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PROBLEM 1.1

SOLUTION

This problem is open-ended and has no unique solution. We suggest that the instructor use this Problem as the basis for an in-class or small group discussion.

Example:

Home heating thermostat (see photo):

Analog type:

Sensor - bimetallic thermometer (round part in center of photo) that lengthens so as to unroll with temperature rise;

Signal conditioning –linkage/motion of sensor that mechanically moves a mercury bulb (seen as the elongated tube) as sensor temperature changes;

Controller- mercury bulb contact switch that turns furnace on/off based on (1) the setpoint temperature (position of long handle)

and (2) position of bulb: changing the setpoint rotates one end of bulb so it is more or is less horizontal, whereas changing sensor temperature rotates bulb so it is more or it is less horizontal. The net effect is to open or close the contact switch;

Output display – room thermometer on outside of thermostat to show actual local temperature (not shown).

Other signal conditioning components: anticipator – adjustable mechanical thermal device that triggers the off signal before the setpoint temperature is reached (anticipating residual heating from furnace): essentially this provides an adjustable temperature offset, it is located in the center of the round part in the figure). The wires in photo connect to the furnace and to power.

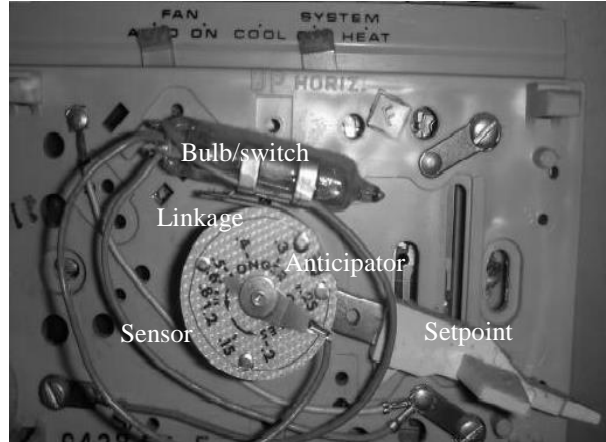


Photo of the insides of a generic analog thermostat

Digital type: **sensor** – thermistor (a type of resistor whose resistance is temperature based). It replaces analog coil in photo; **signal conditioning** – circuit that determines temperature by measuring the current flowing through the thermistor and sends a proportional voltage signal to controller (replaces linkage in photo); **output display** – usually a separate LCD thermometer; **controller** – PID device, a proportional integral device controller that calculates an error value, which is simply the difference between the sensed and setpoint temperatures. The device attempts to minimize the error by turning the furnace on/off. It replaces the bulb in the photo.

PROBLEM 1.2

FIND: Identify measurement stages for each device.

SOLUTION

a) *Microphone/amplifier/speaker system*

Sensor: microphone diaphragm

Transducer: microphone coil/magnet. Diaphragm displacement relative to coil generates a small voltage proportional to diaphragm displacement

Signal conditioning: amplifier. It increases the level of the microphone signal sufficient to drive the output stage

Output: loud speaker. Its voice coil responds to the varying applied voltage output of the amplifier and this moves the speaker cone (opposite of the microphone).

b) *thermostat*

Sensor/transducer: bimetallic thermometer. Temperature changes cause the metal to expand or contract. Coil absorbs prevailing temperature, material expands/contracts changing the thermal energy into a mechanical displacement (transducer)

Output: displacement of thermometer tip

Controller: mercury contact switch (open: furnace off; closed: furnace on)

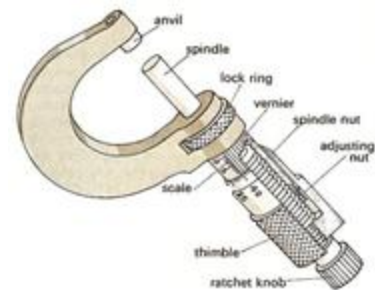


c) *hand-held micrometer*

sensor: space between anvil and spindle

transducer: displacement of spindle via the spindle thimble

Output: vernier scale



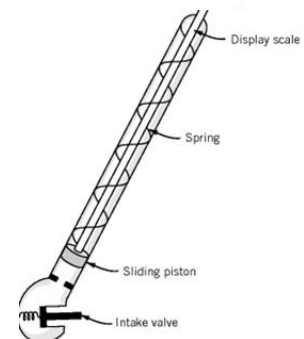
d) *tire pressure gage (pencil-style)*

Sensor: chamber behind the valve and piston equalizes pressure with tire

Transducer: piston

Signal conditioning: piston translates chamber pressure relative to atmospheric pressure into displacement

Output: readout scale



PROBLEM \$1.3

FIND: Identify measurement stages as illustrated in Figure 1.4 for an electronic automotive speedometer.

SOLUTION A toothed wheel creates pulses in a magnetic coil. This signal is transmitted to one of the car's computers.

Sensor stage: toothed wheel

Transducer stage: coil

Signal conditioning stage: On-board computer

Output stage: analog or digital display.

PROBLEM 1.4

FIND: Identify measurement stages as illustrated in Figure 1.4 for an electronic automotive speed control system (cruise control).

SOLUTION

Sensor / transducer stage: Varies by design: speedometer output, driveshaft rotation sensor, wheel sensor

Signal conditioning stage: On-board computer, electronic amplifier

Output stage: Stepper motor controlling throttle cable

PROBLEM 1.5

FIND: Identify measurement stages as illustrated in Figure 1.4 for an electronic range finder.

SOLUTION

The principle of operation is that an infrared laser beam is directed towards a sufficiently reflective surface. A computer chip is used to measure the time required for the reflected beam to return to the rangefinder, and convert that to a distance.

Sensor stage: laser beam

Transducer optical sensor

Signal conditioning stage: On-board computer chip

Output stage: digital display.

PROBLEM 1.6

FIND: Identify measurement stages as illustrated in Figure 1.4 for an automotive parking assist system.

SOLUTION

The principle of operation is that sensors using ultrasound at numerous locations on the car (mainly on the bumpers) provide distance to nearby objects. An electric motor controls the steering. In a fully autonomous condition, brakes are controlled by a modified antilock braking system and the throttle electronically controlled as in the cruise control.

Sensor stage: Ultrasonic sensors

Transducer: Chip to create digital signals from the sensors

Signal conditioning stage: On-board computers

Output stage: digital display, actuators that are computer controlled

PROBLEM 1.7

FIND: Identify measurement stages as illustrated in Figure 1.4 for a dial temperature gauge.

SOLUTION

Sensor stage: Bimetallic strip

Transducer: Linkage to move pointer

Output stage: Dial display with a pointer

PROBLEM 1.8

FIND: Identify measurement stages as illustrated in Figure 1.4 for a tipping bucket rain gauge.

SOLUTION

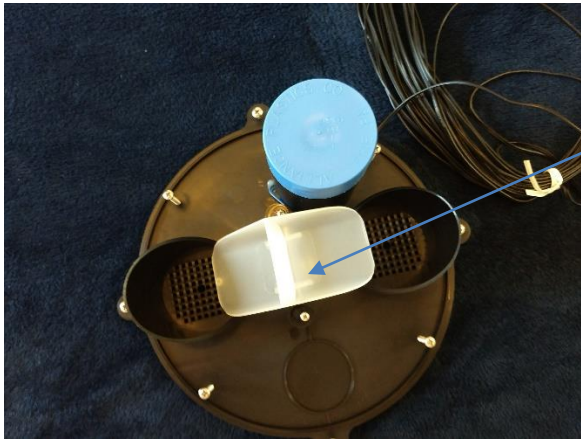
The principle of operation is that a “bucket” (1 in figure below) having two distinct containers to collect water can rotate about a fulcrum. Each time the bucket tips a known amount of water has been collected. The tipping is sensed magnetically.

Sensor stage: Bucket

Transducer: Magnetic sensor

Signal conditioning stage: Chip that counts the number of tips

Output stage: digital display



Funnel of known size to collect rain

PROBLEM 1.9

FIND: Discuss interference in the test of Example 1.1 (Figure 1.5)

SOLUTION :

In the example shown by Figure 1.5, tests were run on different days on which the local barometric pressure had changed. Between any two days of different barometric pressure, the boiling point measured would be different – this offset is due to the interference effect of the pressure.

Consider a test run over several days coincident with the motion of a major weather front through the area. Clearly, this would impose a trend on the dataset. For example, the measured boiling point may be seen as increasing from day to day.

By running tests over random days separated by a sufficient period of days, so as not to allow any one atmospheric front to impose a trend on the data, the effects of atmospheric pressure can be broken up into noise. The measured boiling point might then be high one test but then low on the next, in effect, making it look like random data scatter, i.e. noise.

PROBLEM 1.10

FIND: State whether the following are a continuous variable or a discrete variable:

- a) Time
- b) Temperature
- c) Bank account balance
- d) Pixels in a display

SOLUTION :

- a) Time: continuous
- b) Temperature: continuous
- c) Bank account balance: discrete
- d) Pixels in a display: discrete

PROBLEM 1.11

FIND: Examples of continuous and discrete variables

SOLUTION :

Continuous: These are variables that have possible values that could encompass any (an infinite number) value within a range. In engineering, we usually associate these as variables that vary with time or space.

A value taken from a continuous line graph

Outdoor air temperature

The length of a kitchen appliance cord (think about the different appliances and the range of possible lengths. The length of any one appliance is typically anywhere between 30 and 150 cm)

Discrete: These are variables that have possible values that are distinct and separate, such as 0, 1, 2.

Number of phone (or messages) calls you received in a day.

Number of heads found in a group of coin tosses

Score of a basketball (soccer, baseball, ...) game (think about the possible outcomes of these games. The scores are integers and a direct count)

COMMENT

Most times we can distinguish between the two by deciding if the variable is the result of a measurement or an exact count. Also in most cases, digital displays are discrete; analog displays are continuous.

PROBLEM 1.12

FIND: How accurate is a thermometer? Estimate uncertainty?

SOLUTION

As a check for accuracy:

First, look at the temperature readout. Does the indicated temperature make sense? This is a sanity check. If it fails here, no need to proceed.

The next check would be either: (1) place the thermometer into a condition where the temperature is known (i.e., a known setpoint), or (2) compare it to another temperature indicator of known accuracy at different temperatures that span the desired range of use.

Either of these methods is a form of calibration. An easy temperature test is to use the ice point, which is at 0 °C. This is created by filling an insulated beaker full of ice cubes with pure water, just enough to fill the interstitial volume and allowed the mixture to equalize). In fact, the freezing/melting point of pure solids is used to establish the accuracy of temperature sensors as these temperatures are repeatable and known. Another easy setpoint is to use the boiling water temperature.

The ice point test will indicate any offset in the thermometer. This offset is a measure of the systematic uncertainty. You can correct for the offset but the correction is limited by how well you know the correction. Also, the correction may change with temperature. So doing several (at least two) set point calibrations will help.

If you compare against another thermometer, the systematic uncertainty will also be dependent on the accuracy of the comparison thermometer. Yes, there is a measure of vagueness involved; that is the calibration uncertainty. But you at least gain a measurable amount of confidence in the unknown thermometer reading.

A check for random uncertainty is to place the thermometer into a known temperature environment and read its temperature. Do this repeatedly over a period of time. A statistical analysis of the average and deviation in temperature is a measure of random uncertainty. However: This test will indicate a measure of the repeatability of the thermometer, but it will also indicate how well the known temperature was held constant. The two effects are coupled in the random uncertainty.

PROBLEM 1.13

FIND: How does resolution of a scale affect uncertainty?

SOLUTION

The resolution of a scale is defined by the least significant increment or division on the output display. Resolution affects a user's ability to resolve the output display of an instrument or measuring system, and thereby can introduce error, in this case a resolution error. Thus, there is a source of uncertainty associated with this error. The uncertainty value is the range of possible resolution error.

So, if the indicator has finite resolution, then the measurement has at least some uncertainty based on how well you can resolve a reading. This will show as a type of random uncertainty. Example: take a photograph of a scale reading and show it to twenty people. Record the readings made by the twenty people. The scatter in the data of that sample of twenty will be indicative of the random uncertainty due to resolution error.

The resolution would not contribute to systematic error, as systematic error is a fixed offset.

PROBLEM 1.14

KNOWN: A bulb thermometer is used to measure outside temperature.

FIND: Extraneous variables that might influence thermometer output.

SOLUTION

A thermometer's indicated temperature will be influenced by the temperature of solid objects to which it is in contact, and radiation exchange with bodies at different temperatures (including the sky or sun, buildings, people and ground) within its line of sight. Hence, location should be carefully selected and even randomized. We know that a bulb thermometer does not respond quickly to temperature changes, so that a sufficient period of time needs to be allowed for the instrument to adjust to new temperatures. By replication of the measurement, effects due to instrument hysteresis and instrument and procedural repeatability can be randomized.

Because of limited resolution in such an instrument, different competent temperature observers might record different indicated temperatures even if the instrument output were fixed. Either observers should be randomized or, if not, the test replicated. It is interesting to note that such a randomization will bring about a predictable scatter in recorded data of about $\frac{1}{2}$ the resolution of the instrument scale.

PROBLEM 1.15

KNOWN: Input voltage, (E_i) and Load (τ_L) can be controlled and varied.

Efficiency (η), Winding temperature (T_w), and Current (I) are measured.

FIND: Specify the dependent, independent in the test and suggest any extraneous variables.

SOLUTION

The measured variables are the dependent variables in the test and they depend on the independent variables of input voltage and load. Several influencing extraneous variables include: ambient temperature (T_a) and relative humidity R ; Line voltage fluctuations (e); and each of the individual measuring instruments (m_i). The variation of the independent variables should be performed separately maintaining one independent variable fixed while the other is systematically varied over the test range. A random test procedure for the independent variable will randomize the effects of T_a , R and e . Replication methods using different test instruments would be one way to randomize the effects of the m_i ; alternatively, calibration of all measuring instruments would provide a good degree of control over these variables.

$$\eta = \eta(E_i, \tau_L; T_a, R, e, m_i)$$

$$T_w = T_w(E_i, \tau_L; T_a, R, e, m_i)$$

$$I = I(E_i, \tau_L; T_a, R, e, m_i)$$

PROBLEM 1.16

KNOWN: Specifications Table 1.1
Nominal pressure of 500 cm H₂O to be measured.
Ambient temperature drift between 18 to 25 °C

FIND: Magnitude of each elemental error listed.

SOLUTION

Based on the specifications, the input and output spans (each the difference between the minimum and maximum values of range) are given as

$$\begin{aligned}r_i &= 1000 \text{ cm H}_2\text{O} \\r_o &= 5 \text{ V}\end{aligned}$$

Hence, $K = 5 \text{ V} / 1000 \text{ cm H}_2\text{O} = 5 \text{ mV/cm H}_2\text{O} = 0.005 \text{ V/cm H}_2\text{O}$. This gives a nominal output at 500 cm H₂O input of 2.5 V. This assumes that the input/output relation is linear over range but we are told that it is linear to within some linearity error.

$$\begin{aligned}\text{linearity error uncertainty} &= u_L = (0.005) (1000 \text{ cm H}_2\text{O}) \\&= 5.0 \text{ cm H}_2\text{O} \\&= 0.025 \text{ V}\end{aligned}$$

$$\begin{aligned}\text{hysteresis error uncertainty} &= u_h = (0.0015)(1000 \text{ cm H}_2\text{O}) \\&= 1.5 \text{ cm H}_2\text{O} \\&= 0.0075 \text{ V}\end{aligned}$$

$$\begin{aligned}\text{sensitivity error uncertainty} &= u_K = (0.0025)(500 \text{ cm H}_2\text{O}) \\&= 1.25 \text{ cm H}_2\text{O} = 0.00625 \text{ V}\end{aligned}$$

$$\begin{aligned}\text{thermal sensitivity error uncertainty} &= (0.0002)(7^\circ\text{C})(500 \text{ cm H}_2\text{O}) \\&= 0.7 \text{ cm H}_2\text{O} \\&= 0.0035 \text{ V}\end{aligned}$$

$$\begin{aligned}\text{thermal drift error uncertainty} &= (0.0002)(7^\circ\text{C})(1000 \text{ cm H}_2\text{O}) \\&= 1.4 \text{ cm H}_2\text{O} \\&= 0.007 \text{ V}\end{aligned}$$

$$\text{overall instrument uncertainty} = (5^2 + 1.5^2 + 1.25^2 + 0.7^2 + 1.4^2)^{1/2} = 5.6 \text{ cm H}_2\text{O}$$

COMMENT: When one uncertainty is notably larger than the others, it will dominate the overall uncertainty. Hence, it is important to identify the major sources of error in a measurement. If the uncertainty is smaller by an order of magnitude, you can neglect it.

PROBLEM 1.17

KNOWN: Full scale output = FSO = 1000 N (this is also the value of the output “span”)

FIND: u_c

SOLUTION

From the given specifications, the elemental errors are estimated by:

$$u_L = 0.001 \times 1000\text{N} = 1\text{N}$$

$$u_H = 0.001 \times 1000\text{N} = 1\text{N}$$

$$u_K = 0.0015 \times 1000\text{N} = 1.5\text{N}$$

$$u_z = 0.002 \times 1000\text{N} = 2\text{N}$$

The overall instrument error is estimated as:

$$u_c = (1^2 + 1^2 + 1.5^2 + 2^2)^{1/2} = 2.9 \text{ N}$$

COMMENT

This root-sum-square (RSS) method provides a "probable" estimate (i.e. the most likely estimate) of the uncertainty in the instrument error possible in any given measurement.

"Possible" is a key concept here as the error values will likely change between individual measurements. Uncertainty gives an interval within which the actual error falls with some level of likelihood or probability (such as in 19 measurements out of 20, or 95% of the measurements, we expect the error to be within the interval).

PROBLEM 1.18

SOLUTION

Randomization is used to break-up the effects of interference from either continuous or discrete extraneous (i.e. uncontrolled) variables.

A key independent variable of a particular process is to be increased incrementally over 5 settings of value, ranging from a minimum to a maximum value. Randomizing the order of the settings for the test will break up any potential trends imposed by extraneous effects influenced by the order of application.

PROBLEM 1.19

SOLUTION

Repetition through repeated measurements made under a fixed set of operating conditions provides a measure of the time (or spatial) variation of a measured variable.

Repetition refers to repeating the measurement during a test.

Example: A test is conducted in which a variable is measured multiple times (N) under some condition.

Replication through an independent duplication of tests conducted under similar operating conditions. It provides a measure of the control of the test conditions on the measured variable.

Replication refers to repeating the test (recreating a new data set of repeated measurements).

Example: A test is conducted in which a variable is measured multiple times under some test condition. After reviewing the data, it is decided to repeat the test. The test is setup again and the measurements repeated. This second test is a replication of the first.

In engineering, the term replication is often also applied to tests conducted for purposes of estimating reproducibility of the results – the ability to reproduce a test outcome when conducted by an independent operator, different test lab, or even a different tested device (of the same make and model). The term “reproducibility” explicitly refers to such.

The term replication can have different meanings depending on how it is applied. This is particularly true in statistical studies.

PROBLEM 1.20

FIND: Test matrix to correlate thermostat setting with average room setting

SOLUTION

Although there is no single test matrix, one method of solution follows.

Assume that average room temperature, T , is a function of actual thermostat setting, spatial distribution of temperature, temporal temperature distribution, and thermostat location. We might imagine that for a controlled (fixed) thermostat location, a direct correlation between setting and T could be achieved. However, factors could influence the temperature measured by the thermostat such as sunlight directly hitting the thermostat or the wall on which it is attached or a location directly exposed to furnace forced convection, a condition aggravated by air conditioners or heat pumps in which delivered air temperature is a strong function of outside temperature. Assume a proper location is selected and controlled.

Further, the average room temperature must be defined because local room temperature will vary with position within the room and with time. For the test matrix, the room should be divided into equal areas with temperature sensing devices placed at the center of each area. The output from each sensor will be averaged over a time period that is long compared to the typical furnace on/off cycle.

Select four temperature sensors: A, B, C, D. Select four thermostat settings: s_1, s_2, s_3, s_4 , where $s_1 < s_2 < s_3 < s_4$. Temperatures are to be measured under each setting after the room has adjusted to the new setting. One matrix might be:

BLOCK

- 1 s_1 : A, B, C, D
- 2 s_4 : A, B, C, D
- 3 s_3 : A, B, C, D
- 4 s_2 : A, B, C, D

Note that the order of set temperature has been shuffled to attempt to randomize the test matrix (hysteresis is a common problem in thermostats). The four blocks will yield the average temperatures, T_1, T_4, T_3, T_2 . The data can be presented in a form of T versus s .

PROBLEM 1.21

FIND: Test matrix to evaluate fuel efficiency of a production model of automobile

ASSUMPTIONS: Automobile model design is fixed (i.e. neglect options). Require representative estimate of efficiency.

SOLUTION

Although there is no single test matrix, one method of solution follows. Many variables can affect auto model efficiency: e.g. individual car, driver, terrain, speed, ambient conditions, engine model, fuel, tires, options. Whether these are treated as controlled variables or as extraneous variables depends on the test matrix. Suppose we "control" the options, fuel, tires, and engine model, that is fix these for the test duration. Furthermore, we can fix the terrain and the ambient conditions by using a mechanical chassis dynamometer (a device which drives the wheels with a prescribed mechanical load) in an enclosed, controlled environment. In fact, such a machine and its test conditions have been specified within the U.S.A. by government test standards. By programming the dynamometer to start, accelerate and stop using a preprogrammed routine, we can eliminate the effects of different drivers on different cars. However, this test will fail to randomize the effects of different drivers and terrain as noted in the government statement "... these figures may vary depending on how and where you drive" This leaves the car itself and the test speed as independent variables, x_a and x_b , respectively. We defer considering the effects of the instruments and methods used to compute fuel efficiency until a later chapter, but assume here that this can be done with sufficient accuracy.

With this in mind, we could choose three representative cars and three speeds with the test matrix:

BLOCK

- 1 x_{a1} : x_{b1} , x_{b2} , x_{b3}
- 2 x_{a2} : x_{b1} , x_{b2} , x_{b3}
- 3 x_{a3} : x_{b1} , x_{b2} , x_{b3}

Note that since slight differences will exist between cars that cannot be controlled, the autos are treated as extraneous variables. This matrix randomizes the effects of differences between cars at three different speeds and yields a curve for fuel efficiency versus speed.

As an alternative, we could introduce a driver into the matrix. We could develop a test track of fixed (controlled) terrain. And we could have three drivers drive three cars at three different speeds. This introduces the driver as an extraneous variable, noted as A_1 , A_2 and A_3 for each driver.

Assuming that the tests are run under similar ambient conditions, one test matrix may be

	X _{a1}	X _{a2}	X _{a3}
A ₁	X _{b1}	X _{b2}	X _{b3}
A ₂	X _{b2}	X _{b3}	X _{b1}
A ₃	X _{b3}	X _{b1}	X _{b2}

PROBLEM 1.22

SOLUTION:

Test stand:

Here one would operate the engine under simulated conditions similar to those encountered at the track – such as anticipated engine RPM and engine load (load: estimated mechanical loads on the engine due to mechanical losses, tire rolling resistance, aerodynamic resistance, etc.).

Measure:

- fuel and air consumption
- torque and power output
- exhaust gas temperatures to set air: fuel ratio

Track:

Here one would operate the car at conditions similar to those anticipated during the race.

Measure:

- lap time
- wind and temperature conditions (to normalize lap time)
- depending on team other factors can be measured to estimate loads on the car and car behavior. *Clemson Motorsports Engineering has been active in test method development for professional race teams.*

Obvious major differences:

- Environmental conditions, which effect engine performance, car behavior and tire behavior.
- Engine load on a test stand is well-controlled. On track, the driver does not execute exactly on each lap, hence varies load such as due to differences in drive path 'line' and this affects principally aerodynamic loads and tire rolling resistance. Incidentally, all of these are coupled effects in that a change in one affects the values of the others.
- Ram air effect of moving car can be simulated but difficult to get exactly
- Each engine is an individual. Even slight differences affect handling and therefore, how a driver drives the car (thus changing the engine load).

PROBLEM 1.23

KNOWN: A machining process that utilizes drill bits

FIND: Develop a test plan that could be used to test tool life for three types of drill bits

SOLUTION :

An appropriate test plan would first identify the primary controlled and uncontrolled variables. Some candidate variables would be:

- Variations in life of individual drill bits
- Variations in the material properties of the material being machined
- Differences between machine tools
- Are the machine tool variables operator or numerically controlled

Random blocks would be an appropriate experimental design strategy. This is a good problem for classroom discussion.

PROBLEM 1.24

KNOWN: A golfer wishes to optimize driver performance

FIND: Develop a test plan that could be used to find an optimum club

SOLUTION :

An appropriate test plan would first identify the primary controlled and uncontrolled variables. Some candidate variables would be:

- Swing speed
- Golfer's size
- Handicap

Other variable affecting testing would be:

- Indoor or outdoor testing
- Weather conditions if outdoor
-

Random blocks would be an appropriate experimental design strategy. This is a good problem for classroom discussion.

PROBLEM 1.25

KNOWN: Ideal gas model is applied

FIND: Is it correct?

SOLUTION

Validation refers to determining if an assumed model correctly applied to the real process.

In this case, the assumption is that the particular gas used in a test behaves as an ideal gas:

$$p = \rho RT$$

where absolute pressure is related directly to the gas density and the absolute temperature of the gas. The value 'R' is a gas constant, which is related directly to the molecular weight of the gas.

A test of this model could involve measurements of two of the three independent variables: temperature, pressure, and/or density. The third variable would be measured and compared to the model prediction. This can be done over a range of values to determine suitability of the model for the intended application.

PROBLEM 1.26

KNOWN: Mars Climate Orbiter

FIND: Verification tests to prevent accident

SOLUTION

The Mars Climate Orbiter met its demise due to a conflict in the different unit systems used to develop its on-board thrust propulsion system commands and its ground-based thrust propulsion system commands. The two systems were developed separately by two different vendors and tests had been performed separately by each vendor to ensure their system functioned correctly.

Performing functionality tests on the intended on-board systems using the intended ground-based system prior to the spacecraft being prepared for launch would have identified the problem that the two software algorithms were developed using different unit-systems.

This would be a verification test, one that ensured that the systems were behaving correctly when communicating with each other. Both systems worked and there was no advantage to one unit system over the other. The problem was that they did not work with each other. Systems-level verification would have identified the problem, which was an easy programming fix.

PROBLEM 1.27

KNOWN: Four lathes, 12 machinists are available to produce batches of machine shafts.

FIND: Test matrix to estimate the tolerances held within a batch

SOLUTION

If we assume that batch precision, P , is only a function of lathe and machinist, then

$$P = f(\text{lathe, machinist})$$

We can set up a test matrix using all four lathes, L_1, L_2, L_3, L_4 , and all 12 machinists, A, B, ..., L. The machinists are randomly assigned.

BLOCK

- 1 L_1 : A, B, C
- 2 L_2 : D, E, F
- 3 L_3 : G, H, I
- 4 L_4 : J, K, L

Data from each lathe should be indicative of the precision associated with each lathe and the total ensemble of data indicative of batch precision. However, this test matrix neglects the effects of shift and day of the week.

One method which treats machinist and lathe as extraneous variables and reduces test size selects 4 machinists at random. Suppose more than one shaft size is produced at the plant. We could select 4 shaft diameters, D_1, D_2, D_3, D_4 and set up a Latin square matrix:

	L_1	L_2	L_3	L_4
B	D1	D2	D3	D4
E	D2	D3	D4	D1
G	D3	D4	D1	D2
L	D4	D1	D2	D3

Note that neither matrix includes shift or day of the week effects and these could be incorporated in an expanded test matrix.

PROBLEM 1.28

SOLUTION

Linearity error

A random static calibration over a specified range will provide the input-output relationship between y and x (i.e. $y = f(x)$). A first-order curve fit to this data, for example using a least squares regression analysis, will provide the fit $y_L(x)$. The linearity error is simply the difference between the measured value of y at any value of x and the value of y_L predicted by the fit at that x . The uncertainty value assigned as linearity error is defined by the range of these error values over span. This might be the maximum deviation or some statistical measure.

A manufacturer may wish to keep the linearity error below some target value and, hence, may limit the recommended operating range for the system for this purpose. In your experience, you may notice that some systems can be operated outside of their specification range but be aware their elemental errors may exceed the manufacturer's stated values.

Hysteresis error

A sequential static calibration over a specified range will provide the input-output behavior between y and x during upscale-only and downscale-only operations. This will tend to maximize any hysteresis in the system. The hysteresis error is the difference between the upscale value and the downscale value of y at any given x . The uncertainty value assigned as hysteresis error is defined by the range of these errors over the span.