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SOLUTIONS TO PROBLEMS IN CHAPTER 2

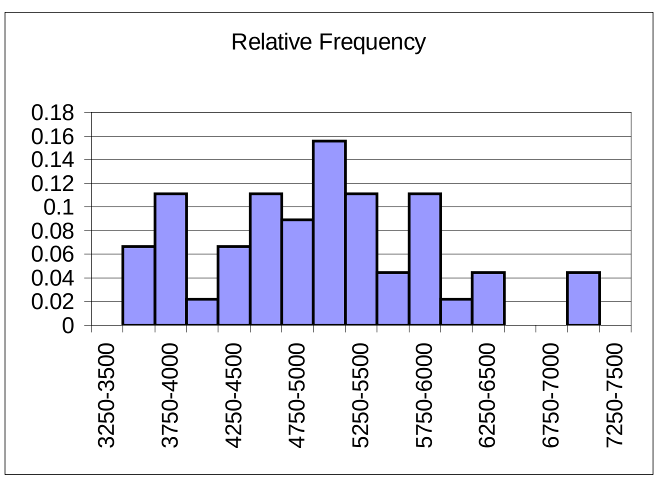
Problem 2.1. The results of tests to determine the modulus of rupture (MOR) for a set of timber beams are shown in Table P2.1.

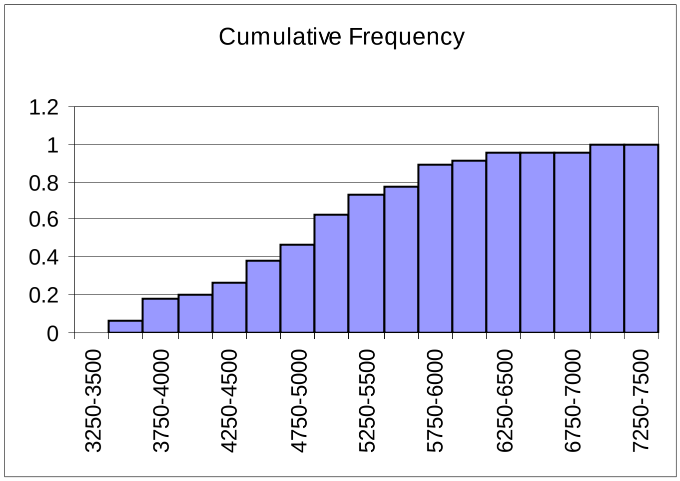
A. Plot the relative frequency and cumulative frequency histograms.
B. Calculate the sample mean, standard deviation, and coefficient of variation.
C. Plot the data on normal probability paper.

Solution:

A. For the histogram plots, the interval size is chosen to be 250. There are 45 data points.

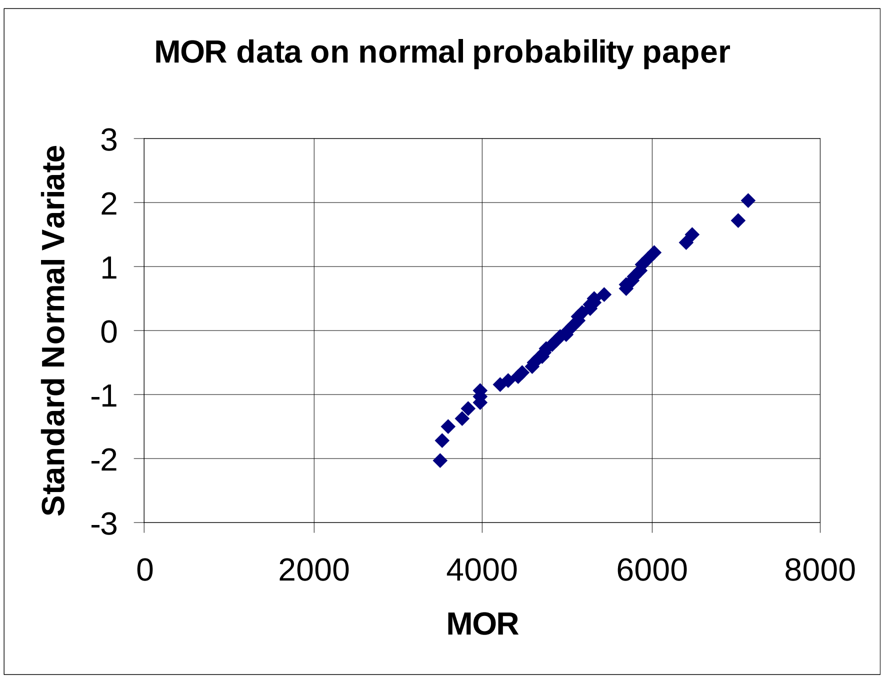
Interval	Relative Frequency	Cumulative Frequency
3250-3500	0	0
3500-3750	0.06667	0.066667
3750-4000	0.11111	0.177778
4000-4250	0.02222	0.200000
4250-4500	0.06667	0.266667
4500-4750	0.11111	0.377778
4750-5000	0.08889	0.466667
5000-5250	0.15556	0.622222
5250-5500	0.11111	0.733333
5500-5750	0.04444	0.777778
5750-6000	0.11111	0.888889
6000-6250	0.02222	0.911111
6250-6500	0.04444	0.955556
6500-6750	0	0.955556
6750-7000	0	0.955556
7000-7250	0.04444	1
7250-7500	0	1





B. Using Eqns. 2.25 and 2.26, sample mean = $\overline{x}$ = 5031 and sample standard deviation = s_X = 880.4. The coefficient of variation based on sample parameters is $s_X / \overline{x}$ = 0.175.

C. The step-by-step procedure described in Section 2.5 is followed to construct the plot on normal probability paper.



<u>Problem 2.2</u>. A set of test data for the load-carrying capacity of a member is shown in Table P2.2.

A. Plot the test data on normal probability paper.

B. Plot a normal distribution on the same probability paper. Use the sample mean and standard deviation as estimates of the true mean and standard deviation.

C. Plot a lognormal distribution on the same normal probability paper. Use the sample mean and standard deviation as estimates of the true mean and standard deviation.

D. Plot the relative frequency and cumulative frequency histograms.

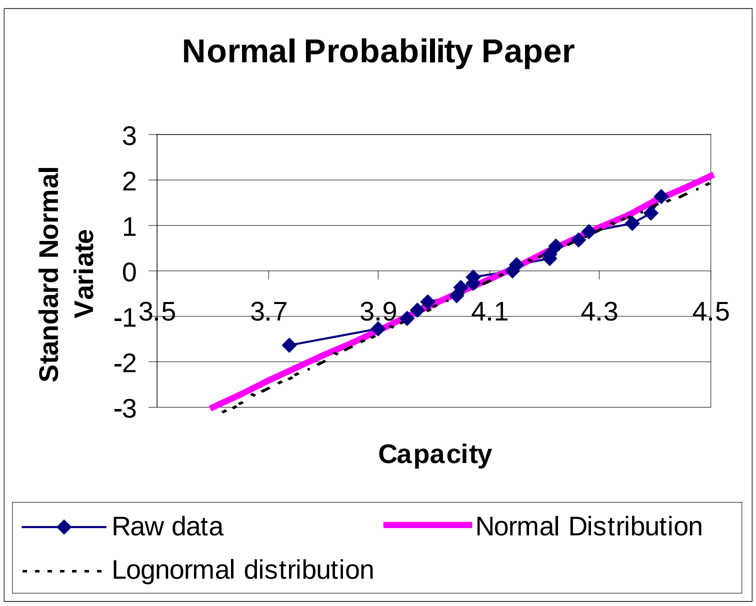
Solution:

A. Using Eqns. 2.25 and 2.26, the sample mean, sample standard deviation, and sample coefficient of variation are

$$\bar{x} = 4.127 \qquad s_X = 0.1770 \qquad CoV = s_X / \bar{x} = 0.04289$$

To plot on normal probability paper, we follow the step-by-step procedure outlined in Section 2.5.

Raw Data	Sorted	i	$i/(N+1)=p_i$	$\Phi^{-1}(p_I)$
3.95	3.74	1	0.05	-1.64485
4.07	3.90	2	0.10	-1.28155
4.14	3.95	3	0.15	-1.03643
3.99	3.97	4	0.20	-0.84162
4.21	3.99	5	0.25	-0.67449
4.39	4.04	6	0.30	-0.5244
4.21	4.05	7	0.35	-0.38532
3.90	4.07	8	0.40	-0.25335
3.74	4.07	9	0.45	-0.12566
4.28	4.14	10	0.50	0
4.15	4.15	11	0.55	0.125661
4.04	4.21	12	0.60	0.253347
4.26	4.21	13	0.65	0.385321
4.41	4.22	14	0.70	0.524401
4.22	4.26	15	0.75	0.67449
4.07	4.28	16	0.80	0.841621
3.97	4.36	17	0.85	1.036433
4.05	4.39	18	0.90	1.281551
4.36	4.41	19	0.95	1.644853



B. See part A. To plot the normal distribution on normal probability paper: (1) Calculate values of the standard normal variable z for some arbitrary values of x (in ascending order) in the range of interest. The formula for z is

$$z = \frac{x - \mu_X}{\sigma_X} = \frac{x - 4.127}{0.1770}$$

(2) Plot z versus x on standard linear graph paper. The plot is shown in the graph in Part A. Note that the relationship between z and x is linear, so only two points are needed to plot the graph.

C. To plot a lognormal distribution, we need the lognormal distribution parameters. We will assume that $\mu_X = \overline{x}$ and $\sigma_X = s_X$.

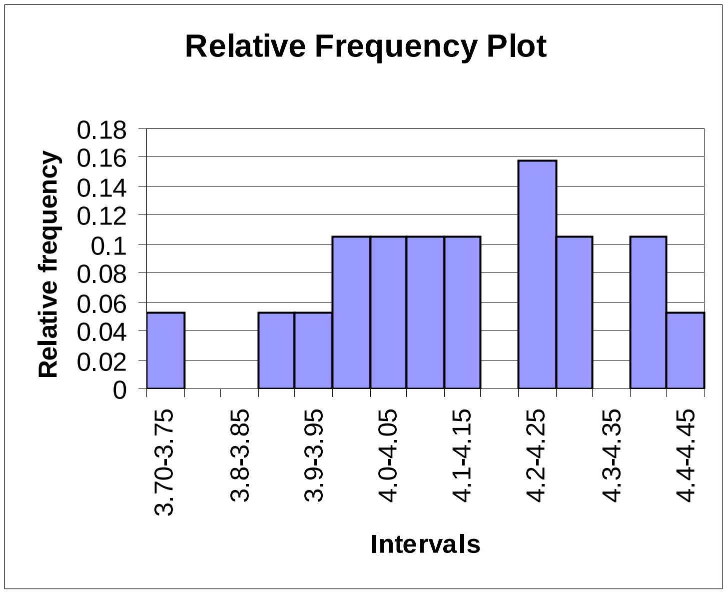
$$\sigma^2_{\ln X} = \ln(1 + V_X^2) = \ln(1 + (0.04289)^2) \quad \Rightarrow \quad \sigma_{\ln X} = 0.04287$$

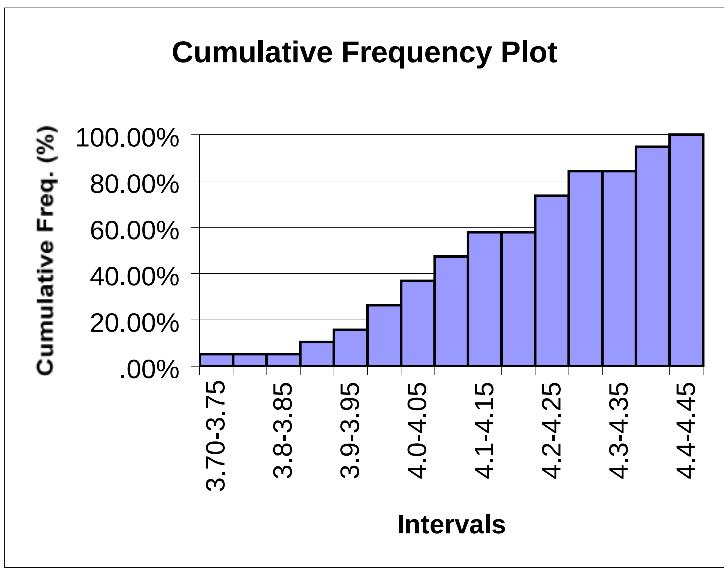
$$\mu_{\ln X} = \ln(\mu_X) - 0.5\sigma^2_{\ln X} = \ln(4.127) - 0.5(0.04287)^2 = 1.417$$

To plot the lognormal distribution on normal probability paper: (1) Calculate $F_X(x)$ for some arbitrary values of x (in ascending order) in the range of interest. $F_X(x)$ is the lognormal distribution, and it can be calculated as shown in Section 2.4.3. (2) Use the values of $F_X(x_i)$ as the values of p_i for plotting points on normal probability paper. (3) Calculate $z_i = \Phi^{-1}(p_i)$. (4) Plot z_i versus x_i. The plot is shown in the graph in Part A.

D. There are 19 data points. The interval size was chosen to be 0.05.

Interval	Number	Cumulative %
3.70-3.75	1	5.26%
3.75-3.8	0	5.26%
3.8-3.85	0	5.26%
3.85-3.9	1	10.53%
3.9-3.95	1	15.79%
3.95-4.0	2	26.32%
4.0-4.05	2	36.84%
4.05-4.1	2	47.37%
4.1-4.15	2	57.89%
4.15-4.2	0	57.89%
4.2-4.25	3	73.68%
4.25-4.3	2	84.21%
4.3-4.35	0	84.21%
4.35-4.4	2	94.74%
4.4-4.45	1	100.00%





Problem 2.3. For the data in Table 2.5, calculate the statistical estimate of the correlation coefficient using Equation (2.99).

Solution:
The formula is

$$\hat{\rho}_{XY} = \frac{1}{n-1}\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{s_X s_Y} = \frac{1}{n-1}\frac{\left(\sum_{i=1}^{n}x_i y_i\right)-n\overline{xy}}{s_X s_Y}$$

Note that is doesn't matter which variable is x and which is y. There are 100 data points. After manipulating the data, you should find:

for f_c' $\quad\bar{x} = 2743.82 \quad s_X = 520.082$

for E $\quad\bar{y} = 2991380 \quad s_Y = 361245$

$\hat{\rho}_{XY} = 0.806$

Problem 2.4. A variable X is to be modelled using a uniform distribution. The lower bound value is 5, and the upper bound value is 36.

A. Calculate the mean and standard deviation of X.

B. What is the probability that the value of X is between 10 and 20?

C. What is the probability that the value of X is greater than 31?

D. Plot the CDF on normal probability paper.

Solution:

A. The value of a is 5, and the value of b is 36. (Refer to Eqn. 2.31.) Using Eqns. 2.32 and 2.33,

$$\mu_X = \frac{a+b}{2} = 20.5$$

$$\sigma_X^2 = \frac{(b-a)^2}{12} = 80.1 \quad \Rightarrow \quad \sigma_X = 8.95$$

B. Using Eqn. 2.31 and the definition of CDF (Eqn. 2.13),

$$F_X(x) = \int_5^x \frac{1}{b-a}\, d\xi = \frac{1}{31}(x-5)$$

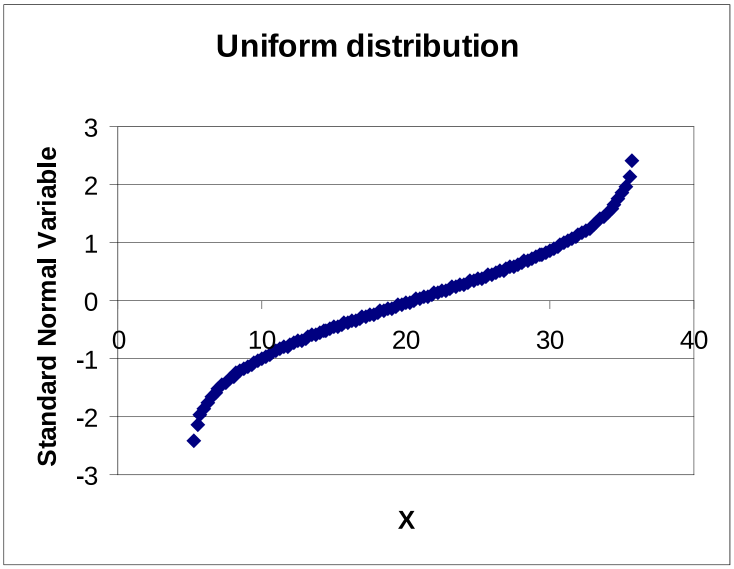
Therefore, using Eqn. 2.15

$$P(10 \le x \le 20) = F_X(20) - F_X(10) = \frac{15}{31} - \frac{5}{31} = 0.3226$$

C.

$$P(X > 31) = 1 - P(X \le 31) = 1 - F_X(31) = 1 - \frac{26}{31} = 0.1613$$

D. To plot the uniform distribution on normal probability paper: (1) Calculate $F_X(x)$ for some arbitrary values of x (in ascending order) in the range of interest. $F_X(x)$ is the uniform distribution, and its range is limited. (2) Use the values of $F_X(x_i)$ as the values of p_i for plotting points on normal probability paper. (3) Calculate $z_i = \Phi^{-1}(p_i)$. (4) Plot z_i versus x_i on standard linear graph paper as shown below.



<u>Problem 2.5</u>. The dead load D on a structure is to be modelled as a normal random variable with a mean value of 100 and a coefficient of variation of 8%.

A. Plot the PDF and CDF on standard graph paper.

B. Plot the CDF on normal probability paper.

C. Determine the probability that D is less than or equal to 95.

D. Determine the probability that D is between 95 and 105.

Solution:

A. The formulas for PDF and CDF are given by Eqns. 2.34 and 2.39. The value of σ_D is $V_D\mu_D$ = 0.08(100)=8. Appendix B can be used to determine values of CDF, or a computer spreadsheet program can be used.