Solutions to Problems in Chapter One

1.1 a) Input is thrust; output is height. Dynamic. b) Input is sunlight; output is temperature. Dynamic. c) Input is rainfall; output is height. Dynamic. d) Input is height; output is pressure. Static.

1.2 a) Only the first. b) Both. c) Both. d) Both.

1.3 $\dot{v} = 0.6t$, $v(t) = 0.3t^2 + 12$, $x(t) = 0.1t^3 + 12t + 5$.

1.4 $\dot{v} = 0.05e^{-3t}$, $v(t) = 10.017 - 0.017e^{-3t}$, $x(t) = 0.0056e^{-3t} + 10.017t + 5.994$.

1.5 Assuming that the mass is moving the the left, the friction force μmg acts to the right. Thus $m\dot{v} = f_1 + \mu mg$.

1.6 a) $m\dot{v} = f_1 - \mu mg \cos\phi - mg \sin\phi$ b) $2\dot{v} = f_1 - 0.5(2)9.81 \cos 30° - 2(9.81) \sin 30° = f_1 - 8.5 - 9.81 = f_1 - 18.31$. For $f_1 = 50$, $\dot{v} = 15.84$ and $v(t) = 15.84t + 3$. Thus, the mass does not reverse direction. For $f_1 = 5$, $\dot{v} = -6.66$ and $v(t) = -6.66t + 3$. Thus, the mass reverses direction at $t = 3/6.66 = 0.45$.

1.7 $(m - q\Delta t)(v + \Delta v) + (q\Delta t)(v - u) - mv = -mg\Delta t$. Expanding the products and cancelling terms gives $m\Delta v - qu\Delta t = -mg\Delta t$. Divide by Δt to obtain

$$m\frac{\Delta v}{\Delta t} - qu = -mg$$

Let $\Delta t \to 0$ to obtain

$$m\frac{dv}{dt} = qu - mg$$

Because $m = m_0 - qt$,

$$(m_0 - qt)\frac{dv}{dt} = qu - (m_0 - qt)g$$

1.8 The equation of motion is

$$(m_0 - qt)\frac{dv}{dt} = qu - (m_0 - qt)g \qquad (1)$$

Separating variables and integrating from $t = 0$ to t, assuming $v(0) = 0$, we obtain

$$\int_0^v dv = \int_0^t \left(\frac{qu}{m_0 - qt} - g\right) dt$$

or

$$v = [-u \ln(m_0 - qt) - gt]_0^t$$

which gives

$$v = u \ln \frac{m_0}{m_0 - qt} - gt \qquad (2)$$

The height is found by integrating $v(t)$. Assuming $h(0) = 0$,

$$h(t) = \int_0^t v(t)\,dt = \int_0^t \left(u \ln \frac{m_0}{m_0 - qt}\right) dt = \int_0^t [u \ln m_0 - u \ln(m_0 - qt) - gt]\,dt$$

Changing variables to $w = m_0 - qt$, $dw = -q\,dt$, we obtain

$$h(t) = \int_0^t (u \ln m_0 - gt)\,dt + \frac{u}{q}\int_{m_0}^{m_0 - qt} \ln w\,dw$$

Use of $\int \ln w\,dw = w \ln w - w$ gives

$$h(t) = (u \ln m_0)t - \frac{gt^2}{2} + \frac{u}{q}\left[(m_0 - qt)\ln(m_0 - qt) - (m_0 - qt) - (m_0 \ln m_0 - m_0)\right]$$

After collecting terms we obtain

$$h(t) = \frac{u}{q}(m_0 - qt)\ln(m_0 - qt) + u(\ln m_0 + 1)t - \frac{gt^2}{2} - \frac{m_0 u}{q}\ln m_0$$

1.9 Let v_0 be the initial vertical speed, v_h be the horizontal speed (which remains constant), and v_f be the final vertical speed. Thus $v_0 = 20$ ft/sec, $v_h = 50(5280)/3600 = 73.3$ ft/sec, and v_f must be found. Let h be the height in feet of the truck platform above the road. From conservation of energy, the energy of the package when it leaves the platform must equal its energy when it strikes the road. Thus

$$\frac{1}{2}mv_0^2 + mgh + \frac{1}{2}mv_h^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}mv_h^2$$

Note that the mass m cancels. The solution for v_f is

$$v_f = \sqrt{400 + 2gh}$$

where $g = 32.2$.

1.10 Let R be the wheel radius; m_b the vehicle's body mass, and m_w the wheel mass. The kinetic energy of the vehicle is

$$KE = \frac{1}{2}\left(m_b + 4m_w + 4\frac{I_w}{R^2}\right)v^2$$

Thus the equivalent mass is

$$m_e = m_b + 4m_w + 4\frac{I_w}{R^2} = \frac{500}{g} + 4\frac{30}{g} + 4\frac{0.1}{R^2} = \frac{620}{g} + \frac{0.4}{R^2} = 19.25 + \frac{0.4}{R^2}$$

Let v be the speed parallel to the surface. The equation of motion is $m_e\dot{v} = (m_b + 4m_w)g \sin 15° = 620 \sin 15°$. Thus

$$v(t) = \frac{620 \sin 15°}{m_e}t$$

and

$$v(30) = \frac{620 \sin 15°}{m_e}30 = \frac{18\,600 \sin 15°}{m_e} = \frac{4814}{m_e}$$

1.11 The equivalent mass is $m_e = m_w + I_w/R^2 = 80 + 3/(0.3)^2 = 113.33$ kg. The equation of motion is $m_e\dot{v} = 400 - m_w g \sin 25° = 68.33$ or $\dot{v} = 0.603$. Thus $v(t) = 0.603t$ and $v(60) = 36.18$ m/s. In addition, $\omega(60) = v(60)/R = 36.18/0.3 = 120.6$ rad/sec, or 1152 rpm.

1.12 Using kinetic energy equivalence, $KE = 0.5mv^2 = 0.5I\omega^2$. The mass translates a distance x when the screw rotates by θ radians. When $\theta = 2\pi$, $x = L$. Thus $x = L\theta/2\pi$ and $\dot{x} = v = L\dot{\theta}/2\pi$. Because $\omega = \dot{\theta}$,

$$KE = 0.5m\left(\frac{L\omega}{2\pi}\right)^2 = 0.5I\omega^2$$

Solve for I to obtain $I = mL^2/4\pi^2$.

1.13 The error is $(\sin\theta - \theta)/\sin\theta$. When $\theta = 0.25$ rad, the error is -0.0105, or -1.05%. When $\theta = 0.54$ rad, the error is -0.005, or 5%.

1.14 Let $f(x) = 3x - 2 + e^{-3x}$. Newton's formula is

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_{i+1})} = x_i - \frac{3x - 2 + e^{-3x}}{3 - 3e^{-3x}}$$

A plot of $f(x)$ shows that there is a zero crossing near $x = 0.6$. So, using $x_0 = 0.6$, the first iteration gives $x_1 = 0.6139$; the second iteration gives $x_2 = 0.6138$, and the third iteration also gives $x_3 = 0.6138$. Thus, to four decimal places, the process has converged to the solution $x = 0.6138$.

1.15 a) $-17 - 51i$ b) $(53 - 9i)/85$

1.16 a) $x = \sqrt{125}\angle 207^\circ$ b) $x = 6e^{8i} = 6(\cos 8 + i \sin 8) = -0.873 + 5.936i$

1.17 a) $x = 9/16$, $y = -29/32$. b) $x = 17/(20a + 15)$. $y = (10 - 15a)/(20a + 15)$. No solution exists if $a = -3/4$. c) $x = -0/(12+12a)$. $y = -0/(12+12a)$. $x = y = 0$ if $a \neq -1$. An infinite number of solutions exist if $a = -1$; these are related by $y = -4x/3$.

1.18 Both properties are true.

1.19

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$|s\mathbf{I} - \mathbf{A}| = s^2 + 5s + 6 = 0$. The roots are $s = -2$ and -3.

1.20 The MATLAB session is

```
>>x = 2;y = 5;
>>y*x^4/(x-y)
ans =
  -26.6667
>>5*x/(3*y)
ans =
  0.6667
>>5*x*y/3
ans =
  16.6667
>>x^3/(x^3-1)
ans =
  1.1429
>>1/(1-1/x^3)
ans =
  1.1429
```

```
≫5*pi*x^2
ans =
  62.8319
≫7*y/(3*x-5)
ans =
  35
≫4*(y-2)/(7*x-6)
ans =
  1.5000
```

1.21 The MATLAB session is

```
≫(-3+5i)*(-6+7i)
ans =
  -17.000 - 51.000i
≫(-3+5i)/(-6+7i)
ans =
  0.625 - 0.1059i
≫3*i/2
ans =
  ≫0 + 1.5000i
≫3/(2i)
ans =
  0 - 1.5000i
```

1.22 The MATLAB session is

```
≫x = -5-7i;y = 6+2i;
≫x+y
ans =
  1.000 - 5.000i
≫x*y
ans =
  -16.0000 - 52.0000i
≫x/y
ans =
  -1.1000 - 0.8000i
```

1.23 The MATLAB session is

```
≫A = [5,-4,2;6,4,-7;3,7,12];
≫B = [5,9,-4;7,4,3;-8,3,1];
≫C = [-8,-3,2;10,6,1;3,-9,8];
≫A*(B+C)-(A*B+A*C)
ans =
  0    0    0
  0    0    0
  0    0    0
≫(A*B)*C-A*(B*C)
ans =
  0    0    0
  0    0    0
  0    0    0
```

1.24 a) compute the time for the ball to hit the ground by finding the roots of $y = 2 + 5.736t - 4.905t^2 = 0$.

```
≫tmax=roots([-4.905,5.736,2])
   tmax =
     1.4505
     -0.2811
```

The time is 1.45 s. Next run the following m-file.

```
t=[0:.01:1.5];
y=2+5.736*t-4.9058*t.^2;
x=8.192*t;
vy=5.736-9.81*t;
plot(t,y,t,vy),xlabel('t (s)'),ylabel('Height and Vertical Velocity'),...
gtext('Height (m)'),gtext('Velocity (m/s)')
```

b) Replace the last two lines in the previous file with

```
plot(x,y),xlabel('x (m)'),ylabel('y (m)'),grid
```

The plot is shown in Figure 1.24b.

1.25 a) The plotting file is

```
x=[0:.01:2];
f=3*x-2+exp(-3*x);
plot(x,f),xlabel('x'),ylabel('f'),grid
```

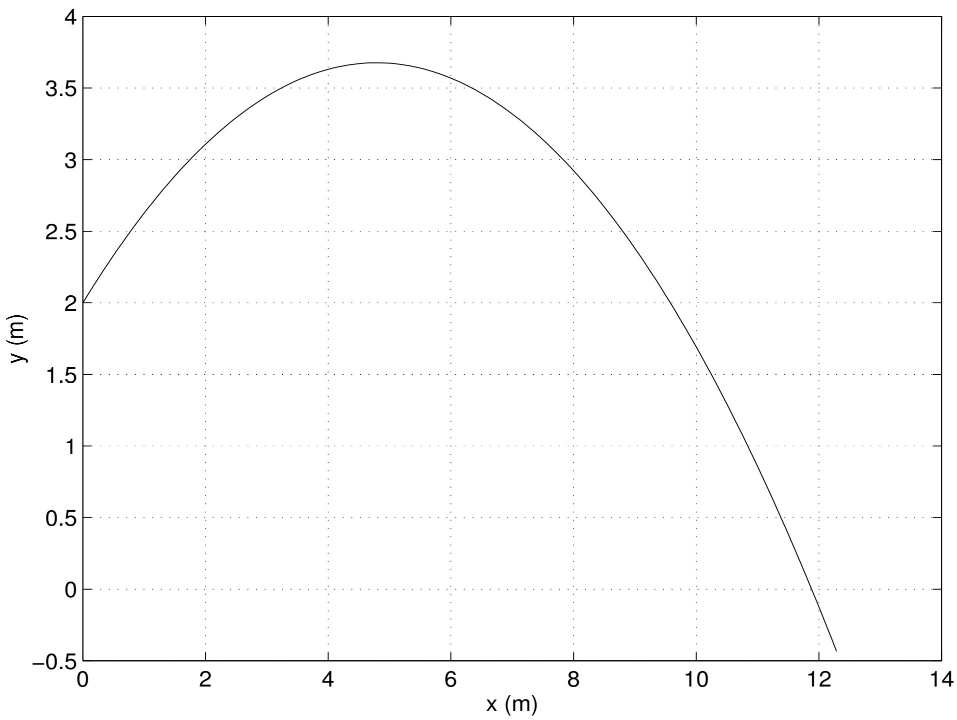

Figure : 1.24b.

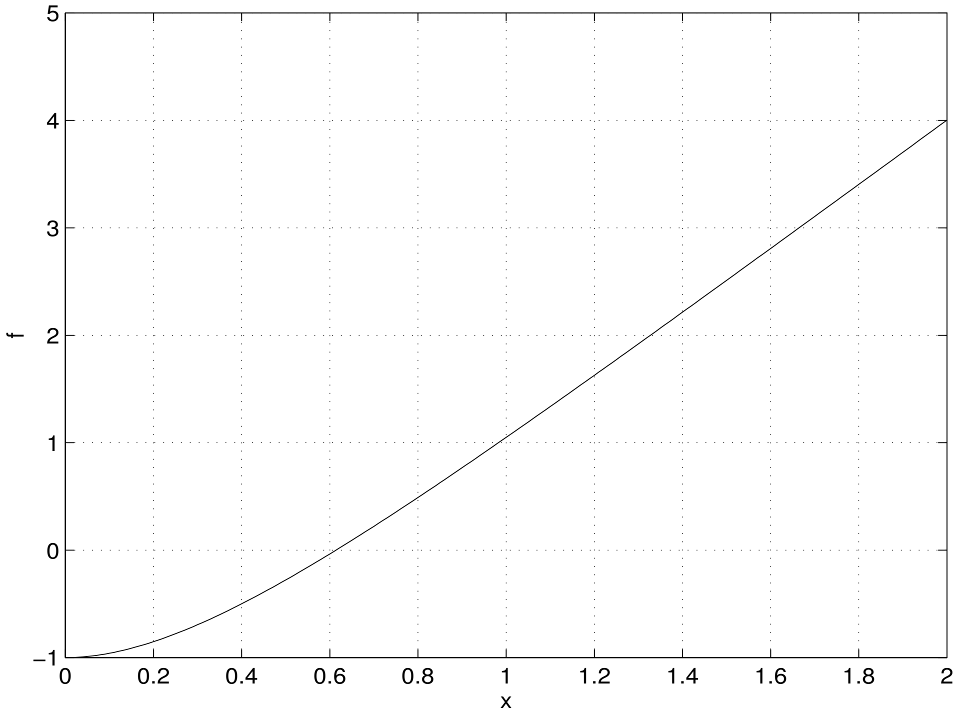

Figure : 1.25a.

The plot is shown in Figure 1.25. The function crosses 0 near $x = 0.6$.

b) First define the function file (named arbitrarily for Problem 25):

```
function f=f25(x)
f=3*x-2+exp(-3*x)
```

Then type in the command window:

```
≫fzero('f25',.6)
  ans =
     0.6138
```

The root is $x = 0.6138$.

1.26 The MATLAB session is

```
≫A=[0,1;-6,-5];
≫p=poly(A)
     p =
     1 5 6
```

```
≫roots(p)
   ans =
      -3
      -2
```

1.27 The MATLAB session is

```
≫roots([1,-2,6,-27])
   ans =
      3.0000
      -0.5000 + 2.9580i
      -0.5000 - 2.9580i
```

1.28 a) $\dot{v} = f/m$. $v(t) = (f/m)t$ if f is constant. Thus, $x(t) = (f/m)t^2/2$, and $f = 2mx_d/t_d^2$. b) $m = 75 + 25f/1200$. From part (a),

$$f = \frac{2(75 + 25f/1200)x_d}{t_d^2}$$

Solve this for f.

$$f = \frac{150}{\frac{t_d^2}{x_d} - \frac{1}{24}}$$

This can be used to plot f versus t_d^2/x_d.

1.29 a) From (1.3-11) and following, $\ddot{\theta} = 5T - 62.5$. Thus $A = 5T - 62.5$ and $y(t) = At^2/2 = (5T - 62.5)t^2/2$, $y_d = At_d^2/2 = (5T - 62.5)t_d^2/$. Solve for T to obtain

$$T = \frac{2y_d}{5T_d^2} + 12.5 \qquad (1)$$

Because $R_2\dot{\theta} = \dot{y}$ and $R_1\omega = R_3\dot{\theta}$, we have

$$\omega = \frac{R_3}{R_1R_2}\dot{y} = 4\dot{y} = 4R_3At$$

Because $R_3 = 1$,

$$\omega = (20T - 250)t$$

and

$$\omega_{max} = (20T - 250)t_d = 20t_d\left(\frac{2y_d}{t_d^2} + 12.5\right) - 250t_d$$

which reduces to

$$\omega_{max} = 40\frac{y_d}{t_d} \qquad (2)$$

The design equations are (1) and (2). For $y_d = 5$ ft and $t_d = 20$ sec, we obtain $T = 12.05$ ft-lb and $\omega_{max} = 10$ rad/sec, or 95.5 rpm.

b) Plot ω_{max} vs. y_d/t_d. It with be a straight line with a zero intercept and a slope of 40. Plot T vs. y_d/t_d^2. It with be a straight line with an intercept of 12.5 and a slope of 2/5.

1.30 a) The equivalent inertia is

$$I_e = (W/g)R_w^2 + 2I_w \qquad (1).$$

Note that $I_e\dot{v} = R_w T$, and that $\dot{v} = (v_d - 0)/t_d = v_d/t_d$. Thus the torque required is

$$T = \frac{I_e v_d}{R_w t_d} \qquad (2)$$

b) $m_e\dot{v} = T/R_w - mg \sin S$. $m_e = I_e/R_w^2$. $mg = W$. Thus,

$$\frac{I_e\dot{v}}{R_w^2} = \frac{T}{R_w} - W \sin S$$

or

$$\dot{v} = \frac{v_d}{t_d} - \frac{WR_w^2}{I_e} \sin S$$

$$v(t) = \left(\frac{v_d}{t_d} - \frac{WR_w^2}{I_e} \sin S\right) t \qquad (3)$$

The program inputs are W, R_w, g, I_w, S, v_d, t_d. Use (1) to compute I_e; use (2) to compute T, and use (3) to compute and plot $v(t)$.

1.31 The vehicle weight is $W = 3700 + E$, where E is the engine weight. From Example 1.4-2, $R_w = 0.75$ ft and $I_w = 13.1$ slug-ft^2. Using the results from Problem 1.30 with $t_d = 10$ sec and $v_d = 15(5280)/3600 = 22$ ft/sec, we have

$$I_e = \frac{3700 + E}{g} R_w^2 + 2I_w = 90.835 + 0.017475E$$

$$T = 22(90.835 + 0.017475E)/[0.75(10)] = 2.933(90.835 + 0.017475E)$$

Substituting the various engine weights into the above formula shows that only engines #4 and #5 require a torque that is less that the maximum available (288 vs. 300 ft-lb for #4, and 292 vs. 350 ft-lb for #5). Therefore, choose engine #4 because it costs less than #5.

1.32 The equivalent inertia of the lead screw is

$$I_e = \frac{mL^2}{4\pi^2} = \frac{0.9(1/96)^2}{4\pi^2} = 2.474 \times 10^{-6}$$

The total inertia felt by the motor is

$$I_{total} = I_e + 8.68 \times 10^{-5} + I_{motor} = 8.927 \times 10^{-5} + I_{motor}$$

The equation of motion is

$$I_{total}\dot{\omega} = T - T_F = T - 0.005$$

The desired linear acceleration is $\dot{v} = 0.05/1 = 0.05$ ft/sec^2. Because $x = L\theta/2\pi = 0.00165778\theta$, the required angular acceleration is $\dot{\omega} = \dot{v}/0.0016578 = 30.159$ rad/sec^2. Thus the required torque is

$$T = I_{total}\dot{\omega} + T_F = (8.927 \times 10^{-5} + I_{motor})(30.159) + 0.005 = 0.0769 + 30.159 I_{motor}$$

Using this equation, we find that the torques required for the four motors are 0.0167, 0.0228, 0.038, and 0.068 ft-lb, respectively. For motors 1 and 2, the required torque is greater than the available motor torque. Thus motors 1 and 2 will not work. For motors 3 and 4, the required torque is less than the available motor torque. Thus motors 3 and 4 will work (all the motors have a maximum speed greater than the required speed of 288 rpm.

Solutions to Problems in Chapter Two©

2.1 a) Note that

$$\int_0^t e^{-at}\,dt = \frac{1}{a}(1 - e^{-at})$$

From Property 5,

$$\mathcal{L}(1 - e^{-at}) = \mathcal{L}(a \int_0^t e^{-at}\,dt) = \frac{a}{s}\left[\frac{1}{s+a} + h(0)\right]$$

where

$$h(0) = \int e^{-at}\,dt\Big|_{t=0} = \frac{1}{a}(1-1) = 0$$

Thus

$$\mathcal{L}(1 - e^{-at}) = \frac{a}{s(s+a)}$$

b) Let $f(t) = t^{n-1}/(n-1)!$. Then $F(s) = 1/s^n$. From Property 7,

$$\mathcal{L}\left(\frac{e^{-at}t^{n-1}}{(n-1)!}\right) = \frac{1}{s^n}$$

c) Let $f(t) = (\sin bt)/b$. Then $F(s) = 1/(s^2 + b^2)$. Note that

$$\frac{1}{b}\frac{d(\sin bt)}{dt} = \cos bt$$

From Property 2,

$$\mathcal{L}(\cos bt) = \frac{s}{s^2+b^2} - \frac{\sin 0}{b} = \frac{s}{s^2+b^2}$$

2.2 Note that $s^2 + 6s + 8 = (s+2)(s+4)$. Let

$$F(s) = \frac{3s+5}{(s+2)(s+4)} = 3\frac{s+5/3}{(s+2)(s+4)}$$