Chapter 2

2.1 From Eq. (2.18),

$$\rho_d = \frac{G_s \rho_w}{1+e} = \frac{2450}{0.925} = \frac{2.80 \times 1000}{1+e}; \; e = 0.0571$$

From Eq. (2.6),

Porosity, $n = \frac{e}{1+e} = \frac{0.0571}{1+0.0571} = \mathbf{0.054}$

2.2 From Eq. (2.13), the dry density

$$\rho_d = \frac{\rho}{1+w} = \frac{2060}{1+0.153} = 1786.6 \text{ kg/m}^3$$

From Eq. (2.18), $\rho_d = \frac{G_s \rho_w}{1+e}$

$$e = \frac{G_s \rho_w}{\rho_d} - 1 = \frac{2.70 \times 1000}{1786.6} - 1 = 0.511$$

Once saturated, from Eq. (2.19),

$$\rho_{\text{sat}} = \frac{(G_s + e)}{(1+e)} \rho_w = \frac{(2.70+0.511)}{(1+0.511)} \times 1000 = \mathbf{2125.1 \ kg/m^3}$$

2.3 Let's consider a 1-m^2 area in plan. The initial volume of this soil is $V = 1 \times 0.5 = 0.5$ m^3. Volume of the solids is V_s.

$$e = 0.9 = \frac{0.5 - V_s}{V_s}; \; V_s = 0.2632 \text{ m}^3$$

$W_s = 0.2632 \times 2.68 \times 9.81 = 6.919$ kN

The new volume after compaction = 1 × 0.455 = 0.455 m^3

The dry unit weight, $\gamma_d = \frac{6.919}{0.455} = 15.21 \text{ kN/m}^3$

From Eq. (2.12), $\gamma_d = \frac{G_s \gamma_w}{1+e}$

$$e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{2.68 \times 9.81}{15.21} - 1 = \mathbf{0.729}$$

From Eq. (2.13), $\gamma = (15.21)(1 + 0.20) = \mathbf{18.25\ kN/m^3}$

2.4 At the compacted road base, the weight of solids,

W_s = 120,000 × 19.5 = 2,340,000 kN

At the borrow pit, the dry unit weight, $\gamma_d = \frac{\gamma}{1+w} = \frac{17.5}{1+0.085} = 16.13\ \text{kN/m}^3$

Volume of the pit, $V = \frac{2{,}340{,}000}{16.13} = \mathbf{145{,}080\ m^3}$

The moisture content has to be increased from 8.5% (at the borrow pit) to 14.0% (at the road base). The quantity of water to add,

2,340,000 × (0.14 – 0.085) = 128,700 kN

Volume of water to be added = $\frac{128{,}700}{9.81} = \mathbf{13{,}119.3\ m^3}$

2.5 $\gamma_d = \frac{\gamma}{1+w} = \frac{110.4}{1+0.105} = 99.9\ \text{lb/ft}^3$

$\gamma_d = \frac{G_s \gamma_w}{1+e}$; $e = \frac{2.65 \times 62.4}{99.9} - 1 = 0.655$

From Eq. (2.23), $D_r = \frac{0.870 - 0.655}{0.870 - 0.515} \times 100 = \mathbf{60.6\%}$

2.6 a. A-1-a c. A-3

b. A-1-b d. A-7-6

2.7 Soil A: % of gravel = 50, % of sand = 13, % of fines = 37

D_{10} = 0.035 mm, D_{30} = 0.061 mm, D_{60} = 9.8 mm ➔ C_u = 280; C_c = 0.02

LL = 58, PL = 34, PI = 24 ➔ plots below the A-line; hence, silt

The soil can be described as **poorly (gap) graded sandy silty gravel with a group symbol of GM.**

Soil B: % of gravel = 24, % of sand = 69, % of fines = 7

D_{10} = 0.17 mm, D_{30} = 0.82 mm, D_{60} = 2.6 mm ➔ C_u = 15.3; C_c = 1.5

LL = 42, PL = 22, PI = 20 ➔ plots above the A-line; hence, clay

The soil can be described as **well graded clayey gravelly sand with a group symbol of SW-SM.**

Soil C: % of gravel = 1, % of sand = 99, % of fines = 0

D_{10} = 0.7 mm, D_{30} = 1.2 mm, D_{60} = 1.6 mm ➔ C_u = 2.3; C_c = 1.3

The soil can be described as **poorly (uniformly) graded sand with a group symbol of SP.**

Soil D: % of gravel = 0, % of sand = 12, % of fines = 88

LL = 75, PL = 31, PI = 44 ➔ plots above the A-line; hence, clay.

The soil can be described as **sandy clay of high plasticity with a group symbol of CH.**

2.8 The head loss from the reservoir to the ditch, $\Delta h = 38 - 28 = 10.0\text{ m}$

The length of the sand seam in the direction of the flow, $L = 200/\cos 10 = 203.1\text{ m}$

The hydraulic gradient, $i = 10.0/203.1 = 0.0492$

By Darcy's law [Eq. (2.35)], $v = (2.6 \times 10^{-5}\text{ m/s})(0.0492) = 0.128 \times 10^{-5}\text{ m/s}$

The cross section of the sand seam through which the flow takes place is

$1.0 \times 500.0 = 500.0\text{ m}^2$

The flow rate $= (0.128 \times 10^{-5}\text{ m/s})(500.0\text{ m}^2) = 64.0 \times 10^{-5}\text{ m}^3\text{/s}$

Volume of water flowing into the ditch per day:

$= (64.0 \times 10^{-5}\text{ m}^3\text{/s})(24)(3600) = \mathbf{55.3\text{ m}^3}$

2.9 For the flow net shown in Figure P2.9, N_f = 3 and N_d = 10

Total head loss from right to left, $h_{\max}$ = 5.0 m

The flow rate is given by [Eq. (2.46)]

$$q = kh_{\max}\frac{N_f}{N_d} = (1.5 \times 10^{-5})(5.0)\left(\frac{3}{10}\right) = 2.25 \times 10^{-5}\text{ m}^3\text{/s/m length}$$

$$= (2.25 \times 10^{-5})(50.0)(24)(3600\text{ m}^3\text{/day}) = \mathbf{97.2\text{ m}^3\text{/day}}$$

2.10 On top of the soft clay layer (i.e., at 10 m depth), initially:

$\sigma' = 1 \times 17.0 + 9(20 - 9.81) = 108.7 \text{ kN/m}^2$

After the water table is lowered,

$\sigma' = 3 \times 17.0 + 7(20 - 9.81) = 122.3 \text{ kN/m}^2$

By lowering the water table, the effective stress has increased by (122.3 – 108.7) = 13.6 kN/m²

2.11 The soil below the water table can be assumed to be fully saturated (i.e., $S = 1$).

$e = wG_s = 0.25 \times 2.70 = \mathbf{0.675}$

The saturated unit weight can be computed as

$$\gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.70 + 0.675) \times 9.81}{1 + 0.675} = \mathbf{19.8 \text{ kN/m}^3}$$

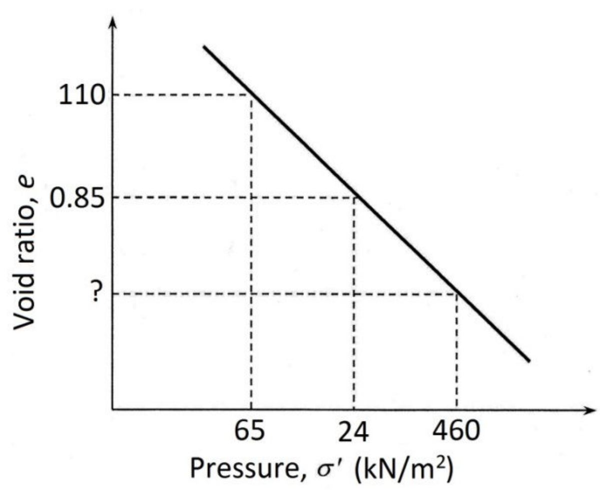
At a depth of 5 m into the sandy clay,

$\sigma = 4 \times 9.81 + 5 \times 19.8 = \mathbf{138.3 \text{ kN/m}^2}$

$u = 9 \times 9.81 = \mathbf{88.3 \text{ kN/m}^2}$

$\sigma' = \sigma - u = 138.3 - 88.3 = \mathbf{50 \text{ kN/m}^2}$

2.12 Refer to the figure.



a. The compression index C_c is given by [Eq. (2.53)],

$$C_c = \frac{e_1 - e_2}{\log\sigma_2' - \log\sigma_1'} = \frac{1.10 - 0.85}{\log 240 - \log 65} = 0.441$$

b. Let the void ratio at 460 kN/m^2 pressure be e_3.

$$e_1 - e_3 = C_c(\log 460 - \log 65) = 0.441 \times \log\left(\frac{460}{65}\right) = 0.375$$

$$e_3 = 1.10 - 0.375 = \mathbf{0.725}$$

2.13 a. The clay is below the water table and, hence, is saturated. The initial void ratio e_o can be determined as

$$e_o = wG_s = 0.225 \times 2.72 = 0.612$$

The saturated unit weight is determined as

$$\gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.72 + 0.612)(9.81)}{1 + 0.612} = 20.3\,\text{kN/m}^3$$

The effective overburden stress at the middle of the clay is

$$\sigma_o' = 2 \times 17.0 + 3(20.2 - 9.81) + 1.5(20.3 - 9.81) = 80.3\,\text{kN/m}^2 < 110.0\,\text{kN/m}^2$$

Since the preconsolidation pressure is greater than the current overburden pressure, the **clay is overconsolidated**. The overconsolidation ratio

$$\text{OCR} = 110.0/80.3 = \mathbf{1.37}$$

b. The 2-m-high compacted fill imposes a surcharge of $2 \times 20 = 40\,\text{kN/m}^2$ (i.e., $\Delta\sigma' = 40.0\,\text{kN/m}^2, \sigma_o' = 80.3\,\text{kN/m}^2, \text{and } \sigma_c' = 110.0\,\text{kN/m}^2$)

Since $\sigma_o' + \Delta\sigma' > \sigma_c'$, the consolidation settlement can be computed from Eq. (2.69) as

$$S_p = \frac{C_s H}{1 + e_o}\log\left(\frac{\sigma_c'}{\sigma_o'}\right) + \frac{C_c H}{1 + e_o}\log\left(\frac{\sigma_o' + \Delta\sigma'}{\sigma_c'}\right)$$

$$= \frac{0.06 \times 3000}{1 + 0.612}\log\left(\frac{110.0}{80.3}\right) + \frac{0.52 \times 3000}{1 + 0.612}\log\left(\frac{80.3 + 40.0}{110}\right)$$

$$= \mathbf{52.9\ mm}$$

2.14 For $U = 75\%$, $T_v = 0.477$ (Table 2.12)

$$T_v = \frac{c_v t}{H_{\text{dr}}^2}$$

From two-way (doubly drained) to one-way (singly drained), H_{dr} is doubled. For the same U and, hence, the same T_v, this would increase the time fourfold. Therefore, it will take **4*t* years**.

2.15 a. The clay layer with one-way drainage has H_c of 6.0 m. After one year,

$$T_v = \frac{c_v t}{H^2} = \frac{0.0014 \times 365 \times 24 \times 3600}{600^2} = 0.123$$; settlement, $S_{c(t)} = 160$ mm

From Table 2.12, $U = 39.6\%$

$$U = \frac{S_{c(t)}}{S_{c(\max)}};\ S_{c(\max)} = 160/0.396 = 404\ \text{mm}$$

When $t = 2$ years, $T_v = 0.246$.

From Table 2.12, $U = 55.8\%$.

Consolidation settlement during the first two years is 0.558×404 = **225.4 mm**

b. The initial effective overburden stress at the middle of the clay is

$$\sigma'_o = 1.5 \times 17.0 + 0.5(18.5 - 9.81) + 3.0(19.0 - 9.81) = 57.4\ \text{kN/m}^2$$

$$\Delta\sigma' = 3 \times 19 = 57\ \text{kN/m}^2$$

$$S_c = \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

$$404 = \frac{C_c \times 6000}{1 + 0.810} \log\left(\frac{57.4 + 57.0}{57.4}\right);\ C_c = \mathbf{0.41}$$

2.16 a. For the clay layer, assuming $S = 100\%$ below the water table,

$$e_o = 0.45 \times 2.70 = 1.215$$

$$\gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{(2.70 + 1.215) \times 9.81}{1 + 1.215} = 17.3\ \text{kN/m}^3$$

The initial effective overburden stress at the middle of the clay layer is

$$\sigma'_o = 1 \times 16 + 1(19.0 - 9.81) + 1.5(17.3 - 9.81) = 36.4\ \text{kN/m}^2$$

With OCR = 1.5, the preconsolidation pressure $\sigma'_c = 1.5 \times 36.4 = 54.6\ \text{kN/m}^2$

When the fill is placed, it imposes a surcharge of $\Delta\sigma' = 20 \times 1.5 = 30.0\ \text{kN/m}^2$

From Eq. (2.69),

$$S_c = \frac{C_s H}{1 + e_o} \log\left(\frac{\sigma'_c}{\sigma'_o}\right) + \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right)$$

$$= \frac{0.08 \times 3000}{1 + 1.215} \log\left(\frac{54.6}{36.4}\right) + \frac{0.65 \times 3000}{1 + 1.215} \log\left(\frac{66.4}{54.6}\right) = \mathbf{93.9\ mm}$$

b. Including the fill load and the warehouse load, $\Delta\sigma' = 30 + 40 = 70\ \text{kN/m}^2$

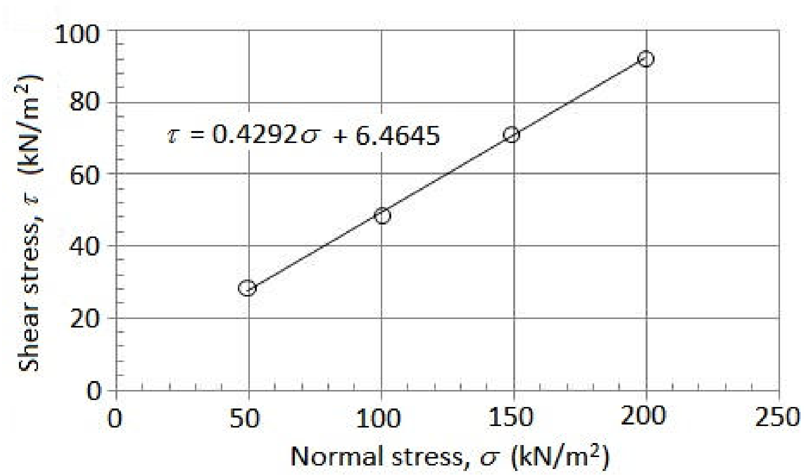
$$S_c = \frac{C_s H}{1+e_o}\log\left(\frac{\sigma'_c}{\sigma'_o}\right) + \frac{C_c H}{1+e_o}\log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right)$$

$$= \frac{0.08 \times 3000}{1+1.215}\log\left(\frac{54.6}{36.4}\right) + \frac{0.65 \times 3000}{1+1.215}\log\left(\frac{106.4}{54.6}\right)$$

$$= 274.2\ \text{mm}$$

Consolidation settlement due to the warehouse alone is $274.2 - 93.9$ = **180.3 mm**

2.17 The direct shear test data are plotted in the figure. From the failure envelope, c' = **6.5 kN/m²** and ϕ' = $\tan^{-1}(0.4292)$ = **23.2°**



2.18 a. $\sigma'_3 = 100\ \text{kN/m}^2$ and $\Delta\sigma_f = 260\ \text{kN/m}^2$

Therefore, $\sigma'_1 = \sigma'_3 + \Delta\sigma_f = 360\ \text{kN/m}^2$

From Eq. (2.91), $\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c'\tan\left(45 + \frac{\phi'}{2}\right)$

For normally consolidated clays, $c' = 0$; hence,

$360 = 100\tan^2\left(45 + \frac{\phi'}{2}\right)$; ϕ' = **34.4°**

b. For the second specimen, $\sigma'_3 = 200$ kN/m^2

$$\sigma'_1 = 200\tan^2\left(45+\frac{34.4}{2}\right) = 719.5 \text{ kN/m}^2$$

$$\Delta\sigma_f = 719.5 - 200 = \mathbf{519.5\ kN/m^2}$$

2.19 a. In normally consolidated clay, $c' = 0$

For the first specimen (consolidated drained test), using Eq. (2.91),

$$260+150 = 410 = 150\tan^2\left(45+\frac{\phi'}{2}\right);\ \phi' = 27.7°$$

In the second specimen (consolidated undrained test), applying the same value of ϕ' in Eq. (2.91),

$$\sigma_3 = 150 \text{ kN/m}^2 \text{ and } \Delta\sigma_f = 115 \text{ kN/m}^2$$

$$\sigma_1 = \sigma_3 + \Delta\sigma_f = 265 \text{ kN/m}^2$$

$$\sigma'_3 = 150 - u_f \text{ and } \sigma'_1 = 265 - u_f$$

$$\sigma'_1 = \sigma'_3\tan^2\left(45+\frac{\phi'}{2}\right) = \sigma'_3\tan^2\left(45+\frac{27.7}{2}\right) = 2.737\sigma'_3$$

$$265 - u_f = (150-u_f)\times 2.737;\ u_f = \mathbf{83.8\ kN/m^2}$$

b. From Eq. (2.96), $A_f = \dfrac{u_f}{\Delta\sigma_f} = \dfrac{83.8}{115} = \mathbf{0.73}$

2.20 At failure the pore water pressure is u_f, $\sigma_3 = 100$ kN/m^2 and $\sigma_1 = 207$ kN/m^2.

$$\sigma'_3 = 100 - u_f \text{ and } \sigma'_1 = 207 - u_f$$

Substituting for σ'_3 and σ'_1 in Eq. (2.91),

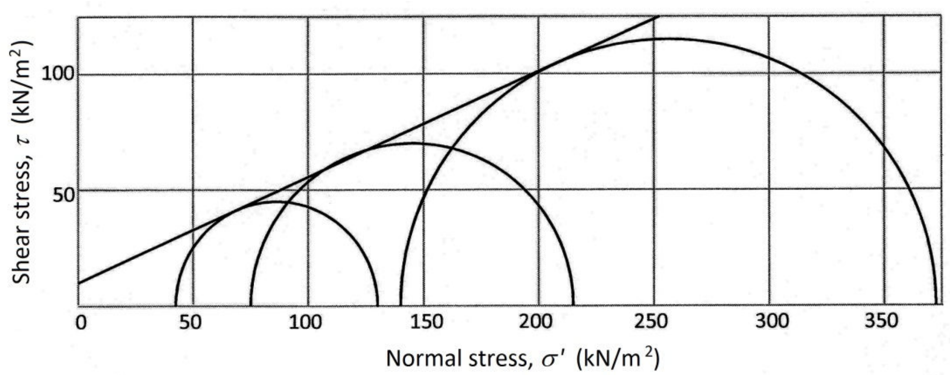
$$\sigma'_1 = \sigma'_3\tan^2\left(45+\frac{\phi'}{2}\right) + 2c'\tan\left(45+\frac{\phi'}{2}\right)$$

$$(207-u_f) = (100-u_f)\tan^2\left(45+\frac{26}{2}\right) + 2\times 10\tan\left(45+\frac{26}{2}\right)$$

u_f = **51.9 kN/m^2**

2.21 The following table can be prepared from the given data, and the Mohr circles are plotted as shown in the figure.

Sample No.	σ_3 (kN/m^2)	$(\Delta\sigma_d)_f$ (kN/m^2)	$(\Delta u_d)_f$ (kN/m^2)	σ_1 (kN/m^2)	σ_3' (kN/m^2)	σ_1' (kN/m^2)
1	100	88.2	57.4	188.2	42.6	130.8
2	200	138.5	123.7	338.5	76.3	214.8
3	350	232.1	208.8	582.1	141.2	373.3



The failure envelope is drawn tangent to the Mohr circles in the figure and, from measurements, c' = **10.0 kN/m^2** and ϕ' = **24.7°**

2.22 The unconfined compressive strength of the clay specimen $q_u = 2c_u = 90\,\text{kN/m}^2$

Cross sectional area of the specimen $= \dfrac{\pi}{4} \times 75^2 = 4417.9\,\text{mm}^2$

Maximum load the specimen can carry $= 90 \times 4417.9 \times 10^{-6} = 0.398\,\text{kN} = 398\,\text{N}$

Weight of one steel plate = 1.5×9.81 N = 7.358 N

Therefore, number of plates that can be stacked on the specimen $= 398/7.358 =$ **54**

With q_u = 90 kN/m^2 (see Table 2.14), it is a **medium clay** (consistency).

2.23 a. From Figure P2.7, D_{10} = 0.7 mm, D_{15} = 0.9 mm, D_{30} = 1.2 mm,

D_{50} = 1.4 mm, D_{60} = 1.6 mm, and D_{85} = 2.05 mm

$$C_u = \frac{D_{60}}{D_{10}} = \frac{1.6}{0.7} = 2.3\,;\;\; C_c = \frac{D_{30}^2}{D_{10} \times D_{60}} = \frac{1.2^2}{0.7 \times 1.6} = 1.3$$

From Eq. (2.87), $\phi' = 26 + (10 \times 0.8) + (0.4 \times 2.3) + 1.6\log(1.4) =$ **35.2°**

b. From Eq. (2.89), $a = 2.101 + 0.097\left(\frac{D_{85}}{D_{15}}\right) = 2.101 + 0.097\left(\frac{2.05}{0.9}\right) = 2.322$

From Eq. (2.90), $b = 0.845 - 0.398a = 0.845 - 0.398 \times 2.322 = -0.0792$

From Eq. (2.88),

$$\phi' = \tan^{-1}\left(\frac{1}{ae+b}\right) = \tan^{-1}\left(\frac{1}{2.322 \times 0.61 - 0.0792}\right) = \mathbf{36.8^\circ}$$