Chapter 2

2.1 $\mathbf{C} = \mathbf{A} + \mathbf{B} = \begin{bmatrix} 17 & -10 & 1 \\ -4 & 4 & 4 \\ -3 & -4 & 0 \end{bmatrix}$; $\mathbf{D} = \mathbf{A} - \mathbf{B} = \begin{bmatrix} 1 & 4 & 3 \\ 10 & -14 & 4 \\ -1 & -4 & 12 \end{bmatrix}$

2.2 $\mathbf{C} = 2\mathbf{A} + \mathbf{B} = \begin{bmatrix} 17 & -4 & -12 & -7 \\ -12 & -3 & 0 & 19 \\ -11 & -2 & 2 & 9 \end{bmatrix}$; $\mathbf{D} = \mathbf{A} - 3\mathbf{B} = \begin{bmatrix} -9 & 5 & -20 & -7 \\ -20 & -19 & 21 & -15 \\ -2 & -8 & -27 & 22 \end{bmatrix}$

2.3 $\mathbf{C}=\mathbf{AB}=\begin{bmatrix}-5\\2\\-8\\4\end{bmatrix}\begin{bmatrix}3 & -7 & 1 & -6\end{bmatrix}=\begin{bmatrix}-15 & 35 & -5 & 30\\6 & -14 & 2 & -12\\-24 & 56 & -8 & 48\\12 & -28 & 4 & -24\end{bmatrix}$

$\mathbf{D}=\mathbf{BA}=\begin{bmatrix}3 & -7 & 1 & -6\end{bmatrix}\begin{bmatrix}-5\\2\\-8\\4\end{bmatrix}=-15-14-8-24=-61$

2.4 $\mathbf{C} = \mathbf{AB} = \begin{bmatrix} 1 & -10 & -2 & -6 \\ 3 & -8 & -12 & 8 \end{bmatrix} \begin{bmatrix} 2 & -10 \\ 6 & -1 \\ -2 & -10 \\ 5 & 9 \end{bmatrix} = \begin{bmatrix} -84 & -74 \\ 22 & -70 \end{bmatrix}$

$$\mathbf{D} = \mathbf{BA} = \begin{bmatrix} 2 & -10 \\ 6 & -1 \\ -2 & 10 \\ 5 & 9 \end{bmatrix} \begin{bmatrix} 1 & -10 & -2 & -6 \\ 3 & -8 & -12 & 8 \end{bmatrix} = \begin{bmatrix} -28 & 60 & 116 & 68 \\ 3 & -52 & 0 & -28 \\ 28 & 60 & -116 & -68 \\ 32 & -122 & -118 & -102 \end{bmatrix}$$

2.5 $\mathbf{C} = \mathbf{AB} = \begin{bmatrix} 5 & 7 & -2 \\ 7 & 9 & 0 \\ -2 & 0 & 8 \end{bmatrix} \begin{bmatrix} 2 & -8 & 1 \\ -8 & 3 & 5 \\ 1 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -48 & -29 & 28 \\ -58 & -29 & 52 \\ 4 & 56 & 46 \end{bmatrix}$

$$\mathbf{D} = \mathbf{BA} = \begin{bmatrix} 2 & -8 & 1 \\ -8 & 3 & 5 \\ 1 & 5 & 6 \end{bmatrix} \begin{bmatrix} 5 & 7 & -2 \\ 7 & 9 & 0 \\ -2 & 0 & 8 \end{bmatrix} = \begin{bmatrix} -48 & -58 & 4 \\ -29 & -29 & 56 \\ 28 & 52 & 46 \end{bmatrix}$$

2.6 $\mathbf{C} = \mathbf{AB} = \begin{bmatrix} 3 & 4 & 5 \\ 6 & 3 & 0 \\ -3 & 0 & -7 \end{bmatrix} \begin{bmatrix} 8 & 1 & 6 & -2 \\ 6 & -2 & 5 & 8 \\ 3 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 54 & -5 & 36 & -26 \\ 10 & -4 & 35 & 20 \\ -6 & -3 & -24 & 6 \end{bmatrix}$

2.8 $\mathbf{AB} = \begin{bmatrix} 14 & -5 & 11 \\ -3 & 18 & -9 \end{bmatrix} \begin{bmatrix} 12 & -7 & 6 & 0 \\ 2 & 9 & 15 & -13 \\ 10 & -4 & -5 & 8 \end{bmatrix} = \begin{bmatrix} 268 & -187 & -46 & 153 \\ -90 & 219 & 297 & -306 \end{bmatrix}$

$$(\mathbf{AB})^T = \begin{bmatrix} 268 & -90 \\ -187 & 219 \\ -46 & 297 \\ 153 & -306 \end{bmatrix} \tag{1}$$

$$\mathbf{B}^T \mathbf{A}^T = \begin{bmatrix} 12 & 2 & 10 \\ -7 & 9 & -4 \\ 6 & 15 & -5 \\ 0 & -13 & 8 \end{bmatrix} \begin{bmatrix} 14 & -3 \\ -5 & 18 \\ 11 & -9 \end{bmatrix} = \begin{bmatrix} 268 & -90 \\ -187 & 219 \\ -46 & 297 \\ 153 & -306 \end{bmatrix} \tag{2}$$

From Eqs. (1) and (2), we can see that $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$.

2.9 $[A][B][C]=\begin{bmatrix}-9 & 0\\ 13 & 20\\ 8 & -3\\ -11 & -5\end{bmatrix}\begin{bmatrix}15 & -1 & -4\\ 6 & 16 & 9\end{bmatrix}\begin{bmatrix}-7 & 10 & 6 & 0\\ -1 & 2 & -8 & -2\\ 16 & 12 & 2 & 8\end{bmatrix}$

$$=\begin{bmatrix}-135 & 9 & 36\\ 315 & 307 & 128\\ 102 & -56 & -59\\ -195 & -69 & -1\end{bmatrix}\begin{bmatrix}-7 & 10 & 6 & 0\\ -1 & 2 & -8 & -2\\ 16 & 12 & 2 & 8\end{bmatrix}$$

$$=\begin{bmatrix}1512 & -900 & -810 & 270\\ -464 & 5300 & -310 & 410\\ -1602 & 200 & 942 & -360\\ 1418 & -2100 & -620 & 130\end{bmatrix}$$

$$\left([A][B][C]\right)^T=\begin{bmatrix}1512 & -464 & -1602 & 1418\\ -900 & 5300 & 200 & -2100\\ -810 & -310 & 942 & -620\\ 270 & 410 & -360 & 130\end{bmatrix} \tag{1}$$

$$[C]^T[B]^T[A]^T=\begin{bmatrix}-7 & -1 & 16\\ 10 & 2 & 12\\ 6 & -8 & 2\\ 0 & -2 & 8\end{bmatrix}\begin{bmatrix}15 & 6\\ -1 & 16\\ -4 & 9\end{bmatrix}\begin{bmatrix}-9 & 13 & 8 & -11\\ 0 & 20 & -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}-168 & 86\\ 100 & 200\\ 90 & -74\\ -30 & 40\end{bmatrix}\begin{bmatrix}-9 & 13 & 8 & -11\\ 0 & 20 & -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}1512 & -464 & -1602 & 1418\\ -900 & 5300 & 200 & -2100\\ -810 & -310 & 942 & -620\\ 270 & 410 & -360 & 130\end{bmatrix} \tag{2}$$

From Eqs. (1) and (2), we can see that

$$\left([A][B][C]\right)^T=[C]^T[B]^T[A]^T$$

$$\textbf{2.10} \quad \mathbf{C} = \mathbf{B}^T\mathbf{AB} = \begin{bmatrix} 5 & -7 & 3 \\ 7 & 8 & -4 \\ -3 & 4 & 9 \end{bmatrix} \begin{bmatrix} 40 & -10 & -25 \\ -10 & 15 & 12 \\ -25 & 12 & 30 \end{bmatrix} \begin{bmatrix} 5 & 7 & -3 \\ -7 & 8 & 4 \\ 3 & -4 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 195 & -119 & -119 \\ 300 & 2 & -199 \\ -385 & 198 & 393 \end{bmatrix} \begin{bmatrix} 5 & 7 & -3 \\ -7 & 8 & 4 \\ 3 & -4 & 9 \end{bmatrix} = \begin{bmatrix} 1,451 & 889 & -2,132 \\ 889 & 2,912 & -2,683 \\ -2,132 & -2,683 & 5,484 \end{bmatrix}$$

2.11 $\mathbf{C} = \mathbf{B}^T\mathbf{AB} = \begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \\ -0.6 & 0.8 \\ -0.8 & -0.6 \end{bmatrix} \begin{bmatrix} 300 & -100 \\ -100 & 200 \end{bmatrix} \begin{bmatrix} 0.6 & 0.8 & -0.6 & -0.8 \\ -0.8 & 0.6 & 0.8 & -0.6 \end{bmatrix}$

$$= \begin{bmatrix} 260 & -220 \\ 180 & 40 \\ -260 & 220 \\ -180 & -40 \end{bmatrix} \begin{bmatrix} 0.6 & 0.8 & -0.6 & -0.8 \\ -0.8 & 0.6 & 0.8 & -0.6 \end{bmatrix} = \begin{bmatrix} 332 & 76 & -332 & -76 \\ 76 & 168 & -76 & -168 \\ -332 & -76 & 332 & 76 \\ -76 & -168 & 76 & 168 \end{bmatrix}$$

2.13 $$\frac{d\mathbf{A}}{dx} = \begin{bmatrix} 2\sin x\cos x & -9x^2 & -6 \\ -12x & -2\cos x\sin x & 0 \\ 0 & 3\cos x & \sec^2 x \end{bmatrix}$$

2.14 $\mathbf{C} = \mathbf{A} + \mathbf{B} = \begin{bmatrix} 2x^2 - 3x & -x + 5 \\ 4x^2 - 12x & -x^3 + 8 \\ 2x^3 - 7 & -3x^2 + 5x \\ 2x^3 - 1 & -x^2 + 6x \end{bmatrix}$; $\dfrac{d\mathbf{C}}{dx} = \begin{bmatrix} 4x - 3 & -1 \\ 8x - 12 & -3x^2 \\ 6x^2 & -6x + 5 \\ 6x^2 & -2x + 6 \end{bmatrix}$

2.15 $[A][B]=\begin{bmatrix} 12-30x^4 & -10x^2-20x^3 \\ -30x^3 & -2x-4x^2+9x^5 \\ -6x+14x^2+25x^5 & 28x+8x^3 \end{bmatrix}$

$$\frac{d[A][B]}{dx}=\begin{bmatrix} -120x^3 & -20x-60x^2 \\ -90x^2 & -2-8x+45x^4 \\ -6+28x+125x^4 & 28+24x^2 \end{bmatrix}$$

2.16 $$\frac{\partial[A]}{\partial x}=\begin{bmatrix}2x & 0 & 0\\ 0 & 3y & 0\\ 0 & 0 & 4z\end{bmatrix}$$

$$\frac{\partial[A]}{\partial y}=\begin{bmatrix}0 & -2y & 0\\ -2y & 3x & -z\\ 0 & -z & 0\end{bmatrix}$$

$$\frac{\partial[A]}{\partial z}=\begin{bmatrix}0 & 0 & 4z\\ 0 & 0 & -y\\ 4z & -y & 4x\end{bmatrix}$$

2.17 $$\int_0^L \mathbf{A}\,dx = \begin{bmatrix} 2L^3 & -\dfrac{L^4}{4} & 9L \\ -\dfrac{L^4}{4} & \dfrac{2L^5}{5} & 2L^2 \\ 9L & 2L^2 & \dfrac{5L^4}{4} \end{bmatrix}$$

2.18 $\int_0^L [A]\,dx = \begin{bmatrix} L^2 & \cos L - 1 & \sin L \cos L + L \\ \cos L - 1 & 5L & -L^4 \\ \sin L \cos L + L & -L^4 & L - \dfrac{L^3}{3} \end{bmatrix}$

2.19 $[A][B]=\begin{bmatrix} 10x^2+9x^3+2x^4 & -9-4x^3-x^5 \\ -9x^2+6x^6 & -2x^3 \end{bmatrix}$

$$\int_0^L([A][B])\,dx=\begin{bmatrix} \dfrac{10L^3}{3}+\dfrac{9L^4}{4}+\dfrac{2L^5}{5} & -9L-L^4-\dfrac{L^6}{6} \\ -3L^3+\dfrac{6L^7}{7} & -\dfrac{L^4}{2} \end{bmatrix}$$

2.20 $\mathbf{AA}^T = \begin{bmatrix} -0.28 & -0.96 & 0 & 0 \\ 0.96 & -0.28 & 0 & 0 \\ 0 & 0 & -0.28 & -0.96 \\ 0 & 0 & 0.96 & -0.28 \end{bmatrix} \begin{bmatrix} -0.28 & 0.96 & 0 & 0 \\ -0.96 & -0.28 & 0 & 0 \\ 0 & 0 & -0.28 & 0.96 \\ 0 & 0 & -0.96 & -0.28 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Thus, **A** is orthogonal.

$$\mathbf{BB}^T = \begin{bmatrix} -0.28 & 0.96 & 0 & 0 \\ 0.96 & -0.28 & 0 & 0 \\ 0 & 0 & -0.28 & 0.96 \\ 0 & 0 & 0.96 & -0.28 \end{bmatrix} \begin{bmatrix} -0.28 & 0.96 & 0 & 0 \\ 0.96 & -0.28 & 0 & 0 \\ 0 & 0 & -0.28 & 0.96 \\ 0 & 0 & 0.96 & -0.28 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -0.5376 & 0 & 0 \\ -0.5376 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.5376 \\ 0 & 0 & -0.5376 & 1 \end{bmatrix}$$

Thus, **B** is not orthogonal.

2.21 $$\left[\begin{array}{ccc|c} 2 & -3 & 1 & -18 \\ -9 & 5 & 3 & 18 \\ 4 & 7 & -8 & 53 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1.5 & 0.5 & -9 \\ 0 & -8.5 & 7.5 & -63 \\ 0 & 13 & -10 & 89 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -0.82353 & 2.1176 \\ 0 & 1 & -0.88235 & 7.4118 \\ 0 & 0 & 1.4706 & -7.3529 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

Thus, $\underline{x_1 = -2 \quad x_2 = 3 \quad x_3 = -5}$

2.22 $\left[\begin{array}{ccc|c} 20 & -9 & 15 & 354 \\ -9 & 16 & -5 & -275 \\ 15 & -5 & 18 & 307 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 1 & -0.45 & 0.75 & 17.7 \\ 0 & 11.95 & 1.75 & -115.7 \\ 0 & 1.75 & 6.75 & 41.5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0.8159 & 13.343 \\ 0 & 1 & 0.14644 & -9.682 \\ 0 & 0 & 6.4937 & 58.444 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

Thus, $x_1 = 6 \quad x_2 = -11 \quad x_3 = 9$