

Chapter 1

Problem Solutions

1.1

- (a) fcc: 8 corner atoms $\times 1/8 = 1$ atom
 6 face atoms $\times 1/2 = 3$ atoms
 Total of 4 atoms per unit cell
- (b) bcc: 8 corner atoms $\times 1/8 = 1$ atom
 1 enclosed atom = 1 atom
 Total of 2 atoms per unit cell
- (c) Diamond: 8 corner atoms $\times 1/8 = 1$ atom
 6 face atoms $\times 1/2 = 3$ atoms
 4 enclosed atoms = 4 atoms
 Total of 8 atoms per unit cell

1.2

- (a) Simple cubic lattice: $a = 2r$
 Unit cell vol = $a^3 = (2r)^3 = 8r^3$
 1 atom per cell, so atom vol = $(1)\left(\frac{4\pi r^3}{3}\right)$
 Then
 Ratio = $\frac{\left(\frac{4\pi r^3}{3}\right)}{8r^3} \times 100\% = 52.4\%$
- (b) Face-centered cubic lattice
 $d = 4r = a\sqrt{2} \Rightarrow a = \frac{d}{\sqrt{2}} = 2\sqrt{2} \cdot r$
 Unit cell vol = $a^3 = (2\sqrt{2} \cdot r)^3 = 16\sqrt{2} \cdot r^3$
 4 atoms per cell, so atom vol = $(4)\left(\frac{4\pi r^3}{3}\right)$
 Then
 Ratio = $\frac{\left(4\right)\left(\frac{4\pi r^3}{3}\right)}{16\sqrt{2} \cdot r^3} \times 100\% = 74\%$
- (c) Body-centered cubic lattice
 $d = 4r = a\sqrt{3} \Rightarrow a = \frac{4}{\sqrt{3}} \cdot r$
 Unit cell vol = $a^3 = \left(\frac{4}{\sqrt{3}} \cdot r\right)^3$
 2 atoms per cell, so atom vol = $(2)\left(\frac{4\pi r^3}{3}\right)$

Then

$$\text{Ratio} = \frac{\left(2\right)\left(\frac{4\pi r^3}{3}\right)}{\left(\frac{4r}{\sqrt{3}}\right)^3} \times 100\% = 68\%$$

(d) Diamond lattice

$$\text{Body diagonal} = d = 8r = a\sqrt{3} \Rightarrow a = \frac{8}{\sqrt{3}} \cdot r$$

$$\text{Unit cell vol} = a^3 = \left(\frac{8r}{\sqrt{3}}\right)^3$$

$$8 \text{ atoms per cell, so atom vol} = (8)\left(\frac{4\pi r^3}{3}\right)$$

Then

$$\text{Ratio} = \frac{\left(8\right)\left(\frac{4\pi r^3}{3}\right)}{\left(\frac{8r}{\sqrt{3}}\right)^3} \times 100\% = 34\%$$

1.3

- (a) $a = 5.43 \text{ \AA}$; From Problem 1.2d,
 $a = \frac{8}{\sqrt{3}} \cdot r$
 Then $r = \frac{a\sqrt{3}}{8} = \frac{(5.43)\sqrt{3}}{8} = 1.176 \text{ \AA}$
 Center of one silicon atom to center of
 nearest neighbor = $2r = 2.35 \text{ \AA}$
- (b) Number density
 $= \frac{8}{(5.43 \times 10^{-8})^3} = 5 \times 10^{22} \text{ cm}^{-3}$
- (c) Mass density
 $= \rho = \frac{N(\text{At.Wt.})}{N_A} = \frac{(5 \times 10^{22})(28.09)}{6.02 \times 10^{23}}$
 $\Rightarrow \rho = 2.33 \text{ grams/cm}^3$

1.4

(a) 4 Ga atoms per unit cell

$$\text{Number density} = \frac{4}{(5.65 \times 10^{-8})^3}$$

$$\Rightarrow \text{Density of Ga atoms} = 2.22 \times 10^{22} \text{ cm}^{-3}$$

4 As atoms per unit cell

$$\Rightarrow \text{Density of As atoms} = 2.22 \times 10^{22} \text{ cm}^{-3}$$

(b) 8 Ge atoms per unit cell

$$\text{Number density} = \frac{8}{(5.65 \times 10^{-8})^3}$$

$$\Rightarrow \text{Density of Ge atoms} = 4.44 \times 10^{22} \text{ cm}^{-3}$$

1.5

From Figure 1.15

$$(a) \quad d = \left(\frac{a}{2}\right) \left(\frac{\sqrt{3}}{2}\right) = (0.4330)a$$

$$= (0.4330)(5.65) \Rightarrow d = 2.447 \text{ \AA}$$

$$(b) \quad d = \left(\frac{a}{2}\right) \sqrt{2} = (0.7071)a$$

$$= (0.7071)(5.65) \Rightarrow d = 3.995 \text{ \AA}$$

1.6

$$\sin\left(\frac{\theta}{2}\right) = \frac{\frac{a}{2}\sqrt{2}}{\frac{a}{2}\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \frac{\theta}{2} = 54.74^\circ$$

$$\Rightarrow \theta = 109.5^\circ$$

1.7

(a) Simple cubic: $a = 2r = 3.9 \text{ \AA}$

(b) fcc: $a = \frac{4r}{\sqrt{2}} = 5.515 \text{ \AA}$

(c) bcc: $a = \frac{4r}{\sqrt{3}} = 4.503 \text{ \AA}$

(d) diamond: $a = \frac{2(4r)}{\sqrt{3}} = 9.007 \text{ \AA}$

1.8

(a) $2(1.035)\sqrt{2} = 2(1.035) + 2r_B$

$$r_B = 0.4287 \text{ \AA}$$

(b) $a = 2(1.035) = 2.07 \text{ \AA}$

(c) A-atoms: # of atoms $= 8 \times \frac{1}{8} = 1$

$$\text{Density} = \frac{1}{(2.07 \times 10^{-8})^3} = 1.13 \times 10^{23} \text{ cm}^{-3}$$

B-atoms: # of atoms $= 6 \times \frac{1}{2} = 3$

$$\text{Density} = \frac{3}{(2.07 \times 10^{-8})^3} = 3.38 \times 10^{23} \text{ cm}^{-3}$$

1.9

(a) $a = 2r = 4.5 \text{ \AA}$

of atoms $= 8 \times \frac{1}{8} = 1$

$$\text{Number density} = \frac{1}{(4.5 \times 10^{-8})^3} = 1.097 \times 10^{22} \text{ cm}^{-3}$$

$$\text{Mass density} = \rho = \frac{N(\text{At. Wt.})}{N_A} = \frac{(1.0974 \times 10^{22})(12.5)}{6.02 \times 10^{23}} = 0.228 \text{ gm/cm}^3$$

(b) $a = \frac{4r}{\sqrt{3}} = 5.196 \text{ \AA}$

of atoms $8 \times \frac{1}{8} + 1 = 2$

$$\text{Number density} = \frac{2}{(5.196 \times 10^{-8})^3} = 1.4257 \times 10^{22} \text{ cm}^{-3}$$

$$\text{Mass density} = \rho = \frac{(1.4257 \times 10^{22})(12.5)}{6.02 \times 10^{23}} = 0.296 \text{ gm/cm}^3$$

1.10

From Problem 1.2, percent volume of fcc atoms is 74%; Therefore after coffee is ground,

$$\text{Volume} = 0.74 \text{ cm}^3$$

1.11

(b) $a = 1.8 + 1.0 = 2.8 \text{ \AA}$

(c) Na: Density = $\frac{(1/2)}{(2.8 \times 10^{-8})^3}$
 $= 2.28 \times 10^{22} \text{ cm}^{-3}$

Cl: Density = $2.28 \times 10^{22} \text{ cm}^{-3}$

(d) Na: At. Wt. = 22.99

Cl: At. Wt. = 35.45

So, mass per unit cell

$$= \frac{\left(\frac{1}{2}\right)(22.99) + \left(\frac{1}{2}\right)(35.45)}{6.02 \times 10^{23}} = 4.85 \times 10^{-23}$$

Then mass density

$$\rho = \frac{4.85 \times 10^{-23}}{(2.8 \times 10^{-8})^3} = 2.21 \text{ grams/cm}^3$$

1.12

(a) $a\sqrt{3} = 2(2.2) + 2(1.8) = 8 \text{ \AA}$

Then $a = 4.62 \text{ \AA}$

Density of A:

$$= \frac{1}{(4.62 \times 10^{-8})^3} = 1.01 \times 10^{22} \text{ cm}^{-3}$$

Density of B:

$$= \frac{1}{(4.62 \times 10^{-8})^3} = 1.01 \times 10^{22} \text{ cm}^{-3}$$

(b) Same as (a)

(c) Same material

1.13

$$a = \frac{2(2.2) + 2(1.8)}{\sqrt{3}} = 4.619 \text{ \AA}$$

(a) For 1.12(a), A-atoms

$$\text{Surface density} = \frac{1}{a^2} = \frac{1}{(4.619 \times 10^{-8})^2}$$

$$= 4.687 \times 10^{14} \text{ cm}^{-2}$$

For 1.12(b), B-atoms: $a = 4.619 \text{ \AA}$

$$\text{Surface density} = \frac{1}{a^2} = 4.687 \times 10^{14} \text{ cm}^{-2}$$

For 1.12(a) and (b), Same material

(b) For 1.12(a), A-atoms; $a = 4.619 \text{ \AA}$

Surface density

$$= \frac{1}{a^2 \sqrt{2}} = 3.315 \times 10^{14} \text{ cm}^{-2}$$

B-atoms;

Surface density

$$= \frac{1}{a^2 \sqrt{2}} = 3.315 \times 10^{14} \text{ cm}^{-2}$$

For 1.12(b), A-atoms; $a = 4.619 \text{ \AA}$

Surface density

$$= \frac{1}{a^2 \sqrt{2}} = 3.315 \times 10^{14} \text{ cm}^{-2}$$

B-atoms;

Surface density

$$= \frac{1}{a^2 \sqrt{2}} = 3.315 \times 10^{14} \text{ cm}^{-2}$$

For 1.12(a) and (b), Same material

1.14

(a) Vol. Density = $\frac{1}{a_o^3}$

$$\text{Surface Density} = \frac{1}{a_o^2 \sqrt{2}}$$

(b) Same as (a)

1.15

(i) (110) plane

(see Figure 1.10(b))

(ii) (111) plane

(see Figure 1.10(c))

(iii) (220) plane $\Rightarrow \left(\frac{1}{2}, \frac{1}{2}, \infty\right) \Rightarrow (1, 1, 0)$

Same as (110) plane and [110] direction

(iv) (321) plane $\Rightarrow \left(\frac{1}{3}, \frac{1}{2}, \frac{1}{1}\right) \Rightarrow (2, 3, 6)$

Intercepts of plane at

$$p = 2, q = 3, s = 6$$

[321] direction is perpendicular to
(321) plane

1.16

(a)

$$\left(\frac{1}{1}, \frac{1}{3}, \frac{1}{1}\right) \Rightarrow (313)$$

(b)

$$\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right) \Rightarrow (121)$$

1.17

$$\text{Intercepts: } 2, 4, 3 \Rightarrow \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{3}\right) \Rightarrow$$

(634) plane

1.18

(a) $d = a = 5.28 \text{ \AA}$

(b) $d = \frac{a\sqrt{2}}{2} = 3.734 \text{ \AA}$

(c) $d = \frac{a\sqrt{3}}{3} = 3.048 \text{ \AA}$

1.19

(a) Simple cubic

(i) (100) plane:

$$\text{Surface density} = \frac{1}{a^2} = \frac{1}{(4.73 \times 10^{-8})^2}$$

$$= 4.47 \times 10^{14} \text{ cm}^{-2}$$

(ii) (110) plane:

$$\text{Surface density} = \frac{1}{a^2 \sqrt{2}}$$

$$= 3.16 \times 10^{14} \text{ cm}^{-2}$$

(iii) (111) plane:

$$\text{Area of plane} = \frac{1}{2}bh$$

$$\text{where } b = a\sqrt{2} = 6.689 \text{ \AA}$$

Now

$$h^2 = (a\sqrt{2})^2 - \left(\frac{a\sqrt{2}}{2}\right)^2 = \frac{3}{4}(a\sqrt{2})^2$$

$$\text{So } h = \frac{\sqrt{6}}{2}(4.73) = 5.793 \text{ \AA}$$

Area of plane

$$= \frac{1}{2}(6.68923 \times 10^{-8})(5.79304 \times 10^{-8})$$

$$= 19.3755 \times 10^{-16} \text{ cm}^2$$

$$\text{Surface density} = \frac{3 \times \frac{1}{6}}{19.3755 \times 10^{-16}}$$

$$= 2.58 \times 10^{14} \text{ cm}^{-2}$$

(b) bcc

(i) (100) plane:

$$\text{Surface density} = \frac{1}{a^2} = 4.47 \times 10^{14} \text{ cm}^{-2}$$

(ii) (110) plane:

$$\text{Surface density} = \frac{2}{a^2 \sqrt{2}}$$

$$= 6.32 \times 10^{14} \text{ cm}^{-2}$$

(iii) (111) plane:

$$\text{Surface density} = \frac{3 \times \frac{1}{6}}{19.3755 \times 10^{-16}}$$

$$= 2.58 \times 10^{14} \text{ cm}^{-2}$$

(c) fcc

(i) (100) plane:

$$\text{Surface density} = \frac{2}{a^2} = 8.94 \times 10^{14} \text{ cm}^{-2}$$

(ii) (110) plane:

$$\text{Surface density} = \frac{2}{a^2 \sqrt{2}}$$

$$= 6.32 \times 10^{14} \text{ cm}^{-2}$$

(iii) (111) plane:

$$\text{Surface density} = \frac{3 \times \frac{1}{6} + 3 \times \frac{1}{2}}{19.3755 \times 10^{-16}}$$

$$= 1.03 \times 10^{15} \text{ cm}^{-2}$$

1.20

(a) (100) plane: - similar to a fcc:

$$\text{Surface density} = \frac{2}{(5.43 \times 10^{-8})^2}$$

$$= 6.78 \times 10^{14} \text{ cm}^{-2}$$

(b) (110) plane:

$$\text{Surface density} = \frac{4}{\sqrt{2}(5.43 \times 10^{-8})^2}$$

$$= 9.59 \times 10^{14} \text{ cm}^{-2}$$

(c) (111) plane:

$$\begin{aligned} \text{Surface density} &= \frac{2}{(\sqrt{3}/2)(5.43 \times 10^{-8})^2} \\ &= 7.83 \times 10^{14} \text{ cm}^{-2} \end{aligned}$$

1.21

$$a = \frac{4r}{\sqrt{2}} = \frac{4(2.37)}{\sqrt{2}} = 6.703 \text{ \AA}$$

$$\begin{aligned} \text{(a) } \#/\text{cm}^3 &= \frac{8 \times \frac{1}{8} + 6 \times \frac{1}{2}}{a^3} = \frac{4}{(6.703 \times 10^{-8})^3} \\ &= 1.328 \times 10^{22} \text{ cm}^{-3} \end{aligned}$$

$$\begin{aligned} \text{(b) } \#/\text{cm}^2 &= \frac{4 \times \frac{1}{4} + 2 \times \frac{1}{2}}{a^2 \sqrt{2}} \\ &= \frac{2}{(6.703 \times 10^{-8})^2 \sqrt{2}} \\ &= 3.148 \times 10^{14} \text{ cm}^{-2} \end{aligned}$$

$$\text{(c) } d = \frac{a\sqrt{2}}{2} = \frac{(6.703)\sqrt{2}}{2} = 4.74 \text{ \AA}$$

$$\text{(d) } \# \text{ of atoms} = 3 \times \frac{1}{6} + 3 \times \frac{1}{2} = 2$$

Area of plane: (see Problem 1.19)

$$b = a\sqrt{2} = 9.4786 \text{ \AA}$$

$$h = \frac{\sqrt{6}a}{2} = 8.2099 \text{ \AA}$$

Area

$$\begin{aligned} &= \frac{1}{2}bh = \frac{1}{2}(9.4786 \times 10^{-8})(8.2099 \times 10^{-8}) \\ &= 3.8909 \times 10^{-15} \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \#/\text{cm}^2 &= \frac{2}{3.8909 \times 10^{-15}} \\ &= 5.14 \times 10^{14} \text{ cm}^{-2} \end{aligned}$$

$$d = \frac{a\sqrt{3}}{3} = \frac{(6.703)\sqrt{3}}{3} = 3.87 \text{ \AA}$$

1.22

Density of silicon atoms = $5 \times 10^{22} \text{ cm}^{-3}$ and
4 valence electrons per atom, so

Density of valence electrons = $2 \times 10^{23} \text{ cm}^{-3}$

1.23

Density of GaAs atoms

$$= \frac{8}{(5.65 \times 10^{-8})^3} = 4.44 \times 10^{22} \text{ cm}^{-3}$$

An average of 4 valence electrons per atom,

So

Density of valence electrons

$$= 1.77 \times 10^{23} \text{ cm}^{-3}$$

1.24

$$\text{(a) } \frac{5 \times 10^{17}}{5 \times 10^{22}} \times 100\% = 10^{-3}\%$$

$$\text{(b) } \frac{2 \times 10^{15}}{5 \times 10^{22}} \times 100\% = 4 \times 10^{-6}\%$$

1.25

(a) Fraction by weight

$$\cong \frac{(2 \times 10^{16})(10.82)}{(5 \times 10^{22})(28.06)} = 1.542 \times 10^{-7}$$

(b) Fraction by weight

$$\cong \frac{(10^{18})(30.98)}{(5 \times 10^{22})(28.06)} = 2.208 \times 10^{-5}$$

1.26

$$\text{Volume density} = \frac{1}{d^3} = 2 \times 10^{16} \text{ cm}^{-3}$$

$$\text{So } d = 3.684 \times 10^{-6} \text{ cm} \Rightarrow d = 368.4 \text{ \AA}$$

We have $a_o = 5.43 \text{ \AA}$

$$\text{Then } \frac{d}{a_o} = \frac{368.4}{5.43} = 67.85$$

1.27

$$\text{Volume density} = \frac{1}{d^3} = 4 \times 10^{15} \text{ cm}^{-3}$$

$$\text{So } d = 6.30 \times 10^{-6} \text{ cm} \Rightarrow d = 630 \text{ \AA}$$

We have $a_o = 5.43 \text{ \AA}$

$$\text{Then } \frac{d}{a_o} = \frac{630}{5.43} = 116$$

Chapter 2

2.1

Sketch

2.2

Sketch

2.3

Sketch

2.4

$$\text{From Problem 2.2, phase} = \frac{2\pi x}{\lambda} - \omega t$$

$$= \text{constant}$$

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} - \omega = 0, \Rightarrow \frac{dx}{dt} = v_p = +\omega \left(\frac{\lambda}{2\pi} \right)$$

$$\text{From Problem 2.3, phase} = \frac{2\pi x}{\lambda} + \omega t$$

$$= \text{constant}$$

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} + \omega = 0, \Rightarrow \frac{dx}{dt} = v_p = -\omega \left(\frac{\lambda}{2\pi} \right)$$

2.5

$$E = hv = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

$$\text{Gold: } E = 4.90 \text{ eV} = (4.90)(1.6 \times 10^{-19}) \text{ J}$$

So,

$$\lambda = \frac{(6.625 \times 10^{-34})(3 \times 10^{10})}{(4.90)(1.6 \times 10^{-19})} = 2.54 \times 10^{-5} \text{ cm}$$

or

$$\lambda = 0.254 \mu\text{m}$$

$$\text{Cesium: } E = 1.90 \text{ eV} = (1.90)(1.6 \times 10^{-19}) \text{ J}$$

So,

$$\lambda = \frac{(6.625 \times 10^{-34})(3 \times 10^{10})}{(1.90)(1.6 \times 10^{-19})} = 6.54 \times 10^{-5} \text{ cm}$$

or

$$\lambda = 0.654 \mu\text{m}$$

2.6

$$\text{(a) } p = \frac{h}{\lambda} = \frac{6.625 \times 10^{-34}}{550 \times 10^{-9}}$$

$$= 1.205 \times 10^{-27} \text{ kg-m/s}$$

$$v = \frac{p}{m} = \frac{1.2045 \times 10^{-27}}{9.11 \times 10^{-31}} = 1.32 \times 10^3 \text{ m/s}$$

$$\text{or } v = 1.32 \times 10^5 \text{ cm/s}$$

$$\text{(b) } p = \frac{h}{\lambda} = \frac{6.625 \times 10^{-34}}{440 \times 10^{-9}}$$

$$= 1.506 \times 10^{-27} \text{ kg-m/s}$$

$$v = \frac{p}{m} = \frac{1.5057 \times 10^{-27}}{9.11 \times 10^{-31}} = 1.65 \times 10^3 \text{ m/s}$$

$$\text{or } v = 1.65 \times 10^5 \text{ cm/s}$$

(c) Yes

2.7

(a) (i)

$$p = \sqrt{2mE} = \sqrt{2(9.11 \times 10^{-31})(1.2)(1.6 \times 10^{-19})}$$

$$= 5.915 \times 10^{-25} \text{ kg-m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{5.915 \times 10^{-25}} = 1.12 \times 10^{-9} \text{ m}$$

$$\text{or } \lambda = 11.2 \text{ \AA}$$

$$\text{(ii) } p = \sqrt{2(9.11 \times 10^{-31})(12)(1.6 \times 10^{-19})}$$

$$= 1.87 \times 10^{-24} \text{ kg-m/s}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{1.8704 \times 10^{-24}} = 3.54 \times 10^{-10} \text{ m}$$

$$\text{or } \lambda = 3.54 \text{ \AA}$$

$$\text{(iii) } p = \sqrt{2(9.11 \times 10^{-31})(120)(1.6 \times 10^{-19})}$$

$$= 5.915 \times 10^{-24} \text{ kg-m/s}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{5.915 \times 10^{-24}} = 1.12 \times 10^{-10} \text{ m}$$

$$\text{or } \lambda = 1.12 \text{ \AA}$$

(b)

$$p = \sqrt{2(1.67 \times 10^{-27})(1.2)(1.6 \times 10^{-19})}$$

$$= 2.532 \times 10^{-23} \text{ kg-m/s}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{2.532 \times 10^{-23}} = 2.62 \times 10^{-11} \text{ m}$$

or $\lambda = 0.262 \text{ \AA}$

2.8

$$E_{avg} = \frac{3}{2} kT = \left(\frac{3}{2}\right)(0.0259) = 0.03885 \text{ eV}$$

Now

$$p_{avg} = \sqrt{2mE_{avg}}$$

$$= \sqrt{2(9.11 \times 10^{-31})(0.03885)(1.6 \times 10^{-19})}$$

or

$$p_{avg} = 1.064 \times 10^{-25} \text{ kg-m/s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{1.064 \times 10^{-25}} = 6.225 \times 10^{-9} \text{ m}$$

or

$$\lambda = 62.25 \text{ \AA}$$

2.9

$$E_p = h\nu_p = \frac{hc}{\lambda_p}$$

Now

$$E_e = \frac{p_e^2}{2m} \text{ and } p_e = \frac{h}{\lambda_e} \Rightarrow E_e = \frac{1}{2m} \left(\frac{h}{\lambda_e}\right)^2$$

Set $E_p = E_e$ and $\lambda_p = 10\lambda_e$

Then

$$\frac{hc}{\lambda_p} = \frac{1}{2m} \left(\frac{h}{\lambda_e}\right)^2 = \frac{1}{2m} \left(\frac{10h}{\lambda_p}\right)^2$$

which yields

$$\lambda_p = \frac{100h}{2mc}$$

$$E_p = E = \frac{hc}{\lambda_p} = \frac{hc}{100h} \cdot 2mc = \frac{2mc^2}{100}$$

$$= \frac{2(9.11 \times 10^{-31})(3 \times 10^8)^2}{100}$$

$$= 1.64 \times 10^{-15} \text{ J} = 10.25 \text{ keV}$$

2.10

(a) $p = \frac{h}{\lambda} = \frac{6.625 \times 10^{-34}}{85 \times 10^{-10}}$

$$= 7.794 \times 10^{-26} \text{ kg-m/s}$$

$$v = \frac{p}{m} = \frac{7.794 \times 10^{-26}}{9.11 \times 10^{-31}} = 8.56 \times 10^4 \text{ m/s}$$

or $v = 8.56 \times 10^6 \text{ cm/s}$

$$E = \frac{1}{2} m v^2 = \frac{1}{2} (9.11 \times 10^{-31}) (8.56 \times 10^4)^2$$

$$= 3.33 \times 10^{-21} \text{ J}$$

or $E = \frac{3.334 \times 10^{-21}}{1.6 \times 10^{-19}} = 2.08 \times 10^{-2} \text{ eV}$

(b) $E = \frac{1}{2} (9.11 \times 10^{-31}) (8 \times 10^3)^2$

$$= 2.915 \times 10^{-23} \text{ J}$$

or $E = \frac{2.915 \times 10^{-23}}{1.6 \times 10^{-19}} = 1.82 \times 10^{-4} \text{ eV}$

$$p = mv = (9.11 \times 10^{-31}) (8 \times 10^3)$$

$$= 7.288 \times 10^{-27} \text{ kg-m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-35}}{7.288 \times 10^{-27}} = 9.09 \times 10^{-8} \text{ m}$$

or $\lambda = 909 \text{ \AA}$

2.11

(a) $E = h\nu = \frac{hc}{\lambda} = \frac{(6.625 \times 10^{-34})(3 \times 10^8)}{1 \times 10^{-10}}$

$$= 1.99 \times 10^{-15} \text{ J}$$

Now

$$E = e \cdot V \Rightarrow V = \frac{E}{e} = \frac{1.99 \times 10^{-15}}{1.6 \times 10^{-19}}$$

$$V = 1.24 \times 10^4 \text{ V} = 12.4 \text{ kV}$$

(b) $p = \sqrt{2mE} = \sqrt{2(9.11 \times 10^{-31})(1.99 \times 10^{-15})}$

$$= 6.02 \times 10^{-23} \text{ kg-m/s}$$

Then

$$\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{6.02 \times 10^{-23}} = 1.10 \times 10^{-11} \text{ m}$$

or

$$\lambda = 0.11 \text{ \AA}$$

2.12

$$\begin{aligned}\Delta p &= \frac{\hbar}{\Delta x} = \frac{1.054 \times 10^{-34}}{10^{-6}} \\ &= 1.054 \times 10^{-28} \text{ kg-m/s}\end{aligned}$$

2.13

(a) (i) $\Delta p \Delta x = \hbar$

$$\Delta p = \frac{1.054 \times 10^{-34}}{12 \times 10^{-10}} = 8.783 \times 10^{-26} \text{ kg-m/s}$$

$$\begin{aligned}\text{(ii) } \Delta E &= \frac{dE}{dp} \cdot \Delta p = \frac{d}{dp} \left(\frac{p^2}{2m} \right) \cdot \Delta p \\ &= \frac{2p}{2m} \cdot \Delta p = \frac{p \Delta p}{m}\end{aligned}$$

Now $p = \sqrt{2mE}$

$$\begin{aligned}&= \sqrt{2(9 \times 10^{-31})(16)(1.6 \times 10^{-19})} \\ &= 2.147 \times 10^{-24} \text{ kg-m/s}\end{aligned}$$

$$\text{so } \Delta E = \frac{(2.1466 \times 10^{-24})(8.783 \times 10^{-26})}{9 \times 10^{-31}}$$

$$= 2.095 \times 10^{-19} \text{ J}$$

$$\text{or } \Delta E = \frac{2.095 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.31 \text{ eV}$$

(b) (i) $\Delta p = 8.783 \times 10^{-26} \text{ kg-m/s}$

$$\text{(ii) } p = \sqrt{2(5 \times 10^{-28})(16)(1.6 \times 10^{-19})}$$

$$= 5.06 \times 10^{-23} \text{ kg-m/s}$$

$$\Delta E = \frac{(5.06 \times 10^{-23})(8.783 \times 10^{-26})}{5 \times 10^{-28}}$$

$$= 8.888 \times 10^{-21} \text{ J}$$

$$\text{or } \Delta E = \frac{8.888 \times 10^{-21}}{1.6 \times 10^{-19}} = 5.55 \times 10^{-2} \text{ eV}$$

2.14

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054 \times 10^{-34}}{10^{-2}} = 1.054 \times 10^{-32} \text{ kg-m/s}$$

$$p = m v \Rightarrow \Delta v = \frac{\Delta p}{m} = \frac{1.054 \times 10^{-32}}{1500}$$

$$\Delta v = 7 \times 10^{-36} \text{ m/s}$$

2.15

(a) $\Delta E \Delta t = \hbar$

$$\Delta t = \frac{1.054 \times 10^{-34}}{(0.8)(1.6 \times 10^{-19})} = 8.23 \times 10^{-16} \text{ s}$$

(b) $\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054 \times 10^{-34}}{1.5 \times 10^{-10}}$
 $= 7.03 \times 10^{-25} \text{ kg-m/s}$

2.16

(a) If $\Psi_1(x, t)$ and $\Psi_2(x, t)$ are solutions to Schrodinger's wave equation, then

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \Psi_1(x, t)}{\partial x^2} + V(x) \Psi_1(x, t) = j\hbar \frac{\partial \Psi_1(x, t)}{\partial t}$$

and

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \Psi_2(x, t)}{\partial x^2} + V(x) \Psi_2(x, t) = j\hbar \frac{\partial \Psi_2(x, t)}{\partial t}$$

Adding the two equations, we obtain

$$\begin{aligned}\frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} [\Psi_1(x, t) + \Psi_2(x, t)] \\ + V(x) [\Psi_1(x, t) + \Psi_2(x, t)] \\ = j\hbar \frac{\partial}{\partial t} [\Psi_1(x, t) + \Psi_2(x, t)]\end{aligned}$$

which is Schrodinger's wave equation. So $\Psi_1(x, t) + \Psi_2(x, t)$ is also a solution.

(b) If $\Psi_1(x, t) \cdot \Psi_2(x, t)$ were a solution to Schrodinger's wave equation, then we could write

$$\begin{aligned}\frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} [\Psi_1 \cdot \Psi_2] + V(x) [\Psi_1 \cdot \Psi_2] \\ = j\hbar \frac{\partial}{\partial t} [\Psi_1 \cdot \Psi_2]\end{aligned}$$

which can be written as

$$\begin{aligned}\frac{-\hbar^2}{2m} \left[\Psi_1 \frac{\partial^2 \Psi_2}{\partial x^2} + \Psi_2 \frac{\partial^2 \Psi_1}{\partial x^2} + 2 \frac{\partial \Psi_1}{\partial x} \cdot \frac{\partial \Psi_2}{\partial x} \right] \\ + V(x) [\Psi_1 \cdot \Psi_2] = j\hbar \left[\Psi_1 \frac{\partial \Psi_2}{\partial t} + \Psi_2 \frac{\partial \Psi_1}{\partial t} \right]\end{aligned}$$

Dividing by $\Psi_1 \cdot \Psi_2$, we find

$$\begin{aligned}\frac{-\hbar^2}{2m} \left[\frac{1}{\Psi_2} \cdot \frac{\partial^2 \Psi_2}{\partial x^2} + \frac{1}{\Psi_1} \cdot \frac{\partial^2 \Psi_1}{\partial x^2} + \frac{2}{\Psi_1 \Psi_2} \frac{\partial \Psi_1}{\partial x} \frac{\partial \Psi_2}{\partial x} \right] \\ + V(x) = j\hbar \left[\frac{1}{\Psi_2} \frac{\partial \Psi_2}{\partial t} + \frac{1}{\Psi_1} \frac{\partial \Psi_1}{\partial t} \right]\end{aligned}$$

Since Ψ_1 is a solution, then

$$\frac{-\hbar^2}{2m} \cdot \frac{1}{\Psi_1} \cdot \frac{\partial^2 \Psi_1}{\partial x^2} + V(x) = j\hbar \cdot \frac{1}{\Psi_1} \cdot \frac{\partial \Psi_1}{\partial t}$$

Subtracting these last two equations, we have

$$\begin{aligned} \frac{-\hbar^2}{2m} \left[\frac{1}{\Psi_2} \cdot \frac{\partial^2 \Psi_2}{\partial x^2} + \frac{2}{\Psi_1 \Psi_2} \frac{\partial \Psi_1}{\partial x} \frac{\partial \Psi_2}{\partial x} \right] \\ = j\hbar \cdot \frac{1}{\Psi_2} \cdot \frac{\partial \Psi_2}{\partial t} \end{aligned}$$

Since Ψ_2 is also a solution, we have

$$\frac{-\hbar^2}{2m} \cdot \frac{1}{\Psi_2} \cdot \frac{\partial^2 \Psi_2}{\partial x^2} + V(x) = j\hbar \cdot \frac{1}{\Psi_2} \cdot \frac{\partial \Psi_2}{\partial t}$$

Subtracting these last two equations, we obtain

$$\frac{-\hbar^2}{2m} \cdot \frac{2}{\Psi_1 \Psi_2} \cdot \frac{\partial \Psi_1}{\partial x} \cdot \frac{\partial \Psi_2}{\partial x} - V(x) = 0$$

This equation is not necessarily valid, which means that $\Psi_1 \Psi_2$ is, in general, not a solution to Schrodinger's wave equation.

2.17

$$\int_{-1}^{+3} A^2 \cos^2\left(\frac{\pi x}{2}\right) dx = 1$$

$$A^2 \left[\frac{x}{2} + \frac{\sin(\pi x)}{2\pi} \right]_{-1}^{+3} = 1$$

$$A^2 \left[\frac{3}{2} - \left(-\frac{1}{2}\right) \right] = 1$$

so $A^2 = \frac{1}{2}$

or $|A| = \frac{1}{\sqrt{2}}$

2.18

$$\int_{-1/2}^{+1/2} A^2 \cos^2(n\pi x) dx = 1$$

$$A^2 \left[\frac{x}{2} + \frac{\sin(2n\pi x)}{4n\pi} \right]_{-1/2}^{+1/2} = 1$$

$$A^2 \left[\frac{1}{4} - \left(-\frac{1}{4}\right) \right] = 1 = A^2 \left(\frac{1}{2}\right)$$

or $|A| = \sqrt{2}$

2.19

Note that $\int_0^{\infty} \Psi \cdot \Psi^* dx = 1$

Function has been normalized.

(a) Now

$$P = \int_0^{a_o/4} \left[\sqrt{\frac{2}{a_o}} \exp\left(\frac{-x}{a_o}\right) \right]^2 dx$$

$$= \frac{2}{a_o} \int_0^{a_o/4} \exp\left(\frac{-2x}{a_o}\right) dx$$

$$= \frac{2}{a_o} \left(\frac{-a_o}{2}\right) \exp\left(\frac{-2x}{a_o}\right) \Big|_0^{a_o/4}$$

or

$$P = (-1) \left[\exp\left(\frac{-2a_o}{4a_o}\right) - 1 \right] = 1 - \exp\left(\frac{-1}{2}\right)$$

which yields

$$P = 0.393$$

(b)

$$P = \int_{a_o/4}^{a_o/2} \left[\sqrt{\frac{2}{a_o}} \exp\left(\frac{-x}{a_o}\right) \right]^2 dx$$

$$= \frac{2}{a_o} \int_{a_o/4}^{a_o/2} \exp\left(\frac{-2x}{a_o}\right) dx$$

$$= \frac{2}{a_o} \left(\frac{-a_o}{2}\right) \exp\left(\frac{-2x}{a_o}\right) \Big|_{a_o/4}^{a_o/2}$$

or

$$P = (-1) \left[\exp(-1) - \exp\left(\frac{-1}{2}\right) \right]$$

which yields

$$P = 0.239$$

(c)

$$P = \int_0^{a_o} \left[\sqrt{\frac{2}{a_o}} \exp\left(\frac{-x}{a_o}\right) \right]^2 dx$$

$$= \frac{2}{a_o} \int_0^{a_o} \exp\left(\frac{-2x}{a_o}\right) dx$$

$$= \frac{2}{a_o} \left(\frac{-a_o}{2}\right) \exp\left(\frac{-2x}{a_o}\right) \Big|_0^{a_o}$$

$$= (-1) [\exp(-2) - 1]$$

which yields

$$P = 0.865$$

2.20

$$P = \int |\psi(x)|^2 dx$$

$$\begin{aligned} \text{(a)} \quad & \int_0^{a/4} \left(\frac{2}{a}\right) \cos^2\left(\frac{\pi x}{2}\right) dx \\ &= \left(\frac{2}{a}\right) \left[\frac{x}{2} + \frac{\sin\left(\frac{2\pi x}{a}\right)}{4\left(\frac{\pi}{a}\right)} \right] \Bigg|_0^{a/4} \\ &= \left(\frac{2}{a}\right) \left[\left(\frac{a}{4}\right) \frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{4\pi}{a}\right)} \right] \\ &= \left(\frac{2}{a}\right) \left[\frac{a}{8} + \frac{(1)(a)}{4\pi} \right] \end{aligned}$$

or $P = 0.409$

$$\begin{aligned} \text{(b)} \quad & P = \int_{a/4}^{a/2} \left(\frac{2}{a}\right) \cos^2\left(\frac{\pi x}{a}\right) dx \\ &= \left(\frac{2}{a}\right) \left[\frac{x}{2} + \frac{\sin\left(\frac{2\pi x}{a}\right)}{4\left(\frac{\pi}{a}\right)} \right] \Bigg|_{a/4}^{a/2} \\ &= \left(\frac{2}{a}\right) \left[\frac{a}{4} + \frac{\sin(\pi)}{\left(\frac{4\pi}{a}\right)} - \frac{a}{8} - \frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{4\pi}{a}\right)} \right] \\ &= 2 \left[\frac{1}{4} + 0 - \frac{1}{8} - \frac{1}{4\pi} \right] \end{aligned}$$

or $P = 0.0908$

$$\begin{aligned} \text{(c)} \quad & P = \int_{-a/2}^{+a/2} \left(\frac{2}{a}\right) \cos^2\left(\frac{\pi x}{a}\right) dx \\ &= \left(\frac{2}{a}\right) \left[\frac{x}{2} + \frac{\sin\left(\frac{2\pi x}{a}\right)}{\left(\frac{4\pi}{a}\right)} \right] \Bigg|_{-a/2}^{+a/2} \\ &= \left(\frac{2}{a}\right) \left[\frac{a}{4} + \frac{\sin(\pi)}{\left(\frac{4\pi}{a}\right)} - \left(\frac{-a}{4}\right) - \frac{\sin(-\pi)}{\left(\frac{4\pi}{a}\right)} \right] \end{aligned}$$

or $P = 1$

2.21

$$\begin{aligned} \text{(a)} \quad & P = \int_0^{a/4} \left(\frac{2}{a}\right) \sin^2\left(\frac{2\pi x}{a}\right) dx \\ &= \left(\frac{2}{a}\right) \left[\frac{x}{2} - \frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)} \right] \Bigg|_0^{a/4} \\ &= \left(\frac{2}{a}\right) \left[\frac{a}{8} - \frac{\sin(\pi)}{\left(\frac{8\pi}{a}\right)} \right] \end{aligned}$$

or $P = 0.25$

$$\begin{aligned} \text{(b)} \quad & P = \int_{a/4}^{a/2} \left(\frac{2}{a}\right) \sin^2\left(\frac{2\pi x}{a}\right) dx \\ &= \left(\frac{2}{a}\right) \left[\frac{x}{2} - \frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)} \right] \Bigg|_{a/4}^{a/2} \\ &= \left(\frac{2}{a}\right) \left[\frac{a}{4} - \frac{\sin(2\pi)}{\left(\frac{8\pi}{a}\right)} - \left(\frac{a}{8}\right) + \frac{\sin(\pi)}{\left(\frac{8\pi}{a}\right)} \right] \end{aligned}$$

or $P = 0.25$

$$\begin{aligned} \text{(c)} \quad & P = \int_{-a/2}^{+a/2} \left(\frac{2}{a}\right) \sin^2\left(\frac{2\pi x}{a}\right) dx \\ &= \left(\frac{2}{a}\right) \left[\frac{x}{2} - \frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)} \right] \Bigg|_{-a/2}^{+a/2} \\ &= \left(\frac{2}{a}\right) \left[\frac{a}{4} - \frac{\sin(2\pi)}{\left(\frac{8\pi}{a}\right)} - \left(\frac{-a}{4}\right) + \frac{\sin(-2\pi)}{\left(\frac{8\pi}{a}\right)} \right] \end{aligned}$$

or $P = 1$

2.22

$$\text{(a) (i)} \quad v_p = \frac{\omega}{k} = \frac{8 \times 10^{12}}{8 \times 10^8} = 10^4 \text{ m/s}$$

$$\text{or } v_p = 10^6 \text{ cm/s}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{8 \times 10^8} = 7.854 \times 10^{-9} \text{ m}$$