

FIFTH EDITION

Modern Digital and
Analog Communication Systems



Solution Manual

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Chapter 2

2.1-1 The step function $u(t)$ is a power signal because its energy is infinite, i.e., $E_u = \int_{-\infty}^{\infty} u^2(t) dt = \infty$.

2.1-2 Let us denote the signal in question by $g(t)$ and its energy by E_g .

(a),(b) For parts **(a)** and **(b)**, we write

$$E_g = \int_0^{2\pi} \sin^2 t dt = \frac{1}{2} \int_0^{2\pi} dt - \frac{1}{2} \int_0^{2\pi} \cos 2t dt = \pi + 0 = \pi$$

(c)

$$E_g = \int_{2\pi}^{4\pi} \sin^2 t dt = \frac{1}{2} \int_{2\pi}^{4\pi} dt - \frac{1}{2} \int_{2\pi}^{4\pi} \cos 2t dt = \pi + 0 = \pi$$

(d)

$$E_g = \int_0^{2\pi} (2 \sin t)^2 dt = 4 \left[\frac{1}{2} \int_0^{2\pi} dt - \frac{1}{2} \int_0^{2\pi} \cos 2t dt \right] = 4[\pi + 0] = 4\pi$$

Sign change and time shift do not affect the signal energy. Doubling the signal quadruples its energy. In the same way, we can show that the energy of $kg(t)$ is $k^2 E_g$.

2.1-3 Both $\varphi(t)$ and $w_0(t)$ are periodic.

The average power of $\varphi(t)$ is $P_g = \frac{1}{T} \int_0^T \varphi^2(t) dt = \frac{1}{\pi} \int_0^{\pi} (e^{-t/2})^2 dt = \frac{1-e^{-\pi}}{\pi}$.

The average power of $w_0(t)$ is $P_g = \frac{1}{T_0} \int_0^{T_0} w_0^2(t) dt = \frac{1}{T_0} \int_0^{T_0} 1 \cdot dt = 1$.

2.1-4

(a) Since $x(t)$ is a real signal, $E_x = \int_0^2 x^2(t) dt$.

Solving for Fig. S2.1-4(a), we have

$$E_x = \int_0^2 (1)^2 dt = 2, \quad E_y = \int_0^1 (1)^2 dt + \int_1^2 (-1)^2 dt = 2$$

$$E_{x+y} = \int_0^1 (2)^2 dt = 4, \quad E_{x-y} = \int_1^2 (2)^2 dt = 4$$

Therefore, $E_{x\pm y} = E_x + E_y$.

Solving for Fig. S2.1-4(b), we have

$$E_x = \int_0^{\pi} (1)^2 dt + \int_{\pi}^{2\pi} (-1)^2 dt = 2\pi, \quad E_y = \int_0^{\pi/2} (1)^2 dt + \int_{\pi/2}^{\pi} (-1)^2 dt + \int_{\pi}^{3\pi/2} (1)^2 dt + \int_{3\pi/2}^{2\pi} (-1)^2 dt = 2\pi$$

$$E_{x+y} = \int_0^{\pi/2} (2)^2 dt + \int_{\pi/2}^{3\pi/2} (0)^2 dt + \int_{3\pi/2}^{2\pi} (-2)^2 dt = 4\pi$$

$$E_{x-y} = \int_0^{\pi/2} (0)^2 dt + \int_{\pi/2}^{\pi} (2)^2 dt + \int_{\pi}^{3\pi/2} (-2)^2 dt + \int_{3\pi/2}^{2\pi} (0)^2 dt = 4\pi$$

Therefore, $E_{x\pm y} = E_x + E_y$.

(b) $E_x = \int_0^{\pi/4} (1)^2 dt + \int_{\pi/4}^{\pi} (-1)^2 dt = \pi, \quad E_y = \int_0^{\pi} (1)^2 dt = \pi$

$$E_{x+y} = \int_0^{\pi/4} (2)^2 dt + \int_{\pi/4}^{\pi} (0)^2 dt = \pi, \quad E_{x-y} = \int_0^{\pi/4} (0)^2 dt + \int_{\pi/4}^{\pi} (-2)^2 dt = 3\pi$$

Therefore, $E_{x\pm y} \neq E_x + E_y$, and $E_{\hat{x}\pm\hat{y}} = E_{\hat{x}} \pm E_{\hat{y}}$ are not true in general.

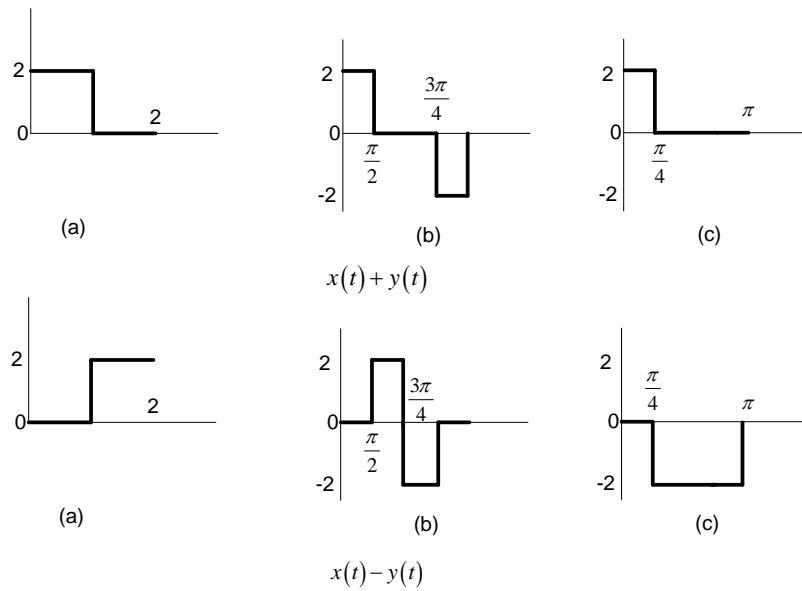


Fig. S2.1-4

2.1-5

$$\begin{aligned}
 P_g &= \frac{1}{4} \int_{-2}^2 (t^3)^2 dt = 64/7 \\
 \text{(a) } P_{-g(t)} &= \frac{1}{4} \int_{-2}^2 (-t^3)^2 dt = 64/7 \\
 \text{(b) } P_{1.5g(t)} &= \frac{1}{4} \int_{-2}^2 (1.5t^3)^2 dt = 144/7 \\
 \text{(c) } P_{g(-t)} &= \frac{1}{4} \int_{-2}^2 (-t^3)^2 dt = 64/7 \\
 \text{(d) } P_{g(1.5t)} &= \frac{3}{8} \int_{-4/3}^{4/3} ((1.5t)^3)^2 dt = 64/7
 \end{aligned}$$

Comment: Changing the sign of a signal does not affect its power. Multiplication of a signal by a constant c increases the power by a factor of c^2 . Time-scaling of a signal does not change its power, though the signal period changes.

2.1-6

$$\begin{aligned}
 P_g &= \frac{1}{T_0} \int_0^{T_0} C^2 \cos^2(\omega_0 t + \theta) dt = \frac{C^2}{2T_0} \int_0^{T_0} [1 + \cos(2\omega_0 t + 2\theta)] dt \\
 &= \frac{C^2}{2T_0} \left[\int_0^{T_0} dt + \int_0^{T_0} \cos(2\omega_0 t + 2\theta) dt \right] = \frac{C^2}{2T_0} [T_0 + 0] = \frac{C^2}{2}
 \end{aligned}$$

2.1-7 If $\omega_1 = \omega_2$, then

$$\begin{aligned}
 g^2(t) &= (C_1 \cos(\omega_1 t + \theta_1) + C_2 \cos(\omega_1 t + \theta_2))^2 \\
 &= C_1^2 \cos^2(\omega_1 t + \theta_1) + C_2^2 \cos^2(\omega_1 t + \theta_2) + 2C_1 C_2 \cos(\omega_1 t + \theta_1) \cos(\omega_1 t + \theta_2)
 \end{aligned}$$

$$\begin{aligned}
P_g &= \lim_{T_0 \rightarrow \infty} \frac{1}{T_0} \int_0^{T_0} (C_1 \cos(\omega_1 t + \theta_1) + C_2 \cos(\omega_1 t + \theta_2))^2 dt \\
&= \frac{C_1^2}{2} + \frac{C_2^2}{2} + \lim_{T \rightarrow \infty} 2C_1 C_2 \frac{1}{T_0} \int_0^{T_0} \cos(\omega_1 t + \theta_1) \cos(\omega_1 t + \theta_2) dt \\
&= \frac{C_1^2}{2} + \frac{C_2^2}{2} + \lim_{T \rightarrow \infty} 2C_1 C_2 \frac{1}{T_0} \int_0^{T_0} \frac{1}{2} [\cos(2\omega_1 t + \theta_1 + \theta_2) + \cos(\theta_1 - \theta_2)] dt \\
&= \frac{C_1^2}{2} + \frac{C_2^2}{2} + 0 + \frac{2C_1 C_2}{2} \cos(\theta_1 - \theta_2) \\
&= \frac{C_1^2 + C_2^2 + 2C_1 C_2 \cos(\theta_1 - \theta_2)}{2}
\end{aligned}$$

2.1-8

$$\begin{aligned}
P_g &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t)g^*(t) dt \\
&= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=m}^n \sum_{r=m}^n D_k D_r^* e^{j(\omega_k - \omega_r)t} dt \\
&= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=m}^n \sum_{r=m, r \neq k}^n D_k D_r^* e^{j(\omega_k - \omega_r)t} dt + \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=m}^n |D_k|^2 dt
\end{aligned}$$

The integrals of the cross-product terms (when $k \neq r$) are finite because the integrands (functions to be integrated) are periodic signals (made up of sinusoids). These terms, when divided by $T \rightarrow \infty$, yield zero. The remaining terms ($k = r$) yield

$$P_g = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=m}^n |D_k|^2 dt = \sum_{k=m}^n |D_k|^2$$

2.1-9

(a) From Eq. (2.5a), the power of a signal of amplitude C is $P_g = \frac{C^2}{2}$, regardless of phase and frequency; therefore, $P_g = 25/2$; the rms value is $\sqrt{P_g} = 5/\sqrt{2}$.

(b) From Eq. (2.5b), the power of the sum of two sinusoids of different frequencies is the sum of the power of individual sinusoids, regardless of the phase, $\frac{C_1^2}{2} + \frac{C_2^2}{2}$, therefore, $P_g = 25/2 + 4/2 = 12.5 + 2 = 14.5$; the rms value is $\sqrt{P_g} = \sqrt{14.5}$.

(c) Same as (b), $P_g = 25/2 + 4/2 = 12.5 + 2 = 14.5$; the rms value is $\sqrt{P_g} = \sqrt{14.5}$.

(d) $g(t) = 5 \sin(55t) \sin(\pi t) = \frac{5(\cos(55t - \pi t) - \cos(55t + \pi t))}{2}$

Therefore, $P_g = 25/8 + 25/8 = 25/4$; the rms value is $\sqrt{P_g} = 5/2$.

(e) Given $g(t) = 10 \sin(5t) \cos(10t) u(t)$. By definition,

$$\begin{aligned}
P_g &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^{T/2} 100 \sin^2(5t) \cos^2(10t) dt \\
&= \lim_{T \rightarrow \infty} \frac{100}{T} \int_0^{T/2} \left(\frac{1 - \cos(10t)}{2} \right) \left(\frac{1 + \cos(20t)}{2} \right) dt = \frac{100}{4T} \frac{T}{2} = 25/2.
\end{aligned}$$

The rms value is $\sqrt{P_g} = 5/\sqrt{2}$.

(f) $|g(t)|^2 = \sin^2(\omega_0 t)$

Therefore, $P_g = 1/2 = 0.5$; the rms value is $\sqrt{P_g} = \sqrt{0.5}$.

2.1-10

(a) $P_g = \frac{1}{4} \int_{-2}^2 1^2 dt = 1$; the rms value is $\sqrt{P_g} = 1$.

(b) $P_g = \frac{1}{10\pi} \int_{-\pi}^{\pi} 1^2 dt = \frac{1}{5}$; the rms value is $\sqrt{P_g} = 1/\sqrt{5}$.

(c) $P_g = \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{t}{2\pi}\right)^2 dt = \frac{1}{3}$; the rms value is $\sqrt{P_g} = 1/\sqrt{3}$.

(d) $P_g = \frac{1}{\pi} \int_{-\pi/4}^{\pi/4} \left(\frac{4t}{\pi}\right)^2 dt = \frac{1}{6}$; the rms value is $\sqrt{P_g} = 1/\sqrt{6}$.

(e) $P_g = \frac{1}{3} \int_0^1 t^2 dt = \frac{1}{9}$; the rms value is $\sqrt{P_g} = 1/3$.

(f) $P_g = \frac{1}{6} \left(\int_{-2}^{-1} (t+2)^2 dt + \int_{-1}^1 dt + \int_1^2 (t-2)^2 dt \right) = \frac{4}{9}$; the rms value is $\sqrt{P_g} = 2/3$.

2.2-1 If a is real, then both $E_g = \int_{-\infty}^{\infty} |e^{-at}|^2 \cdot dt = \infty$ and $P_g = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |e^{-at}|^2 \cdot dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} e^{-2at} \cdot dt = \infty$.

If a is purely imaginary, $a = i\alpha$; then, $g(t) = e^{-i\alpha t}$ and $|g(t)|^2 = 1$. $P_g = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} 1 \cdot dt = \lim_{T \rightarrow \infty} \frac{1}{T} T = 1$. It is not an energy signal since $E_g = \int_{-\infty}^{\infty} |g(t)|^2 \cdot dt = \infty$. Hence it is a power signal.

2.2-2

(a) $P_g = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} t^4 dt = \infty$. Hence, it is not a power signal.

(b) $E_g = \int_{-\infty}^{\infty} t^2 dt = \infty$. Hence, it is not an energy signal.

2.3-1

$$g_2(t) = g(t-1) + g_1(t-1), \quad g_3(t) = g(t-1) + g_1(t+1), \quad g_4(t) = g(t-0.5) + g_1(t+0.5)$$

The signal $g_5(t)$ can be obtained by (i) delaying $g(t)$ by 1 second (replace t with $t-1$), (ii) then time-expanding by a factor 2 (replace t with $t/2$), (iii) then multiplying by 1.5. Thus $g_5(t) = 1.5g(\frac{t}{2}-1)$.

2.3-2 See Fig. S2.3-2.

2.3-3

(a) See Fig. S2.3-3.

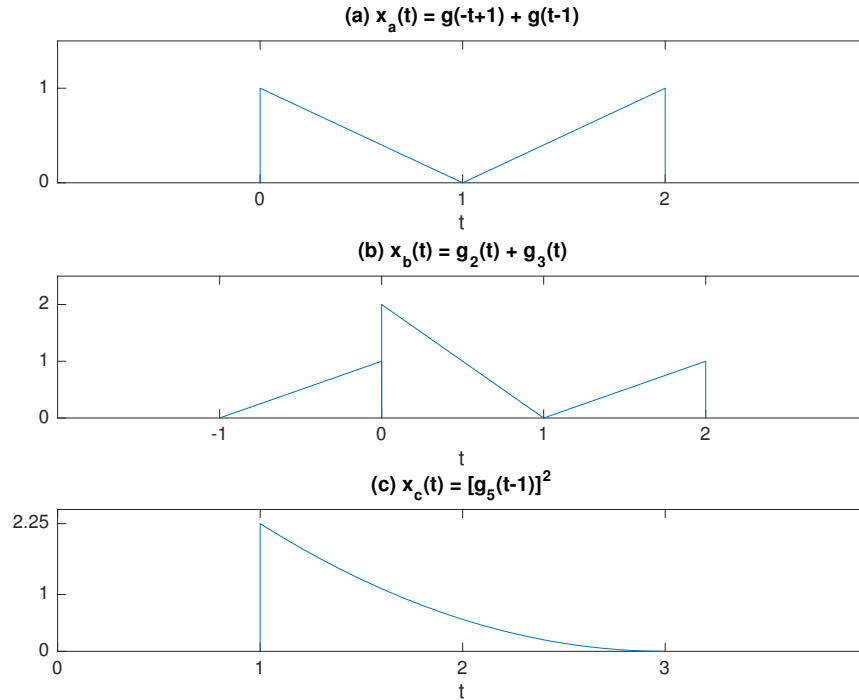


Fig. S2.3-2

$$(b) \quad E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_6^{15} \left[\frac{1}{6}(t-12)\right]^2 dt + \int_{15}^{24} \left[-\frac{1}{18}(t-24)\right]^2 dt = 3.$$

Based on the properties derived in Prob. 2.3-6, the energies are

- (i) $E_{g(-t)} = 3$
- (ii) $E_{g(t+2)} = 3$
- (iii) $E_{g(-3t)} = E_g/3 = 1$
- (iv) $E_{g(t/3)} = 3E_g = 9$
- (v) $E_{g(2t+1)} = E_g/2 = 1.5$
- (vi) $E_{g(2t+2)} = E_g/2 = 1.5$

2.3-4 Denote $g(at) = f(t)$. Since $g(t)$ is periodic with period T ,

$$\begin{aligned} g(t) &= g(t+T) \\ g(at) &= g(at+T) = g\left(a\left(t+\frac{T}{a}\right)\right) \\ f(t) &= f\left(t+\frac{T}{a}\right) \end{aligned}$$

Therefore, $g(at)$ is periodic with period T/a .

The average power of $g(at)$ is

$$P_{g(at)} = \lim_{T \rightarrow \infty} \frac{a}{T} \int_{-T/2a}^{T/2a} g^2(at) dt = \lim_{T \rightarrow \infty} \frac{a}{T} \int_{-T/2}^{T/2} g^2(z) \frac{dz}{a} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(z) dz = P_g$$

Therefore, the average power remains the same.

2.3-5 See Fig. S2.3-5.

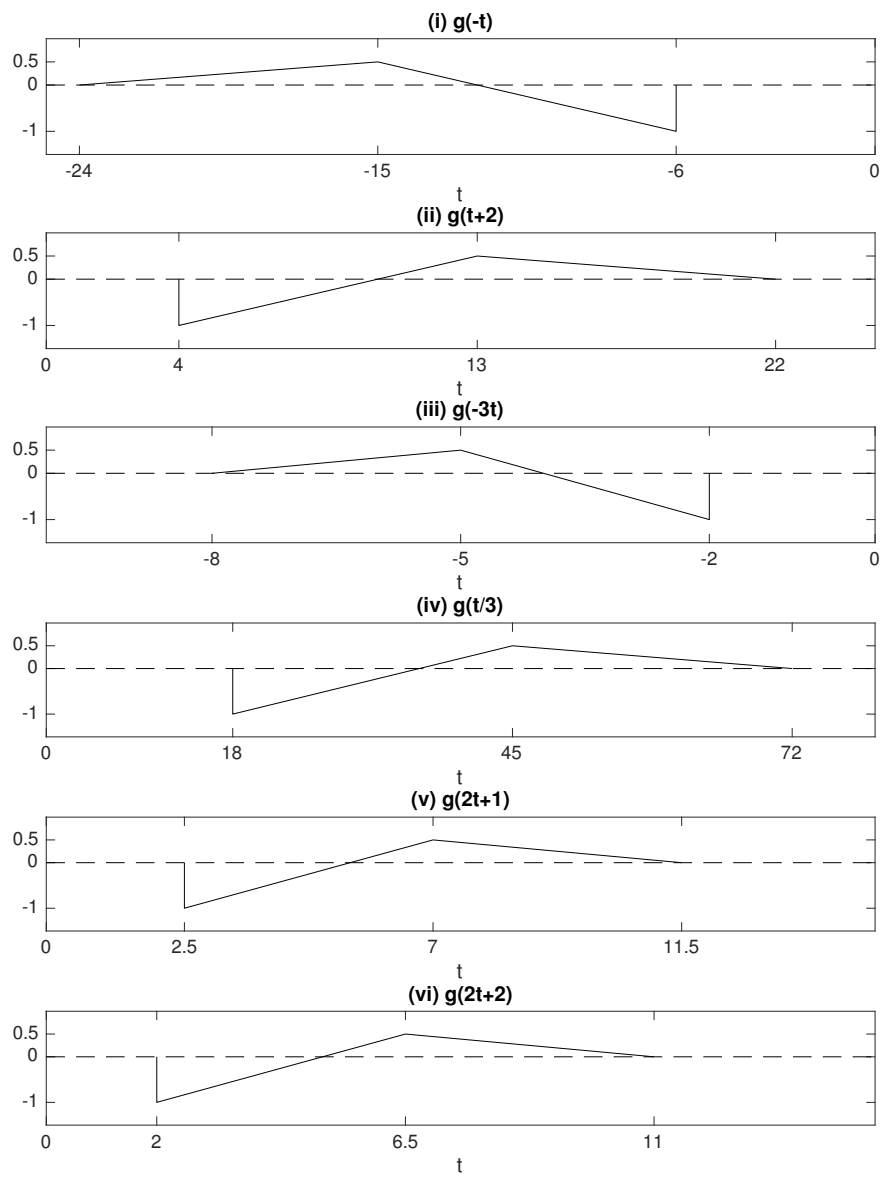


Fig. S2.3-3

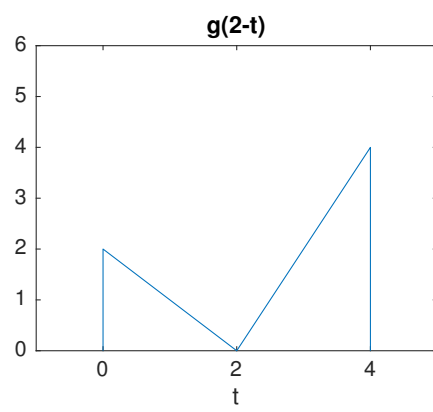
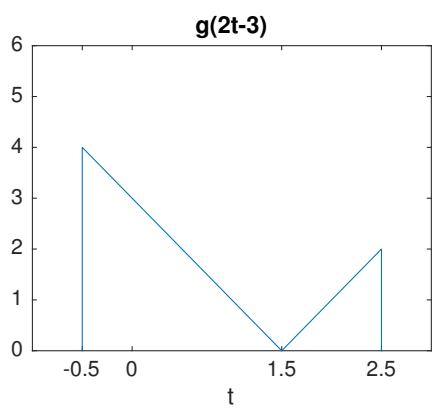
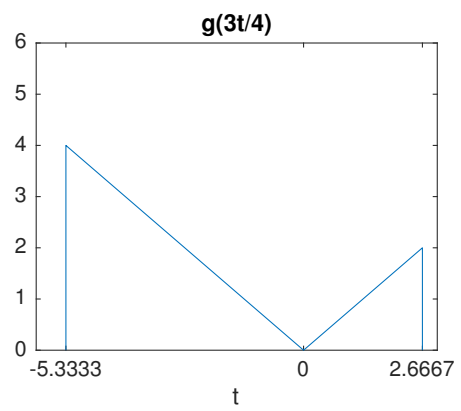
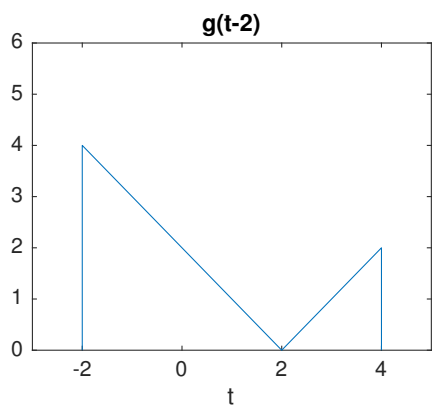


Fig. S2.3-5

2.3-6

$$E_{-g} = \int_{-\infty}^{\infty} [-g(t)]^2 dt = \int_{-\infty}^{\infty} g^2(t) dt = E_g, \quad E_{g(-t)} = \int_{-\infty}^{\infty} [g(-t)]^2 dt = \int_{-\infty}^{\infty} g^2(x) dx = E_g$$

$$E_{g(t-T)} = \int_{-\infty}^{\infty} [g(t-T)]^2 dt = \int_{-\infty}^{\infty} g^2(x) dx = E_g, \quad E_{g(at)} = \int_{-\infty}^{\infty} [g(at)]^2 dt = \frac{1}{a} \int_{-\infty}^{\infty} g^2(x) dx = E_g/a$$

$$E_{g(at-b)} = \int_{-\infty}^{\infty} [g(at-b)]^2 dt = \frac{1}{a} \int_{-\infty}^{\infty} g^2(x) dx = E_g/a, \quad E_{g(t/a)} = \int_{-\infty}^{\infty} [g(t/a)]^2 dt = a \int_{-\infty}^{\infty} g^2(x) dx = aE_g$$

$$E_{ag(t)} = \int_{-\infty}^{\infty} [ag(t)]^2 dt = a^2 \int_{-\infty}^{\infty} g^2(t) dt = a^2 E_g$$

2.4-1 Using the facts that $\phi(t)\delta(t) = \phi(0)\delta(t)$ and $\phi(t)\delta(t-T) = \phi(T)\delta(t-T)$, we have

- (a) $\frac{\tan(3\pi/4)}{\pi^2/8+1} \delta(t - \frac{\pi}{4}) = \frac{-8}{\pi^2+8} \delta(t - \frac{\pi}{4})$
- (b) $\frac{1-j}{10\pi} \delta(\omega + \pi)$
- (c) $-e^{\pi/15} \delta(t + \pi/15)$
- (d) $\frac{\sin(1.5\pi)}{1-4} \delta(t-1) = \frac{1}{3} \delta(t-1)$
- (e) $\frac{\cos(-3\pi/2)}{1/2} = 0$
- (f) $k^2 \delta(\omega) - \frac{4 \sin^2(k\pi/2)}{\pi^2} \delta(\omega + \pi/2)$ (use L'Hôpital's rule $\lim_{\omega \rightarrow 0} \frac{\sin k\omega}{k\omega} = 1$)

2.4-2

- (a) $\frac{-8}{\pi^2+8}$
- (b) $\frac{1-j}{10\pi}$
- (c) $-e^{\pi/15}$
- (d) $\frac{1}{3}$
- (e) 0
- (f) $k^2 - \frac{4 \sin^2(k\pi/2)}{\pi^2}$

2.4-3 Letting $at = x$, we obtain (for $a > 0$)

$$\int_{-\infty}^{\infty} \phi(t)\delta(at) dt = \frac{1}{a} \int_{-\infty}^{\infty} \phi\left(\frac{x}{a}\right) \delta(x) dx = \frac{1}{a} \phi(0)$$

Similarly for $a < 0$, we show that this integral is $-\frac{1}{a}\phi(0)$. Therefore

$$\int_{-\infty}^{\infty} \phi(t)\delta(at) dt = \frac{1}{|a|} \phi(0) = \frac{1}{|a|} \int_{-\infty}^{\infty} \phi(t)\delta(t) dt$$

$$\delta(at) = \frac{1}{|a|} \delta(t)$$

Therefore,

$$\delta(\omega) = \delta(2\pi f) = \frac{1}{2\pi} \delta(f)$$

2.4-4 Using the fact that $\delta(at) = \frac{1}{|a|}\delta(t)$ (see Problem **2.4-3**), and the equality $\int_a^b \phi(t)\delta(t-T) dt = \phi(T)$, we get

(a) $g(-3t + a)$

(b) $g(t)$

(c) $e^{j2\omega}$

(d) 0

(e) $e^6/2$

(f) 5

(g) $2g(-4)$

(h) $\frac{\cos(7\pi/3)}{3}$

2.5-1

(a) In this case $E_x = \int_0^1 dt = 1$, and

$$c = \frac{1}{E_x} \int_0^1 g(t)x(t) dt = \frac{1}{1} \int_0^1 t dt = 0.5$$

(b) Thus, $g(t) \approx 0.5x(t)$, and the error $e(t) = t - 0.5$ over $(0 \leq t \leq 1)$, and zero outside this interval. Also E_g and E_e (the energy of the error) are

$$E_g = \int_0^1 g^2(t) dt = \int_0^1 t^2 dt = 1/3 \quad \text{and} \quad E_e = \int_0^1 (t - 0.5)^2 dt = 1/12$$

The error $(t - 0.5)$ is orthogonal to $x(t)$ because

$$\int_0^1 (t - 0.5)(1) dt = 0$$

Note that $E_g = c^2 E_x + E_e$. To explain these results in terms of vector concepts, we observe from Fig. 2.13 that the error vector \mathbf{e} is orthogonal to the component $c\mathbf{x}$. Because of this orthogonality, the length-square of \mathbf{g} [energy of $g(t)$] is equal to the sum of the square of the lengths of $c \cdot \mathbf{x}$ and \mathbf{e} [sum of the energies of $cx(t)$ and $e(t)$].

2.5-2 In this case $E_g = \int_0^1 g^2(t) dt = \int_0^1 t^2 dt = 1/3$, and

$$c = \frac{1}{E_g} \int_0^1 x(t)g(t) dt = 3 \int_0^1 t dt = 1.5$$

Thus, $x(t) \approx 1.5g(t)$, and the error $e(t) = x(t) - 1.5g(t) = 1 - 1.5t$ over $(0 \leq t \leq 1)$, and zero outside this interval. Also E_e (the energy of the error) is $E_e = \int_0^1 (1 - 1.5t)^2 dt = 1/4$.

2.5-3

$$|\mathbf{e}|^2 = |\mathbf{g}|^2 + c^2|\mathbf{x}|^2 - 2c\mathbf{g} \cdot \mathbf{x}$$

To minimize error, set $\frac{d|\mathbf{e}|^2}{dc} = 0$:

$$2c|\mathbf{x}|^2 - 2\mathbf{g} \cdot \mathbf{x} = 0$$

$$c = \frac{\mathbf{g} \cdot \mathbf{x}}{|\mathbf{x}|^2} = \frac{\langle \mathbf{g}, \mathbf{x} \rangle}{|\mathbf{x}|^2}$$

2.5-4

(a) In this case $E_x = \int_0^1 \sin^2 2\pi t dt = 0.5$, and

$$c = \frac{1}{E_x} \int_0^1 g(t)x(t) dt = \frac{1}{0.5} \int_0^1 t \sin 2\pi t dt = -1/\pi$$

(b) Thus, $g(t) \approx -(1/\pi)x(t)$, and the error $e(t) = t + (1/\pi) \sin 2\pi t$ over $(0 \leq t \leq 1)$, and zero outside this interval.

Also E_g and E_e (the energy of the error) are

$$E_g = \int_0^1 g^2(t) dt = \int_0^1 t^2 dt = 1/3$$

and

$$E_e = \int_0^1 [t + (1/\pi) \sin 2\pi t]^2 dt = \frac{1}{3} - \frac{1}{2\pi^2}$$

The error $[t + (1/\pi) \sin 2\pi t]$ is orthogonal to $x(t)$ because

$$\int_0^1 \sin 2\pi t [t + (1/\pi) \sin 2\pi t] dt = 0$$

Note that $E_g = c^2 E_x + E_e$. To explain these results in terms of vector concept we observe from Fig. 2.13 that the error vector \mathbf{e} is orthogonal to the component $c\mathbf{x}$. Because of this orthogonality, the square of length of \mathbf{g} [energy of $g(t)$] is equal to the sum of squares of the lengths $c\mathbf{x}$ and \mathbf{f} [sum of the energies of $c x(t)$ and $e(t)$].

2.5-5

(a) If $x(t)$ and $y(t)$ are orthogonal, then we can show that the energy of $x(t) \pm y(t)$ is $E_x + E_y$.

$$\begin{aligned} \int |x(t) \pm y(t)|^2 dt &= \int_{-\infty}^{\infty} |x(t)|^2 dt + \int_{-\infty}^{\infty} |y(t)|^2 dt \pm \int_{-\infty}^{\infty} x(t)y^*(t) dt \pm \int_{-\infty}^{\infty} x^*(t)y(t) dt \\ &= \int_{-\infty}^{\infty} |x(t)|^2 dt + \int_{-\infty}^{\infty} |y(t)|^2 dt \end{aligned}$$

The last result follows from the fact that because of orthogonality, the two integrals of the cross products $x(t)y^*(t)$ and $x^*(t)y(t)$ are zero. Thus the energy of $x(t) + y(t)$ is equal to that of $x(t) - y(t)$ if $x(t)$ and $y(t)$ are orthogonal.

(b) We can use a similar argument to show that the energy of $c_1x(t)+c_2y(t)$ is equal to that of $c_1x(t)-c_2y(t)$ if $x(t)$ and $y(t)$ are orthogonal. This energy is given by $|c_1|^2E_x + |c_2|^2E_y$.

(c) If $z(t) = x(t) \pm y(t)$, then it follows from **part (a)** in the preceding derivation that

$$E_z = E_x + E_y \pm (E_{xy} + E_{yx})$$

2.6-1 We shall use Eq. (2.51) to compute ρ_n for each of the four cases. Let us first compute the energies of all the signals:

$$E_x = \int_0^1 \sin^2 2\pi t dt = 0.5$$

In the same way, we find $E_{g_1} = E_{g_2} = E_{g_3} = E_{g_4} = 0.5$.

From Eq. (2.51), the correlation coefficients for four cases are found as follows:

$$(1) \quad \frac{1}{\sqrt{(0.5)(0.5)}} \int_0^1 \sin 2\pi t \sin 4\pi t dt = 0$$

$$(2) \quad \frac{1}{\sqrt{(0.5)(0.5)}} \int (\sin 2\pi t) (-\sin 2\pi t) dt = -1$$

$$(3) \quad \frac{1}{\sqrt{(0.5)(0.5)}} \int_0^1 0.707 \sin 2\pi t dt = 0$$

$$(4) \quad \frac{1}{\sqrt{(0.5)(0.5)}} \left[\int_0^{0.5} 0.707 \sin 2\pi t dt - \int_{0.5}^1 0.707 \sin 2\pi t dt \right] = 2.828/\pi$$

Signals $x(t)$ and $g_2(t)$ provide the maximum protection against noise.

2.6-2 $E_{g_1} = \int_{-\infty}^{\infty} g_1^2(t)dt = \int_0^2 1dt = 2$ and $E_{g_2} = \int_{-\infty}^{\infty} g_2^2(t)dt = \int_0^{\infty} e^{-t} = 1$.

Therefore, $\rho = \frac{1}{\sqrt{2 \cdot 1}} \int_0^2 e^{-0.5t} dt = \sqrt{2}(1 - e^{-1})$.

2.6-3 Given $g(t) = \exp(-2t) \cos(\pi t)u(t)$, the autocorrelation function is

$$\begin{aligned} \psi_g(\tau) &= \int_{-\infty}^{\infty} g(t)g(t+\tau)dt \\ &= \int_{-\infty}^{\infty} \exp(-2t) \cos(\pi t)u(t) \exp(-2(t+\tau)) \cos(\pi(t+\tau))u(t+\tau)dt \end{aligned}$$

If $\tau \geq 0$,

$$\begin{aligned} \psi_g(\tau) &= \int_0^{\infty} \exp(-2t) \cos(\pi t) \exp(-2(t+\tau)) \cos(\pi(t+\tau))dt \\ &= \int_0^{\infty} \exp(-4t - 2\tau) \left[\frac{1}{2} \cos(2\pi t + \pi\tau) + \frac{1}{2} \cos(\pi\tau) \right] dt \\ &= \frac{1}{2} \int_0^{\infty} \exp(-4(t + \frac{\tau}{2})) \cdot \cos(2\pi(t + \frac{\tau}{2})) dt + \frac{1}{2} \exp(-2\tau) \cos(\pi\tau) \int_0^{\infty} \exp(-4t) dt \\ &= \frac{1}{8} \exp(-2\tau) \cos(\pi\tau) + \frac{1}{8\pi^2 + 32} \exp(-2\tau)(4 \cos(\pi\tau) - 2\pi \sin(\pi\tau)), \quad \tau \geq 0. \end{aligned}$$

Because of symmetry,

$$\psi_g(\tau) = \frac{1}{8} \exp(-2|\tau|) \left[\frac{\pi^2 + 8}{\pi^2 + 4} \cos(\pi\tau) - \frac{2\pi}{\pi^2 + 4} \sin(\pi|\tau|) \right]$$

2.7-1

(a) $\mathbf{g}_1 = (2, -1)$, $\mathbf{g}_2 = (-1, 2)$, $\mathbf{g}_3 = (0, -1)$, $\mathbf{g}_4 = (1, 2)$, $\mathbf{g}_5 = (2, 1)$, and $\mathbf{g}_6 = (3, 0)$.

(b) Signal pairs $(\mathbf{g}_3, \mathbf{g}_6)$, $(\mathbf{g}_1, \mathbf{g}_4)$ and $(\mathbf{g}_2, \mathbf{g}_5)$ are orthogonal. We can verify this analytically.

$$\begin{aligned}\langle \mathbf{g}_3, \mathbf{g}_6 \rangle &= (0 \times 3) + (-2 \times 0) = 0, \\ \langle \mathbf{g}_1, \mathbf{g}_4 \rangle &= (2 \times 1) + (-1 \times 2) = 0, \\ \langle \mathbf{g}_2, \mathbf{g}_5 \rangle &= (-1 \times 2) + (2 \times 1) = 0\end{aligned}$$

We can show that the corresponding signal pairs are also orthogonal.

$$\begin{aligned}\int_{-\infty}^{\infty} g_3(t)g_6(t) dt &= \int_{-\infty}^{\infty} [-x_2(t)][3x_1(t)] dt = 0 \\ \int_{-\infty}^{\infty} g_1(t)g_4(t) dt &= \int_{-\infty}^{\infty} [2x_1(t) - x_2(t)][x_1(t) + 2x_2(t)] dt = 0 \\ \int_{-\infty}^{\infty} g_2(t)g_5(t) dt &= \int_{-\infty}^{\infty} [-x_1(t) + 2x_2(t)][2x_1(t) + x_2(t)] dt = 0\end{aligned}$$

In deriving these results, we used the fact that $\int_{-\infty}^{\infty} x_1^2 dt = \int_{-\infty}^{\infty} x_2^2(t) dt = 1$ and $\int_{-\infty}^{\infty} x_1(t)x_2(t) dt = 0$.

(c) Because $\int_{-\infty}^{\infty} x_1^2 dt = \int_{-\infty}^{\infty} x_2^2(t) dt = 1$ and $\int_{-\infty}^{\infty} x_1(t)x_2(t) dt = 0$, signal energy $E_g = c_1^2 + c_2^2$ for $g(t) = c_1x_1(t) + c_2x_2(t)$. Therefore, $E_{g_1} = 5$, $E_{g_2} = 5$, $E_{g_3} = 4$, $E_{g_4} = 5$, $E_{g_5} = 5$, $E_{g_6} = 9$.

2.7-2

(a) Since $x(t)$ and $e(t)$ are mutually orthogonal, we can choose these two functions as the basis functions. After normalizing, we obtain $\psi_1(t) = x(t)/\sqrt{E_x} = x(t)$, $0 \leq t \leq 1$ and $\psi_2(t) = e(t)/\sqrt{E_e} = 2\sqrt{3}e(t) = 2\sqrt{3}(t - 0.5)$, $0 \leq t \leq 1$.

(b) Through orthogonal decomposition, we know that $g(t) = 0.5x(t) + e(t) = 0.5\psi_1(t) + \frac{1}{2\sqrt{3}}\psi_2(t)$ and $x(t) = \psi_1(t) + 0\psi_2(t)$. Thus, the vector representations are $\mathbf{g} = [\frac{1}{2} \quad \frac{1}{2\sqrt{3}}]$ and $\mathbf{x} = [1 \quad 0]$, respectively.

(c) Similarly, this time $\psi_1(t) = x(t)/\sqrt{E_x} = \sqrt{2}x(t) = \sqrt{2}\sin(2\pi t)$, $0 \leq t \leq 1$ and $\psi_2(t) = e(t)/\sqrt{E_e} = \frac{1}{\sqrt{1/3 - 1/(2\pi^2)}}(t + \sin(2\pi t)/\pi)$, $0 \leq t \leq 1$.

$g(t) = -\frac{1}{\pi}x(t) + e(t) = -\frac{1}{\pi\sqrt{2}}\psi_1(t) + \sqrt{\frac{1}{3} - \frac{1}{2\pi^2}}\psi_2(t)$ and $x(t) = \frac{1}{\sqrt{2}}\psi_1(t) + 0\psi_2(t)$. Thus, the vector representations are $\mathbf{g} = [-\frac{1}{\pi\sqrt{2}} \quad \sqrt{\frac{1}{3} - \frac{1}{2\pi^2}}]$ and $\mathbf{x} = [\frac{1}{\sqrt{2}} \quad 0]$, respectively.

2.7-3

(a) We can choose the normalized $x(t), g_1(t), g_3(t)$ as the first three orthonormal bases for the set of signals, since all the correlations between these signals are zeros. Therefore,

$$\begin{aligned}\phi_1(t) &= \sqrt{2}x(t) \\ \phi_2(t) &= \sqrt{2}g_1(t) \\ \phi_3(t) &= \sqrt{2}g_3(t).\end{aligned}$$

Signal $g_2(t)$ is the negative of signal $x(t)$. Therefore, $g_2(t) = -\frac{1}{\sqrt{2}}\phi_1(t)$.

To represent $g_4(t)$, we need an additional basis function $\phi_4(t)$. We can represent $g_4(t)$ as $g_4(t) = c_1\phi_1(t) + c_2\phi_2(t) + c_3\phi_3(t) + c_4\phi_4(t)$, where,

$$\begin{aligned}c_1 &= \int_0^1 g_4(t)\phi_1^*(t) dt = \sqrt{2} \int_0^1 g_4(t)x(t) dt = \frac{2}{\pi} \\ c_2 &= \int_0^1 g_4(t)\phi_2^*(t) dt = \sqrt{2} \int_0^1 g_4(t)g_1(t) dt = \sqrt{2} \left[\int_0^{0.5} 0.707 \sin 4\pi t dt - \int_{0.5}^1 0.707 \sin 4\pi t dt \right] = 0 \\ c_3 &= \int_0^1 g_4(t)\phi_3^*(t) dt = \sqrt{2} \int_0^1 g_4(t)g_3(t) dt = \sqrt{2} (0.707) \int_0^1 g_4(t) dt = 0\end{aligned}$$

Therefore,

$$c_4\phi_4(t) = g_4(t) - c_1\phi_1(t) = g_4(t) - \frac{2\sqrt{2}}{\pi}x(t)$$

The energy of the signal is

$$\begin{aligned}E_{c_4\phi_4} &= \int_0^1 \left[g_4(t) - \frac{2\sqrt{2}}{\pi}x(t) \right]^2 dt = \int_0^1 \left[g_4^2(t) + \frac{8}{\pi^2}x^2(t) - \frac{4\sqrt{2}}{\pi}g_4(t)x(t) \right] dt \\ &= \frac{1}{2} + \frac{4}{\pi^2} - \frac{4\sqrt{2}}{\pi} \cdot \frac{\sqrt{2}}{\pi} = \frac{1}{2} + \frac{4}{\pi^2} - \frac{8}{\pi^2} = \frac{1}{2} - \frac{4}{\pi^2} = 0.0946\end{aligned}$$

Therefore,

$$\phi_4(t) = \frac{g_4(t) - \frac{2\sqrt{2}}{\pi}x(t)}{\sqrt{E_{c_4\phi_4}}} = \frac{g_4(t) - \frac{2\sqrt{2}}{\pi}x(t)}{0.3076}$$

(b)

$$\begin{aligned}x_1(t) &= \left[\frac{1}{\sqrt{2}} \quad 0 \quad 0 \quad 0 \right]^T \\ g_1(t) &= \left[0 \quad \frac{1}{\sqrt{2}} \quad 0 \quad 0 \right]^T \\ g_2(t) &= \left[-\frac{1}{\sqrt{2}} \quad 0 \quad 0 \quad 0 \right]^T \\ g_3(t) &= \left[0 \quad 0 \quad \frac{1}{\sqrt{2}} \quad 0 \right]^T \\ g_4(t) &= \left[\frac{2}{\pi} \quad 0 \quad 0 \quad 0.3076 \right]^T\end{aligned}$$

2.8-1

(a) $T_0 = 4$, $\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$. Because of even symmetry, all sine terms are zero.

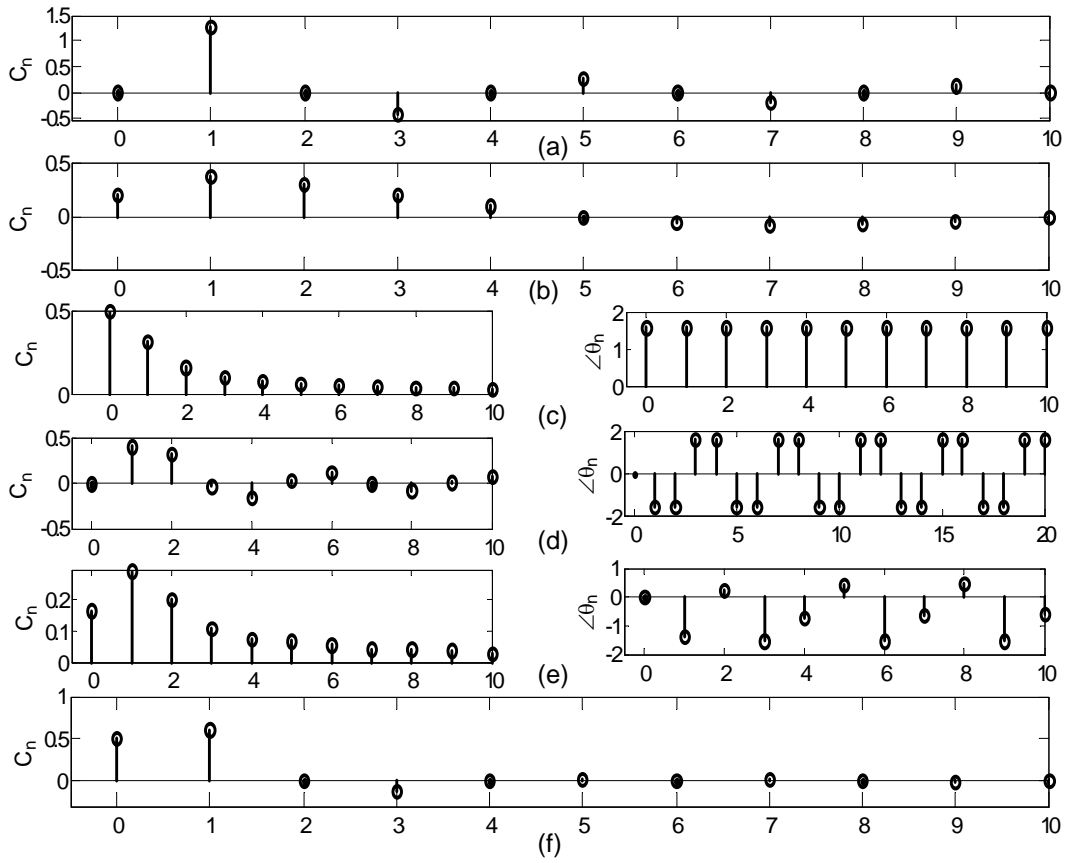


Fig. S2.8-1

$$\begin{aligned}
 g(t) &= a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{2}t\right) \\
 a_0 &= 0 \text{ (by inspection of its lack of dc)} \\
 a_n &= \frac{4}{4} \left[\int_0^1 \cos\left(\frac{n\pi}{2}t\right) dt - \int_1^2 \cos\left(\frac{n\pi}{2}t\right) dt \right] \\
 &= \frac{4}{n\pi} \sin \frac{n\pi}{2}
 \end{aligned}$$

Therefore, the Fourier series for $g(t)$ is

$$g(t) = \frac{4}{\pi} \left(\cos \frac{\pi t}{2} - \frac{1}{3} \cos \frac{3\pi t}{2} + \frac{1}{5} \cos \frac{5\pi t}{2} - \frac{1}{7} \cos \frac{7\pi t}{2} + \dots \right)$$

Here $b_n = 0$, and we allow C_n to take negative values. Figure S2.8-1(a) shows the plot of C_n which is real-valued.

(b) $T_0 = 10\pi$, $\omega_0 = \frac{2\pi}{T_0} = \frac{1}{5}$. Because of even symmetry, all the sine terms are zero.

$$\begin{aligned}
 g(t) &= a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n}{5}t\right) + b_n \sin\left(\frac{n}{5}t\right) \\
 a_0 &= \frac{1}{5} \quad (\text{by inspection}) \\
 a_n &= \frac{2}{10\pi} \int_{-\pi}^{\pi} \cos\left(\frac{n}{5}t\right) dt \\
 &= \frac{1}{5\pi} \left(\frac{5}{n}\right) \sin\left(\frac{n}{5}t\right) \Big|_{-\pi}^{\pi} = \frac{2}{\pi n} \sin\left(\frac{n\pi}{5}\right) \\
 b_n &= \frac{2}{10\pi} \int_{-\pi}^{\pi} \sin\left(\frac{n}{5}t\right) dt \\
 &= 0 \quad (\text{integrand is an odd function of } t)
 \end{aligned}$$

Here $b_n = 0$, and we allow C_n to take negative values. Note that $C_n = a_n$ for $n = 0, 1, 2, 3, \dots$. Fig. S2.8-1(b) shows the plot of C_n which is real-valued.

(c) $T_0 = 2\pi$, $\omega_0 = 1$, and

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt$$

with

$$a_0 = 0.5 \quad (\text{by inspection of the dc or average})$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} \frac{t}{2\pi} \cos nt dt = 0, \quad \text{and} \quad b_n = \frac{1}{\pi} \int_0^{2\pi} \frac{t}{2\pi} \sin nt dt = -\frac{1}{\pi n}$$

and

$$\begin{aligned}
 g(t) &= 0.5 - \frac{1}{\pi} \left(\sin t + \frac{1}{2} \sin 2t + \frac{1}{3} \sin 3t + \frac{1}{4} \sin 4t + \dots \right) \\
 &= 0.5 + \frac{1}{\pi} \left[\cos\left(t + \frac{\pi}{2}\right) + \frac{1}{2} \cos\left(2t + \frac{\pi}{2}\right) + \frac{1}{3} \cos\left(3t + \frac{\pi}{2}\right) + \dots \right]
 \end{aligned}$$

The cosine terms vanish because when 0.5 (the dc component) is subtracted from $g(t)$, the remaining function has odd symmetry. Hence, the Fourier series would contain dc and sine terms only. Figure S2.8-1(c) shows the plots of $|C_n|$ and θ_n . Note that C_n is purely imaginary.

(d) $T_0 = \pi$, $\omega_0 = 2$ and $g(t) = \frac{4}{\pi}t$.

$$a_0 = 0 \quad (\text{by inspection})$$

$$a_n = 0 \quad (n > 0) \quad (\text{because of odd symmetry})$$

$$b_n = \frac{4}{\pi} \int_0^{\pi/4} \frac{4}{\pi} t \sin 2nt dt = \frac{2}{\pi n} \left(\frac{2}{\pi n} \sin \frac{\pi n}{2} - \cos \frac{\pi n}{2} \right)$$