1.7 Plate with width change, Fig. A.11(c).

$P = 3600\ N$, $w_2 = 24$, $w_1 = 16$, $t = 5\ mm$

Polycarbonate, $\sigma_o = 62\ MPa$, $\varepsilon_f = 110$ to 150%

$X_1 = ?$ adequate?

$$S = \frac{P}{w_1 t} = \frac{3600\ N}{16(5)\ mm^2} = 45\ MPa$$

$$X_1 = \frac{\sigma_o}{S} = \frac{62\ MPa}{45\ MPa} = 1.38$$ ◀

The value is a bit low but may be suitable under ideal circumstances. Note that the material is quite ductile. ◀

1.8 Shaft with circumferential groove, Fig. A.12 (c). $d_2 = 25$, $d_1 = 20$, $\rho = 2.5$ mm
Aluminum alloy, $\sigma_o = 303$ MPa, $\varepsilon_f = 20\%$
$M = 120$ N·m. $X_1 = ?$ adequate?

$$S = \frac{32M}{\pi d_1^3} = \frac{32\,(120{,}000\ \text{N·mm}}{\pi\,(20\ \text{mm})^3} = 152.8\ \text{MPa}$$

$$X_1 = \frac{\sigma_o}{S} = \frac{303\ \text{MPa}}{152.8\ \text{MPa}} = 1.98 \qquad ◀$$

The value is quite adequate in view of the ductile material behavior. ◀

3.11 Cantilever beam, circular cross sec.

$$v_{max} = \frac{PL^3}{3EI}, \quad I = \frac{\pi r^4}{4} \quad \text{(Figs. A.4, A.2)}$$

Requirements: L, P, v_{max}

Geometry: r Material: ρ, E

Minimize: (a) $m = \pi r^2 L \rho$

(b) cost, $C_m m$

$$v_{max} = \frac{PL^3}{3E}\frac{4}{\pi r^4}, \quad r^2 = \left(\frac{4PL^3}{3\pi E v_{max}}\right)^{0.5}$$

$$m = \pi L \rho \left(\frac{4PL^3}{3\pi E v_{max}}\right)^{0.5} = f_1(\text{Req.})\, f_2(\text{Mat'l.})$$

$$m = \left[2L^{2.5}\left(\frac{\pi P}{3 v_{max}}\right)^{0.5}\right]\left[\frac{\rho}{\sqrt{E}}\right] = f_1 f_2$$

For the Table 3.13 materials, use the properties given to calculate:

(a) $f_2 = \rho/\sqrt{E}$, (b) $f_2 = C_m \rho/\sqrt{E}$

(a)

Material	Modulus E, GPa	Density ρ, g/cm^3	Mass f_2 $\rho/E^{0.5}$	Mass Rank
1020 steel	203	7.9	0.554	7
4340 steel	207	7.9	0.549	6
7075 Al	71	2.7	0.320	3
Ti-6-4	117	4.5	0.416	4
PC	2.4	1.2	0.775	8
Pine	12.3	0.51	0.145	1
GFRP	21	2.0	0.436	5
CFRP	76	1.6	0.184	2

(3.11, p.2)

Pine has the lowest mass, and CFRP the second lowest. ◀

(b)

Material	Rel Cost C_m	Cost f_2 $C_m \rho/E^{0.5}$	Cost Rank
1020 steel	1	0.554	2
4340 steel	3	1.647	3
7075 Al	6	1.923	4
Ti-6-4	45	18.721	7
PC	5	3.873	5
Pine	1.5	0.218	1
GFRP	10	4.364	6
CFRP	200	36.707	8

Pine also has the lowest cost, but now 1020 steel is second. ◀

(c) If pine is suitable, it is the clear choice. If not, then 7075 Al or 4340 steel might be reasonable. ◀

3.12 Tension mbr., square with side h. Required X and maximum ΔL.

First, look at mass and cost for given X.

$$\sigma = \frac{\sigma_c}{X} = \frac{P}{h^2}, \quad h^2 = \frac{PX}{\sigma_c} \quad \left(\begin{array}{c} h \text{ is geom.} \\ \text{that varies} \end{array}\right)$$

$$m = h^2 L \rho = \frac{PXL\rho}{\sigma_c} = \left[PXL\right]\left[\rho/\sigma_c\right] = f_1 f_2$$

Compare materials: $f_2 = \frac{\rho}{\sigma_c}$, $\frac{C_m \rho}{\sigma_c}$

Material	Modulus E, GPa	Strength σ_c, MPa	Density ρ, g/cm^3	Rel Cost C_m
1020 steel	203	260	7.9	1
4340 steel	207	1103	7.9	3
7075 Al	71	469	2.7	6
Ti-6-4	117	1185	4.5	45
PC	2.4	62	1.2	5
Pine	12.3	88	0.51	1.5
GFRP	21	380	2	10
CFRP	76	930	1.6	200

Material	Mass f_2 ρ/σ_c	Mass Rank	Cost f_2 $C_m\rho/\sigma_c$	Cost Rank
1020 steel	0.03038	8	0.0304	3
4340 steel	0.00716	6	0.0215	2
7075 Al	0.00576	4	0.0345	4
Ti-6-4	0.00380	2	0.1709	7
PC	0.01935	7	0.0968	6
Pine	0.00580	5	0.0087	1
GFRP	0.00526	3	0.0526	5
CFRP	0.00172	1	0.3441	8

(3.12, p.2)

Second, look at mass and cost for given maximum ΔL. From Fig. A.1(a):

$$\Delta L = \frac{PL}{AE} = \frac{PL}{h^2 E}, \quad h^2 = \frac{PL}{\Delta L E}$$

h is again the geometry that varies.

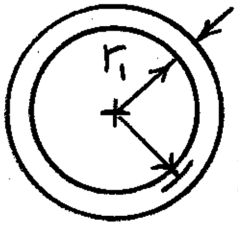
$$m = h^2 L \rho = \frac{PL^2 \rho}{\Delta L E} = \left[\frac{PL^2}{\Delta L}\right]\left[\frac{\rho}{E}\right] = f_1 f_2$$

Compare materials: $f_2 = \frac{\rho}{E}, \ \frac{C_m \rho}{E}$

Material	Mass f_2 ρ/E	Mass Rank	Cost f_2 $C_m \rho/E$	Cost Rank
1020 steel	0.0389	5	0.0389	1
4340 steel	0.0382	3	0.1145	3
7075 Al	0.0380	2	0.2282	4
Ti-6-4	0.0385	4	1.7308	6
PC	0.5000	8	2.5000	7
Pine	0.0415	6	0.0622	2
GFRP	0.0952	7	0.9524	5
CFRP	0.0211	1	4.2105	8

If cost is unimportant, CFRP is the clear choice. 7075 Al is a good compromise considering cost, and is resistant to corrosion and rot. ◀

3.13 Column with a tubular section.



$t = 0.2 r_1$

$r_{avg} = r_1 + t/2$

$r_{avg} = 1.1 r_1$

From Fig. A.2(c):

$A = 2\pi r_{avg} t$

$I \approx \pi r_{avg}{}^3 t$

$$A = 2\pi (1.1 r_1)(0.2 r_1) = 0.44 \pi r_1^2$$

$$I = \pi (1.1 r_1)^3 (0.2 r_1) = 0.2662 \pi r_1^4$$

Requirements: L, P, $X = P_{cr}/P$

Geometry: r_1 Material: ρ, E, (C_m)

Minimize: m, (cost)

$$P_{cr} = \frac{\pi^2 E I}{L^2}, \quad m = AL\rho = 0.44\pi r_1^2 L \rho$$

$$P_{cr} = XP = \frac{\pi^2 E (0.2662 \pi r_1^4)}{L^2}$$

$$r_1^2 = \left(\frac{XPL^2}{0.2662 \pi^3 E}\right)^{0.5}$$

$$m = 0.44 \pi L \rho \left(\frac{XPL^2}{0.2662 \pi^3 E}\right)^{0.5} = f_1 f_2$$

$$m = \left[0.8528 L^2 \left(\frac{XP}{\pi}\right)^{0.5}\right] \left[\frac{\rho}{E^{0.5}}\right]$$

Minimize: $f_2 = \frac{\rho}{E^{0.5}}, \quad \frac{C_m \rho}{E^{0.5}}$

(3.13, p.2)

Material	Modulus E, GPa	Density ρ, g/cm^3	Rel Cost C_m	Mass f_2 $\rho/E^{0.5}$	Mass Rank	Cost f_2 $C_m \rho/E^{0.5}$	Cost Rank
1020 steel	203	7.9	1	0.554	7	0.554	2
4340 steel	207	7.9	3	0.549	6	1.647	3
7075 Al	71	2.7	6	0.320	3	1.923	4
Ti-6-4	117	4.5	45	0.416	4	18.721	7
PC	2.4	1.2	5	0.775	8	3.873	5
Pine	12.3	0.51	1.5	0.145	1	0.218	1
GFRP	21	2	10	0.436	5	4.364	6
CFRP	76	1.6	200	0.184	2	36.707	8

(a) For the space station, light weight is paramont, and the cost of the material unimportant. CFRP is the best choice. Pine may have difficulty with planes of weakness in the material that can be overcome in CFRF by laminating or winding the fibers such that there is no weak plane. ◀

(b) Pine is a good choice, as cost is now important. It is not conveniently made into a tube, but a box section would work. If rot due to moisture or the size of the column is a problem use 1020 steel, as weight does not matter in the garage case. ◀

3.14 Spherical pressure vessel: r_1, t

Requirements: pressure p, $X = \sigma_c / \sigma$

Geometry: t Material: ρ, σ_c, (C_m)

Minimize: m, (cost)

(a) $\sigma_t \approx \dfrac{p r_1}{2t}$, $\sigma_r = -p \approx 0$ (Fig. A.7)

(assume $r_1 / t >> 1$)

$$m = \rho V = 4 \pi r_1^2 t \rho \quad \text{(from surface area)}$$

$$\sigma = \frac{p r_1}{2t} = \frac{\sigma_c}{X}, \quad t = \frac{p r_1 X}{2 \sigma_c}$$

$$m = 4 \pi r_1^2 \left(\frac{p r_1 X}{2 \sigma_c} \right) \rho = \left[2 \pi r_1^3 p X \right] \left[\frac{\rho}{\sigma_c} \right]$$

Minimize: $f_2 = \rho / \sigma_c$, $C_m \rho / \sigma_c$

Material	Strength σ_c, MPa	Density ρ, g/cm^3	Rel Cost C_m	Mass f_2 ρ/σ_c	Mass Rank	Cost f_2 $C_m \rho/\sigma_c$	Cost Rank	Thickness t, mm
1020 steel	260	7.9	1	0.03038	8	0.0304	3	8.08
4340 steel	1103	7.9	3	0.00716	6	0.0215	2	1.90
7075 Al	469	2.7	6	0.00576	4	0.0345	4	4.48
Ti-6-4	1185	4.5	45	0.00380	2	0.1709	7	1.77
PC	62	1.2	5	0.01935	7	0.0968	6	33.87
Pine	88	0.51	1.5	0.00580	5	0.0087	1	23.86
GFRP	380	2	10	0.00526	3	0.0526	5	5.53
CFRP	930	1.6	200	0.00172	1	0.3441	8	2.26

If both light weight and cost are important, 7075 Al or GFRP are possibilities.

(3.14, p.2)

Other possibilities exist, such as a steel if weight is not important, or CFRP or Ti-6Al-4V if weight is critical.

(b) $t = ?$ if $r_1 = 2\,m$, $p = 0.7\,MPa$, $X = 3$

$$t = \frac{p r_1 X}{2\sigma_c} = \frac{(0.7MPa)(2000\,mm)(3)}{2(260\,MPa)}$$

$t = 8.08$ mm for mild steel, others similarly - see table above. (The $r_1/t \gg 1$ assumption is satisfied for all, so the original $\sigma_r \approx 0$ assumption is valid.) The stronger materials have the lower thickness, due to $t \propto 1/\sigma_c$ for given p, r_1, X. ◀

3.15 Leaf spring as simple beam.

$L = 0.5\,m$, $t = 60\,mm$, $h = 5\,mm$, P at ctr.

Made from low-alloy (assume 4340) steel.

Required: $h \le 12\,mm$, $k = P/v = 50\,kN/m$

at $v_{max} = 30\,mm$, $X = 1.4$

(a) For $k = 50\,kN/m$, which Table 3.13 materials give lighter weight?

$$v = \frac{PL^3}{48EI}, \quad I = \frac{th^3}{12} \qquad \text{(Figs. A.4, A.2)}$$

Requirements: $k = P/v = 50\,kN/m$

Geometry: h Material: ρ, E, (C_m)

Minimize: m, (cost)

$$k = \frac{P}{v} = \frac{48EI}{L^3} = \frac{4Eth^3}{L^3}, \quad h = L\left(\frac{k}{4Et}\right)^{1/3}$$

$$m = thL\rho = tL^2\left(\frac{k}{4Et}\right)^{1/3}\rho$$

$$m = \left[tL^2\left(\frac{k}{4t}\right)^{1/3}\right]\left[\frac{\rho}{E^{1/3}}\right] = f_1 f_2$$

$$\text{Minimize } f_2 = \frac{\rho}{E^{1/3}}, \frac{C_m\rho}{E^{1/3}}$$

From the table (next page) all but 1020 steel would give a lighter component, but Ti-6-4 and CFRP would be very expensive. ◀

(3.15, p. 2)

Material	Modulus E, GPa	Density ρ, g/cm^3	Rel Cost C_m	Mass f_2 $\rho/E^{1/3}$	Mass Rank	Cost f_2 $C_m\rho/E^{1/3}$	Cost Rank
1020 steel	203	7.9	1	1.344	8	1.34	2
4340 steel	207	7.9	3	1.335	7	4.01	4
7075 Al	71	2.7	6	0.652	3	3.91	3
Ti-6-4	117	4.5	45	0.920	6	41.40	7
PC	2.4	1.2	5	0.896	5	4.48	5
Pine	12.3	0.51	1.5	0.221	1	0.33	1
GFRP	21	2	10	0.725	4	7.25	6
CFRP	76	1.6	200	0.378	2	75.55	8

(b) $h = L\left(\frac{k}{4Et}\right)^{1/3}$ For 1020 steel:

$$h = 500\,\text{mm}\left(\frac{50{,}000\,\text{N}}{1000\,\text{mm}}\;\frac{1}{4(203{,}000\,\text{MPa})(60\,\text{mm})}\right)^{1/3}$$

$h = 5.04$ mm (others similarly; see 2nd table)

$$P_{max} = k\,v_{max} = \frac{50{,}000\,\text{N}}{1000\,\text{mm}}\,30\,\text{mm} = 1500\,\text{N}$$

$$\sigma = \frac{\sigma_c}{X} = \frac{Mc}{I}, \quad c = \frac{h}{2}, \quad I = \frac{th^3}{12}, \quad M = \frac{PL}{4}$$

(Figs. A.1, A.2, and A.4) $X \geq 1.4$

$$\frac{\sigma_c}{X} = \frac{PL}{4}\,\frac{h}{2}\,\frac{12}{th^3}, \quad X = \frac{2\sigma_c t h^2}{3PL}$$

$$X = \frac{2(260\,\text{MPa})(60\,\text{mm})(5.04\,\text{mm})^2}{3(1500\,\text{N})(500\,\text{mm})} = 0.352$$

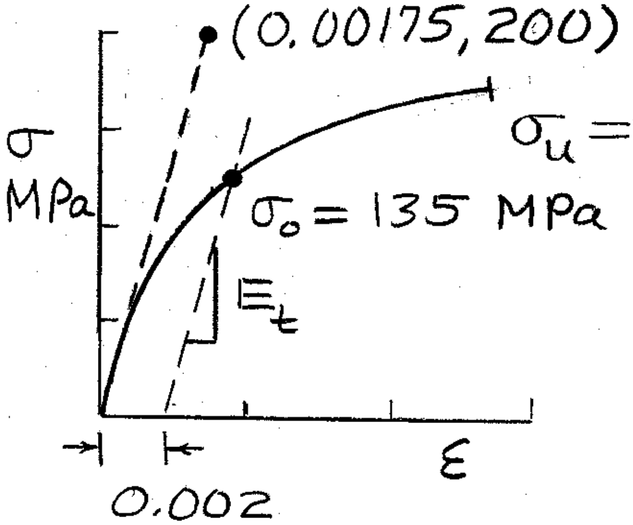
(for 1020 steel; others similarly)

(3.15, p.3)

Material	Strength σ_c, MPa	Depth h, mm	Safety Fac, X	Comment
1020 steel	260	5.04	0.35	fails X
4340 steel	1103	5.01	1.48	old design
7075 Al	469	7.16	1.28	fails X
Ti-6-4	1185	6.06	2.32	passes
PC	62	22.14	1.62	fails h
Pine	88	12.84	0.77	fails h, X
GFRP	380	10.74	2.34	passes
CFRP	930	7.00	2.43	passes

(c) All but Ti-6-4, GFRP, and CFRP fail due to h too large or $X < 1.4$. All of these involve a cost increase, by a factor of 7.25/4.01 = 1.8 for GFRP, and much more for the other two. For GFRP, the weight is reduced by a factor of 7.25/1.335 = 0.54. Hence, GFRP is a reasonable choice. CFRP is about half the weight of GFRP, but costs 10X more, and so seems an unlikely choice. ◀

4.4 From Fig. 4.8 for gray cast iron, estimate E_t, σ_o, σ_u, and $100\,\varepsilon_f$.



$\sigma_u = 160$ MPa, $100\,\varepsilon_f = 0.91\%$

$$E_t = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{200}{0.00175}$$

$$E_t = 114{,}000 \text{ MPa}$$ ◀

This gray iron has lower E, σ_o, and σ_u, and much lower elongation (vs. 15%) than the ductile iron in Table 4.2. This is due to the graphite flakes in gray iron acting as cracks to cause brittle behavior.

Comment: $100\varepsilon_f = 0.91\%$ is the value at fracture. The value after fracture is approximately:

$$\varepsilon_{pf} \approx \varepsilon_f - \frac{\sigma_f}{E_t} = 0.0091 - \frac{160 \text{ MPa}}{114{,}000 \text{ MPa}}$$

$$\varepsilon_{pf} = 0.0077, \quad 100\,\varepsilon_{pf} \approx 0.77\%$$ ◀

4.5 Initial portion of a tension test on 7075-T651 Al. $d_i = 9.07$, $L_i = 50.8\text{ mm}$

(a) $$\sigma = \frac{P}{A_i} = \frac{4P}{\pi d_i^2} = \frac{4\,(7.22\times10^3\text{ N})}{\pi\,(9.07\text{ mm})^2} = 112\text{ MPa}$$

$$\varepsilon = \frac{\Delta L}{L_i} = \frac{0.0839\text{ mm}}{50.8\text{ mm}} = 0.00165$$

P, kN	ΔL, mm	σ, MPa	ε
0	0	0	0
7.22	0.0839	112	0.00165
14.34	0.1636	222	0.00322
21.06	0.241	326	0.00474
26.8	0.308	415	0.00606
31.7	0.380	491	0.00748
34.1	0.484	528	0.00953
35.0	0.614	542	0.01209
36.0	0.924	557	0.01819
36.5	1.279	565	0.02518
36.9	1.622	571	0.03193
37.2	1.994	576	0.03925

Other values similarly ◀

(b) On graph at $\varepsilon_{po} = 0.002$:

$\sigma_o = 530\text{ MPa}$ ◀