

CHAPTER 3

AEROTHERMODYNAMICS OF TURBOMACHINES AND DESIGN-RELATED TOPICS

PROBLEM # 1

With a constant density, the continuity equation becomes one of conserving the volumetric flow rate (Q), as follows:

$$Q_{in} = V_s r b d\theta + V_\theta ds \left(b + \frac{db}{2}\right)$$

$$Q_{out} = [(V_s + dV_s)(r + dr)(b + db)d\theta] + (V_\theta + dV_\theta)ds \left(b + \frac{db}{2}\right)$$

Expanding the preceding two expressions, ignoring higher-order terms and dividing by $(r b d s d\theta)$, the equality of Q_{in} and Q_{out} yields:

$$\frac{1}{b} \frac{\partial}{\partial s} (r b V_s) + \frac{\partial V_\theta}{\partial \theta} = 0$$

PROBLEM # 2

Applying the continuity-equation version 3.40, we obtain:

$$\dot{m} = 7.07 \text{ kg/s}$$

As the stator reaches the choking status ($M_{cr,1} = 1$), we now reapply the same equation at the stator exit station. This step yields:

$$(\alpha_1)_{new} = 68.8^\circ$$

Thus:

$$\Delta\alpha = 1.8^\circ$$

PROBLEM # 3

With the friction coefficient (f) being 0.11, application of eqn 3.76 leads to the following non-linear equation:

$$0.876[\ln(1 + 0.165M_2^2)] - \frac{0.75}{M_2^2} = -0.769$$

Iterative solution of this equation yields:

$$M_2 \approx 0.92$$

Applying eqn 3.75, we get the following:

$$p_{t2} = 6.97 \text{ bars}$$

Now, application of the continuity equation yields:

$$h_2 = 0.02188 \text{ m}$$

Finally:

$$\frac{\Delta h}{h_1} = \frac{h_1 - h_2}{h_1} = 0.53\%$$

PROBLEM # 4

Part 1:

The two velocity components V_{r1} & $V_{\theta1}$ can be computed as follows:

$$V_{r1} = V_1 \cos \alpha_1 = 23.5 \text{ m/s}$$

$$V_{\theta1} = V_1 \sin \alpha_1 = 72.3 \text{ m/s}$$

Now, applying the continuity at station 2, and utilizing the free-vortex flow condition across the radial gap, we get:

$$V_{r2} = 28.1 \text{ m/s}$$

$$V_{\theta2} = 86.5 \text{ m/s}$$

Finally, the magnitude of V_2 is:

$$V_2 = 90.9 \text{ m/s}$$

Part 2:

With the flow process being adiabatic, we have:

$$T_{t2} = T_{t1} = T_1 + \frac{V_1^2}{2c_p} = T_2 + \frac{V_2^2}{2c_p}$$

which, upon substitution, yields:

$$T_{t2} = 1224.5 \text{ K}$$

$$T_2 = 1220.9 \text{ K}$$

Part c:

Applying the continuity equation at station 1, we get:

$$\dot{m} = (2\pi r_1 b) \left(\frac{p_1}{RT_1} \right) V_{r1} = 0.80 \text{ kg/s}$$

Note that low mass-flow rate is typically a feature of a radial-flow turbomachine.

PROBLEM # 5

Part a:

$$T_{t1} = T_{t0} = 446 \text{ K} = T_1 + \frac{V_1^2}{2c_p}$$

which gives:

$$V_1 = 268.9 \text{ m/s}$$

We can now calculate M_{cr1} as follows:

$$V_{cr1} = \sqrt{\frac{2\gamma}{\gamma+1} RT_{t1}} = 386.4 \text{ m/s}$$

$$M_{cr1} = \frac{V_1}{V_{cr1}} = 0.696$$

The flow angle β_1 can be computed as follows:

$$\beta_1 = \beta_1' - |i_R| = 56^\circ$$

The process leading to the shaft speed is as follows:

$$V_{z1} = V_1 = 268.9 \text{ m/s} = W_1 \cos \beta_1$$

$$W_1 = 480.9 \text{ m/s}$$

$$U_m = W_1 \sin \beta_1 = 398.7 \text{ m/s}$$

$$\omega = \frac{U_m}{r_m} = 3,986.6 \text{ rad/s}$$

$$N = 38,069 \text{ rpm}$$

Part b:

Calculation of the inlet total pressure p_{t0} follows:

$$p_{t1} = \frac{p_1}{[1 - \frac{\gamma-1}{\gamma+1} M_{cr1}^2]} = 4.70 \text{ bars}$$

$$p_{t0} = p_{t1} + \Delta p_t = 5.0 \text{ bars}$$

PROBLEM # 6

Part Ia:

Applying the continuity equation at the stator-inlet station, we get:

$$\dot{m} = 7.83 \text{ kg/s}$$

Part Ib:

The procedure to compute the axial-velocity-component rise includes application of the continuity equation at the stator-exit station, and is as follows:

$$p_{t2} = p_{t1} + 0.06p_{t1} = 10.62 \text{ bars}$$

$$\alpha_2 = 69^\circ \dots \text{ (obtained through the continuity eqn.)}$$

$$V_1 = 276.4 \text{ m/s} \dots \text{ (computed in example 8)}$$

$$V_2 = V_{cr2} = 658.1 \text{ m/s} \dots \text{ (computed in example 8)}$$

$$V_{z1} = V_1 \cos \alpha_1 = 214.8 \text{ m/s}$$

$$V_{z2} = V_2 \cos \alpha_2 = 235.8 \text{ m/s}$$

$$\frac{\Delta V_z}{V_{z1}} = 9.8\%$$

Part II

First, we calculate the arithmetic average of the swirl angle (α_{av}):

$$\alpha_{av} = \frac{1}{2}(\alpha_1 + \alpha_2) = \frac{1}{2}(-39.0 + 69.0) = 15.0^\circ$$

With all of the given and computed flow properties in example 8, we can now substitute in the provided \bar{e}_s expression:

$$\bar{e}_s \equiv 1 - \frac{V_{act.,1}^2}{V_{id.,1}^2} = 0.122 \dots \text{ (higher by comparison)}$$

The procedure to convert \bar{e}_s into the pressure loss coefficient $\bar{\omega}$ is as follows:

$$V_{act.,1} = 259.0 \text{ m/s}$$

$$T_{act.,1} = T_{t1} - \frac{V_{act.,1}^2}{2c_p} = 1293.0 \text{ K}$$

$$\Delta s = c_p \ln\left(\frac{T_{act.,1}}{T_1}\right) - R \ln\left(\frac{p_{act.,1}}{p_1}\right) = 3.58 \text{ J/(kg K)}$$

$$\Delta s = -R \ln\left(\frac{p_{t2}}{p_{t1}}\right) = 3.58 \text{ J/(kg K)}$$

$$p_{t2} = 0.988 p_{t1} = 11.16 \text{ bars}$$

$$\bar{\omega} \equiv \frac{\Delta p_t}{p_{t1}} = 1.24\%$$

PROBLEM # 9

Part a:

$$V_{cr1} = \sqrt{\frac{2\gamma}{\gamma+1} R T_{t1}} = 673.4 \text{ m/s}$$

$$V_1 = V_{cr1} M_{cr1} = 612.8 \text{ m/s}$$

$$V_{\theta 1} = U_1 = \omega r_1 = 578.1 \text{ m/s}$$

$$\alpha_1 = \sin^{-1}\left(\frac{U_1}{V_1}\right) = 70.6^\circ$$

Applying the continuity equation (3.40) at the stator-exit station, we get:

$$\dot{m} = 4.07 \text{ kg/s}$$

Part b:

The procedure leading to the total-relative pressure decline across the rotor, is as follows:

$$\begin{aligned}W_1 &= V_1 \cos \alpha_1 = 203.5 \text{ m/s} \\T_{tr1} &= T_{t1} + \left(\frac{W_1^2 - V_1^2}{2c_p}\right) = 1239.6 \text{ K} \\p_{tr1} &= p_{t1} \left(\frac{T_{tr1}}{T_{t1}}\right)^{\frac{\gamma}{\gamma-1}} = 8.21 \text{ bars} \\U_2 &= \omega r_2 = 216.8 \text{ m/s} \\p_2 &= p_{t1} \left\{1 - \frac{1}{\eta_{t-s}} \left[1 - \left(\frac{T_{t2}}{T_{t1}}\right)\right]\right\}^{\frac{\gamma}{\gamma-1}} = 3.45 \text{ bars} \\V_{\theta 2} &= \frac{U_1 V_{\theta 1} - c_p (T_{t1} - T_{t2})}{U_2} = -160.5 \text{ m/s} \\\rho_2 &= \left(\frac{p_{t2}}{RT_{t2}}\right) \left[1 - \frac{\gamma-1}{\gamma+1} M_{cr2}^2\right]^{\frac{1}{\gamma-1}} = 1.164 \text{ kg/m}^3 \\V_{z2} &= \frac{\dot{m}}{\rho_2 (2\pi r_2 h_2)} = 309.2 \text{ m/s} \\V_2 &= 348.3 \text{ m/s} \\W_{\theta 2} &= V_{\theta 2} - U_2 = -377.3 \text{ m/s} \\W_2 &= \sqrt{W_{\theta 2}^2 + V_{z2}^2} = 487.8 \text{ m/s} \\T_{tr2} &= 1115.4 \text{ K} \\p_{tr2} &= p_{t2} \left(\frac{T_{tr2}}{T_{t2}}\right) = 4.70 \text{ bars} \\(\Delta p_{tr})_{rotor} &= 3.51 \text{ bars}\end{aligned}$$

In order to compute $(\Delta p_{tr})_{rotor}$ due to the radius change only, we set the total-to-total efficiency (η_{t-t}) to 100%. In the following, the variables which are influenced by this change are re-calculated:

$$\begin{aligned}V_{\theta 2} &= -344.0 \text{ m/s} \\\rho_2 &= 1.202 \text{ kg/m}^3 \\V_{z2} &= 299.3 \text{ m/s} \\W_{\theta 2} &= -560.8 \text{ m/s} \\V_2 &= 456.0 \text{ m/s} \\W_2 &= 635.7 \text{ m/s} \\T_{tr2} &= 1115.4 \text{ K}\end{aligned}$$

$$p_{tr2} = 5.36 \text{ bars}$$

$$(\Delta p_{tr})_{rad.change} = 2.85 \text{ bars}$$

Finally, we can compute Δp_{tr} due to the flow-process irreversibility sources only as follows:

$$(\Delta p_{tr})_{irrev.} = (\Delta p_{tr})_{tot.} - (\Delta p_{tr})_{rad.change} = 0.66 \text{ bars}$$

PROBLEM # 10

Part a:

Application of the continuity equation at the stator-exit station yields the local swirl angle, as follows:

$$\alpha_1 = 71.8^\circ$$

Part b:

Referring to Sovran & Klomp chart (Fig. 3.37), and for an L/h_{in} ratio of 3.0, we get:

$$c_{pr} \equiv \frac{p_3 - p_2}{\frac{\gamma}{2} p_2 M_2^2} = 0.45$$

For the back flow not to occur, the discharge-station pressure (p_3) must, at least, be equal to the ambient pressure, i.e.

$$p_3 = 1.06 \text{ bars}$$

Using the total-to-total efficiency definition, we get:

$$p_{t2} = 1.17 \text{ bars}$$

Substituting in the c_{pr} -expression (above), we obtain the following non-linear equation:

$$p_2 = 1.06 - 0.30(p_2 M_2^2)$$

The iterative procedure to solve this equation consists of repeatedly assuming M_2 , then calculating p_2 towards convergence. The final result is:

$$M_2 = 0.52 \text{ bars}$$

In words, the rotor-exit Mach no. will have to be less than, or equal to, 0.52 bars in order to avoid any diffuser-exit flow reversal.

PROBLEM # 11

Part a:

Application of the continuity equation at the stator-exit station yields:

$$\alpha_1 = +20.3^\circ$$

Remainder of the procedure, which leads to the rotor-exit relative-critical Mach no., is as follows:

$$V_1 = M_{cr1} V_{cr1} = 173.1 \text{ m/s}$$

$$U_m = \omega r_m = 428.8 \text{ m/s}$$

$$V_{\theta 1} = V_1 \sin \alpha_1 = 60.05 \text{ m/s}$$

$$V_z = V_1 \cos \alpha_1 = 162.3 \text{ m/s}$$

$$W_{\theta 1} = V_{\theta 1} - U_m = -368.8 \text{ m/s}$$

$$W_1 = 402.9 \text{ m/s}$$

$$T_{tr1} = T_{t1} - \left(\frac{W_1^2 - V_1^2}{2c_p} \right) = 488.9 \text{ K}$$

$$W_{cr1} = 404.6 \text{ m/s}$$

$$\frac{W_1}{W_{cr1}} = 0.996 \dots \text{ (high but, nevertheless, subsonic)}$$

Part b:

$$V_{\theta 2} = V_{\theta 1} + \left[\frac{c_p(T_{t2} - T_{t1})}{U_m} \right] = 313.0 \text{ m/s}$$

$$V_2 = 352.6 \text{ m/s}$$

$$V_{cr2} = 421.7 \text{ m/s}$$

$$M_{cr2} = 0.836$$

$$M_2 = 0.812 \dots \text{(obtained by using expression 3.78)}$$

$$p_{t2} = 6.64 \text{ bars (obtained by applying the } \eta_{t-t} \text{ definition)}$$

$$\left(\frac{\Delta p_t}{p_{t2}}\right)_{int.gap} = 3.16\%$$

Part c:

$$\alpha_2 = \sin^{-1}\left(\frac{V_{\theta 2}}{V_2}\right) = 62.6^\circ$$

Substituting in equation 3.76, we get:

$$\frac{fL}{D_h} = 0.062$$

where:

$$L = \frac{\Delta z}{\cos \alpha_2} = 12.2 \text{ cm}$$

$$D_h = 2(\Delta r)_{gap} = 6.4 \text{ cm}$$

With the preceding two variables, the endwall friction coefficient (f) across the interstage gap is:

$$f = 0.033$$

PROBLEM # 12

Part a:

In the following, the intermediate steps leading to the sidewall spacing (b) are summarized:

$$V_{cr1} = 668.8 \text{ m/s}$$

$$V_1 = 628.7 \text{ m/s}$$

$$U_1 = \omega r_1 = 546.6 \text{ m/s}$$

$$V_{\theta 1} = U_1 = 546.6 \text{ m/s} \dots \text{(radial entry to the rotor)}$$

$$\rho_1 = \frac{p_{t1}}{RT_{t1}} \left[1 - \frac{\gamma-1}{\gamma+1} M_{cr1}^2\right] = 2.383 \text{ kg/m}^3$$

$$V_{r1} = \sqrt{V_1^2 - V_{\theta 1}^2} = 310.6 \text{ m/s}$$

$$b_1 = \frac{\dot{m}}{2\pi r_1 \rho_1 V_{r1}} = 1.30 \text{ cm}$$

Part b:

$$W_1 = V_{r1} = 310.6 \text{ m/s} \dots \text{ (explained above)}$$

$$T_{tr1} = 1235.8 \text{ K}$$

$$p_{tr1} = p_{t1} \left(\frac{T_{tr1}}{T_{t1}} \right)^{\frac{\gamma}{\gamma-1}} = 9.38 \text{ bars}$$

$$p_1 = 8.17 \text{ bars}$$

$$T_1 = 1194.2 \text{ K}$$

$$T_{t2} = \frac{T_2}{[1 - \frac{\gamma-1}{\gamma+1} M_{cr2}^2]} = 1080.1 \text{ K}$$

$$V_{cr2} = 615.9 \text{ m/s}$$

$$U_2 = 205.0 \text{ m/s}$$

$$V_2 = 357.2 \text{ m/s}$$

$$V_{\theta 2} = \frac{U_1 V_{\theta 1} - c_p (T_{t1} - T_{t2})}{U_2} = -150.0 \text{ m/s}$$

$$V_{z2} = \sqrt{V_2^2 - V_{\theta 2}^2} = 324.2 \text{ m/s}$$

$$W_{\theta 2} = V_{\theta 2} - U_2 = -355.0 \text{ m/s}$$

$$W_2 = 480.8 \text{ m/s}$$

$$T_{tr2} = T_{t2} + \left(\frac{W_2^2 - V_2^2}{2c_p} \right) = 1124.9 \text{ K}$$

$$p_{tr2} = p_{t2} \left(\frac{T_{tr2}}{T_{t2}} \right)^{\frac{\gamma}{\gamma-1}} = 5.38 \text{ bars}$$

$$p_2 = p_{t2} [1 - \frac{\gamma-1}{\gamma+1} M_{cr2}^2]^{\frac{\gamma}{\gamma-1}} = 3.75 \text{ bars}$$

With all of the rotor-exit static, total and total-relative flow properties known, we can proceed to compute the required efficiencies as follows:

$$\eta_{t-s} = \frac{1 - \left(\frac{T_{t2}}{T_{t1}} \right)}{1 - \left(\frac{p_2}{p_{t1}} \right)^{\frac{\gamma}{\gamma-1}}} = 74.9\%$$

$$\eta_{tr-tr} = \frac{1 - \left(\frac{T_{tr2}}{T_{tr1}} \right)}{1 - \left(\frac{p_{tr2}}{p_{tr1}} \right)^{\frac{\gamma}{\gamma-1}}} = 69.7\%$$

$$\eta_{s-s} = \frac{1 - \left(\frac{T_2}{T_1} \right)}{1 - \left(\frac{p_2}{p_1} \right)^{\frac{\gamma}{\gamma-1}}} = 71.1\%$$