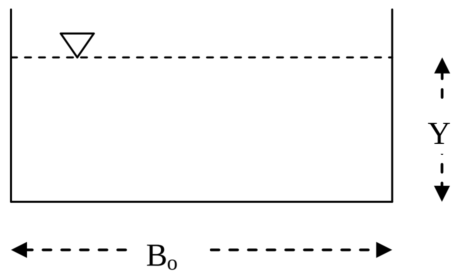
Chapter 1
BASIC CONCEPTS

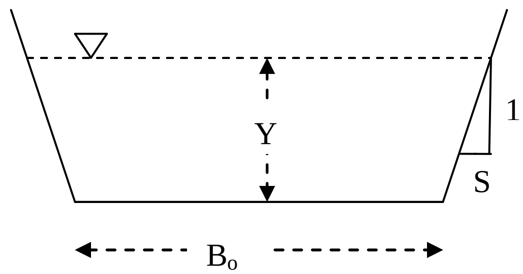
1.1

(i) Rectangular section



$A = B_0Y$
$P = 2Y+B_0$
$B = B_0$
$R = A/P = B_0Y/(2Y+B_0)$
$D = A/B = B_0Y/ B_0 = Y$

(ii) Trapezoidal section



$A = B_0Y+ 2Y(SY/2)$

$= Y(B_0 + SY)$

$P = B0 +2Y\, S(S^2+1)$

$R = A/P = Y(B_0 + SY)/ [B0 +2Y\, S(S^2+1)]$

$B = B_0 +2SY$

$D = A/B = Y(B_0 + SY)/(B_0 + 2SY)$

(iii) Triangular section

We may use the same equation as that in the case of trapezoidal section with $B_0 = 0$.

Thus, $D = Y/2$

(iv) Partially full circular section

$A = r^2\theta/2 +2\, (r\cos\alpha)/2\, (Y\text{-}D_0/2)$

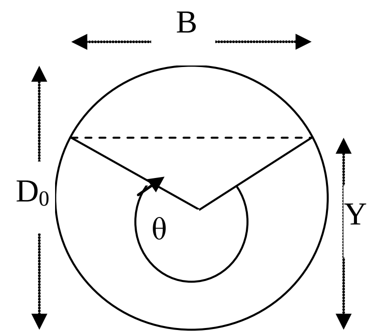
$= D_0^2\theta/8 + (Y - D_0/2)(r\cos\alpha)$

$Y = D_0/2 + (D_0/2) \sin\alpha$

$A = D_o^2\, \theta/8 + D_o^2 / 4 \sin\alpha \cos\alpha$

$$A = \frac{D_o^2\theta}{8} + \frac{D_o^2}{4}\sin\alpha\cos\alpha$$

$$= \frac{D_o^2}{8}(\theta + \sin 2\alpha)$$



But $\theta = 2\alpha + \pi$

and $\sin\theta = \sin(2\alpha + \pi) = -\sin 2\alpha$

$$A = \frac{D_o^2}{8}(\theta - \sin\theta) \qquad 0 \le \theta \le 2\pi$$

$$P = r\theta = \frac{D_o\theta}{4}\left(1 - \frac{\sin\theta}{\theta}\right)$$

$$B = 2r\cos\alpha = 2r\cos\left(\frac{\theta}{2} - \frac{\pi}{2}\right) = D_o\sin\frac{\theta}{2}$$

$$D = \frac{A}{B} = \frac{D_o}{8}\left(1 - \frac{\theta - \sin\theta}{\sin\theta/2}\right)$$

(v) Standard horseshoe section:

Length KB

$$\overline{OM} = \overline{MC} - \overline{OM}$$

$$\overline{OM} = d_0/2\sqrt{2}$$

$$\overline{MC} = \sqrt{\left(\overline{CG}\right)^2 - \left(\overline{GM}\right)^2} = \sqrt{d_0^2 - \left(\frac{d_0}{2\sqrt{2}}\right)^2} = d_0\left(\sqrt{1 - \frac{1}{8}}\right)$$

$$\overline{MC} = \sqrt{\frac{7}{8}}\ d_0$$

$$\overline{OC} = d_0\left(\sqrt{7/8} - \frac{1}{2\sqrt{2}}\right) = 0.58186\,d_0 \text{ --------(1)}$$

$$\overline{KC} = \overline{FC}^2 - \overline{KF}^2 = d_0^2 - \left(d_0 - \overline{KB}\right)^2 \text{ -------(2)}$$

$$\overline{KC}^2 = \overline{FC}^2 - \overline{OK}^2 = \overline{OC}^2 - \left(\overline{OB} - \overline{KB}\right)^2$$

$$\overline{KC}^2 = \left(0.58186\,d_0\right)^2 - \left(\frac{d_0}{2} - KB\right)^2 \text{ ---------(3)}$$

(2) *and* (3) $$d_0^{\,2} - \left(d_0 - \overline{KB}\right)^2 = 0.33856\ d_0^{\,2} - \left(\frac{d_0}{2} - \overline{KB}\right)^2$$

$$d_0^{\,2} - d_0^{\,2} + 2d_0\,\overline{KB} - \overline{KB}^2 = 0.38856\ d_0^{\,2} - \frac{d_0^{\,2}}{4} + d_0\,\overline{KB} - \left(\overline{KB}\right)^2$$

$$\overline{KB} = 0.088562\ d_0 \qquad \overline{OK} = \frac{d_0}{2} - 0.088562\ d_0 = 0.411438\ d_0$$

$$\overline{KC}^2 = d_0^{\,2} - \left(d_0 - 0.088562 d_0\right)^2 = 0.169281\ d_0^{\,2}$$

$$KC = 04114377\ d_0 \qquad CC = 0.822875\, d_0$$

$$Sin\frac{\theta_L}{2} = 0.4114377 \qquad \frac{\theta_L}{2} = 24.295^0 \qquad \theta_L = 48.59^0$$

The standard horse shoe section is divided into three sections, i.e., upper section, middle section and lower section.

(a) Upper section

$\pi \le \theta_u \le 2\pi$

Flow area, $A = \frac{D_0^2}{8}\left(\theta_u - Sin\theta_u\right) - \frac{\pi D_0^2}{8}$

Wetted perimeter, $P = \frac{D_0\theta}{2}$

Hydraulic radius, $R = A/P = \frac{D_0}{4}\left(1 - \frac{Sin\theta}{\theta}\right)$

Top water surface, $B = D_0\ Sin\ (\theta/2)$

Hydraulic depth, $D = A/B = \frac{D_0}{8}\left(\frac{\theta - Sin\theta}{Sin(\theta/2)}\right)$

(b) Lower section

$0 \le \theta_L \le 48.59^0$

Flow area, $A = \frac{D_0^2}{8}\left(\theta_L - Sin\theta_L\right) = \frac{d_0^2}{2}\left(\theta - Sin\theta\right)$

Wetted perimeter, $P = \frac{D_0\theta}{2} = \theta d_0$

Hydraulic radius, $R = A/P = \frac{d_0}{2}\left(1 - \frac{Sin\theta}{\theta}\right)$

Top water surface, $B = 2d_0\ Sin\ (\theta/2)$

Hydraulic depth, $D = A/B = \frac{d_0}{4}\left(\frac{\theta - Sin\theta}{Sin(\theta/2)}\right)$

(c) Middle section

Assume trapezoidal section, S = 0.215

Area, $A = Y(B_0 + SY)$

$A = Y(0.8229\ d_0 + 0.215Y)$

$P = B_0 + 2Y\ \sqrt{S^2 - 1}$

$$P = 0.8229\, d_0 + 2Y\sqrt{0.215^2 + 1} = 0.8229\, d_0 + 2.05Y$$

$$R = A/P = \frac{Y(0.8229 d_0 + 0.2154)}{0.8229\, d_0 + 2.05Y}$$

$$B = B_0 - 254 = 0.8229 d_0 + 2 \times 0.215Y = 0.8229 d_0 + 0.43Y$$

$$D = A/B = \frac{Y(0.8229 d_0 + 0.215Y)}{0.8229 d_0 + 0.43Y}$$

1.2

$Q = K\ A\ R^{2/3}$

$A = (\theta - Sin\theta)\ D^2/8$

$R = A/P$

$P = D\theta/2$

$$Q = \frac{KD^{\frac{10}{3}}}{8^{\frac{5}{3}}} \frac{(\theta - Sin\theta)^{\frac{5}{3}}}{(D/2)^{\frac{2}{3}}\theta^{\frac{2}{3}}} = C\frac{(\theta - Sin\theta)^{\frac{5}{3}}}{\theta^{2/3}}$$

Where $C = \frac{KD^{10/3}}{8^{5/3}(D/2)^{2/3}}$

$$\frac{dQ}{d\theta} = 0\ \ will\ \ give\ the\ angle\ \theta\ corresponding\ to\ Q_{max.}$$

$$\frac{dQ}{d\theta} = C\left[-\frac{2}{3}\theta^{-5/3}\ (\theta - Sin\theta)^{5/3} + \frac{5}{3}\theta^{-2/3}(\theta - Sin\theta)^{2/3}(1 - Cos\theta)\right] = 0$$

$$\frac{dQ}{d\theta} = C\frac{\theta^{-2/3}}{3}(\theta - Sin\theta)^{2/3}\left[-2\theta^{-1}(\theta - Sin\theta) + 5(1 - Cos\theta)\right] = 0$$

$\therefore\ 2(\theta - Sin\theta) = 5\theta(1 - Cos\theta)$

θ – Sinθ = 5/2θ – 5/2θ Cosθ

5/2 Cos θ – Sin θ / θ – 1.5 = 0

Solving by trial and error or numerically θ = 302.41

From the figure $Y = \frac{D}{2} + \frac{D}{2}Sin\alpha$ and $\alpha = \frac{\theta}{2} - \pi/2$

$$Y = \frac{D}{2} + \frac{D}{2}Sin\left(\frac{\theta}{2} - \pi/2\right)$$

$$Y = \frac{D}{2}\left[1 - Cos\frac{\theta}{2}\right]$$

Substituting θ value for the Q_{max}

$$Y = \frac{D}{2}\left[1 - Cos\frac{302.41}{2}\right]$$

Y= 0.938 D

1.3

$$(AR^{2/3})_F = \frac{\pi D^2}{4}\left(\frac{D}{4}\right)^{2/3} = \frac{\pi}{4^{5/3}}D^{8/3}$$

$$AR^{2/3} = \frac{D^2}{8}(\theta - Sin\theta)\left(\frac{D}{4}\right)^{2/3}\left(\frac{\theta - Sin\theta}{\theta^{2/3}}\right)^{2/3}$$

$$= \frac{2^{2/3}}{8^{5/3}}\frac{D^{8/3}}{\theta^{2/3}}(\theta - Sin\theta)^{5/3}$$

$$\frac{AR^{2/3}}{\left(AR^{2/3}\right)_F} = \frac{1}{2\pi}\left(\frac{\theta - Sin\theta}{\theta^{2/3}}\right)^{5/3}$$

$$\frac{R}{R_F} = \frac{\theta - Sin\theta}{\theta}$$

$$Y = \frac{D}{2} + \frac{D}{2}Sin\alpha \qquad \alpha = \frac{\theta}{2} - \pi/2$$

$$Y = \frac{D}{2} + \frac{D}{2}Sin\left(\frac{\theta}{2} - \frac{\pi}{2}\right)$$

$$Y = \frac{D}{2}\left[1 - Cos\frac{\theta}{2}\right]$$

$$\frac{Y}{D} = \frac{1}{2}\left[1 - Cos\frac{\theta}{2}\right]$$

Using trial and error

$\frac{Y}{D} = 0.938$ gives maximum value for $\frac{AR^{2/3}}{\left(AR^{2/3}\right)_F}$

$\frac{Y}{D} = 0.81$ gives maximum value for $\frac{R^{2/3}}{\left(R^{2/3}\right)_F}$

1.4

$$V = 5.75\, V_0 \log\frac{30y}{K}$$

$$V_m = \frac{\int VdA}{dA} = \frac{\int Vdy}{dy}$$

$$V_m = \frac{5.75\int_0^{y_0} V_0 \log 30y/K\, dy}{\int_0^{y_0} dy} = \frac{5.75V_0}{y_0}\int_0^{y_0}\log\frac{30y}{k}dy$$

Let x = 30y/K dx = $\frac{30}{K}dy$ dy = $\frac{K}{30}dx$

$V_m = 5.75\ V_0/Y_0\ \frac{K}{30}\int \log\times dx$

$= 5.75\ V_0/Y_0\ \frac{K}{30}\left[x\log x - x\right]_0^{y_0}$

$= 5.75\ V_0/Y_0\ \frac{K}{30}\left[\frac{30y}{K}\log\frac{30y}{K} - \frac{30y}{K}\right]_0^{y_0}$

$= 5.75V_0/Y_0\ \left[y_0\log\frac{30y_0}{K} - y_0\right]$

$V_m = 5.75\ V_0\ \left[\log\frac{30y_0}{K} - 1\right]$

Energy coefficient, α :

$$\alpha = \frac{\int V^3 dA}{V_m^3 A} = \frac{\int V^3 B dy}{V_m^3 B y_0} = \frac{\int \left[5.75 V_0 \log \frac{30y}{K}\right]^3 dy}{V_m^3 y_0}$$

$$\alpha = \frac{(5.75V_0)^3 \int \left(\log \frac{30y}{K}\right)^3 dy}{V_m^3 y_0}$$

$$V_m^3 \alpha = \frac{b}{y_0} \int (\log x)^3 dx$$ where, b = $(5.75\ V_0)^3K/30$

But, $\int (\log x)^n dx = x(\log x)^n - n\int (\log x)^{n-1} dx$

$$V_m^3 \alpha = \frac{b}{y_0}\left[\frac{30y}{K}\left(\log\frac{30y}{K}\right)^3 - 3\left(\frac{30y}{K}\left(\log\frac{30y}{K}\right)^2 - 2\frac{30y}{K}\log\frac{30y}{K} + 2\frac{30y}{K}\right)\right]_0^{y_0}$$

$$= \frac{(5.75V_0)^3}{y_0}\frac{K}{30}\left[\frac{30y_0}{K}\left(\log\frac{30y_0}{K} - 3\left\{\frac{30y_0}{K}\left(\log\frac{30y_0}{K} - 2\frac{30y_0}{K}\log\frac{30y_0}{K} + 2\frac{30y_0}{K}\right)\right\}\right)\right]$$

$$\alpha = \frac{(5.75V_0)^3\left[\left(\log\frac{30y_0}{K}\right)^3 - 3\left(\log\frac{30y_0}{K}\right)^2 + 6\log\left(\frac{30y_0}{K}\right) - 6\right]}{(5.75V_0)^3\left[\log\frac{30y_0}{K} - 1\right]^3}$$

Let, $(V_m)^3 = \left[\log\frac{30y_0}{K} - 1\right]^3 = \left(\log\frac{30y_0}{K}\right)^3 - 3\left(\log\frac{30y_0}{K}\right)^2 + \log(30y_0) - 1$

$$\alpha = \frac{V_m^{*3} + 3\left(\log\frac{30y_0}{K} - 1\right) - 2}{V_m^{*3}} = 1 + \frac{3}{V_m^{*2}} - \frac{2}{V_m^{*3}}$$

$$\alpha = 1 + 3\in^2 - 2\in^3$$ where $\in = \frac{1}{V_m^*}$

Momentum coefficient, β :

$$\beta = \frac{\int V^2 dA}{V_m^2 A} = \frac{\int V^2 B dy}{V_m^2 B y_0} = \frac{\int V^2 dy}{V_m^2 y_0}$$

$$= \frac{\int \left[5.75V_0 \log\frac{30y}{K}\right]^2 dy}{V_m^2 y_0}$$

$$V_m^2 \beta = \frac{b}{y_0} \int (\log x)^2 dx$$ where b = $(5.75V_0)^2K/30$

$$\int (\log \pi)^n dx = x(\log x)^n - n\int (\log x)^{n-1} dx$$

$$V_m^2 \beta = \frac{b}{y_0}\left[\frac{30y}{K}\left(\log\frac{30y}{K}\right)^2 - 2\frac{30y}{k}\log\frac{30y}{k} + 2\frac{30y}{K}\right]_0^{y_0}$$

$$\beta = \frac{\dfrac{(5.75V_0)^2}{y_0}\dfrac{K}{30}\left[\dfrac{30y_0}{K}\left(\log 30\dfrac{y_0}{K}\right)^2 - 2\dfrac{30y_0}{K}\log\dfrac{30y_0}{K} + 2\dfrac{30y_0}{K}\right]}{(5.75V_0)^2\left[\log\dfrac{30y_0}{K} - 1\right]^2}$$

Let $V_m^{*2} = \left(\log\frac{30y_o}{K} - 1\right)^2 = \left(\log\frac{30y_0}{K}\right)^2 - 2\log\frac{30y_0}{K} + 1$

$$\beta = \frac{\left(\log\dfrac{30y_0}{K}\right)^2 - 2\log\dfrac{30y_0}{K} + 1 + 1}{\left(\log\dfrac{30y_0}{K}\right)^2 - 2\log\dfrac{30y_0}{K} + 1}$$

$$\beta = \frac{V_m^{*2} + 1}{V_m^{*2}} = 1 + \frac{1}{V_m^{*2}}$$

$$\beta = 1 + \in^2 \qquad where \in = \frac{1}{V_m^*}$$

1.6

$$\alpha = \frac{\sum V_i^3 A_i \quad \sum A_i^2}{\left(\sum V_i A_i\right)^3}$$

For i = 3

$$\alpha = \frac{\left(V_1^3 A_1 + V_2^3 A_2 + V_3^3 A_3\right)\left(A_1 + A_2 + A_3\right)^2}{\left(V_1 A_1 + V_2 A_2 + V_3 A_3\right)^3}$$

$$\beta = \frac{\left(V_1^2 A_1 + V_2^2 A_2 + V_3^2 A_3\right)\left(A_1 + A_2 + A_3\right)}{\left(V_1 A_1 + V_2 A_2 + V_3 A_3\right)^2}$$

Table 1

	A	V	VA	V^2A	V^3A
1	40	3	120	360	1080
2	80	3	240	720	2160
3	80	3.1	248	768.8	2383.28
4	80	3.2	256	819.2	2621.44
5	80	3.3	264	871.2	2874.96
6	80	3.3	264	871.2	2874.96
7	80	3.2	256	819.2	2621.44
8	80	3.1	248	768.8	2383.28
9	40	3	120	360	1080
$\sum$	640		2016	6358.4	20079.36

The calculation is shown in Table -1.

$\alpha = (20079.36)(640)^2/2016^3 = 1.0038$

$\beta = (6358.4)(640)/2016^2 = 1.00126$

1.9

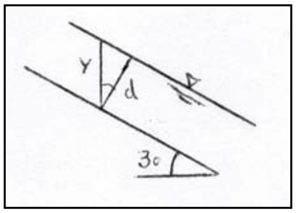
Using hydrostatic pressure distribution and depth = y = 5m

$T = \gamma y^2/2 = 9810\ (5^2/1000)2 = 122.6$ KN

$M = TY/3 = 122.6x5/3 = 204.3$ KN.m

But, $d = yCos\theta = 5Cos30 = 4.33$ m

In this case the pressure distribution is not hydrostatic



Correction:

$T_t = Pd/2 = \gamma d^2 Cos\theta/2 = 79.65$KN

$M_t = T_t\ d/3 = 79.65\ (4.33)/3 = 114.96$ KN m

% error in the shearing force:

$100(T_t - T)/T_t = (79.65 - 122.6)/79.65 = 53.9\ \%$

% error in the moment:

$100(M_t - M)/M_t = (114.96 - 204.38) / 114.96 = 77\ \%$

1.10

Centrifugal acceleration = V^2/R

Centrifugal force = $\rho y_s \Delta A\ V^2/R$

Pressure head due to centrifugal acceleration = $(1/g) y_s\ V^2/R$

Total pressure head, $y_t = y_s + (1/g) y_s\ V^2/R = y_s\ (1 + V^2/gR)$

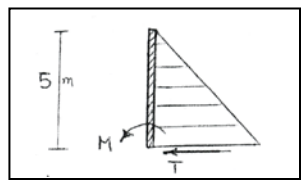
$y_t = 5\ (1 + 20^2/9.81/20) = 15.19$ m

Pressure intensity at point $C = \gamma\ y_t = (9.81)\ (1000)(15.19/1000) = 149$ kPa

1.11

$Q = K\, A\, R^{\,2/3}$
$A = [B - (h/\sqrt{3})]h$
$P = B + (4h/\sqrt{3})$
$Q = K\ [B - h/\sqrt{3}]^{5/3} h^{5/3} [B + 4\ h/\sqrt{3}]^{-2/3}$
$\quad = K[hB - h^2/\sqrt{3}]^{5/3} [B + 4\ h/\sqrt{3}]^{-2/3}$
Q is maximum or minimum if $dQ/dh = 0$

$dQ/dh = \frac{5}{3} K \left[hB - h^2/\sqrt{3}\right]^{2/3} \left[B - 2h/\sqrt{3}\right]\left[B + 4h/\sqrt{3}\right]^{-2/3}$



1.12

(i) nonuniform

(ii) nonuniform

(iii) nonuniform

(iv) uniform

1.13

(i) unsteady

(ii) unsteady

(iii) steady

(iv) unsteady

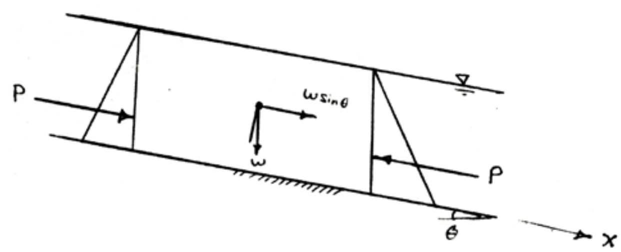
1.14

(i) $R_e = Vy/\nu = (1)(1)/(0.11 * 10^{-5}) = 9.0 * 10^5$ Turbulent

(ii) $R_e = Vy/\nu = \dfrac{0.1\times 2\times 10^{-3}}{0.11\times 10^{-5}}$ = 181.8 Laminar

1.15

It is not possible to have uniform flow in a frictionless sloping channel. The forces in the x direction will not be balanced.



1.16

It is not possible to have uniform flow in a horizontal channel.

There is no acceleration force.

1.17

y = flow depth measured vertically; θ = angle between water surface and horizontal; φ = angle between channel bed and horizontal. Let d = perpendicular on channel bed.

Thus, pressure at channel bed is

$p = \rho g d Cos\phi$

From the geometry, $y = dCos\phi + dSin\phi \tan\theta$

Or, $dCos\phi = \dfrac{y}{1+Tan\phi Tan\theta}$

Or, $p = \rho g \dfrac{y}{1+Tan\phi Tan\theta}$

1.18

$V = 5.75V_f \log (30y/K)$ (1)

$$V_m = \frac{\int VdA}{\int dA} = \frac{\int Vdy}{\int dy} = \frac{5.75\int_0^{y_0} V_f \log(30y/K)\,dy}{\int_0^{y_0} dy} = \frac{5.75V_f}{y_0}\int_0^{y_0}\log(30y/K)\,dy$$

Let *x = 30y/K, dx = (30/K)dy* or, *dy = (K/30)dx*

$$V_m = \frac{\int VdA}{\int dA} = \frac{5.75V_f}{y_0}\frac{K}{30}\int_0^{y_0}\log x\,dx = \frac{5.75V_f}{y_0}\frac{K}{30}\left[x\log x - x\right]_0^{y_0}$$

$$V_m = \frac{5.75V_f}{y_0}\frac{K}{30}\left[\frac{30y}{K}\log\frac{30y}{K} - \frac{30y}{K}\right]_0^{y_0}$$

$$V_m = 5.75V_f\left[\log(30y_0/K) - 1\right]$$

In Eq 1, at $y = y_0$, $V = V_{max}$

$V_{max} = 5.75V_f \log(30y_0/K)$

Let $\gamma = (V_{max}/V_m) - 1 = 5.75V_f \log(30y_0/K)/\ 5.75V_f \log(30y_0/K-1)-1$

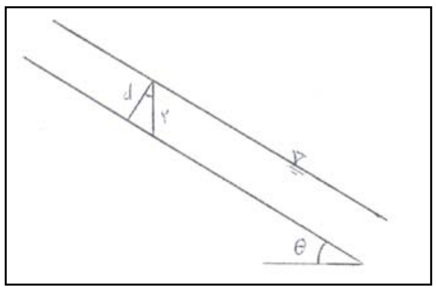
$\gamma = 1/[\log(30y_0/K)-1]$

Similar to the solution of problem 1.4 this will lead to

$\alpha = 1 + 3\gamma^2 - 2\gamma^3$

$\beta = 1 + \gamma^2$

1.19



$d = y\cos\theta$

$p = (\gamma d^2/2)(\cos\theta)$

$p = (\gamma y^2/2)(\cos^2\theta\ \cos\theta)$

$p = (\gamma y^2/2)(\cos^3\theta)$

$M = pd^3/3 = (py\cos\theta)/3$

$= (\gamma/2)(y^2\cos^3\theta)(y\cos\theta/3)$

$M = (\gamma y^3/6)(\cos^4\theta)$

Shearing force = $p = (\gamma y^2/2)(\cos^3\theta)$