotropy is important in cold-working processes, especially sheet-metal forming where the material's properties should preferably be uniform in the plane of the sheet and stronger in the thickness direction. As shown in Section 16.7, these characteristics allow for deep drawing of parts (like beverage cans) without earing, tearing, or cracking in the forming operations involved. In a finished part, anisotropy is important so that the strongest direction of the part can be designed to support the largest load in service. Also, the efficiency of transformers can be improved by using a sheet steel with anisotropy that can reduce *magnetic hysteresis* losses. Hysteresis is well known in ferromagnetic materials. When an external magnetic field is applied to a ferromagnet, the ferromagnet absorbs some of the external field. When sheet steel is highly anisotropic, it contains small grains and a crystallographic orientation that is far more uniform than for isotropic materials, and this orientation will reduce magnetic hysteresis losses.

1.34. What is the difference between an interstitial atom and a substitutional atom?

The difference can be seen in Fig. 1.8. A substitutional atom replaces an atom in the repeating lattice without distortion. An interstitial does not fit in the normal lattice; it can fit in the gap between atoms, or else it distorts the lattice. Examples of substitutionally are copper-nickel, gold-silver, and molybdenum-tungsten. Interstitials can be self-interstitials, but common other examples are carbon, lithium, sodium, and nitrogen.

1.35. Explain why the strength of a polycrystalline metal at room temperature decreases as its grain size increases.

Strength increases as more entanglements of dislocations occur with grain boundaries (Section 1.4.2). Metals with larger grains have less grain-boundary area per unit volume, and hence will not be able to generate as many entanglements at grain boundaries, thus the strength will be lower.

1.36. Describe the technique you would use to reduce the orange-peel effect on the surface of workpieces.

Orange peel is surface roughening induced by plastic strain. There are a number of ways of reducing the orange peel effect, including:

- Performing all forming operations without a lubricant, or else a very thin lubricant film (smaller than the desired roughness) and very smooth tooling. The goal is to have the surface roughness of the tooling imparted onto the workpiece.
- Large grains exacerbate orange peel, so the use of small grained materials would reduce orange peel.
- If deformation processes can be designed so that the surfaces see no deformation, then there would be no orange peel. For example, upsetting beneath flat dies can lead to a reduction in thickness with very little surface strains beneath the platen (see Fig. 14.3).
- Finishing operations can remove orange peel effects.

1.37. What is the significance of the fact that such metals as lead and tin have a recrystallization temperature that is about room temperature?

Recrystallization around room temperature prevents these metals from work hardening when cold worked. This characteristic prevents their strengthening and hardening, thus requiring a recrystallization cycle to restore their ductility. This behavior is also useful in experimental verification of analytical results concerning force and energy requirements in metalworking processes (see Part III of the text).

1.38. It was stated in this chapter that twinning usually occurs in hcp materials, but Fig. 1.6b shows twinning in a rectangular array of atoms. Can you explain the discrepancy?

The hcp unit cell shown in Fig. 1.6a has a hexagon on the top and bottom surfaces. However, an intersecting plane that is vertical in this figure would intersect atoms in a rectangular array as depicted in Fig. 1.6b. Thus, twinning occurs in hcp materials, but not in the hexagonal (close packed) plane such as in the top of the unit cell.

- 1.39. It has been noted that the more a metal has been cold worked, the less it strain hardens. Explain why.

This phenomenon can be observed in stress-strain curves, such as those shown in Figs. 2.2 and 2.5. Recall that the main effects of cold working are that grains become elongated and that the average grain size becomes smaller (as grains break down) with strain. Strain hardening occurs when dislocations interfere with each other and with grain boundaries. When a metal is annealed, the grains are large, and a small strain results in grains moving relatively easily at first, but they increasingly interfere with each other as strain increases. This explains that there is strain hardening for annealed materials at low strain. To understand why there is less strain hardening at higher levels of cold work, consider the extreme case of a very highly cold-worked material, with very small grains and very many dislocations that already interfere with each other. For this highly cold-worked material, the stress cannot be increased much more with strain, because the dislocations have nowhere else to go - they already interfere with each other and are pinned at grain boundaries.

- 1.40. Is it possible to cold work a metal at temperatures above the boiling point of water? Explain.

The metallurgical distinction between cold and hot working is associated with the *homologous temperature*. Cold working is associated with plastic deformation of a metal when it is below one-third of its melting temperature on an absolute scale. At the boiling point of water, the temperature is 100 °C, or 373 K. If this value is one-third the melting temperature, then a metal would have to have a melting temperature of 1119 K, or 846 °C. As can be seen in Table 3.1, there are many such metals.

- 1.41. Comment on your observations regarding Fig. 1.14.

This is an open-ended problem with many potential answers. Students may choose to address this problem by focusing on the shape of individual curves or their relation to each other. The instructor may wish to focus the students on a curve or two, or ask if the figure would give the same trends for a material that is quickly heated, held at that temperature for a few seconds, and then quenched, or alternatively for one that is maintained at the temperatures for very long times.

- 1.42. Is it possible for a metal to be completely isotropic? Explain.

This answer can be answered only if isotropy is defined within limits. For example:

- A single crystal of a metal has an inherent an unavoidable anisotropy. Thus, at a length scale that is on the order of a material's grain size, a metal will always be anisotropic.
- A metal with elongated grains will have a lower strength and hardness in one direction than in others, and this is unavoidable.
- However, a metal that contains a large number of small and equiaxed grains will have the first two effects essentially made very small; the metal may be isotropic within measurement limits.
- Annealing can lead to equiaxed grains, and depending on the measurement limits, this can essentially result in an isotropic metal.
- A metal with a very small grain size (i.e., a metal glass) can have no apparent crystal structure or slip systems, and can be essentially isotropic.

- 1.43. Referring to Fig. 1.1, assume you can make a ball bearing from a single crystal. What advantages and disadvantages would such a bearing have?

This is a challenging problem, and is one that can be reintroduced at the end of Chapters 1-4. Students should be encouraged to provide answers that are based on their understanding and experience; these are intended to start a discussion.

The advantages of a single crystal bearing include:

- Corrosion generally starts at a grain boundary, where loosely packed atoms are more susceptible to chemical attack; therefore, the bearing could be more corrosion resistant.
- Fatigue cracks may be more difficult to form, but with some materials are more difficult to propagate through a grain than along a grain boundary.
- The strength could be higher than a polycrystalline metal.
- Distortion of the bearing at elevated temperatures may not be an issue.

Disadvantages include:

- The ductility of the bearing may be suspect, since plastic deformation associated with dislocation motion are not present.
- Because of the anisotropy of the material, producing a perfect sphere may be very difficult.
- Cost may be a serious concern.

1.44. Referring to Fig. 1.10, explain why edge dislocations cannot cross grain boundaries using appropriate sketches.

Figure 1.10 shows that dislocations can move inside a lattice in close-packed directions. At grain boundaries, it should be recognized that the direction of the lattice changes on each side; there is then no natural direction for a dislocation to travel. This is shown below for a simple cubic arrangement. Note that the grain on the left (blue) and the grain on the right (red) are rotated  $15^\circ$  from each other. The grain boundary in between has only occasional direct contact between atoms, and has a large gap where dislocations would have a difficult time moving and translating across. Note that grain boundaries often have impurities, which are not shown.

## Quantitative Problems

1.45. How many atoms are in a single repeating cell of an bcc crystal structure? How many in a repeating cell of an hcp structure?

For an fcc structure, refer to Fig. 1.4a. The atoms at each corner are shared by eight unit cells, and there are eight of these atoms. Therefore, the corners contribute one total atom. The atoms on the faces are each shared by two cells, and there are six of these atoms. Therefore, the atoms on the faces contribute a total of three atoms to the unit cell. Therefore, the total number of atoms in an fcc unit cell is four atoms.

For the hcp, refer to Fig. 1.5. The atoms on the periphery of the top and bottom are each shared by six cells, and there are 12 of these atoms (on top and bottom), for a contribution of two atoms. The atoms in the center of the hexagon are shared by two cells, and there are two of these atoms, for a net contribution of one atom. There are also three atoms fully contained in the unit cell. Therefore, there are six atoms in an hcp unit cell.

1.46. The atomic weight of gold is 196.97, meaning that  $6.023 \times 10^{23}$  atoms weigh 196.97 grams. The density of gold is  $19,320 \text{ kg/m}^3$ , and pure gold forms fcc crystals. Estimate the diameter of a gold atom.

Consider the face of the fcc unit cell, which consists of a right triangle with side length  $a$  and hypotenuse of  $4r$ . From the Pythagorean theorem,  $a = 4r/\sqrt{2}$ . Therefore, the volume of the unit cell is

$$V = a^3 = \left(\frac{4r}{\sqrt{2}}\right)^3 = 22.63r^3$$

Each fcc unit cell has four atoms (see Prob. 1.34), and each atom has a mass of

$$\text{Mass} = \frac{196.97 \text{ g}}{6.023 \times 10^{23}} = 3.27 \times 10^{-22} \text{ g} = 3.27 \times 10^{-25} \text{ kg}$$

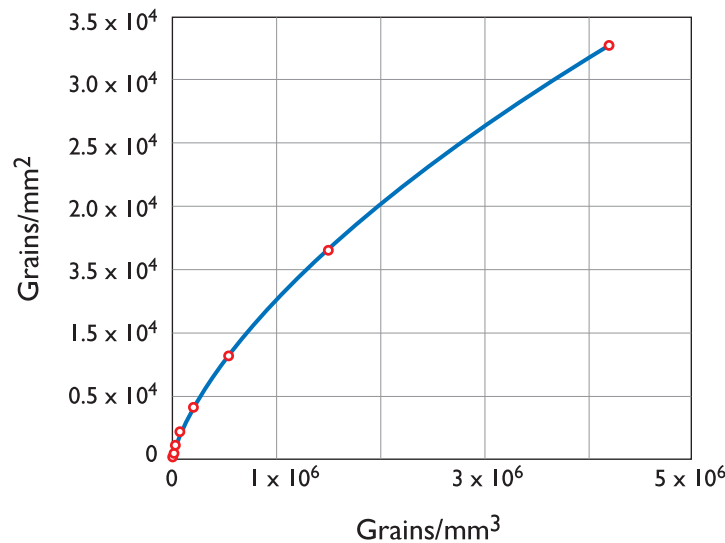
So that the density inside a fcc unit cell is

$$\rho = \frac{\text{Mass}}{V} = 19,320 \text{ kg/m}^3 = \frac{4(3.27 \times 10^{-25}) \text{ kg}}{22.63r^3}$$

Solving for  $r$  yields  $r = 1.444 \times 10^{-10} \text{ m}$ , or  $1.44 \text{ \AA}$ . Note that the accepted value is  $1.6 \text{ \AA}$ ; the difference is attributable to a number of factors, including impurities in crystal structure and a concentration of mass in the nucleus of the atom.

1.47. Plot the data given in Table 1.1 in terms of grains/mm<sup>2</sup> vs. grains/mm<sup>3</sup>, and discuss your observations.

The plot is shown below. It can be seen that the grains per cubic millimeter increases faster than the grains per square millimeter. This relationship is to be expected since the volume of an equiaxed grain depends on the diameter cubed, whereas its area depends on the diameter squared.



1.48. A strip of metal is reduced from 50 mm in thickness to 25 mm by cold working; a similar strip is reduced from 50 mm to 30 mm. Which of these cold-worked strips will recrystallize at a lower temperature? Why?

The metal that is reduced to 25 mm by cold working will recrystallize at a lower temperature. The more the cold work the lower the temperature required for recrystallization. This is because the number of dislocations and energy stored in the material increases with cold work. Thus, when recrystallizing a more highly cold worked material, this energy can be recovered and less energy needs to be imparted to the material.

- 1.49. The ball of a ballpoint pen is 1.6 mm in diameter and has an ASTM grain size of 12. How many grains are there in the ball?

From Table 1.1, a metal with an ASTM grain size of 10 has about 520,000 grains/mm<sup>3</sup>. The volume of the ball is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.8)^3 = 2.143 \text{ mm}^3$$

Multiplying the volume by the grains per cubic millimeter gives the number of grains in the paper clip as about 2.18 million.

- 1.50. How many grains are on the surface of the head of a pin? Assume that the head of a pin is spherical with a 2-mm diameter and has an ASTM grain size of 12.

Note that the surface area of a sphere is given by  $A = 4\pi r^2$ . Therefore, a 2 mm diameter head has a surface area of

$$A = 4\pi r^2 = 4\pi(1)^2 = 4\pi \text{ mm}^2$$

From Eq. (1.2), the number of grains per area is

$$N = 2^{11} = 2048$$

This is the number of grains per 0.0645 mm<sup>2</sup> of actual area; therefore the number of grains on the surface is

$$N_g = \frac{2048 (4\pi)}{0.0645} = 398804 \text{ grains}$$

- 1.51. The unit cells shown in Figs. 1.3 through 1.5 can be represented by tennis balls arranged in various configurations in a box. In such an arrangement, the *atomic packing factor* (APF) is defined as the ratio of the sum of the volumes of the atoms to the volume of the unit cell. Show that the APF is 0.68 for the bcc structure and 0.74 for the fcc structure.

Note that the bcc unit cell in Fig. 1.3a has 2 atoms inside of it; one inside the unit cell and eight atoms that have one-eighth of their volume inside the unit cell. Therefore the volume of atoms inside the cell is  $8\pi r^3/3$ , since the volume of a sphere is  $4\pi r^3/3$ . Note that the diagonal of a face of a unit cell has a length of  $a\sqrt{2}$ , which can be easily determined from the Pythagorean theorem. Using that diagonal and the height of  $a$  results in the determination of the diagonal of the cube as  $a\sqrt{3}$ . Since there are four radii across that diagonal, it can be deduced that

$$a\sqrt{3} = 4r \quad \rightarrow \quad a = \frac{4r}{\sqrt{3}}$$

The volume of the unit cell is  $a^3$ , so

$$V = a^3 = \left(\frac{4r}{\sqrt{3}}\right)^3 = \left(\frac{4}{\sqrt{3}}\right)^3 r^3.$$

Therefore, the atomic packing factor is

$$\text{APF}_{\text{bcc}} = \frac{\frac{8}{3}\pi r^3}{\left(\frac{4}{\sqrt{3}}\right)^3 r^3} = 0.68$$

For the fcc cell, there are four atoms in the cell, so the volume of atoms inside the fcc unit cell is  $16\pi r^3/3$ . On a face of the fcc cell, it can be shown from the Pythagorean theorem that the hypotenuse is  $a\sqrt{2}$ . Also, there are four radii across the diameter, so that

$$a\sqrt{2} = 4r \quad \rightarrow \quad a = 2r\sqrt{2}$$

Therefore, the volume of the unit cell is

$$V = a^3 = (2r\sqrt{2})^3$$

so that the atomic packing factor is

$$\text{APF}_{\text{fcc}} = \frac{\frac{16}{3}\pi r^3}{(2\sqrt{2})^3 r^3} = 0.74$$

- 1.52. Show that the lattice constant  $a$  in Fig. 1.4a is related to the atomic radius by the formula  $a = 2\sqrt{2}R$ , where  $R$  is the radius of the atom as depicted by the tennis-ball model.

For a face centered cubic unit cell as shown in Fig. 1.4a, the Pythagorean theorem yields

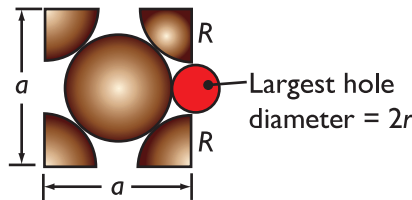
$$a^2 + a^2 = (4r)^2$$

Therefore,

$$2a^2 = 16r^2 \rightarrow a = 2\sqrt{2}r$$

- 1.53. Show that, for the fcc unit cell, the radius  $r$  of the largest hole is given by  $r = 0.414R$ . Determine the size of the largest hole for the iron atoms in the fcc structure.

The largest hole is shown in the sketch below. Note that this hole occurs in other locations, in fact in three other locations of this sketch.



For a face centered cubic unit cell as shown in Fig. 1.4a, the Pythagorean theorem yields

$$a^2 + a^2 = (4R)^2$$

Therefore,

$$2a^2 = 16R^2 \rightarrow a = 2\sqrt{2}R$$

Also, the side dimension  $a$  is

$$a = 2R + 2r$$

Therefore, substituting for  $a$ ,

$$2\sqrt{2}R = 2R + 2r \rightarrow r = (\sqrt{2} - 1)R = 0.414R$$

- 1.54. A technician determines that the grain size of a certain etched specimen is 8. Upon further checking, it is found that the magnification used was  $150\times$ , instead of the  $100\times$  that is required by the ASTM standards. Determine the correct grain size.

If the grain size is 8, then there are 2048 grains per square millimeter (see Table 1.1). However, the magnification was too large, meaning that too small of an area was examined. For a magnification of  $100\times$ , the area is reduced by a factor of  $1/1.82=0.309$ . Therefore, there really are 632 grains per  $\text{mm}^2$ , which corresponds to a grain size between 6 and 7.

- 1.55. If the diameter of the aluminum atom is 0.28 nm, how many atoms are there in a grain of ASTM grain size 10?

If the grain size is 8, there are 65,000 grains per cubic millimeter of aluminum - see Table 1.1. Each grain has a volume of  $1/65,000 = 1.538 \times 10^{-5} \text{ mm}^3$ . Note that for an fcc material there are four atoms per unit cell (see solution to Prob. 1.43), with a total volume of  $16\pi R^3/3$ , and that the diagonal,  $a$ , of the unit cell is given by

$$a = (2\sqrt{2}) R$$

Hence,

$$\text{APF}_{\text{fcc}} = \frac{(16\pi R^3/3)}{(2R\sqrt{2})^3} = 0.74$$

Note that as long as all the atoms in the unit cell have the same size, the atomic packing factors do not depend on the atomic radius. Therefore, the volume of the grain which is taken up by atoms is  $(4.88 \times 10^{-4})(0.74) = 3.61 \times 10^{-4} \text{ mm}^3$ . (Recall that  $1 \text{ mm} = 10^6 \text{ nm}$ .) If the diameter of an aluminum atom is 0.5 nm, then its radius is 0.25 nm or  $0.25 \times 10^{-6} \text{ mm}$ . The volume of an aluminum atom is then

$$V = 4\pi R^3/3 = 4\pi(0.25 \times 10^{-6})^3/3 = 6.54 \times 10^{-20} \text{ mm}^3$$

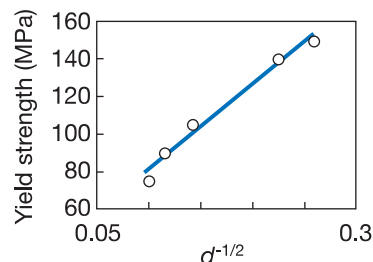
Dividing the volume of aluminum in the grain by the volume of an aluminum atom yields the total number of atoms in the grain as  $(1.538 \times 10^{-5})/(6.54 \times 10^{-20}) = 2.35 \times 10^{14}$ .

- 1.56. The following data are obtained in tension tests of brass:

Grain size ( $\mu\text{m}$ )	Yield stress (MPa)
30	150
40	140
100	105
150	90
200	75

Does the material follow the Hall-Petch effect? If so, what is the value of  $k$ ?

First, it is obvious from this table that the material becomes stronger as the grain size decreases, which is the expected result. However, it is not clear whether Eq. (1.1) is applicable. It is possible to plot the yield stress as a function of grain diameter, but it is better to plot it as a function of  $d^{-1/2}$ , as follows:



The least-squares curve fit for a straight line is

$$Y = 35.22 + 458d^{-1/2}$$

with an  $R$  factor of 0.990. This suggests that a linear curve fit is proper, and it can be concluded that the material does follow the Hall-Petch effect, with a value of  $k = 458 \text{ MPa}\cdot\sqrt{\mu\text{m}}$ .

- 1.57. Does water have a homologous temperature? What is the highest temperature where water (ice) can get cold worked?

All materials with a melting point have a homologous temperature, defined as the ratio of the working temperature to the melting temperature on an absolute scale. If water had strengthening mechanisms like metals, it could be cold worked up to one-third of its melting temperature. Since water melts at 0°C or 273 K, it could be cold worked up to 273/3=91 K = -182 °C. However, water is a complicated material with many possible crystal structures, and does not in general display cold working like metals.

- 1.58. The atomic radius of iron is 0.125 nm, while that of a carbon atom is 0.070 nm. Can a carbon atom fit inside a steel bcc structure without distorting the neighboring atoms?

Consider the bcc cell shown in Fig. 1.3a. The diagonal is  $4R$ . The length of a side of the bcc unit cell is  $a$ , so that

$$a^2 + a^2 + a^2 = (4R)^2$$

or  $a = 2.31R$ . Since there are two radii on an edge, the size of the opening is  $0.31R$ . For iron, this would be  $(0.31)(0.125)=0.0387$  nm, which is too small for a carbon atom to fit without distortion.

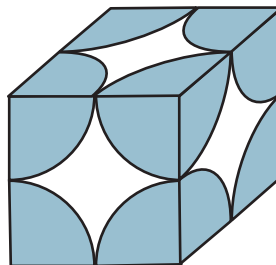
- 1.59. Estimate the atomic radius for the following materials and data: (a) Aluminum (atomic weight = 26.98 grams per mole, density = 2700 kg/m<sup>3</sup>) (b) silver (atomic weight=107.87 grams/mole, density = 10,500 kg/m<sup>3</sup>); (c) titanium (atomic weight = 47.87 grams per mole, density = 4506 kg/m<sup>3</sup>).

Refer to Problem 1.46. Aluminum and silver are fcc materials; titanium can be fcc or hcp. The radii for aluminum is found as  $r = 1.44\text{Å}$ ; 1.45 Å for silver, and 1.46 for Titanium. The following Matlab code can be used to evaluate other materials:

```
M=47.87/(6.023e23)/1000;
rho=4506;
r=(4*M/rho/22.63)^0.3333;
```

- 1.60. A simple cubic structure involves atoms located at the cube corners that are in contact with each other along the cube edges. Make a sketch of a simple cubic structure, and calculate its atomic packing factor.

The sketch is as follows:



The edge has a length of two times the atom radius, so that the volume of the cube is  $8a^3$ . There is one atom in the structure, which would take a volume of  $\pi a^3$ . Therefore, the atomic packing factor is

$$APF = \frac{\pi a^3}{8a^3} = 39\%.$$

- 1.61. Estimate the ASTM grain size number for a 300 mm silicon wafer used to produce computer chips.

The notion of a grain is not useful for silicon wafers, but it provides an interesting demonstration of the ASTM grain size number. If a cross section of a 300-mm diameter is considered, it would have an area of  $A = \pi d^2/4 = \pi(300)^2/4 = 70,680 \text{ mm}^2$ .  $0.0645 \text{ mm}$  would represent  $9.12 \times 10^{-7}$  grains. Therefore,

$$9.12 \times 10^{-7} = 2^{n-1}$$

$$\log 9.12 \times 10^{-7} = (n - 1) \log 2$$

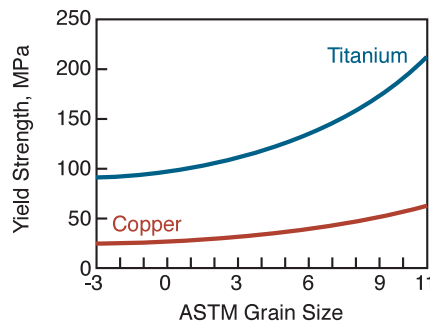
or  $n = -19$ .

- 1.62. Pure copper and pure titanium follow the Hall-Petch equation. For copper,  $S_{yi} = 24 \text{ MPa}$  and  $k = 0.12 \text{ MPa}\cdot\text{m}^{1/2}$ . For titanium,  $S_{yi} = 80 \text{ MPa}$  and  $k = 0.40 \text{ MPa}\cdot\text{m}^{1/2}$ . (a) Plot the yield strength of these metals as a function of grain size for ASTM grain sizes of -3 to 11. (b) Explain which material would see greater strengthening from a reduction in grain size, as in cold working.

Note from Eq. (1.2) that for an ASTM grain size of -3, there would be  $N = 2^{-4} = 0.0625$  grains in  $0.0645 \text{ mm}^2$ . Hence, each grain would have an area of around  $1.03 \text{ mm}^2$ . Using  $A = \pi d^2/4$  would give an estimate of a grain of diameter  $1.15 \text{ mm}$ . For a grain size of 11, the grain diameter is similarly estimated as  $d = 0.008955 \text{ mm} = 8.955 \times 10^{-6} \text{ m}$ . The Hall-Petch equation is given by

$$S_y = S_{yi} + kd^{-1/2}$$

The desired plot is as follows:



As can be seen, titanium strengthens more with a decrease in grain size (higher ASTM grain size number).

## Synthesis, Design, and Projects

- 1.63. By stretching a thin strip of polished metal, as in a tension-testing machine, demonstrate and comment on what happens to its reflectivity as the strip is being stretched.

The polished surface is initially smooth, which allows light to be reflected uniformly across the surface. As the metal is stretched, the reflective surface of the polished sheet metal will begin to become dull. The slip and twin bands developed at the surface cause roughening (see Fig. 1.7), which tends to scatter the reflected light.

- 1.64. Draw some analogies to mechanical fibering—for example, layers of thin dough sprinkled with flour or melted butter between each layer.

A wide variety of acceptable answers are possible based on the student's experience and creativity. Some examples of mechanical fibering include: (a) food products such as lasagna, where layers of noodles bound sauce, or pastries with many thin layers, such as baklava; (b) log cabins, where tree trunks are oriented to construct walls and then sealed with a matrix; and (c) straw-reinforced mud.

1.65. Draw some analogies to the phenomenon of hot shortness.

Some analogies to hot shortness include: (a) a brick wall with deteriorating mortar between the bricks, (b) time-released medicine, where a slowly soluble matrix surrounds doses of quickly soluble medicine, and (c) an Oreo cookie at room temperature compared to a frozen cookie.

1.66. Obtain a number of small balls made of plastic, wood, marble, or metal, and arrange them with your hands or glue them together to represent the crystal structures shown in Figs. 1.3–1.5. Comment on your observations.

By the student. There are many possible comments, including the relative densities of the three crystal structures (hcp is clearly densest). Also, the ingenious and simple solid-ball models are striking when performing such demonstrations.

1.67. Take a deck of playing cards, place a rubber band around it, and then slip the cards against each other to represent Figs. 1.6a and 1.7. If you repeat the same experiment with more and more rubber bands around the same deck, what are you accomplishing as far as the behavior of the deck is concerned?

By the student. With an increased number of rubber bands, you are physically increasing the friction force between each card. This is analogous to increasing the magnitude of the shear stress required to cause slip. Furthermore, the greater the number of rubber bands, the higher the shear or elastic modulus of the material (see Section 2.4). This problem can be taken to a very effective extreme by using small C-clamps to highly compress the cards; the result is an object that acts like one solid, with much higher stiffness than the loose cards.

1.68. Give examples in which anisotropy is scale dependent. For example, a wire rope can contain annealed wires that are isotropic on a microscopic scale, but the rope as a whole is anisotropic.

All materials may behave in an anisotropic manner when considered at atomic scales, but when taken as a continuum, many materials are isotropic. Other examples include:

- Clothing, which overall appears to be isotropic, but clearly has anisotropy defined by the direction of the threads in the cloth. This anisotropic behavior can be verified by pulling small patches of the cloth in different directions.
- Wood has directionality (orthotropic) but it can be ignored for many applications.
- Human skin: it appears isotropic at large length scales, but microscopically it consists of cells with varying strengths within the cell.

1.69. The movement of an edge dislocation was described in Section 1.4.1 by means of an analogy involving a hump in a carpet on the floor and how the whole carpet can eventually be moved by moving the hump forward. Recall that the entanglement of dislocations was described in terms of two humps at different angles. Use a piece of cloth placed on a flat table to demonstrate these phenomena.

By the student. This can be clearly demonstrated, especially with a cloth that is compliant (flexible) but has high friction with a flat surface. Two methods of ensuring this is the case are (a) to use a cotton material (as found in T-shirts) and wetting it before conducting the experiments, or (b) spraying the bottom side of the fabric with temporary adhesives, as found in most arts and office supply stores. The experiments (single and two lumps) can then be conducted and observations made.

1.70. If you wanted to strengthen a material, would you wish to have it consist of one grain, or would you want it to have grains that contain the minimum number of atoms? Explain.

Most likely, the desire would be to have grains with the minimum number of atoms, as suggested by Eq. (1.1) (The Hall-Petch equation). This is the rationale for the use of metallic glasses, with no identifiable grain structure.

