Chapter 1

1.1. a. $x(t) = e^{.5t} \quad t \geq 0 \Rightarrow$

$$E = \lim_{T\to\infty} \int_0^T (e^{.5t})^2 dt = \lim_{T\to\infty} \int_0^T e^t dt \Rightarrow E = \lim_{T\to\infty} (e^T - 1)$$

$\Rightarrow E = \infty \Rightarrow$ not an energy signal

$$P = \lim_{T\to\infty} \frac{1}{T}\int_0^{T/2} e^t dt = \lim_{T\to\infty} \frac{1}{T}(e^{T/2} - 1) = \infty \Rightarrow \text{not a power signal}$$

$\Rightarrow x(t) = e^{.5t}$ is not stable

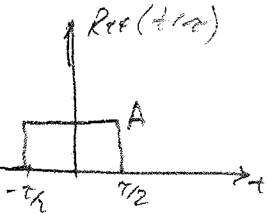
b. $x(t) = e^{-.5t} \quad t \geq 0 \Rightarrow E = \lim_{T\to\infty} \int_0^T e^{-T} dt = \lim_{T\to\infty} (e^{-T} + 1) = 1$ Joule $\Rightarrow$ Energy signal

c. $x(t) = \cos t + \cos 2t \Rightarrow P = \frac{1}{T}\int_{-T/2}^{T/2} (\cos t + \cos 2t)^2 dt = 0.5 + 0.5 = 1^W \Rightarrow$ power signal

d. $x(t) = e^{-a|t|} \Rightarrow$

$$E = \int_{-\infty}^{\infty} (e^{-a|t|})^2 dt = 2\int_0^{\infty} e^{-2at} dt = \frac{1}{a}J \Rightarrow \text{energy signal.}$$

1.2 $x(t) = A\, Rect(t/\tau)$



$$\Rightarrow E = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-\tau/2}^{\tau/2} A^2 dt = A^2(\tau/2 + \tau/2) = A^2\tau \text{ Joules}$$

1.4. a. $\int_{-2}^{2} \varphi_1(t)\varphi_1^*(t) dt = \int_0^2 (1) dt = 2$

$$\int_{-2}^{2} \varphi_1(t)\varphi_2^*(t) dt = \int_0^{.5} 1\, dt + \int_{.5}^{1} (-1) dt = \phi$$

Finally

$$\int_{-2}^{2} \varphi_2(t)\varphi_2^*(t) dt = \int_{-1}^{1} (1) dt = 2 \Rightarrow \text{Then } \varphi_1 \;\&\; \varphi_2 \text{ are orthogonal}$$

b. $x(t) = \sum_{n=1}^{2} X_n \phi_n(t) = X_1\varphi_1(t) + X_2\varphi_2(t)$, where

$$X_1 = \frac{1}{2}\int_{-2}^{2} (t)\, \varphi_1^*(t)\, dt = \frac{1}{2}\int_0^1 t\, dt + \frac{1}{2}\int_1^2 (-t) dt = -1/2$$

$$X_2 = \frac{1}{2}\int_{-2}^{2} t\; \varphi_2^*(t) = \frac{1}{2}\left[2\int_0^{.5} t\, dt + 2\int_{.5}^{1} (-t) dt\right] = -1/4.$$

1.3. a. From Fourier transform tables

$$\mathfrak{F}\left[2\Delta\left(\frac{t-3}{5}\right)\right] = 5e^{-j3\omega}\, Sinc^2\left(\frac{5\omega}{4}\right)$$

b. $x_p(t) = \sum_{n=-\infty}^{\infty} X_n e^{j2n\pi t/T}$ where $X_n = \frac{5}{10} Sinc^2\left(\frac{n\pi}{4}\right)$

1.5.

$$y(t) = \sum_{n=-\infty}^{\infty} Y_n e^{j2n\pi t/T} \quad \text{where } Y_n = \frac{1}{T}\int_0^T y(t) e^{-j\frac{2n\pi t}{T}} dt = \frac{1}{T}\int_0^T x(t-t_0) e^{-j2n\pi t/T} dt$$

let $u = t - t_0 \Rightarrow du = dt$

$$\Rightarrow Y_n = \frac{1}{T}\int_{t_0}^{T+t_0} x(u) e^{j2n\frac{\pi}{T}(u-t_0)} du = e^{-j2n\pi\frac{t_0}{T}} \cdot \frac{1}{T}\int_{t_0}^{T-t_0} x(u) e^{-j2n\frac{\pi u}{T}} du = X_n e^{-j2n\pi t_0/T}$$

1.6. a. $R_u = \frac{c}{2f_r}$; $f_r = 200\ Hz \Rightarrow R_u = \frac{3\times10^8}{2\times200} = 750\ Km$

for $f_r = 750\ Hz \Rightarrow R_u = \frac{3\times10^8}{2\times750} = 200\ Km$

b. $PRI = \frac{1}{f_r} \Rightarrow PRI_1 = \frac{1}{200} = 5.0\ msec$

$PRI_2 = \frac{1}{750} = 1.33\ msec$

1.7. a. for $f_r = 200\ Hz$; $d_t = \frac{\tau}{T}$; $T = 5.0\ msec \Rightarrow \tau = 0.3\times5.0\times10^{-3} = 1.5\ msec$

The average transmitted power is $P_{av} = d_t \cdot P_t = .3\times5\times10^3 = 1.5\ KW$

In 20 msec we have 4-periods $\Rightarrow$ The average transmitted power in 20 msec is

$P_{av_{20ms}} = 4\times1.5\ KW = 6\ KW$

The single pulse energy is $E_p = P_t\,\tau = 5\ KW \times 1.5\ msec = 7.5$ joules

and in 20 msec $\Rightarrow E_{20msec} = 4 \cdot \times 7.5 = 30$ joules

b. for $f_r = 750\ Hz \Rightarrow \tau = \frac{.3}{750} = 0.4\ msec$

As in 20 msec $\Rightarrow \frac{20}{1.33} = 15$ periods $\Rightarrow P_{av} = 15\times1.5 = 22.5\ KW$

& $E_{20} = 15 * E_p = 15\times5\ KW\times.4 = 30$ joules.

1.8. $\Delta R = \frac{c\tau}{2} \Rightarrow \Delta R = \frac{3\times10^8 \times 1\times10^{-6}}{2} = 150\,m$

1.9.

$R_u = \frac{c}{2f_r} = \frac{3\times10^8}{2\times3\times10^3} = 50\,km$

$\Delta R = \frac{c}{2B} \Rightarrow B = \frac{c}{2\Delta R} = \frac{3\times10^8}{2\times30} = 5\,MHz$

$d_t = \frac{\tau}{T} = \tau f_r = f_r/B = \frac{3\times10^3}{5\times10^6} = 0.006$

1.10.

$f_d = \frac{2V_r}{\lambda} \Rightarrow$ for $v_r = 100\frac{m}{s} \Rightarrow f_{d_1} = \frac{2\times100}{0.3} = 666.67\,kHz$

for $v_r = 200\,\frac{m}{s} \Rightarrow f_{d_2} = \frac{2\times200}{.3} = 1.33\,kHz$

for $v_r = 350\,\frac{m}{s} \Rightarrow f_{d_3} = \frac{2\times350}{.3} = 2.33\,kHz$

1.11.

$\Delta t = \frac{2R}{c} \Rightarrow$ for $R_1 = 30\,km \Rightarrow \Delta t_1 = \frac{2\times30\times10^3}{3\times10^8} = 200\,\mu sec$

for $R_2 = 80\,Km \Rightarrow \Delta t_2 = \frac{2\times80\times10^3}{3\times10^8} = 533.33\,\mu sec$

for $R_3 = 150\,Km \Rightarrow \Delta t_3 = \frac{2\times150\times10^3}{3\times10^8} = 1.0\,msec$

1.12.

for S-Band radar where $f = 3\,GHz \Rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 0.1\,m$

$\Rightarrow$ for $v_r = 50\,m/sec \Rightarrow f_{d_1} = \frac{2\times50}{0.1} = 1\,KHz$

for $v_r = 200\,\frac{m}{sec} \Rightarrow f_{d_2} = \frac{2\times200}{0.1} = 4\,KHz$

for $v_r = 250\,\frac{m}{sec} \Rightarrow f_{d_3} = \frac{2\times250}{0.1} = 5\,KHz$

1.13. for $f = 10\,GHz \Rightarrow \lambda = \frac{3\times10^8}{10\times10^9} = 0.03\,m = 30\,mm$

$\Rightarrow$ for $v_r = 50\frac{m}{sec} \Rightarrow f_{d_1} = \frac{2\times50}{.03} = 3.33\,KHz$

for $v_r = 200 \frac{m}{sec} \Rightarrow f_{d_2} = \frac{2 \times 200}{.03} = 13.33\,KHz$

for $v_r = 250 \frac{m}{sec} \Rightarrow f_{d_3} = \frac{2 \times 250}{.03} = 22.0\,KHz$

1.14.

$$f_{d_{max}} = \frac{2 v_{r_{max}}}{\lambda} \; ; \text{ we know that } f_{r_{min}} \geq 2 \frac{2 v_{r_{max}}}{\lambda}$$

$$\Rightarrow v_{r_{max}} = \frac{f_{r_{min}}}{4} \lambda \; ; \; \& \; \lambda = \frac{c}{f} = \frac{3 \times 10^8}{1.5 \times 10^9} = 0.2\,m$$

$$\Rightarrow v_{r_{max}} = 10 \times 10^3 \cdot \frac{.2}{4} = 500 \frac{m}{sec} \Rightarrow f_{d_{max}} = \frac{2 \times 500}{.2} = 5\,KHz$$

1.15. Start with

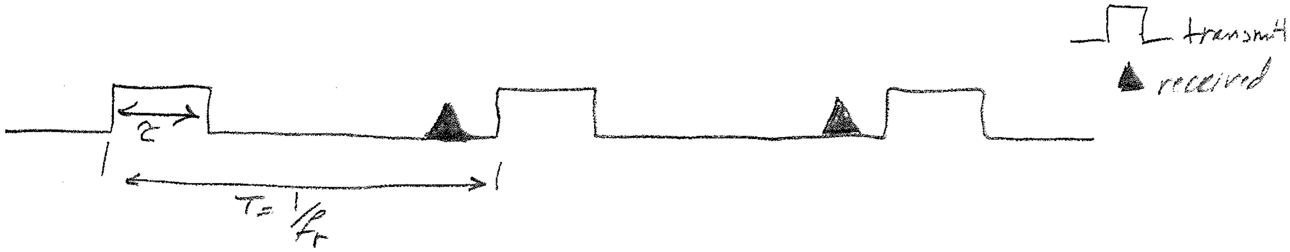
$R(t) = R_0 + v(t - t_0)$ & follow same steps outlined in text.

1.16.

$$f_d = \frac{2 v_r}{\lambda} \cos\theta_e \cos\theta_a \Rightarrow f_d = \frac{2 \times 150}{0.1} \cos 30^\circ \cos 15^\circ = 2.51\,KHz$$

$$\Delta R = \frac{c\tau}{2} \Rightarrow 0.3 = \frac{3 \times 10^8 \tau}{2} \Rightarrow \tau = 2\,nsec \Rightarrow B = \frac{1}{\tau} = 500\,MHz$$

1.18



The minimum PRF must be chosen so that the target range is unambigous $\Rightarrow$

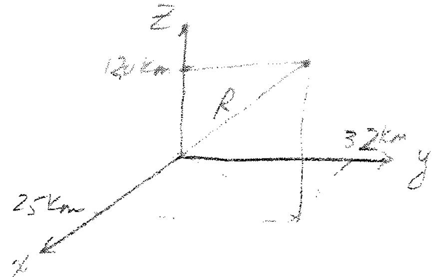
$$f_{r_{min}} \geq 2 \frac{2 v_{r_{max}}}{\lambda} \geq 2 f_{d_{max}} \Rightarrow$$

$$f_{r_{min}} = 4 \frac{400}{.2} = 8\,KHz$$

$$\& \; R_u = \frac{c}{2 f_r} = \frac{3 \times 10^8}{2 \times 8 \times 10^3} = 18.75\,KHz$$

1.19.

$R = \sqrt{x^2+y^2+z^2}$



$\Rightarrow$

a. $R = \sqrt{12.0^2 + 25^2 + 32^2}$

$= \sqrt{179.3} \qquad = 42.344 \text{ Km}.$

b. $v_x = -250$ m/sec ; $v_z = v_y = 0$.

$\dot{R}$ = Range rate $= \dfrac{dR}{dt} = \dfrac{\Delta R}{\Delta t} = \dfrac{\Delta v \cdot R}{\Delta R} = \dfrac{\Delta v_x \cdot R_x + \Delta v_y \cdot R_y + \Delta v_z \cdot R_z}{\Delta R}$

$= \dfrac{-250 * 25 + 32 * 0 + 12 * 0}{42.344} \Rightarrow \dot{R} = -147 \text{ m/sec}.$

c. Round trip delay is $\Delta t = \dfrac{2 * 42.344 \times 10^3}{3 \times 10^8} = 282.29\ \mu\text{sec}$

$\lambda = \dfrac{c}{f} = \dfrac{3 \times 10^8}{9 \times 10^9} = 33.33 \times 10^{-3}$ m

$\Rightarrow$

$f_d = \dfrac{2\dot{R}}{\lambda} = \dfrac{2 * -147}{33.33 \times 10^{-3}} = -8.82$ KHz (openning target).

1.20

$\Delta R = \dfrac{c\tau}{2} \Rightarrow \tau = \dfrac{2 \times 10^2}{3 \times 10^8} = 66.67\ \mu\text{sec}$

$R_u = \dfrac{c}{2 f_r} \Rightarrow f_r = \dfrac{2 \times 100 \times 10^3}{3 \times 10^8} = 1.5 \text{ KHz}$

$P_{av} = P_t d_t = P_t \tau f_r = 500 \times 66.667 \times 10^{-6} \times 1.5 \times 10^3 = 0.5 \text{ KW} \Leftrightarrow 27 \text{dB}$

2.1. $x(t) = Rect(\frac{t}{\tau}) \cos(\omega_0 t + Bt^2/2\tau)$

$$x_I(t) = Rect(t/\tau) \cos \frac{B}{2\tau} t^2 \quad \& \quad x_Q(t) = Rect(\tfrac{t}{\tau}) \sin(\tfrac{Bt^2}{2\tau})$$

2.2. for $\tau = 15\,\mu sec$ & $B = 10\,MHz$ $\Rightarrow$

$$x_I(t) = Rect(\tfrac{t}{15\mu}) \cos(333.336 t^2) \quad \& \quad x_Q(t) = Rect(\tfrac{t}{15\mu}) \sin(333.336 t^2)$$

2.3.

$f_i(t) = \frac{1}{2\pi} Im\{\frac{d}{dt} \ln \psi(t)\}$, $\psi(t)$ is the analytic signal

$\Rightarrow f_i(t) = \frac{1}{2\pi} Im\{\frac{\psi'(t)}{|\psi(t)|^2}\}$ where ψ' indicates derivative wrt t

but $\psi(t) = \tilde{x}(t)\, e^{j2\pi f_0 t}$ $\quad \tilde{x}(t)$ is the complex envelope for $x(t)$

$\Rightarrow$

$f_i(t) = \frac{1}{2\pi|\psi(t)|^2}\left[\hat{x}'(t)\, x(t) - x'(t)\, \hat{x}(t)\right]$ where $\hat{x}$ is the Hilbert transform for x

Recall $\tilde{x} = x_I + j x_Q$ & $x = x_I \cos 2\pi f_0 t - x_Q \sin 2\pi f_0 t$

so when $\psi(t) = Rect(\frac{t}{T}) \cos(2\pi f_0 t + \frac{B}{2\tau} t^2)$ we get

$$\ln[\psi(t)] = C + j(2\pi f_0 t + \frac{B}{2\tau} t^2)$$

$$\Rightarrow f_i(t) = \frac{1}{2\pi} Im\{\frac{d}{dt} \ln \psi(t)\} = f_0 + \frac{1}{2\pi}\frac{B}{\tau} t$$

2.4.

$h(t) = \delta(t) - \frac{\omega_0}{\omega_d} e^{-2t} \sin \omega_0 t \quad t \geq 0$

In general $h(t) = h_I(t) \cos \omega_0 t - h_Q(t) \sin \omega_0 t$

$\Rightarrow$ by inspection

$$h_I(t) = \delta(t) \quad \& \quad h_Q(t) = \frac{\omega_0}{\omega_d} e^{-2t}$$

2.8 a) REPLACE $D(y)$ BY THE CONSTANT d_0 $\Longrightarrow$

$$E(\xi) = d_0 \int_{-r/2}^{r/2} \mathrm{EXP}\left(2\pi y j \frac{\sin\xi}{\lambda}\right) dy$$

$$= d_0 \left[\frac{\mathrm{EXP}\left(2\pi y j \frac{\sin\xi}{\lambda}\right)}{j 2\pi \frac{\sin\xi}{\lambda}} \right] \Bigg|_{-r/2}^{r/2}$$

$\Longrightarrow$

$$E[\xi] = \frac{d_0}{j 2\pi \frac{\sin\xi}{\lambda}} \left[\mathrm{EXP}\left(\pi j \frac{\sin\xi}{\lambda} r\right) - \mathrm{EXP}\left(-\pi j \frac{\sin\xi}{\lambda} r\right) \right]$$

USING THE RELATION $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$ AND $Sa(\theta) = \frac{\sin\theta}{\theta}$ $\Longrightarrow$

$$E(\xi) = d_0 r\, Sa\left(\pi \frac{\sin\xi}{\lambda} r\right) \Rightarrow |E(\xi)|^2 = d_0^2 a^2 Sa^2\left(\pi \frac{\sin\xi}{\lambda} r\right)$$

b) $P(\xi) = \frac{E(\xi)}{E(0)} = Sa^2\left(\frac{\pi \sin\xi\, r}{\lambda}\right) = Sa^2(x)$, $x = \frac{\pi \sin\xi\, r}{\lambda}$

x	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$	$5\pi/8$	$6\pi/8$	$7\pi/8$	π
$P(x)$	1	0.949	0.8105	0.61499	0.40528	0.2214	0.090	0.019	0
$P(x)_{dB}$	0	-0.2244	-0.9121	-2.111	-3.9223	-6.548	-10.45	-17.12	$-\infty$

2.9 a.

$$P_{rad} = \iint |F(\theta,\varphi)|^2 d\Omega = \int_0^{\pi}\int_0^{\pi} |\cos^n\theta| \sin\theta\, d\theta\, d\varphi = 2\pi(2)\int_0^{2\pi} \cos^n\theta \sin\theta\, d\theta$$

$\Rightarrow$ for $n=1$

$$P_{rad} = 4\pi \int_0^{\pi/2} \cos\theta \sin\theta\, d\theta = 4\pi \left(\frac{1}{2} \sin^2\theta \Big|_0^{\pi/2} \right) = 2\pi$$

for $n=2$

$$P_{rad} = 4\pi \int_0^{\pi/2} \cos^2\theta \sin\theta\, d\theta = 4\pi \left(-\frac{\cos^3\theta}{3} \Big|_0^{\pi/2} \right) = \frac{4\pi}{3}$$