P1.1 A steel bar of rectangular cross section, 15 mm by 60 mm, is loaded by a compressive force of 110 kN that acts in the longitudinal direction of the bar. Compute the average normal stress in the bar.

Solution

The cross-sectional area of the steel bar is

$$A = (15 \text{ mm})(60 \text{ mm}) = 900 \text{ mm}^2$$

The normal stress in the bar is

$$\sigma = \frac{F}{A} = \frac{(110 \text{ kN})(1{,}000 \text{ N/kN})}{900 \text{ mm}^2} = 122.222 \text{ MPa} = \boxed{122.2 \text{ MPa}}$$ **Ans.**

P1.2 A circular pipe with outside diameter of 4.5 in. and wall thickness of 0.375 in. is subjected to an axial tensile force of 42,000 lb. Compute the average normal stress in the pipe.

Solution

The outside diameter *D*, the inside diameter *d*, and the wall thickness *t* are related by

$$D = d + 2t$$

Therefore, the inside diameter of the pipe is

$$d = D - 2t = 4.5 \text{ in.} - 2(0.375 \text{ in.}) = 3.75 \text{ in.}$$

The cross-sectional area of the pipe is

$$A = \frac{\pi}{4}\left(D^2 - d^2\right) = \frac{\pi}{4}\left[(4.5 \text{ in.})^2 - (3.75 \text{ in.})^2\right] = 4.8597 \text{ in.}^2$$

The average normal stress in the pipe is

$$\sigma = \frac{F}{A} = \frac{42{,}000 \text{ lb}}{4.8597 \text{ in.}^2} = 8{,}642.6 \text{ psi} = \boxed{8{,}640 \text{ psi}}$$ **Ans.**

P1.3 A circular pipe with an outside diameter of 80 mm is subjected to an axial compressive force of 420 kN. The average normal stress may not exceed 130 MPa. Compute the minimum wall thickness required for the pipe.

Solution

From the definition of normal stress, solve for the minimum area required to support a 420 kN load without exceeding a normal stress of 130 MPa

$$\sigma = \frac{F}{A} \qquad \therefore A_{min} \geq \frac{F}{\sigma} = \frac{(420 \text{ kN})(1{,}000 \text{ N/kN})}{130 \text{ N/mm}^2} = 3{,}230.77 \text{ mm}^2$$

The cross-sectional area of the pipe is given by

$$A = \frac{\pi}{4}(D^2 - d^2)$$

Set this expression equal to the minimum area and solve for the maximum inside diameter d

$$\frac{\pi}{4}\left[(80 \text{ mm})^2 - d^2\right] \geq 3{,}230.77 \text{ mm}^2$$

$$(80 \text{ mm})^2 - d^2 \geq \frac{4}{\pi}\left(3{,}230.77 \text{ mm}^2\right)$$

$$\therefore d_{max} \leq 47.8169 \text{ mm}$$

The outside diameter D, the inside diameter d, and the wall thickness t are related by

$$D = d + 2t$$

Therefore, the minimum wall thickness required for the aluminum tube is

$$t_{min} \geq \frac{D-d}{2} = \frac{80 \text{ mm} - 47.8169 \text{ mm}}{2} = 16.092 \text{ mm} = \boxed{16.09 \text{ mm}}$$ **Ans.**

P1.4 Three solid bars, each with square cross sections, make up the axial assembly shown in Figure P1.4/5. Two loads of P = 30 kN are applied to the assembly at flange B, two loads of Q = 18 kN are applied at C, and one load of R = 42 kN is applied at end D. The bar dimensions are b_1 = 60 mm, b_2 = 20 mm, and b_3 = 40 mm. Determine the normal stress in each bar.

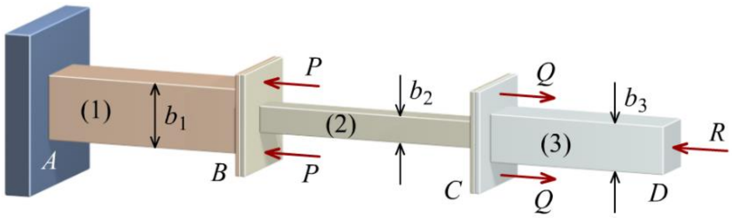


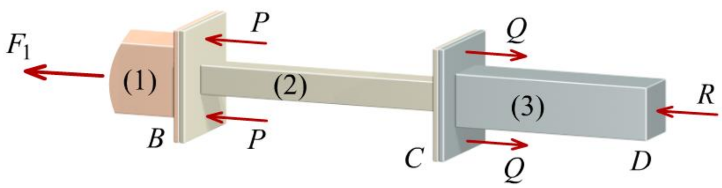
FIGURE P1.4/5

Solution

Cut an FBD through bar (1). The FBD should include the free end of the assembly at D. We will assume that the internal force in bar (1) is tension. From equilibrium, the force in bar (1) is

$$\Sigma F_x = -F_1 - 2P + 2Q - R = 0$$

$$\therefore F_1 = -2P + 2Q - R = -2(30 \text{ kN}) + 2(18 \text{ kN}) - 42 \text{ kN} = -66 \text{ kN} = 66 \text{ kN (C)}$$



From the given width of bar (1), the cross-sectional area of bar (1) is

$$A_1 = b_1^2 = (60 \text{ mm})^2 = 3{,}600 \text{ mm}^2$$

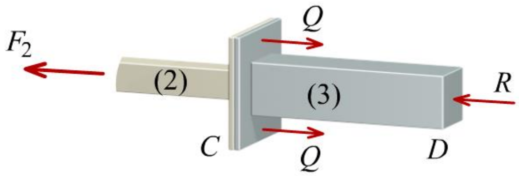
and thus, the normal stress in bar (1) is

$$\sigma_1 = \frac{F_1}{A_1} = \frac{(-66 \text{ kN})(1{,}000 \text{ N/kN})}{3{,}600 \text{ mm}^2} = -18.333 \text{ MPa} = \boxed{18.33 \text{ MPa (C)}} \qquad \textbf{Ans.}$$

Cut an FBD through bar (2). The FBD should include the free end of the assembly at D. We will assume that the internal force in bar (2) is tension. From equilibrium, the force in bar (2) is

$$\Sigma F_x = -F_2 + 2Q - R = 0$$

$$\therefore F_2 = 2Q - R = 2(18 \text{ kN}) - 42 \text{ kN} = -6 \text{ kN} = 6 \text{ kN (C)}$$



From the given width of bar (2), the cross-sectional area of bar (2) is

$$A_2 = b_2^2 = (20 \text{ mm})^2 = 400 \text{ mm}^2$$

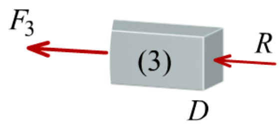
The normal stress in bar (2) is

$$\sigma_2 = \frac{F_2}{A_2} = \frac{(-6\text{ kN})(1{,}000\text{ N/kN})}{400\text{ mm}^2} = -15.000\text{ MPa} = \boxed{15.00\text{ MPa (C)}}$$ **Ans.**

Cut an FBD through bar (3). The FBD should include the free end of the assembly at *D*. We will assume that the internal force in bar (3) is tension. From equilibrium, the force in bar (3) is

$$\Sigma F_x = -F_3 - R = 0$$

$$\therefore F_3 = -R = -42\text{ kN} = 42\text{ kN (C)}$$



The cross-sectional area of bar (3) is

$$A_3 = b_3^2 = (40\text{ mm})^2 = 1{,}600\text{ mm}^2$$

The normal stress in bar (3) is

$$\sigma_2 = \frac{F_2}{A_2} = \frac{(-42\text{ kN})(1{,}000\text{ N/kN})}{1{,}600\text{ mm}^2} = -26.250\text{ MPa} = \boxed{26.3\text{ MPa (C)}}$$ **Ans.**

P1.5 Three solid bars, each with square cross sections, make up the axial assembly shown in Figure P1.4/5. Two loads of P = 25 kN are applied to the assembly at flange B, two loads of Q = 15 kN are applied at C, and one load of R = 35 kN is applied at end D. Bar (1) has a width of b_1 = 90 mm. Calculate the width b_2 required for bar (2) if the normal stress magnitude in bar (2) must equal the normal stress magnitude in bar (1).

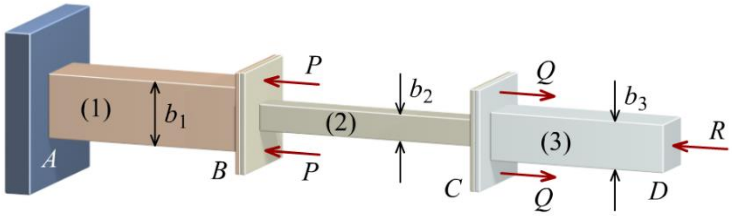


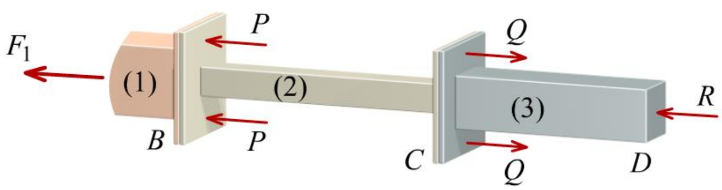
FIGURE P1.4/5

Solution

Cut an FBD through bar (1). The FBD should include the free end of the assembly at D. We will assume that the internal force in bar (1) is tension. From equilibrium, the force in bar (1) is

$$\Sigma F_x = -F_1 - 2P + 2Q - R = 0$$

$$\therefore F_1 = -2P + 2Q - R = -2(25 \text{ kN}) + 2(15 \text{ kN}) - 35 \text{ kN} = -55 \text{ kN} = 55 \text{ kN (C)}$$



From the given width of bar (1), the cross-sectional area of bar (1) is

$$A_1 = b_1^2 = (90 \text{ mm})^2 = 8{,}100 \text{ mm}^2$$

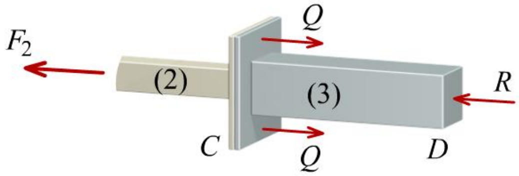
and thus, the normal stress in bar (1) is

$$\sigma_1 = \frac{F_1}{A_1} = \frac{(-55 \text{ kN})(1{,}000 \text{ N/kN})}{8{,}100 \text{ mm}^2} = -6.7901 \text{ MPa}$$

Cut an FBD through bar (2). The FBD should include the free end of the assembly at D. We will assume that the internal force in bar (2) is tension. From equilibrium, the force in bar (2) is

$$\Sigma F_x = -F_2 + 2Q - R = 0$$

$$\therefore F_2 = 2Q - R = 2(15 \text{ kN}) - 35 \text{ kN} = -5 \text{ kN}$$



The normal stress in bar (2) must equal the normal stress in bar (1). Thus,

$$\sigma_2 = \sigma_1 = -6.7901 \text{ MPa}$$

Solve for the required area of bar (2):

$$\sigma_2 = \frac{F_2}{A_2}$$

$$\therefore A_2 = \frac{F_2}{\sigma_2} \frac{(-5 \text{ kN})(1{,}000 \text{ N/kN})}{-6.7901 \text{ N/mm}^2} = 736.364 \text{ mm}^2$$

The width of bar (2) is therefore:

$$b_2 = \sqrt{736.364 \text{ mm}^2} = 27.136 \text{ mm} = \boxed{27.1 \text{ mm}}$$ **Ans.**

P1.6 Axial loads are applied with rigid bearing plates to the solid cylindrical rods shown in Figure P1.6/7. One load of P = 1,500 lb is applied to the assembly at A, two loads of Q = 900 lb are applied at B, and two loads of R = 1,300 lb are applied at C. The diameters of rods (1), (2), and (3) are d_1 = 0.625 in., d_2 = 0.500 in., and d_3 = 0.875 in. Determine the axial normal stress in each of the three rods.

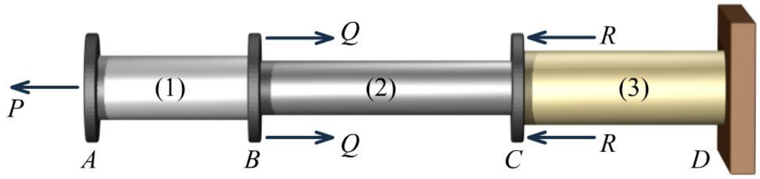


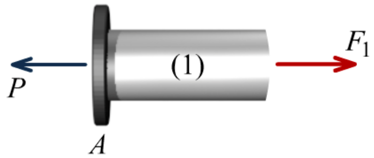
FIGURE P1.6/7

Solution

Cut an FBD through rod (1). The FBD should include the free end of the assembly at A. We will assume that the internal force in rod (1) is tension. From equilibrium, the force in rod (1) is

$$\Sigma F_x = -P + F_1 = 0$$

$$\therefore F_1 = P = 1{,}500 \text{ lb} = 1{,}500 \text{ lb (T)}$$



Use the given diameter to calculate the cross-sectional area of rod (1):

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(0.625 \text{ in.})^2 = 0.3068 \text{ in.}^2$$

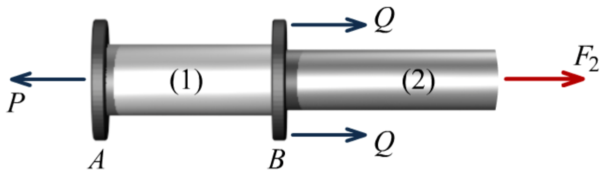
The normal stress in rod (1) is

$$\sigma_1 = \frac{F_1}{A_1} = \frac{1{,}500 \text{ lb}}{0.3068 \text{ in.}^2} = 4{,}889.24 \text{ psi} = \boxed{4{,}890 \text{ psi (T)}}$$ **Ans.**

Cut an FBD through rod (2). The FBD should include the free end of the assembly at A. We will assume that the internal force in rod (2) is tension. From equilibrium, the force in rod (2) is

$$\Sigma F_x = -P + 2Q + F_2 = 0$$

$$\therefore F_2 = P - 2Q = 1{,}500 \text{ lb} - 2(900 \text{ lb}) = -300 \text{ lb} = 300 \text{ lb (C)}$$



Use the given diameter to calculate the cross-sectional area of rod (2):

$$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(0.500 \text{ in.})^2 = 0.1963 \text{ in.}^2$$

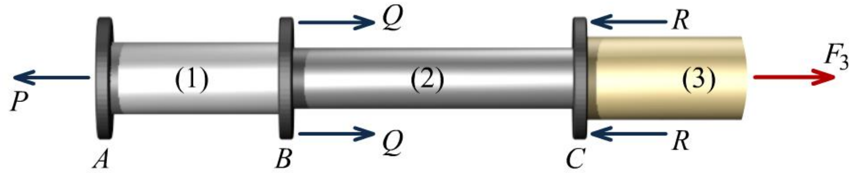
The normal stress in rod (2) is

$$\sigma_2 = \frac{F_2}{A_2} = \frac{-300 \text{ lb}}{0.1963 \text{ in.}^2} = -1{,}527.89 \text{ psi} = \boxed{1{,}528 \text{ psi (C)}}$$ **Ans.**

Cut an FBD through rod (3). The FBD should include the free end of the assembly at *A*. We will assume that the internal force in rod (3) is tension. From equilibrium, the force in rod (3) is

$$\Sigma F_x = -P + 2Q - 2R + F_3 = 0$$

$$\therefore F_3 = P - 2Q + 2R = 1{,}500 \text{ lb} - 2(900 \text{ lb}) + 2(1{,}300 \text{ lb}) = 2{,}300 \text{ lb} = 2{,}300 \text{ lb (T)}$$



Use the given diameter to calculate the cross-sectional area of rod (3):

$$A_3 = \frac{\pi}{4} d_3^2 = \frac{\pi}{4}(0.8750 \text{ in.})^2 = 0.6013 \text{ in.}^2$$

The normal stress in rod (3) is

$$\sigma_3 = \frac{F_3}{A_3} = \frac{2{,}300 \text{ lb}}{0.6013 \text{ in.}^2} = 3{,}824.92 \text{ psi} = \boxed{3{,}820 \text{ psi (T)}}$$ **Ans.**

P1.7 Axial loads are applied with rigid bearing plates to the solid cylindrical rods shown in Figure P1.6/7. One load of P = 30 kips is applied to the assembly at A, two loads of Q = 25 kips are applied at B, and two loads of R = 35 kips are applied at C. The normal stress magnitude in aluminum rod (1) must be limited to 20 ksi. The normal stress magnitude in steel rod (2) must be limited to 35 ksi. The normal stress magnitude in brass rod (3) must be limited to 25 ksi. Determine the minimum diameter required for each of the three rods.

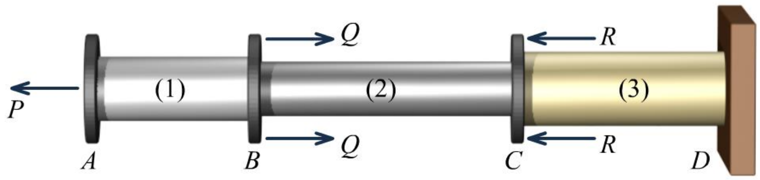


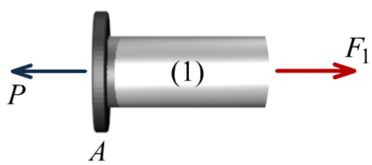
FIGURE P1.6/7

Solution

Cut an FBD through aluminum rod (1). The FBD should include the free end of the assembly at A. We will assume that the internal force in rod (1) is tension. From equilibrium, the force in rod (1) is

$$\Sigma F_x = -P + F_1 = 0$$

$$\therefore F_1 = P = 30 \text{ kips} = 30 \text{ kips (T)}$$



The normal stress magnitude in aluminum rod (1) must be limited to 20 ksi. Therefore, the minimum cross-sectional area of rod (1) must be

$$A_1 \geq \frac{|F_1|}{\sigma_1} = \frac{|30 \text{ kips}|}{20 \text{ ksi}} = 1.500 \text{ in.}^2$$

The diameter must be

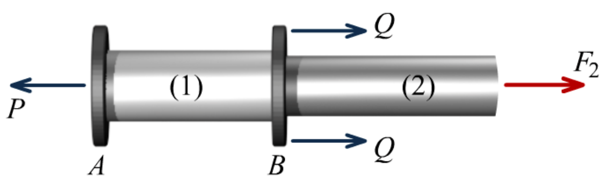
$$A_1 \leq \frac{\pi}{4} d_1^2$$

$$\therefore d_1 \geq \sqrt{\frac{4}{\pi}\left(1.500 \text{ in.}^2\right)} = \boxed{1.382 \text{ in.}}$$ **Ans.**

Cut an FBD through steel rod (2). The FBD should include the free end of the assembly at A. We will assume that the internal force in rod (2) is tension. From equilibrium, the force in rod (2) is

$$\Sigma F_x = -P + 2Q + F_2 = 0$$

$$\therefore F_2 = P - 2Q = 30 \text{ kips} - 2(25 \text{ kips}) = -20 \text{ kips} = 20 \text{ kips (C)}$$



The normal stress magnitude in steel rod (2) must be limited to 35 ksi. Therefore, the minimum cross-sectional area of rod (2) must be

$$A_2 \geq \frac{|F_2|}{\sigma_2} = \frac{|-20 \text{ kips}|}{35 \text{ ksi}} = 0.5714 \text{ in.}^2$$

The diameter of rod (2) must be

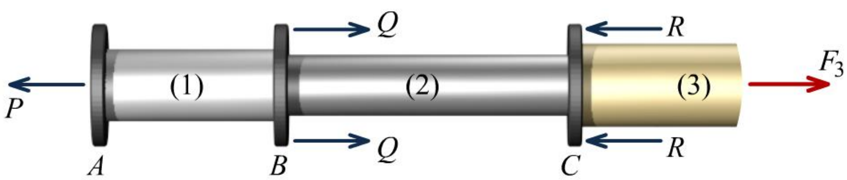
$$A_2 \leq \frac{\pi}{4} d_2^2$$

$$\therefore d_2 \geq \sqrt{\frac{4}{\pi}\left(0.5714 \text{ in.}^2\right)} = \boxed{0.853 \text{ in.}}$$ **Ans.**

Cut an FBD through brass rod (3). The FBD should include the free end of the assembly at *A*. We will assume that the internal force in rod (3) is tension. From equilibrium, the force in rod (3) is

$$\Sigma F_x = -P + 2Q - 2R + F_3 = 0$$

$$\therefore F_3 = P - 2Q + 2R = 30 \text{ kips} - 2(25 \text{ kips}) + 2(35 \text{ kips}) = 50 \text{ kips} = 50 \text{ kips (T)}$$



The normal stress magnitude in brass rod (3) must be limited to 25 ksi. Therefore, the minimum cross-sectional area of rod (3) must be

$$A_3 \geq \frac{|F_3|}{\sigma_3} = \frac{|50 \text{ kips}|}{25 \text{ ksi}} = 2.0000 \text{ in.}^2$$

The diameter of rod (3) must be

$$A_3 \leq \frac{\pi}{4} d_3^2$$

$$\therefore d_3 \geq \sqrt{\frac{4}{\pi}\left(2.0000 \text{ in.}^2\right)} = \boxed{1.596 \text{ in.}}$$ **Ans.**

P1.8 Determine the normal stress in rod (1) for the mechanism shown in Figure P1.8. The diameter of rod (1) is 8 mm, and load P = 2,300 N. Use the following dimensions: a = 120 mm, b = 200 mm, c = 170 mm, and d = 90 mm.

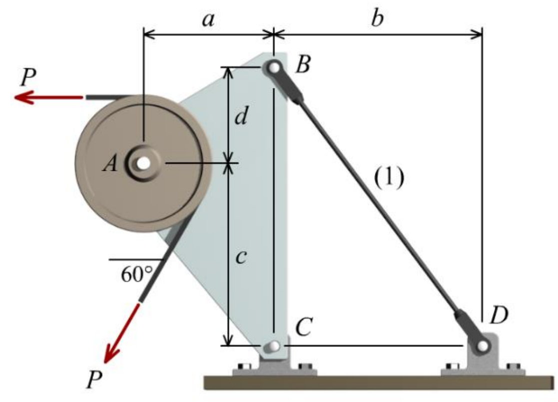


FIGURE P1.8

Solution

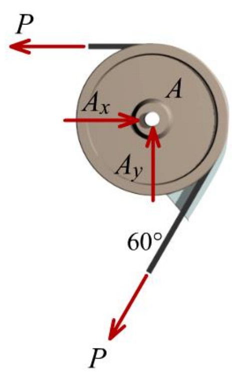
First, consider an FBD of the pulley to determine the reaction forces exerted on the pulley by the mechanism.

$$\Sigma F_x = A_x - P - P\cos(60°) = 0$$

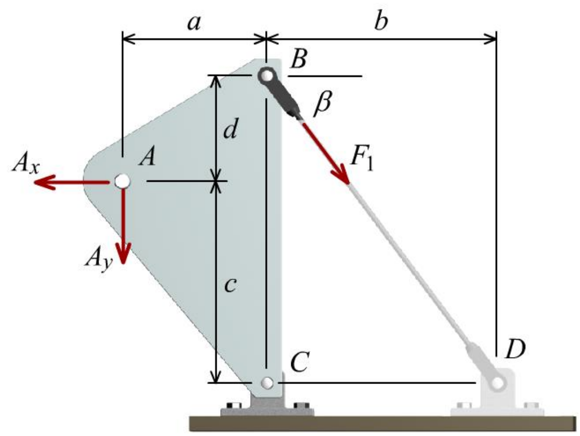
$$\therefore A_x = (2{,}300\text{ N}) + (2{,}300\text{ N})\cos(60°) = 3{,}450.000\text{ N}$$

$$\Sigma F_y = A_y - P\sin(60°) = 0$$

$$\therefore A_y = (2{,}300\text{ N})\sin(60°) = 1{,}991.858\text{ N}$$



FBD of pulley



FBD of mechanism

Next, consider an FBD of the mechanism to determine the force in rod (1). Rod (1) is oriented at an angle of:

$$\tan\beta = \frac{c+d}{b} = \frac{170\text{ mm} + 90\text{ mm}}{200\text{ mm}} = 1.30$$

$$\therefore \beta = 52.431°$$

Rod (1) is a two-force member, and its axial force can be calculated from:

$$\Sigma M_C = A_x c + A_y a - (F_1 \cos\beta)(c+d) = 0$$

$$\therefore F_1 = \frac{A_x c + A_y a}{(c+d)\cos\beta} = \frac{(3{,}450.000\text{ N})(170\text{ mm}) + (1{,}991.858\text{ N})(120\text{ mm})}{(170\text{ mm} + 90\text{ mm})\cos(52.431°)} = 5{,}207.523\text{ N}$$

The area of rod (1) is

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(8\text{ mm})^2 = 50.265\text{ mm}^2$$

The normal stress in the rod is thus

$$\sigma_1 = \frac{F_1}{A_1} = \frac{5{,}207.532\text{ N}}{50.265\text{ mm}^2} = 103.601\text{ MPa} = \boxed{103.6\text{ MPa}}$$ **Ans.**

P1.9 Determine the normal stress in bar (1) for the mechanism shown in Figure P1.9. The area of bar (1) is 2,600 mm². The distributed load intensities are w_C = 12 kN/m and w_D = 30 kN/m. Use the following dimensions: a = 7.5 m and b = 3.0 m.

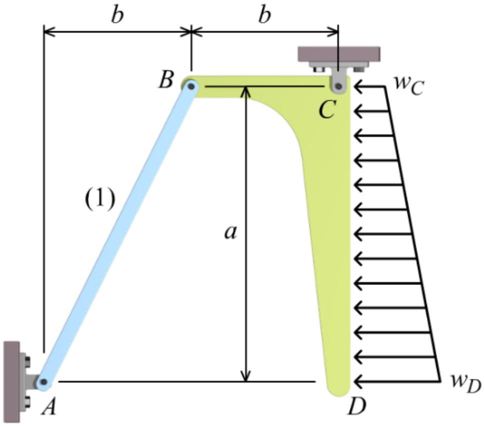


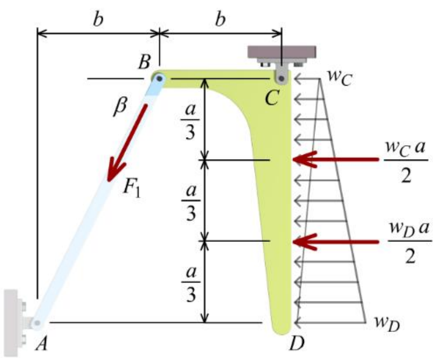
FIGURE P1.9

Solution

Consider an FBD of the mechanism. Determine the angle β between rod (1) and the horizontal axis:

$$\tan\beta = \frac{a}{b} = \frac{7.5 \text{ m}}{3.0 \text{ m}} = 2.5$$

$$\therefore \beta = 68.199°$$



Write an equilibrium equation for the sum of moments about C to compute the force in bar (1). Note: Bar (1) is a two-force member.

$$\Sigma M_C = (F_1 \sin\beta)b - \frac{w_C a}{2} \times \frac{a}{3} - \frac{w_D a}{2} \times \frac{2a}{3} = 0$$

$$\therefore F_1 = \frac{\frac{w_C a^2}{6} + \frac{2 w_D a^2}{6}}{b \sin\beta} = \frac{a^2 (w_C + 2w_D)}{6b\sin\beta} = \frac{(7.5 \text{ m})^2 \left[12 \text{ kN/m} + 2(30 \text{ kN/m})\right]}{6(3.0 \text{ m})\sin(68.199°)} = 242.332 \text{ kN}$$

The normal stress in bar (1) is thus:

$$\sigma_1 = \frac{F_1}{A_1} = \frac{(242.332 \text{ kN})(1{,}000 \text{ N/kN})}{2{,}600 \text{ mm}^2} = 93.205 \text{ N/mm}^2 = \boxed{93.2 \text{ MPa (T)}}$$

Ans.

P1.10 The rigid beam *BC* shown in Figure P1.10 is supported by rods (1) and (2) that have diameters of 0.875 in. and 1.125 in., respectively. For a uniformly distributed load of w = 4,200 lb/ft, determine the normal stress in each rod. Assume L = 14 ft and a = 9 ft.

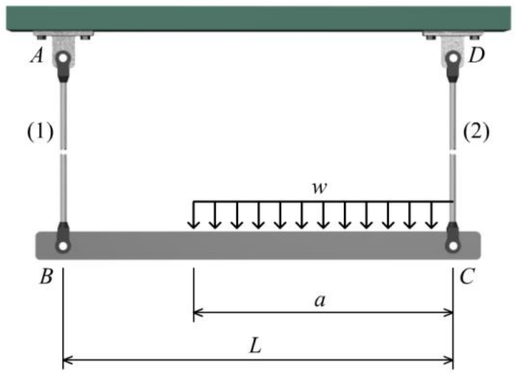


FIGURE P1.10

Solution

Equilibrium: Calculate the internal forces in rods (1) and (2).

$$\Sigma M_C = -F_1(14 \text{ ft}) + (4{,}200 \text{ lb/ft})(9 \text{ ft})\left(\frac{9 \text{ ft}}{2}\right) = 0$$

$$\therefore F_1 = 12.150 \text{ kips}$$

$$\Sigma M_B = F_2(14 \text{ ft}) - (4{,}200 \text{ lb/ft})(9 \text{ ft})\left(14 \text{ ft} - \frac{9 \text{ ft}}{2}\right) = 0$$

$$\therefore F_2 = 25.650 \text{ kips}$$

Areas:

$$A_1 = \frac{\pi}{4}(0.875 \text{ in.})^2 = 0.601 \text{ in.}^2$$

$$A_2 = \frac{\pi}{4}(1.125 \text{ in.})^2 = 0.994 \text{ in.}^2$$

Stresses:

$$\sigma_1 = \frac{F_1}{A_1} = \frac{12.150 \text{ kips}}{0.601 \text{ in.}^2} = 20.206 \text{ ksi} = \boxed{20.2 \text{ ksi}}$$ **Ans.**

$$\sigma_2 = \frac{F_2}{A_2} = \frac{25.650 \text{ kips}}{0.994 \text{ in.}^2} = 25.804 \text{ ksi} = \boxed{25.8 \text{ ksi}}$$ **Ans.**

P1.11 The rigid beam *ABC* shown in Figure P1.11 is supported by a pin connection at *C* and by steel rod (1), which has a diameter of 10 mm. If the normal stress in rod (1) must not exceed 225 MPa, what is the maximum uniformly distributed load *w* that may be applied to beam *ABC*? Use dimensions of *a* = 340 mm, *b* = 760 mm, and *c* = 550 mm.

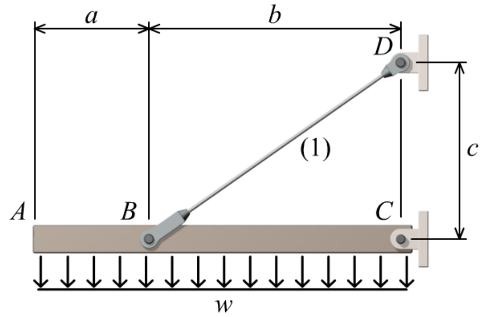


FIGURE P1.11

Solution

The cross-sectional area of rod (1) is

$$A_1 = \frac{\pi}{4}(10 \text{ mm})^2 = 78.540 \text{ mm}^2$$

Since the normal stress in rod (1) must not exceed 225 MPa, the allowable force that can be applied to rod (1) is:

$$F_{1,\text{allow}} = \sigma_1 A_1 = \left(225 \text{ N/mm}^2\right)\left(78.540 \text{ mm}^2\right) = 17{,}671.459 \text{ N}$$

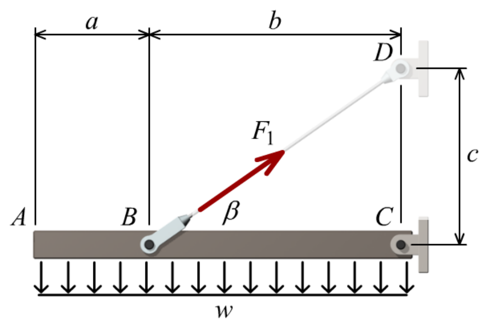
Rod (1) is oriented at an angle of β with respect to the horizontal direction:

$$\tan\beta = \frac{c}{b} = \frac{550 \text{ mm}}{760 \text{ mm}} = 0.7237 \qquad \therefore \beta = 35.893°$$

Consider an FBD of rigid beam *ABC*. From the moment equilibrium equation about joint *C*, the relationship between the force in rod (1) and the distributed load *w* is:

$$\Sigma M_C = w(a+b)\left(\frac{a+b}{2}\right) - (F_1 \sin\beta)b = 0$$

$$\therefore w = \frac{2b(F_1 \sin\beta)}{(a+b)^2}$$



Substitute the allowable force $F_{1,\text{allow}}$ into this relationship to obtain the maximum distributed load that may be applied to the structure:

$$w = \frac{2b(F_1 \sin\beta)}{(a+b)^2}$$

$$= \frac{2(760 \text{ mm})(17{,}671.459 \text{ N})\sin(35.893°)}{(340 \text{ mm} + 760 \text{ mm})^2}$$

$$= 13.014 \text{ N/mm} = \boxed{13.01 \text{ kN/m}}$$ **Ans.**

P1.12 A simple pin-connected truss is loaded and supported as shown in Figure P1.12. The load P is 200 kN. All members of the truss are aluminum pipes that have an outside diameter of 115 mm and a wall thickness of 6 mm. Determine the normal stress in each truss member. Assume truss dimensions of a = 12.0 m, b = 7.5 m, and c = 6.0 m.

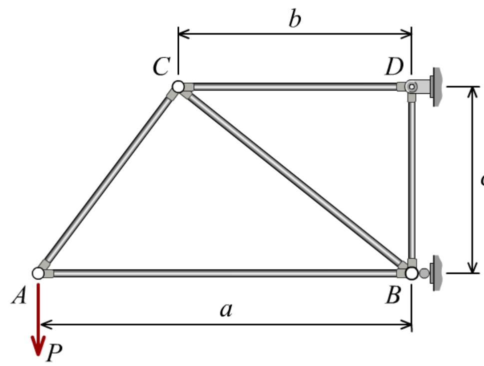
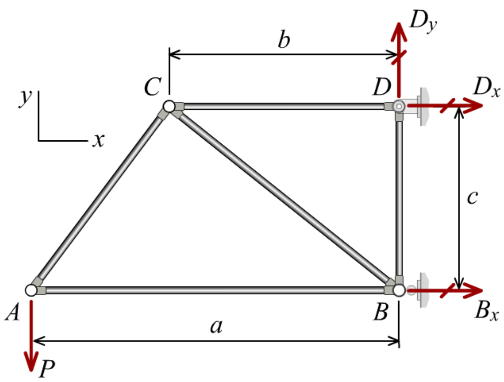


FIGURE P1.12

Solution

Overall equilibrium:

Begin the solution by determining the external reaction forces acting on the truss at supports B and D. Write equilibrium equations that include all *external* forces. Note that only the external forces (i.e., loads and reaction forces) are considered at this time. The internal forces acting in the truss members will be considered after the external reactions have been computed. The free-body diagram (FBD) of the entire truss is shown. The following equilibrium equations can be written for this structure:



$$\Sigma F_y = D_y - P = 0$$

$$\therefore D_y = P = 200 \text{ kN}$$

$$\Sigma M_D = Pa + B_x c = 0$$

$$\therefore B_x = -\frac{Pa}{c} = -\frac{P(12 \text{ m})}{6 \text{ m}} = -2P = -400 \text{ kN}$$

$$\Sigma M_B = Pa - D_x c = 0$$

$$\therefore D_x = \frac{Pa}{c} = \frac{P(12 \text{ m})}{6 \text{ m}} = 2P = 400 \text{ kN}$$

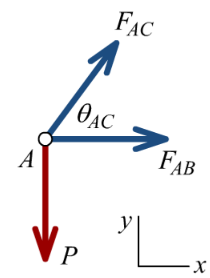
Method of joints:

Before beginning the process of determining the internal forces in the axial members, the geometry of the truss will be used to determine the magnitude of the inclination angles of members AC and BC. Use the definition of the tangent function to determine θ_{AC} and θ_{BC}:

$$\tan\theta_{AC} = \frac{c}{a-b} = \frac{6.0 \text{ m}}{12.0 \text{ m} - 7.5 \text{ m}} = 1.3333 \qquad \therefore \theta_{AC} = 53.130°$$

$$\tan\theta_{BC} = \frac{c}{b} = \frac{6.0 \text{ m}}{7.5 \text{ m}} = 0.8 \qquad \therefore \theta_{BC} = 38.660°$$

Joint A:
Begin the solution process by considering an FBD of joint *A*. Consider only those forces acting directly on joint *A*. In this instance, two axial members, *AB* and *AC*, are connected at joint *A*. Tension forces will be assumed in each truss member.



$$\Sigma F_x = F_{AB} + F_{AC}\cos\theta_{AC} = 0 \qquad \text{(a)}$$

$$\Sigma F_y = F_{AC}\sin\theta_{AC} - P = 0 \qquad \text{(b)}$$

Solve Eq. (b) for F_{AC}:

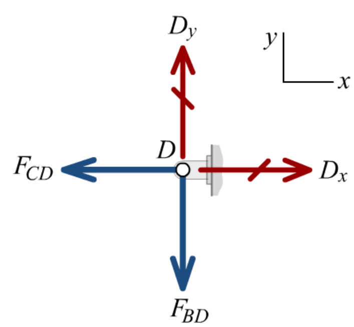
$$F_{AC} = \frac{P}{\sin\theta_{AC}} = \frac{200 \text{ kN}}{\sin(53.130°)} = 250.0 \text{ kN}$$

and then compute F_{AB} using Eq. (a):

$$F_{AB} = -F_{AC}\cos\theta_{AC}$$

$$= -(250.0 \text{ kN})\cos(53.130°) = -150.0 \text{ kN}$$

Joint D:
Next, consider an FBD of joint *D*. As before, tension forces will be assumed in each truss member.



$$\Sigma F_x = D_x - F_{CD} = 0 \qquad \text{(c)}$$

$$\Sigma F_y = D_y - F_{BD} = 0 \qquad \text{(d)}$$

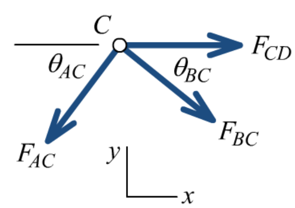
Solve Eq. (c) for F_{CD}:

$$F_{CD} = D_x = 400.0 \text{ kN}$$

and solve Eq. (d) for F_{BD}:

$$F_{BD} = D_y = 200.0 \text{ kN}$$

Joint C:
Next, consider an FBD of joint *C*. As before, tension forces will be assumed in each truss member.



$$\Sigma F_x = F_{CD} + F_{BC}\cos\theta_{BC} - F_{AC}\cos\theta_{AC} = 0 \qquad \text{(e)}$$

$$\Sigma F_y = -F_{BC}\sin\theta_{BC} - F_{AC}\sin\theta_{AC} = 0 \qquad \text{(f)}$$

Solve Eq. (e) for F_{BC}:

$$F_{BC} = -F_{AC}\frac{\sin\theta_{AC}}{\sin\theta_{BC}} = -(250 \text{ kN})\frac{\sin(53.130°)}{\sin(38.660°)} = -320.1562 \text{ kN}$$

Eq. (f) can be used as a check on our calculations:

$$\Sigma F_y = -F_{BC}\sin\theta_{BC} - F_{AC}\sin\theta_{AC}$$

$$= -(-320.1562 \text{ kN})\sin(38.660°) - (250.0 \text{ kN})\sin(53.130°) = 0$$ **Checks!**

Section properties:
For each of the five truss members:

$$d = 115\text{ mm} - 2(6\text{ mm}) = 103\text{ mm} \qquad A = \frac{\pi}{4}\left[(115\text{ mm})^2 - (103\text{ mm})^2\right] = 2{,}054.602\text{ mm}^2$$

Normal stress in each truss member:

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{(-150\text{ kN})(1{,}000\text{ N/kN})}{2{,}054.602\text{ mm}^2} = -73.007\text{ MPa} = \boxed{73.0\text{ MPa (C)}}$$ **Ans.**

$$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{(250.0\text{ kN})(1{,}000\text{ N/kN})}{2{,}054.602\text{ mm}^2} = 121.678\text{ MPa} = \boxed{121.7\text{ MPa (T)}}$$ **Ans.**

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{(-320.156\text{ kN})(1{,}000\text{ N/kN})}{2{,}054.602\text{ mm}^2} = -155.824\text{ MPa} = \boxed{155.8\text{ MPa (C)}}$$ **Ans.**

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{(200.0\text{ kN})(1{,}000\text{ N/kN})}{2{,}054.602\text{ mm}^2} = 97.342\text{ MPa} = \boxed{97.3\text{ MPa (T)}}$$ **Ans.**

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{(400.0\text{ kN})(1{,}000\text{ N/kN})}{2{,}054.602\text{ mm}^2} = 194.685\text{ MPa} = \boxed{194.7\text{ MPa (T)}}$$ **Ans.**

P1.13 A horizontal load P is applied to an assembly consisting of two inclined bars, as shown in Figure 1.13. The cross-sectional area of bar (1) is 1.5 in.^{2}, and the cross-sectional area of bar (2) is 1.8 in.^{2}. The normal stress in either bar may not exceed 24 ksi. Determine the maximum load P that may be applied to this assembly. Assume dimensions of a = 16 ft, b = 8 ft, and c = 13 ft.

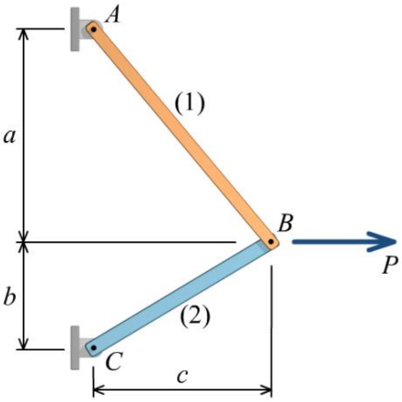


FIGURE P1.13

Solution

Allowable member forces:

Using the allowable stresses and the member areas, we can determine the allowable force for each member:

$$F_{1,\text{allow}} = \sigma_{1,\text{allow}} A_1 = (24 \text{ ksi})(1.5 \text{ in.}^2) = 36 \text{ kips} \qquad \text{(a)}$$

$$F_{2,\text{allow}} = \sigma_{2,\text{allow}} A_2 = (24 \text{ ksi})(1.8 \text{ in.}^2) = 43.2 \text{ kips} \qquad \text{(b)}$$

Equilibrium:

The geometry of the two-bar assembly will be used to determine the magnitude of the inclination angles for members AB and BC. We can use the definition of the tangent function to determine θ_{AB} and θ_{BC}:

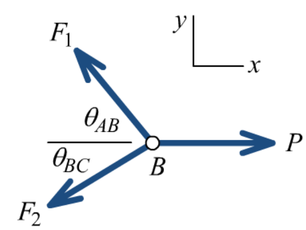
$$\tan\theta_{AB} = \frac{a}{c} = \frac{16 \text{ ft}}{13 \text{ ft}} = 1.2308 \qquad \therefore \theta_{AB} = 50.906°$$

$$\tan\theta_{BC} = \frac{b}{c} = \frac{8 \text{ ft}}{13 \text{ ft}} = 0.6154 \qquad \therefore \theta_{BC} = 31.608°$$

Consider a free-body diagram (FBD) of joint B. The following equilibrium equations can be written for this joint:

$$\Sigma F_x = P - F_1 \cos\theta_{AB} - F_2 \cos\theta_{BC} = 0 \qquad \text{(c)}$$

$$\Sigma F_y = F_1 \sin\theta_{AB} - F_2 \sin\theta_{BC} = 0 \qquad \text{(d)}$$



Erroneous approach for finding maximum load *P*:

Since we are trying to calculate P, the temptation at this point in the solution is to substitute the values from Equations (a) and (b) into Eq. (c) and simply solve for P:

$$\begin{aligned} P &= F_1 \cos\theta_{AB} + F_2 \cos\theta_{BC} \\ &= (36 \text{ kips})\cos(50.906°) + (43.2 \text{ kips})\cos(31.608°) \\ &= 59.493 \text{ kips} \end{aligned} \qquad \text{(e)}$$

However, if we use the values from Equations (a) and (b) in Eq. (d), we find that equilibrium is not satisfied:

$$\begin{aligned} \Sigma F_y &= F_1 \sin\theta_{AB} - F_2 \sin\theta_{BC} \\ &= (36 \text{ kips})\sin(50.906°) - (43.2 \text{ kips})\sin(31.608°) \\ &= 5.299 \text{ kips} \neq 0 \end{aligned}$$

Equilibrium must always be satisfied; therefore, we must conclude that F_1 and F_2 will not have the allowable values of Equations (a) and (b). **The answer obtained in Eq. (e) is incorrect** because equilibrium is not satisfied.

Correct method for calculating the capacity of the two-bar assembly:
The allowable load that can be applied to this two-bar assembly will be the load P that produces the allowable load in either member (1) or member (2). Let's return to Eq. (d), only this time, we are going to make an assumption. We will assume that the force in member (1) will control the capacity of the two-bar assembly. If this assumption is true, then the force in member (1) will equal its allowable force as given in Eq. (a), and the force in member (2) will be less than its allowable force as given in Eq. (b).

$$\begin{aligned} F_2 &= F_1 \frac{\sin\theta_{AB}}{\sin\theta_{BC}} = F_1 \frac{\sin(50.906°)}{\sin(31.608°)} = 1.4809 F_1 \\ &= 1.4809(36 \text{ kips}) \\ &= 53.311 \text{ kips} > F_{2,\text{allow}} = 43.2 \text{ kips} \qquad \textbf{N.G.} \end{aligned}$$

This calculation shows that the force in member (2) will exceed its allowable force when the force in member (1) equals its allowable force. Therefore, our assumption is proved incorrect. This result shows us that the force in member (2) will control the capacity of the two-bar assembly. We'll return to Eq. (d), only this time, we know that member (2) will control. Set the force in member (2) to its allowable force from Eq. (b) and solve for the force in member (1) that is required to satisfy equilibrium.

$$\begin{aligned} F_1 &= F_2 \frac{\sin\theta_{BC}}{\sin\theta_{AB}} = F_2 \frac{\sin(31.608°)}{\sin(50.906°)} = 0.6753 F_2 \\ &= 0.6753(43.2 \text{ kips}) \\ &= 29.172 \text{ kips} < F_{1,\text{allow}} = 36 \text{ kips} \qquad \textbf{O.K.} \end{aligned}$$

We now know the forces in members (1) and (2) that will satisfy the equilibrium equations without exceeding the allowable force in either member. Finally, we use these values to determine the load P from Eq. (c):

$$\begin{aligned} P &= F_1 \cos\theta_{AB} + F_2 \cos\theta_{BC} \\ &= (29.172 \text{ kips})\cos(50.906°) + (43.2 \text{ kips})\cos(31.608°) \\ &= 55.188 \text{ kips} \\ &= \boxed{55.2 \text{ kips}} \qquad \textbf{Ans.} \end{aligned}$$

P1.14 The rectangular bar shown in Figure P1.14 is subjected to a uniformly distributed axial loading of w = 13 kN/m and a concentrated force of P = 9 kN at B. Determine the magnitude of the maximum normal stress in the bar and its location x. Assume a = 0.5 m, b = 0.7 m, c = 15 mm, and d = 40 mm.

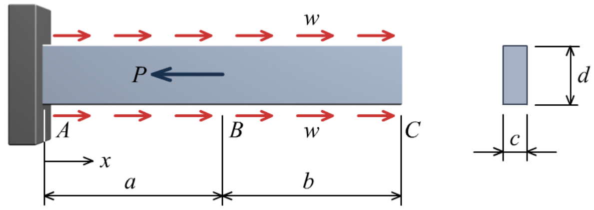
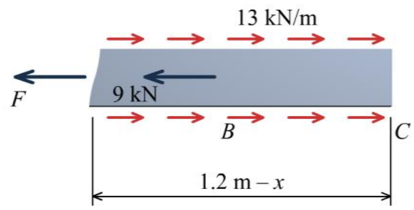


FIGURE P1.14

Solution

Equilibrium:

Draw an FBD for the interval between A and B where $0 \le x < a$. Write the following equilibrium equation:

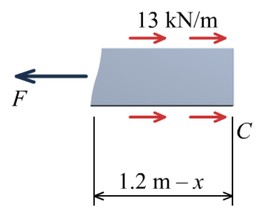


$$\xrightarrow{+} \Sigma F_x = (13\text{ kN/m})(1.2\text{ m} - x) - (9\text{ kN}) - F = 0$$

$$\therefore F = (13\text{ kN/m})(1.2\text{ m} - x) - (9\text{ kN})$$

The largest force in this interval occurs at x = 0 where F = 6.6 kN.

In the interval between B and C where $a \le x < a + b$, and write the following equilibrium equation:



$$\xrightarrow{+} \Sigma F_x = (13\text{ kN/m})(1.2\text{ m} - x) - F = 0$$

$$\therefore F = (13\text{ kN/m})(1.2\text{ m} - x)$$

The largest force in this interval occurs at $x = a$ where F = 9.1 kN.

Maximum Normal Stress:

$$\sigma_{\max} = \frac{(9.1\text{ kN})(1{,}000\text{ N/kN})}{(15\text{ mm})(40\text{ mm})} = \boxed{15.17\text{ MPa at } x = 0.5\text{ m}}$$

Ans.

P1.15 The solid 1.25 in. diameter rod shown in Figure P1.15 is subjected to a uniform axial distributed loading along its length of w = 750 lb/ft. Two concentrated loads also act on the rod: P = 2,000 lb and Q = 1,000 lb. Assume a = 16 in. and b = 32 in. Determine the normal stress in the rod at the following locations:
(a) x = 10 in.
(b) x = 30 in.

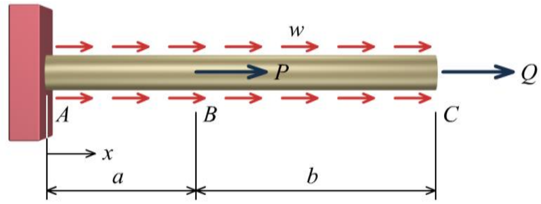
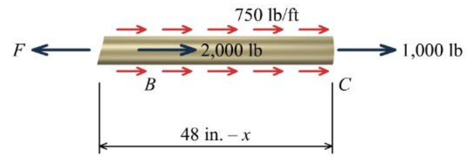


FIGURE P1.15

Solution

(a) *x* = 10 in.

Equilibrium: Draw an FBD for the interval between A and B where $0 \leq x < a$, and write the following equilibrium equation:



$$\xrightarrow{+} \Sigma F_x = (750 \text{ lb/ft})(1 \text{ ft}/12 \text{ in.})(48 \text{ in.} - x) + (2{,}000 \text{ lb}) + (1{,}000 \text{ lb}) - F = 0$$

$$\therefore F = (62.5 \text{ lb/in.})(48 \text{ in.} - x) + 3{,}000 \text{ lb}$$

At x = 10 in., F = 5,375 lb.

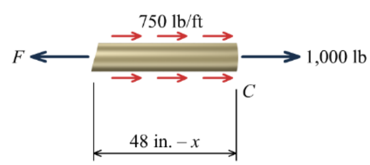
Stress: The normal stress at this location can be calculated as follows.

$$A = \frac{\pi}{4}(1.25 \text{ in.})^2 = 1.227185 \text{ in.}^2$$

$$\sigma = \frac{5{,}375 \text{ lb}}{1.227185 \text{ in.}^2} = 4{,}379.944 \text{ psi} = \boxed{4{,}380 \text{ psi}}$$ **Ans.**

(b) *x* = 30 in.

Equilibrium: Draw an FBD for the interval between B and C where $a \leq x < a + b$, and write the following equilibrium equation:



$$\xrightarrow{+} \Sigma F_x = (750 \text{ lb/ft})(1 \text{ ft}/12 \text{ in.})(48 \text{ in.} - x) + (1{,}000 \text{ lb}) - F = 0$$

$$\therefore F = (62.5 \text{ lb/in.})(48 \text{ in.} - x) + 1{,}000 \text{ lb}$$

At x = 30 in., F = 2,125 lb.

Stress: The normal stress at this location can be calculated as follows.

$$\sigma = \frac{2{,}125 \text{ lb}}{1.227185 \text{ in.}^2} = 1{,}731.606 \text{ psi} = \boxed{1{,}730 \text{ psi}}$$ **Ans.**

P1.16 A block of wood is tested in direct shear using the test fixture shown below. The dimensions of the test specimen are a = 3.75 in., b = 1.25 in., c = 2.50 in., and d = 6.0 in. During the test, a load of P = 1,590 lb produces a shear failure in the wood specimen. What is the magnitude of the average shear stress in the wood specimen at failure?

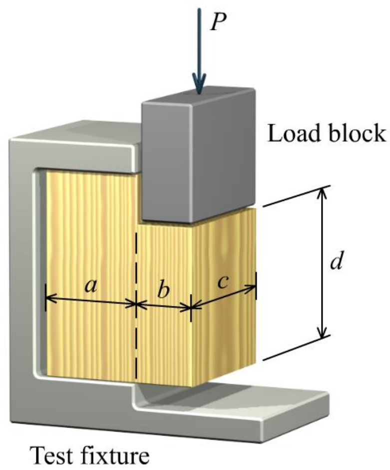
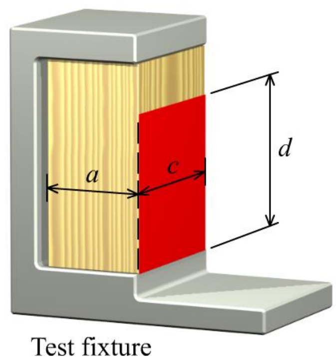


FIGURE P1.16

Solution

Visualize the surface that will be exposed when the specimen fails. The area of this surface will be

$$A_V = cd = (2.50\text{ in.})(6.0\text{ in.}) = 15.0\text{ in.}^2$$



The average shear stress in the specimen at failure is thus

$$\tau_{\text{avg}} = \frac{P}{A_V} = \frac{1{,}590\text{ lb}}{15.0\text{ in.}^2} = \boxed{106.0\text{ psi}}$$

Ans.

P1.17 A cylindrical rod of diameter d = 0.625 in. is attached to a plate by a cylindrical rubber grommet. The plate has a thickness of t = 0.875 in. If the axial load on the rod is P = 175 lb, what is the average shear stress on the cylindrical surface of contact between the rod and the grommet?

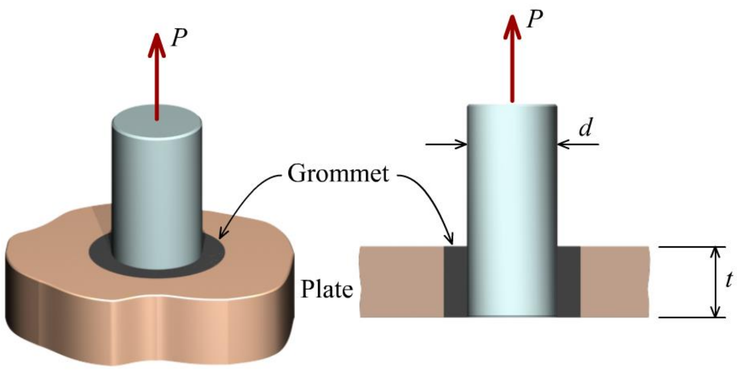


FIGURE P1.17

Solution

Visualize the contact surface between the rod and the grommet. It will be a cylinder with a diameter of d and a height of t. The area of this cylinder will be

$$A_V = \pi d t = \pi(0.625\text{ in.})(0.875\text{ in.}) = 1.718\text{ in.}^2$$

The average shear stress between the rod and the grommet is thus

$$\tau_{\text{avg}} = \frac{P}{A_V} = \frac{175\text{ lb}}{1.718\text{ in.}^2} = \boxed{101.9\text{ psi}}$$ **Ans.**

P1.18 Two wood boards, each 19 mm thick, are joined by the glued finger joint shown in Figure P1.18. The finger joint will fail when the average shear stress in the glue reaches 940 kPa. Determine the shortest allowable length *d* of the cuts if the joint is to withstand an axial load of *P* = 5.5 kN. Use *a* = 23 mm and *b* = 184 mm.

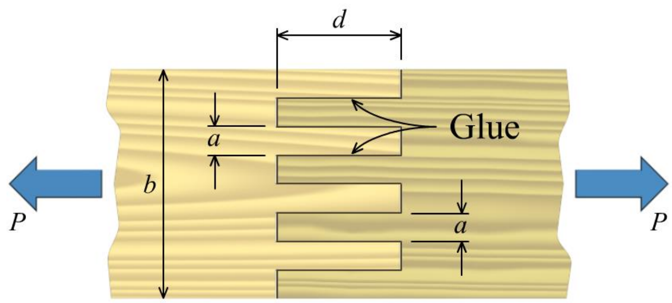


FIGURE P1.18

Solution

We are considering the shear strength of the glued joint. The minimum shear area that is required for this connection can be determined from the load *P* and the shear strength of the glue. Consequently, we will need at least this much area

$$A_{V,\min} = \frac{P}{\tau} = \frac{(5.5\text{ kN})(1{,}000\text{ N/kN})}{0.940\text{ N/mm}^2} = 5{,}851.064\text{ mm}^2$$

to transmit the load *P* through the joint, based on the shear strength of the glue.

For this particular joint, there are seven surfaces that will be glued. Each of these surfaces has a length of *d* and a thickness of 19 mm. Accordingly, the minimum length *d* required for each of the finger joints is

$$7dt \geq 5{,}851.064\text{ mm}^2$$

$$\therefore d \geq \frac{5{,}851.064\text{ mm}^2}{7(19\text{ mm})} = \boxed{44.0\text{ mm}}$$

Ans.

P1.19 For the connection shown in Figure P1.19, determine the average shear stress produced in the 7/8 in. diameter bolts if the applied load is P = 32,000 lb.

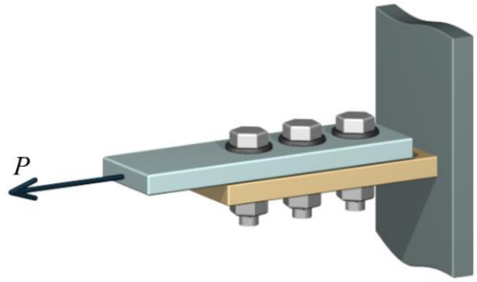


FIGURE P1.19

Solution

There are three bolts, and it is always assumed that each bolt supports an equal portion of the external load P. Therefore, the shear force V carried by each bolt is

$$V = \frac{32{,}000 \text{ lb}}{3 \text{ bolts}} = 10{,}666.667 \text{ lb}$$

The bolts in this connection act in single shear. The cross-sectional area of a single bolt is

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \frac{\pi}{4}(7/8 \text{ in.})^2 = \frac{\pi}{4}(0.875 \text{ in.})^2 = 0.6013 \text{ in.}^2$$

Therefore, the average shear stress in each bolt is

$$\tau = \frac{V}{A_{\text{bolt}}} = \frac{10{,}666.667 \text{ lb}}{0.6013 \text{ in.}^2} = 17{,}738.739 \text{ psi} = \boxed{17{,}740 \text{ psi}}$$ **Ans.**

P1.20 For the clevis connection shown in Figure P1.20, determine the maximum applied load P that can be supported by the 15 mm diameter pin if the average shear stress in the pin must not exceed 130 MPa.

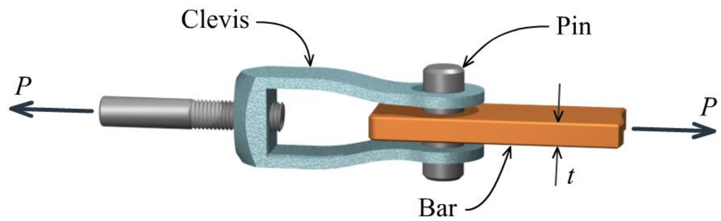
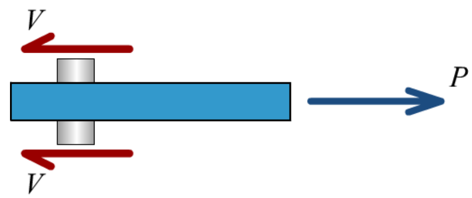


FIGURE P1.20

Solution

Consider an FBD of the bar that is connected by the clevis, including a portion of the pin. If the shear force acting on each exposed surface of the pin is denoted by V, then the shear force on each pin surface is related to the load P by:

$$\Sigma F_x = P - V - V = 0 \qquad \therefore P = 2V$$



The area of the pin surface exposed by the FBD is simply the cross-sectional area of the pin:

$$A_{\text{pin}} = \frac{\pi}{4} d_{\text{pin}}^2 = \frac{\pi}{4}(15\text{ mm})^2 = 176.715\text{ mm}^2$$

If the average shear stress in the pin must be limited to 130 MPa, the maximum shear force V on a single cross-sectional surface must be limited to

$$V = \tau A_{\text{bolt}} = \left(130\text{ N/mm}^2\right)\left(176.715\text{ mm}^2\right) = 22{,}972.95\text{ N}$$

Therefore, the maximum load P that may be applied to the connection is

$$P = 2V = 2\left(22{,}972.95\text{ N}\right) = 45{,}945.9\text{ N} = \boxed{45.9\text{ kN}}$$ **Ans.**

P1.21 The five-bolt connection shown in Figure P1.21 must support an applied load of P = 160 kips. If the average shear stress in the bolts must be limited to 30 ksi, what is the minimum bolt diameter that may be used for this connection?

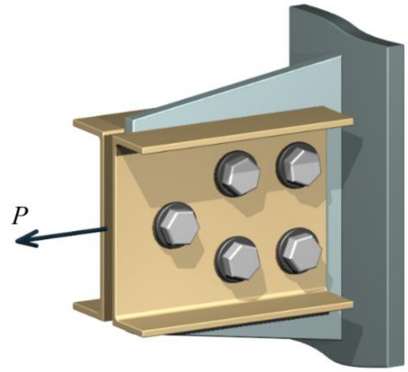


FIGURE P1.21

Solution

There are five bolts, and it is assumed that each bolt supports an equal portion of the external load P. Therefore, the shear force carried by each bolt is

$$V = \frac{160 \text{ kips}}{5 \text{ bolts}} = 32 \text{ kips}$$

Since the average shear stress must be limited to 30 ksi, each bolt must provide a shear area of at least:

$$A_V = \frac{32 \text{ kips/bolt}}{30 \text{ ksi}} = 1.0667 \text{ in.}^2\text{/bolt}$$

Each bolt in this connection acts in double shear; therefore, two cross-sectional bolt surfaces are available to transmit shear stress in each bolt.

$$A_{\text{bolt}} = \frac{A_V}{2 \text{ surfaces per bolt}} = \frac{1.0667 \text{ in.}^2\text{/bolt}}{2 \text{ surfaces/bolt}} = 0.5333 \text{ in.}^2 \text{ per bolt surface}$$

The minimum bolt diameter must be

$$\frac{\pi}{4} d_{\text{bolt}}^2 \geq 0.5333 \text{ in.}^2 \qquad \therefore d_{\text{bolt}} \geq \boxed{0.824 \text{ in.}}$$

Ans.

P1.22 The handle shown in Figure P1.22 is attached to a 40 mm diameter shaft with a square shear key. The forces applied to the lever are P = 1,300 N. If the average shear stress in the key must not exceed 150 MPa, determine the minimum dimension a that must be used if the key is 25 mm long. The overall length of the handle is L = 0.70 m.

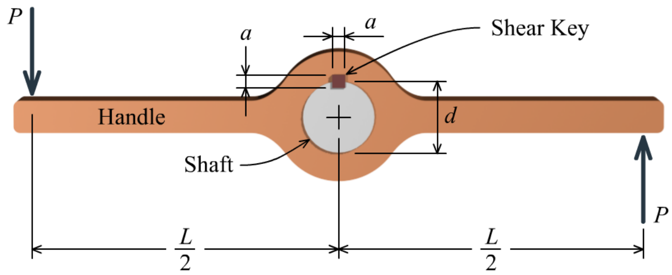


FIGURE P1.22

Solution

To determine the shear force V that must be resisted by the shear key, sum moments about the center of the shaft (which will be denoted O):

$$\Sigma M_O = (1{,}300\text{ N})\left(\frac{700\text{ mm}}{2}\right) + (1{,}300\text{ N})\left(\frac{700\text{ mm}}{2}\right) - \left(\frac{40\text{ mm}}{2}\right)V = 0$$

$$\therefore V = 45{,}500\text{ N}$$

Since the average shear stress in the key must not exceed 150 MPa, the shear area required is

$$A_V \geq \frac{V}{\tau} = \frac{45{,}500\text{ N}}{150\text{ N/mm}^2} = 303.3333\text{ mm}^2$$

The shear area in the key is given by the product of its length L (i.e., 25 mm) and its width a. Therefore, the minimum key width a is

$$a \geq \frac{A_V}{L} = \frac{303.3333\text{ mm}^2}{25\text{ mm}} = 12.1333\text{ mm} = \boxed{12.13\text{ mm}}$$ **Ans.**

P1.23 An axial load *P* is supported by the short steel column shown in Figure P1.23. The column has a cross-sectional area of 14,500 mm^2. If the average normal stress in the steel column must not exceed 75 MPa, determine the minimum required dimension *a* so that the bearing stress between the base plate and the concrete slab does not exceed 8 MPa. Assume *b* = 420 mm.

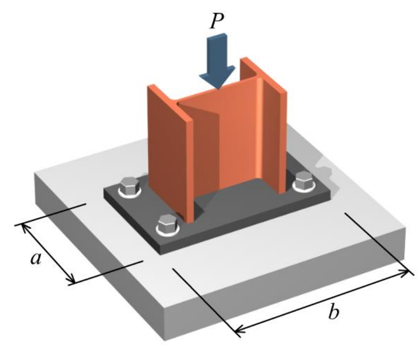


FIGURE P1.23

Solution

Since the normal stress in the steel column must not exceed 75 MPa, the maximum column load is

$$P_{max} = \sigma A = (75 \text{ N/mm}^2)(14{,}500 \text{ mm}^2) = 1{,}087{,}500 \text{ N}$$

The maximum column load must be distributed over a large enough area so that the bearing stress between the base plate and the concrete slab does not exceed 8 MPa; therefore, the minimum plate area is

$$A_{min} = \frac{P}{\sigma_b} = \frac{1{,}087{,}500 \text{ N}}{8 \text{ N/mm}^2} = 135{,}937.5 \text{ mm}^2$$

The area of the plate is $a \times b$. Since $b = 420$, the minimum length of *a* must be

$$A_{min} = 135{,}937.5 \text{ mm}^2 = a \times b$$

$$\therefore a \geq \frac{135{,}937.5 \text{ mm}^2}{420 \text{ mm}} = \boxed{324 \text{ mm}}$$

Ans.

P1.24 The two wooden boards shown in Figure P1.24 are connected by a 0.5 in. diameter bolt. Washers are installed under the head of the bolt and under the nut. The washer dimensions are D = 2 in. and d = 5/8 in. The nut is tightened to cause a tensile stress of 9,000 psi in the bolt. Determine the bearing stress between the washer and the wood.

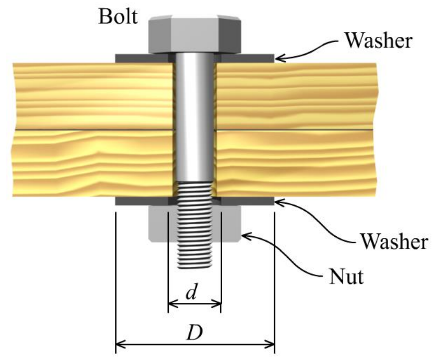


FIGURE P1.24

Solution

The tensile stress in the bolt is 9,000 psi; therefore, the tension force that acts in the bolt is

$$F_{\text{bolt}} = \sigma_{\text{bolt}} A_{\text{bolt}} = (9{,}000\text{ psi})\frac{\pi}{4}(0.5\text{ in.})^2 = (9{,}000\text{ psi})(0.196350\text{ in.}^2) = 1{,}767.146\text{ lb}$$

The contact area between the washer and the wood is

$$A_{\text{washer}} = \frac{\pi}{4}\left[(2\text{ in.})^2 - (0.625\text{ in.})^2\right] = 2.834796\text{ in.}^2$$

Thus, the bearing stress between the washer and the wood is

$$\sigma_b = \frac{1{,}767.146\text{ lb}}{2.834796\text{ in.}^2} = \boxed{623\text{ psi}}$$ **Ans.**

P1.25 For the beam shown in Figure P1.25, the allowable bearing stress for the material under the supports at *A* and *B* is σ_b = 800 psi. Assume w = 2,100 lb/ft, P = 4,600 lb, a = 20 ft, and b = 8 ft. Determine the size of *square* bearing plates required to support the loading shown. Dimension the plates to the nearest ½ in.

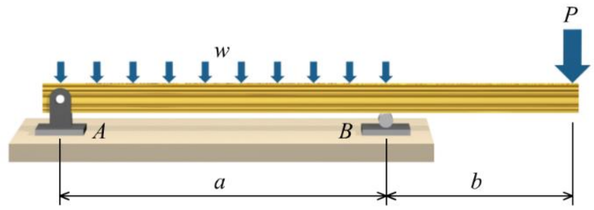
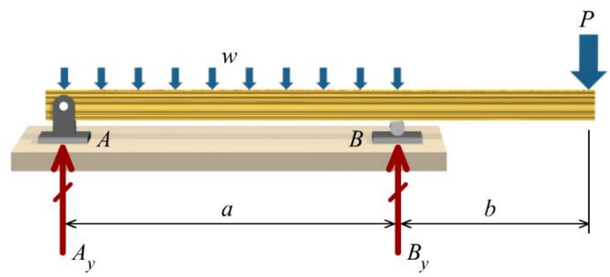


FIGURE P1.25



Solution

Equilibrium: Using the FBD shown, calculate the beam reaction forces.

$$\Sigma M_A = B_y(20\text{ ft}) - (2{,}100\text{ lb/ft})(20\text{ ft})\left(\frac{20\text{ ft}}{2}\right) - (4{,}600\text{ lb})(28\text{ ft}) = 0$$

$$\therefore B_y = 27{,}440\text{ lb}$$

$$\Sigma M_B = -A_y(20\text{ ft}) + (2{,}100\text{ lb/ft})(20\text{ ft})\left(\frac{20\text{ ft}}{2}\right) - (4{,}600\text{ lb})(8\text{ ft}) = 0$$

$$\therefore A_y = 19{,}160\text{ lb}$$

Bearing plate at *A*: The area of the bearing plate required for support *A* is

$$A_A \geq \frac{19{,}160\text{ lb}}{800\text{ psi}} = 23.950\text{ in.}^2$$

Since the plate is to be square, its dimensions must be

$$width \geq \sqrt{23.950\text{ in.}^2} = 4.894\text{ in.}$$

use 5 in. × 5 in. bearing plate at *A* **Ans.**

Bearing plate at *B*: The area of the bearing plate required for support *B* is

$$A_B \geq \frac{27{,}440\text{ lb}}{800\text{ psi}} = 34.300\text{ in.}^2$$

Since the plate is to be square, its dimensions must be

$$width \geq \sqrt{34.300\text{ in.}^2} = 5.857\text{ in.}$$

use 6 in. × 6 in. bearing plate at *B* **Ans.**

P1.26 A wood beam rests on a square post. The vertical reaction force of the beam at the post is P = 1,300 lb. The square post has cross-sectional dimensions of a = 6.25 in. The beam has a width of b = 1.50 in. and a depth of d = 7.50 in. What is the average bearing stress in the wood beam?

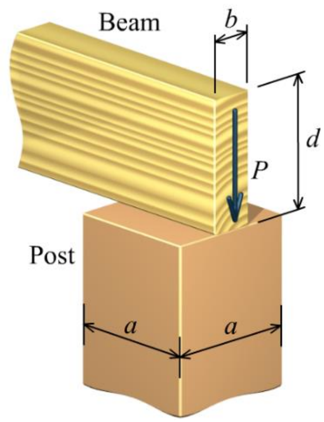


FIGURE P1.26

Solution

Contact area: Visualize the contact area between the beam and the post. The contact area is

$$A_b = ab = (6.25\text{ in.})(1.50\text{ in.}) = 9.375\text{ in.}^2$$

Bearing stress: The average bearing stress in the wood beam is

$$\sigma_b = \frac{P}{A_b} = \frac{1{,}300\text{ lb}}{9.375\text{ in.}^2} = \boxed{138.7\text{ psi}}$$ **Ans.**

P1.27 The pulley shown in Figure P1.27 is connected to a bracket with a circular pin of diameter d = 6 mm. Each vertical side of the bracket has a width of b = 25 mm and a thickness of t = 4 mm. If the pulley belt tension is P = 570 N, what is the average bearing stress produced in the bracket by the pin?

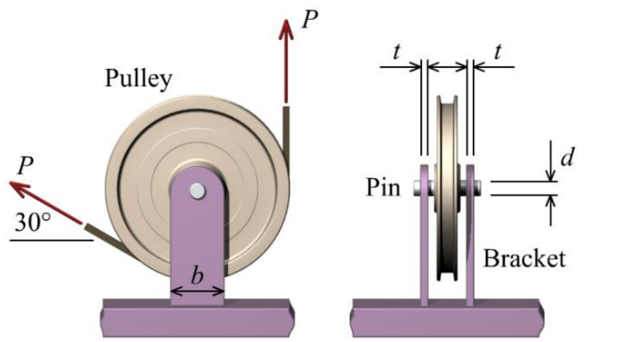
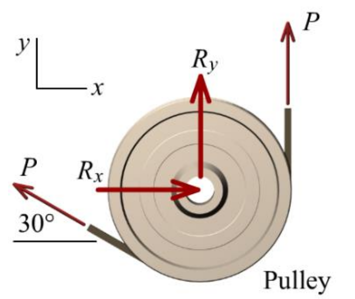


FIGURE P1.27

Solution

Pulley FBD: Consider an FBD of the pulley with the belt tensions. From equilibrium, the bracket exerts horizontal and vertical reaction forces R_x and R_y, respectively, on the pulley.



$$\Sigma F_x = R_x - P\cos(30°) = 0$$

$$\therefore R_x = P\cos(30°) = (570 \text{ N})\cos(30°) = 493.634 \text{ N}$$

$$\Sigma F_y = R_y + P + P\sin(30°) = 0$$

$$\therefore R_y = -P - P\sin(30°) = -570 \text{ N} - (570 \text{ N})\sin(30°) = -855.0 \text{ N}$$

The resultant force exerted on the pulley by the bracket is thus

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(493.634 \text{ N})^2 + (-855.0 \text{ N})^2}$$

$$= 987.269 \text{ N}$$

Bearing stress in the bracket: From Newton's Third Law, the pulley pin exerts an equal force R on the bracket. The bracket has two vertical pieces (i.e., a plate on each side of the pulley). The resultant force R is divided equally between these two vertical pieces. Therefore, the force exerted by the pin on one of the vertical bracket pieces is 493.635 N. The average bearing stress in the bracket is based on the projected area of the pin. Therefore, the average bearing stress produced in the bracket by the pin is

$$\sigma_b = \frac{R/2}{A_b} = \frac{R/2}{dt} = \frac{493.635 \text{ N}}{(6 \text{ mm})(4 \text{ mm})} = 20.568 \text{ MPa} = \boxed{20.6 \text{ MPa}}$$ **Ans.**

P1.28 The d = 15 mm diameter solid rod shown in Figure P1.28 passes through a D = 20 mm diameter hole in the support plate. When a load P is applied to the rod, the rod head rests on the support plate. The support plate has a thickness of b = 12 mm. The rod head has a diameter of a = 30 mm and the head has a thickness of t = 10 mm. If the normal stress produced in the rod by load P is 225 MPa, determine:

(a) the average bearing stress acting between the support plate and the rod head.
(b) the average shear stress produced in the rod head.
(c) the punching shear stress produced in the support plate by the rod head.

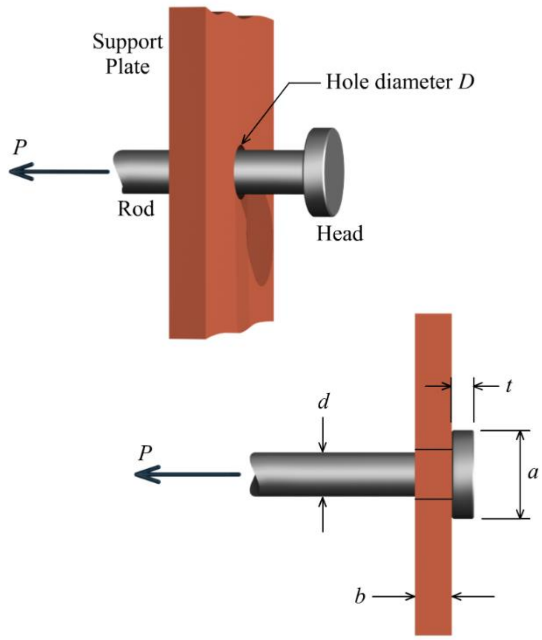


FIGURE P1.28

Solution

The cross-sectional area of the rod is:

$$A_{rod} = \frac{\pi}{4}(15 \text{ mm})^2 = 176.715 \text{ mm}^2$$

The tensile stress in the rod is 225 MPa; therefore, the tension force in the rod is

$$F_{rod} = \sigma_{rod} A_{rod} = (225 \text{ N/mm}^2)(176.715 \text{ mm}^2) = 39{,}760.782 \text{ N}$$

(a) The contact area between the support plate and the rod head is

$$A_{contact} = \frac{\pi}{4}\left[(30 \text{ mm})^2 - (20 \text{ mm})^2\right] = 392.699 \text{ mm}^2$$

Thus, the bearing stress between the support plate and the rod head is

$$\sigma_b = \frac{39{,}760.782 \text{ N}}{392.699 \text{ mm}^2} = \boxed{101.3 \text{ MPa}}$$ **Ans.**

(b) In the rod head, the area subjected to shear stress is equal to the perimeter of the rod times the thickness of the head.

$$A_V = \pi(15 \text{ mm})(10 \text{ mm}) = 471.239 \text{ mm}^2$$

and therefore, the average shear stress in the rod head is

$$\tau = \frac{39{,}760.782 \text{ N}}{471.239 \text{ mm}^2} = \boxed{84.4 \text{ MPa}}$$ **Ans.**

(c) In the support plate, the area subjected to shear stress is equal to the product of the rod head perimeter and the thickness of the plate.

$$A_V = \pi(30 \text{ mm})(12 \text{ mm}) = 1{,}130.973 \text{ mm}^2$$

and therefore, the average punching shear stress in the support plate is

$$\tau = \frac{39{,}760.782 \text{ N}}{1{,}130.973 \text{ mm}^2} = \boxed{35.2 \text{ MPa}}$$ **Ans.**

P1.29 A hollow box beam *ABCD* is supported at *A* by a pin that passes through the beam as shown in Figure P1.29. The box beam is also supported by a roller that is located at *B*. The beam dimensions are a = 2.5 ft, b = 5.5 ft, and c = 3.5 ft. Two equal concentrated loads of P = 2,750 lb are placed on the box beam at points *C* and *D*. The box beam has a wall thickness of t = 0.375 in., and the pin at *A* has a diameter of 0.750 in. Determine:
(a) the average shear stress in the pin at *A*.
(b) the average bearing stress in the box beam at *A*.

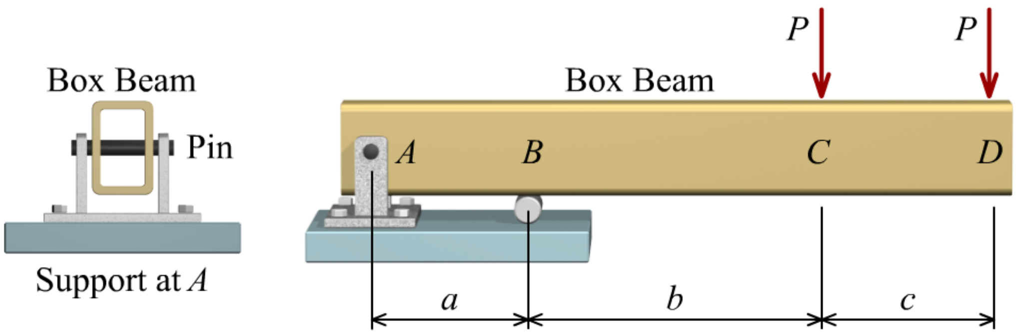


FIGURE P1.29

Solution

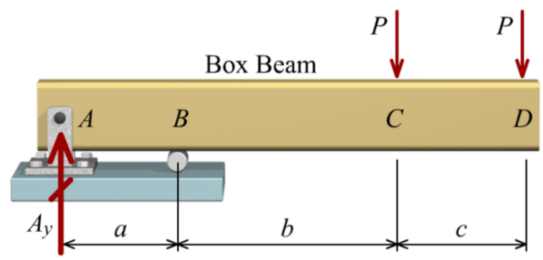
Equilibrium: Determine the reaction force exerted on the beam by the pin at *A*.

$$\Sigma M_B = -A_y a - Pb - P(b+c) = 0$$

$$A_y = -P\frac{b+(b+c)}{a} = -P\frac{2b+c}{a}$$

$$= -(2,750 \text{ lb})\frac{2(5.5 \text{ ft})+3.5 \text{ ft}}{2.5 \text{ ft}}$$

$$= -15,950 \text{ lb}$$



Average shear stress in the pin at *A*: The pin diameter is 0.750 in. The cross-sectional area of the pin is

$$A_{\text{pin}} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.750 \text{ in.})^2 = 0.4418 \text{ in.}^2$$

From the support detail figure, we observe that this pin acts in double shear; therefore, the shear area of the pin is

$$A_V = 2A_{\text{pin}} = 2(0.4418 \text{ in.}^2) = 0.8836 \text{ in.}^2$$

The average shear stress in the pin at *A* is thus

$$\tau = \frac{V}{A_V} = \frac{|A_y|}{A_V} = \frac{15,950 \text{ lb}}{0.8836 \text{ in.}^2} = 18,051.71 \text{ psi} = \boxed{18,050 \text{ psi}}$$ **Ans.**

Average bearing stress in the box beam at *A*: The average bearing stress produced in the box beam by the pin is based on the **projected area** of the pin. The projected area is equal to the pin diameter times the wall thickness of the box beam, taking into account that there are two walls that contact the pin. Therefore, the average bearing stress in the box beam is

$$\sigma_b = \frac{|A_y|}{2dt} = \frac{15,950 \text{ lb}}{2(0.750 \text{ in.})(0.375 \text{ in.})} = 28,355.56 \text{ psi} = \boxed{28,400 \text{ psi}}$$ **Ans.**

P1.30 Rigid bar *ABC* shown in Figure P1.30 is supported by a pin at bracket *A* and by tie rod (1). Tie rod (1) has a diameter of 5 mm, and it is supported by double-shear pin connections at *B* and *D*. The pin at bracket *A* is a single-shear connection. All pins are 7 mm in diameter. Assume a = 600 mm, b = 300 mm, h = 450 mm, P = 900 N, and θ = 55°. Determine the following:
(a) the normal stress in rod (1)
(b) the average shear stress in pin *B*
(c) the average shear stress in pin *A*

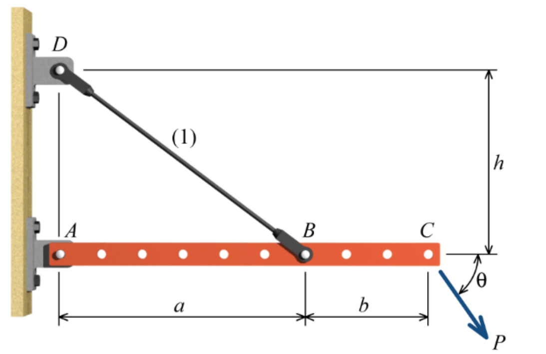
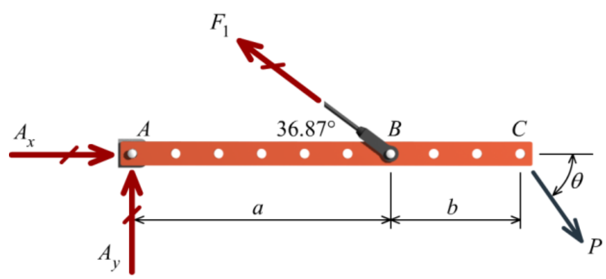


FIGURE P1.30

Solution

Equilibrium: Using the FBD shown, calculate the reaction forces that act on rigid bar *ABC*.



$$\Sigma M_A = F_1 \sin(36.87°)(600 \text{ mm}) - (900 \text{ N})\sin(55°)(900 \text{ mm}) = 0$$

$$\therefore F_1 = 1{,}843.092 \text{ N}$$

$$\Sigma F_x = A_x - (1{,}843.092 \text{ N})\cos(36.87°) + (900 \text{ N})\cos(55°) = 0$$

$$\therefore A_x = 958.255 \text{ N}$$

$$\Sigma F_y = A_y + (1{,}843.092 \text{ N})\sin(36.87°) - (900 \text{ N})\sin(55°) = 0$$

$$\therefore A_y = -368.618 \text{ N}$$

The resultant force at *A* is

$$|A| = \sqrt{(958.255 \text{ N})^2 + (-368.618 \text{ N})^2} = 1{,}026.709 \text{ N}$$

(a) Normal stress in rod (1).

$$A_{\text{rod}} = \frac{\pi}{4}(5 \text{ mm})^2 = 19.635 \text{ mm}^2$$

$$\sigma_{\text{rod}} = \frac{1{,}843.092 \text{ N}}{19.635 \text{ mm}^2} = \boxed{93.9 \text{ MPa}}$$ **Ans.**

(b) Shear stress in pin *B*. The cross-sectional area of a 7 mm diameter pin is:

$$A_{\text{pin}} = \frac{\pi}{4}(7 \text{ mm})^2 = 38.485 \text{ mm}^2$$

Pin *B* is a double shear connection; therefore, its average shear stress is

$$\tau_{\text{pin }B} = \frac{1{,}843.092 \text{ N}}{2(38.485 \text{ mm}^2)} = \boxed{23.9 \text{ MPa}}$$ **Ans.**

(c) Shear stress in pin *A*.
Pin *A* is a single shear connection; therefore, its average shear stress is

$$\tau_{\text{pin }A} = \frac{1{,}026.709 \text{ N}}{38.485 \text{ mm}^2} = \boxed{26.7 \text{ MPa}}$$ **Ans.**

P1.31 The bell crank shown in Figure P1.31 is in equilibrium for the forces acting in rods (1) and (2). The bell crank is supported by a 10 mm diameter pin at B that acts in single shear. The thickness of the bell crank is 5 mm. Assume a = 65 mm, b = 150 mm, F_1 = 1,100 N, and θ = 50°. Determine the following:
(a) the average shear stress in pin B
(b) the average bearing stress in the bell crank at B

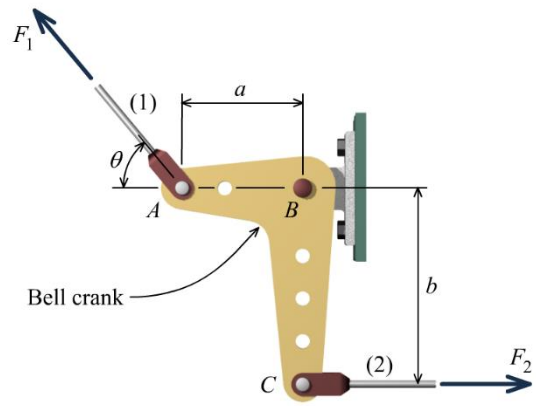


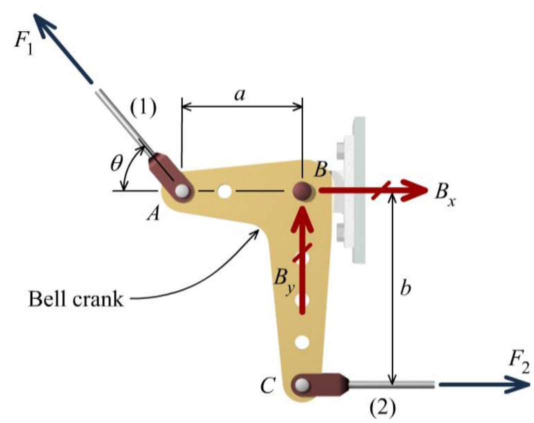
FIGURE P1.31

Solution

Equilibrium: Using the FBD shown, calculate the reaction forces that act on the bell crank.

$$\Sigma M_B = -(1{,}100 \text{ N})\sin(50°)(65 \text{ mm}) + F_2(150 \text{ mm}) = 0$$

$$\therefore F_2 = 365.148 \text{ N}$$



$$\Sigma F_x = B_x - (1{,}100 \text{ N})\cos(50°) + 365.148 \text{ N} = 0$$

$$\therefore B_x = 341.919 \text{ N}$$

$$\Sigma F_y = B_y + (1{,}100 \text{ N})\sin(50°) = 0$$

$$\therefore B_y = -842.649 \text{ N}$$

The resultant force at B is

$$|B| = \sqrt{(341.919 \text{ N})^2 + (-842.649 \text{ N})^2} = 909.376 \text{ N}$$

(a) Shear stress in pin *B*. The cross-sectional area of the 10 mm diameter pin is:

$$A_{\text{pin}} = \frac{\pi}{4}(10 \text{ mm})^2 = 78.540 \text{ mm}^2$$

Pin B is a single shear connection; therefore, its average shear stress is

$$\tau_{\text{pin } B} = \frac{909.376 \text{ N}}{78.540 \text{ mm}^2} = \boxed{11.58 \text{ MPa}}$$ **Ans.**

(b) Bearing stress in the bell crank at *B*. The average bearing stress produced in the bell crank by the pin is based on the **projected area** of the pin. The projected area is equal to the pin diameter times the bell crank thickness. Therefore, the average bearing stress in the bell crank is

$$\sigma_b = \frac{909.376 \text{ N}}{(10 \text{ mm})(5 \text{ mm})} = \boxed{18.19 \text{ MPa}}$$ **Ans.**

P1.32 The beam shown in Figure P1.32 is supported by a pin at *C* and by a short link *AB*. If w = 30 kN/m, determine the average shear stress in the pins at *A* and *C*. Each pin has a diameter of 25 mm. Assume L = 1.8 m and θ = 35°.

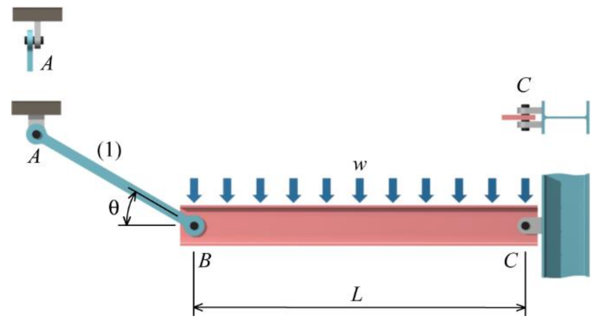
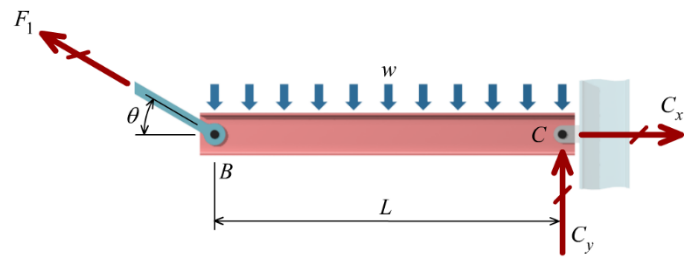


FIGURE P1.32

Solution

Equilibrium: Using the FBD shown, calculate the reaction forces that act on the beam.



$$\Sigma M_C = -F_1 \sin(35°)(1.8\text{ m}) + (30\text{ kN/m})(1.8\text{ m})\left(\frac{1.8\text{ m}}{2}\right) = 0$$

$$\therefore F_1 = 47.0731\text{ kN}$$

$$\Sigma F_x = C_x - (47.0731\text{ kN})\cos(35°) = 0$$

$$\therefore C_x = 38.5600\text{ kN}$$

$$\Sigma M_B = C_y(1.8\text{ m}) - (30\text{ kN/m})(1.8\text{ m})\left(\frac{1.8\text{ m}}{2}\right) = 0$$

$$\therefore C_y = 27.0000\text{ kN}$$

The resultant force at *C* is

$$|C| = \sqrt{(38.5600\text{ kN})^2 + (27.0000\text{ kN})^2} = 47.0731\text{ kN}$$

Shear stress in pin *A*. The cross-sectional area of a 25 mm diameter pin is:

$$A_{pin} = \frac{\pi}{4}(25\text{ mm})^2 = 490.8739\text{ mm}^2$$

Pin *A* is a single shear connection; therefore, its average shear stress is

$$\tau_{\text{pin }A} = \frac{47{,}073.1\text{ N}}{490.8739\text{ mm}^2} = \boxed{95.9\text{ MPa}}$$ **Ans.**

Shear stress in pin *C*.

Pin *C* is a double shear connection; therefore, its average shear stress is

$$\tau_{\text{pin }C} = \frac{47{,}073.1\text{ N}}{2(490.8739\text{ mm}^2)} = \boxed{47.9\text{ MPa}}$$ **Ans.**

P1.33 The bell-crank mechanism shown in Figure P1.33 is in equilibrium for an applied load of *P* = 7 kN applied at *A*. Assume *a* = 200 mm, *b* = 150 mm, and θ = 65°. Determine the minimum diameter *d* required for pin *B* for each of the following conditions:

(a) The average shear stress in the pin may not exceed 40 MPa.
(b) The bearing stress in the bell crank may not exceed 100 MPa.
(c) The bearing stress in the support bracket may not exceed 165 MPa.

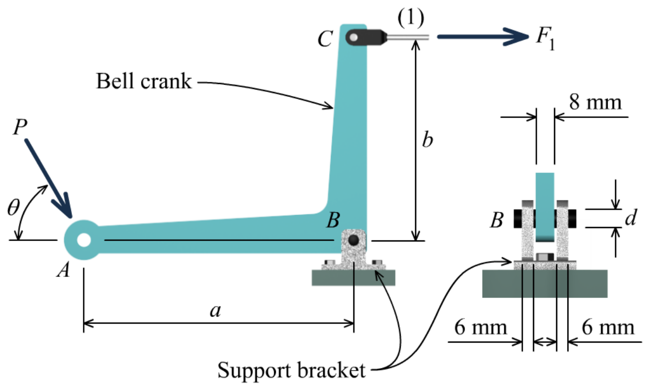


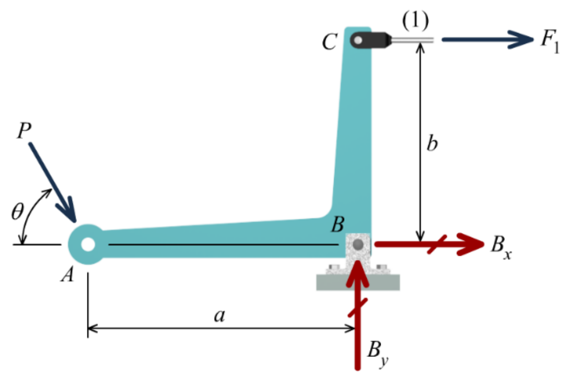
FIGURE P1.33

Solution

Equilibrium: Using the FBD shown, calculate the reaction forces that act on the bell crank.

$$\Sigma M_B = (7{,}000\text{ N})\sin(65°)(200\text{ mm}) - F_1(150\text{ mm}) = 0$$

$$\therefore F_1 = 8{,}458.873\text{ N}$$



$$\Sigma F_x = B_x + (7{,}000\text{ N})\cos(65°) + 8{,}458.873\text{ N} = 0$$

$$\therefore B_x = -11{,}417.201\text{ N}$$

$$\Sigma F_y = B_y - (7{,}000\text{ N})\sin(65°) = 0$$

$$\therefore B_y = 6{,}344.155\text{ N}$$

The resultant force at *B* is

$$|B| = \sqrt{(-11{,}417.201\text{ N})^2 + (6{,}344.155\text{ N})^2} = 13{,}061.423\text{ N}$$

(a) The average shear stress in the pin may not exceed 40 MPa. The shear area required for the pin at *B* is

$$A_V \geq \frac{13{,}061.423\text{ N}}{40\text{ N/mm}^2} = 326.536\text{ mm}^2$$

Since the pin at *B* is supported in a double shear connection, the required cross-sectional area for the pin is

$$A_{\text{pin}} = \frac{A_V}{2} = 163.268\text{ mm}^2$$

and therefore, the pin must have a diameter of

$$d \geq \sqrt{\frac{4}{\pi}(163.268\text{ mm}^2)} = \boxed{14.42\text{ mm}}$$ **Ans.**

(b) The bearing stress in the bell crank may not exceed 100 MPa. The projected area of pin *B* on the bell crank must equal or exceed

$$A_b \geq \frac{13{,}061.423 \text{ N}}{100 \text{ N/mm}^2} = 130.614 \text{ mm}^2$$

The bell crank thickness is 8 mm; therefore, the projected area of the pin is A_b = (8 mm)*d*. Calculate the required pin diameter *d*:

$$d \geq \frac{130.614 \text{ mm}^2}{8 \text{ mm}} = \boxed{16.33 \text{ mm}}$$ **Ans.**

(c) The bearing stress in the support bracket may not exceed 165 MPa. The pin at *B* bears on two 6 mm thick support brackets. Thus, the minimum pin diameter required to satisfy the bearing stress limit on the support bracket is

$$A_b \geq \frac{13{,}061.423 \text{ N}}{165 \text{ N/mm}^2} = 79.160 \text{ mm}^2$$

$$d \geq \frac{79.160 \text{ mm}^2}{2(6 \text{ mm})} = \boxed{6.60 \text{ mm}}$$ **Ans.**

P1.34 A structural steel bar with a 4.0 in. × 0.875 in. rectangular cross section is subjected to an axial load of 45 kips. Determine the maximum normal and shear stresses in the bar.

Solution

The maximum normal stress in the steel bar is

$$\sigma_{\text{max}} = \frac{F}{A} = \frac{45\text{ kips}}{(4.0\text{ in.})(0.875\text{ in.})} = \boxed{12.86\text{ ksi}}$$ **Ans.**

The maximum shear stress is one-half of the maximum normal stress

$$\tau_{\text{max}} = \frac{\sigma_{\text{max}}}{2} = \boxed{6.43\text{ ksi}}$$ **Ans.**

P1.35 A stainless steel rod of circular cross section will be used to carry an axial load of 30 kN. The maximum stresses in the rod must be limited to 100 MPa in tension and 60 MPa in shear. Determine the required minimum diameter for the rod.

Solution

Based on the allowable 100 MPa tension stress limit, the minimum cross-sectional area of the rod must equal or exceed

$$A_{\text{min}} \geq \frac{F}{\sigma_{\text{max}}} = \frac{(30\text{ kN})(1{,}000\text{ N/kN})}{100\text{ N/mm}^2} = 300\text{ mm}^2$$

For the 60 MPa shear stress limit, the minimum cross-sectional area of the rod must be equal or exceed

$$A_{\text{min}} \geq \frac{F}{2\tau_{\text{max}}} = \frac{(30\text{ kN})(1{,}000\text{ N/kN})}{2(60\text{ N/mm}^2)} = 250\text{ mm}^2$$

Therefore, the rod must have a cross-sectional area of at least 300 mm^2 to satisfy both the normal and shear stress limits.

The minimum rod diameter D is therefore

$$\frac{\pi}{4} d_{\text{min}}^2 \geq 300\text{ mm}^2 \qquad \therefore d_{\text{min}} \geq \boxed{19.54\text{ mm}}$$

Ans.

P1.36 Two wooden members, each having a width of b = 1.50 in. and a depth of d = 0.5 in., are joined by the simple glued scarf joint shown in Figure P1.36/37. Assume β = 40°. If the allowable shear stress for the glue used in the joint is 90 psi, what is the largest axial load P that may be applied?

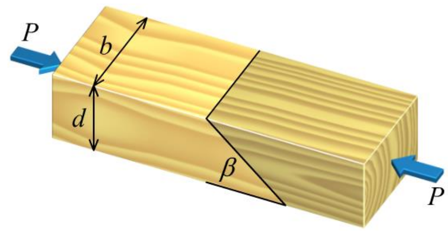


FIGURE P1.36

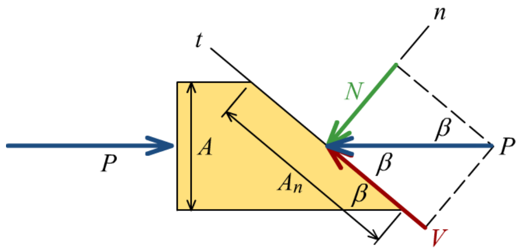
Solution

The angle β shown for the scarf joint is 40°. The normal force N perpendicular to the scarf joint can be expressed as

$$N = P\sin\beta$$

and the shear force V parallel to the scarf joint can be expressed as

$$V = P\cos\beta$$



The cross-sectional area of the bar is

$$A = bd$$

but the area along the inclined scarf joint is

$$A_n = \left(\frac{d}{\sin\beta}\right)b = \frac{A}{\sin\beta}$$

Consequently, the shear stress τ_{nt} parallel to the scarf joint can be expressed as

$$\tau_{nt} = \frac{V}{A_n} = \frac{P\cos\beta}{A/\sin\beta} = \frac{P}{A}\sin\beta\cos\beta$$

Given that the shear stress τ_{nt} must be limited to 90 psi, solve for the maximum load P as:

$$\tau_{nt} \geq \frac{P}{A}\sin\beta\cos\beta$$

$$90\text{ psi} \geq \frac{P}{bd}\sin 40°\cos 40°$$

$$P \leq \frac{(90\text{ psi})(1.50\text{ in.})(0.5\text{ in.})}{\sin 40°\cos 40°} = 137.083\text{ lb} = \boxed{137.1\text{ lb}}$$ **Ans.**

P1.37 Two wooden members, each having a width of b = 4.50 in. and a depth of d = 1.75 in., are joined by the simple glued scarf joint shown in Figure P1.36/37. Assume β = 35°. Given that the compressive axial load is P = 900 lb, what are the normal stress and shear stress magnitudes in the glued joint?

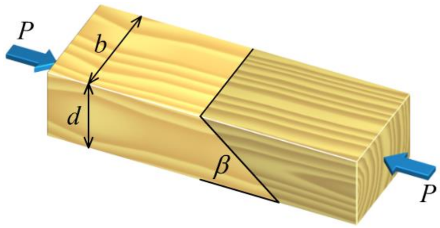


FIGURE P1.37

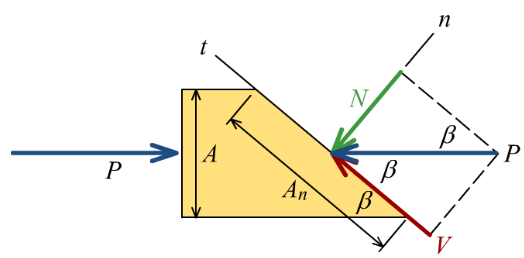
Solution

The angle β shown for the scarf joint is 35°. The normal force N perpendicular to the scarf joint can be expressed as

$$N = P\sin\beta$$

and the shear force V parallel to the scarf joint can be expressed as

$$V = P\cos\beta$$



The cross-sectional area of the bar is

$$A = bd$$

but the area along the inclined scarf joint is

$$A_n = \left(\frac{d}{\sin\beta}\right)b = \frac{A}{\sin\beta}$$

Consequently, the normal stress σ_n magnitude perpendicular to the inclined scarf joint can be expressed as

$$\sigma_n = \frac{N}{A_n} = \frac{P\sin\beta}{A/\sin\beta} = \frac{P}{A}\sin^2\beta$$

$$= \frac{900\text{ lb}}{(4.50\text{ in.})(1.75\text{ in.})}\sin^2 35° = \boxed{37.6\text{ psi}}$$ **Ans.**

and the shear stress τ_{nt} magnitude parallel to the scarf joint can be expressed as

$$\tau_{nt} = \frac{V}{A_n} = \frac{P\cos\beta}{A/\sin\beta} = \frac{P}{A}\sin\beta\cos\beta$$

$$= \frac{900\text{ lb}}{(4.50\text{ in.})(1.75\text{ in.})}\sin 35°\cos 35° = \boxed{53.7\text{ psi}}$$ **Ans.**

P1.38 Two aluminum plates, each having a width of b = 7.0 in. and a thickness of t = 0.625 in., are welded together as shown in Figure P1.38/39. Assume a = 4.0 in. For a load of P = 115 kips, determine (a) the normal stress that acts perpendicular to the weld and (b) the shear stress that acts parallel to the weld.

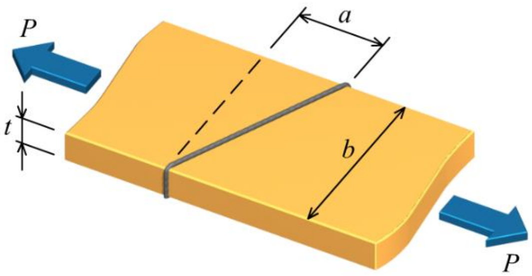


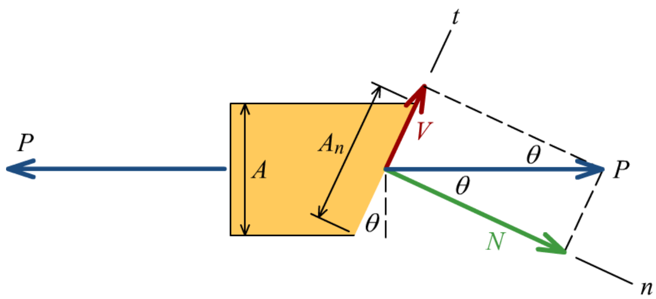
FIGURE P1.38/39

Solution

Begin by calculating the angle θ for the weld joint.

$$\tan\theta = \frac{a}{b} = \frac{4.0\text{ in.}}{7.0\text{ in.}} = 0.5714$$

$$\therefore \theta = 29.745°$$



The normal force N perpendicular to the weld joint can be expressed as

$$N = P\cos\theta$$

and the shear force V parallel to the weld joint can be expressed as

$$V = P\sin\theta$$

The cross-sectional area of the bar is

$$A = bt$$

but the area along the inclined weld joint is

$$A_n = \left(\frac{b}{\cos\theta}\right)t = \frac{A}{\cos\theta}$$

(a) Normal stress perpendicular to the weld: The normal stress σ_n magnitude perpendicular to the inclined weld joint can be expressed as

$$\sigma_n = \frac{N}{A_n} = \frac{P\cos\theta}{A/\cos\theta} = \frac{P}{A}\cos^2\theta$$

$$= \frac{115\text{ kips}}{(7.0\text{ in.})(0.625\text{ in.})}\cos^2 29.745° = \boxed{19.82\text{ ksi}}$$ **Ans.**

(b) Shear stress parallel to the weld: The shear stress τ_{nt} magnitude parallel to the weld joint can be expressed as

$$\tau_{nt} = \frac{V}{A_n} = \frac{P\sin\theta}{A/\cos\theta} = \frac{P}{A}\sin\theta\cos\theta$$

$$= \frac{115\text{ kips}}{(7.0\text{ in.})(0.625\text{ in.})}\sin 29.745^\circ \cos 29.745^\circ = \boxed{11.32\text{ ksi}}$$

Ans.

P1.39 Two aluminum plates, each having a width of b = 5.0 in. and a thickness of t = 0.75 in., are welded together as shown in Figure P1.38/39. Assume a = 2.0 in. Specifications require that the normal and shear stress magnitudes acting in the weld material may not exceed 35 ksi and 24 ksi, respectively. Determine the largest axial load P that can be applied to the aluminum plates.

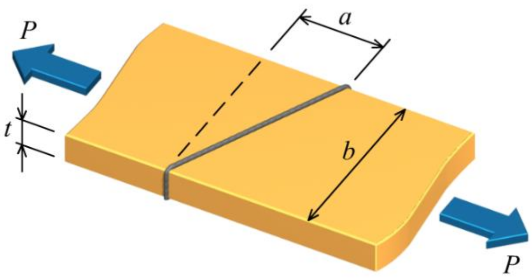


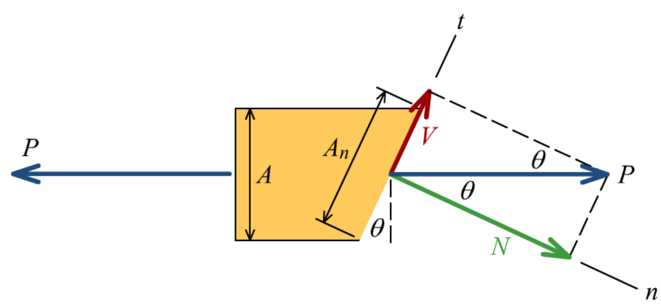
FIGURE P1.38/39

Solution

Begin by calculating the angle θ for the weld joint.

$$\tan\theta = \frac{a}{b} = \frac{2.0\text{ in.}}{5.0\text{ in.}} = 0.4$$

$$\therefore \theta = 21.801°$$



The normal force N perpendicular to the weld joint can be expressed as

$$N = P\cos\theta$$

and the shear force V parallel to the weld joint can be expressed as

$$V = P\sin\theta$$

The cross-sectional area of the bar is

$$A = bt$$

but the area along the inclined weld joint is

$$A_n = \left(\frac{b}{\cos\theta}\right)t = \frac{A}{\cos\theta}$$

Normal stress perpendicular to the weld: The normal stress σ_n magnitude perpendicular to the inclined weld joint can be expressed as

$$\sigma_n = \frac{N}{A_n} = \frac{P\cos\theta}{A/\cos\theta} = \frac{P}{A}\cos^2\theta$$

The normal stress perpendicular to the weld joint may not exceed 35 ksi. The allowable load P that satisfies this constraint is

$$P \le \frac{\sigma_n A}{\cos^2\theta} = \frac{(35\text{ ksi})(5.0\text{ in.})(0.75\text{ in.})}{\cos^2(21.801°)} = 152.25\text{ kips}$$

Shear stress parallel to the weld: The shear stress τ_{nt} magnitude parallel to the weld joint can be expressed as

$$\tau_{nt} = \frac{V}{A_n} = \frac{P\sin\theta}{A/\cos\theta} = \frac{P}{A}\sin\theta\cos\theta$$

The shear stress parallel to the weld joint may not exceed 24 ksi. The allowable load *P* that satisfies this requirement is

$$P \leq \frac{\tau_{nt} A}{\sin\theta\cos\theta} = \frac{(24\text{ ksi})(5.0\text{ in.})(0.75\text{ in.})}{\sin(21.801°)\cos(21.801°)} = 261.00\text{ kips}$$

Allowable load *P*: The largest axial load *P* that can be applied to the aluminum plates is thus

$$P \leq \boxed{152.3\text{ kips}}$$ **Ans.**

P1.40 Two wooden member are glued together as shown in Figure P1.40. Each member has a width of b = 1.50 in. and a depth of d = 3.50 in. Use β = 75°. Determine the average shear stress magnitude in the glue joint if P = 1,300 lb.

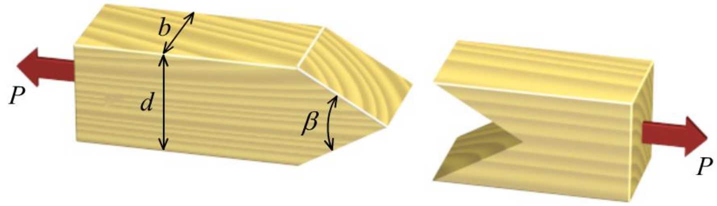
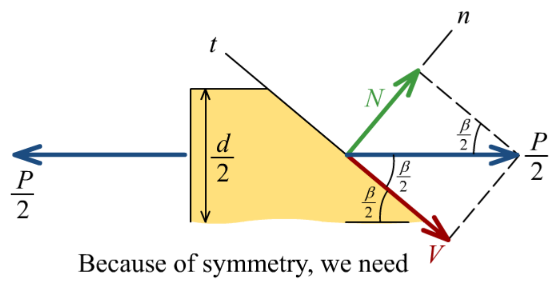


FIGURE P1.40

Solution

Using the notion of symmetry, we will consider an FBD for only the upper half of the left-hand wood piece. The central angle β for the joint is 75°.



The shear force V parallel to the upper half joint can be expressed as

$$V = \frac{P}{2}\cos\frac{\beta}{2}$$

The cross-sectional area of the upper half member is

$$A = b\left(\frac{d}{2}\right)$$

but the area along the inclined upper half joint is

$$A_n = \frac{bd}{2\sin\dfrac{\beta}{2}}$$

Consequently, the shear stress τ_{nt} magnitude parallel to the joint can be calculated as

$$\tau_{nt} = \frac{V}{A_n} = \frac{\dfrac{P}{2}\cos\dfrac{\beta}{2}}{\dfrac{bd}{2\sin\dfrac{\beta}{2}}} = \frac{P}{bd}\sin\frac{\beta}{2}\cos\frac{\beta}{2}$$

$$= \frac{1{,}300 \text{ lb}}{(1.50 \text{ in.})(3.50 \text{ in.})}\sin\frac{75°}{2}\cos\frac{75°}{2} = \boxed{119.6 \text{ psi}}$$ **Ans.**

P1.41 Two bars are connected with a welded butt joint as shown in Figure P1.41. The bar dimensions are b = 200 mm and t = 50 mm, and the angle of the weld is $\alpha = 35°$. The bars transmit a force of P = 250 kN. What is the magnitude of the average shear stress that acts on plane AB?

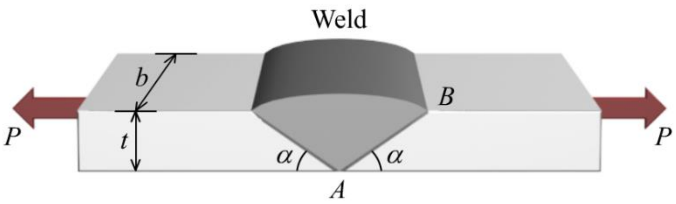
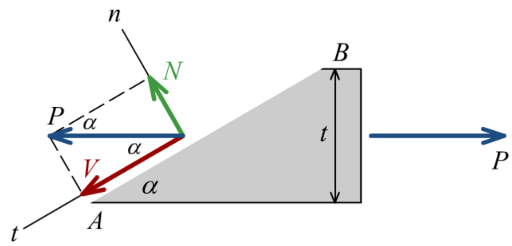


FIGURE P1.41

Solution

The angle α shown for the weld joint is 35°.



The normal force N perpendicular to the weld joint can be expressed as

$$N = P\sin\alpha$$

and the shear force V parallel to the weld joint can be expressed as

$$V = P\cos\alpha$$

The cross-sectional area of the bar is

$$A = bt$$

but the area along the inclined weld joint is

$$A_n = \left(\frac{t}{\sin\alpha}\right)b = \frac{A}{\sin\alpha}$$

Shear stress parallel to the weld: The shear stress τ_{nt} magnitude parallel to the weld joint can be expressed as

$$\tau_{nt} = \frac{V}{A_n} = \frac{P\cos\alpha}{A/\sin\alpha} = \frac{P}{A}\sin a\cos\alpha$$

$$= \frac{(250\text{ kN})(1{,}000\text{ N/kN})}{(200\text{ mm})(50\text{ mm})}\sin 35°\cos 35° = \boxed{11.75\text{ MPa}}$$ **Ans.**