PROBLEM SOLUTIONS

CHAPTER 1. *PRELIMINARY CONCEPTS*

1-1 A 1.4-cm diameter sphere placed in a freestream at 18 m/s at 20°C and 1 atm. Compute the diameter Reynolds number for 3 cases:

(a) Air: Table A-2 - at 20°C, $\rho = 1.205\ \text{kg/m}^3$, $\mu = 1.81$ E-5 Pa-s. Then

$$Re_D = \rho VD/\mu = \frac{(1.205)(18)(0.014)}{1.81\text{E-5}} = \mathbf{16,800} \qquad \text{(Ans.)}$$

(b) Water: Table A-1 - at 20°C, $\rho = 998\ \text{kg/m}^3$, $\mu = 1.002$ mPa-s:

$$Re_D = (998)(18)(0.014)/(0.001002) = \mathbf{251,000} \qquad \text{(Ans.)}$$

(c) Hydrogen: Table A-3, $M = 2.016$, then $R = 8313/M = 4124\ \text{m}^2/\text{s}^2\text{-°K}$. Thus estimate $\rho = p/RT = (101350)/(4124)(293) = 0.0838\ \text{kg/m}^3$. From Table 1-2 for hydrogen,

$$\mu \approx \mu_o (T/T_o)^n = (8.411\text{E-6})(293/273)^{068} = 8.83\ \text{E-6 Pa-s}$$

$$\text{Then } Re_D = (0.0838)(18)(0.014)/(8.83\ \text{E-6}) = \mathbf{2,400} \qquad \text{(Ans.)}$$

1-2 At what wind velocity will an 8-mm-diameter wire "sing" at middle C (256 Hz)?

For air at 20°C, assume $v \approx 1.5\text{E-5}\ \text{m}^2/\text{s}$. From Fig. 1-8 guess a vortex-shedding Strouhal number of 0.2 [check the Reynolds number afterward]. Then

$fD/U \approx 0.2 = (256)(0.008)/U$, or $U \approx 10.24$ m/s. At this speed the Reynolds number is $Re_D = UD/v = (10.24)(0.008)/1.5\text{E-5} = 5400$. This is nicely in the range where $fD/U = 0.2$. Perhaps we could iterate just a little more closely to obtain

$$fD/U \approx 0.205,\ Re = UD/v \approx 5300,\ \text{or } U = \mathbf{10.0\ m/s} \qquad \text{(Ans.)}$$

1-3 If $U = 12$ m/s in Prob. 1-2 above, what is the wire drag in N/m?

For air assume $\rho = 1.205\ \text{kg/m}^3$ and $v = 1.5\text{E-5}\ \text{m}^2/\text{s}$. The Reynolds number is

$$Re_D = UD/v = (12)(0.008)/(1.5\text{E-5}) = 6400$$

From Fig. 1-9 at this Reynolds number, estimate a drag coefficient of 1.1. Then

$$F_{drag} = C_D\left(\frac{1}{2}\right)\rho V^2(DL) = 1.1(0.5)(1.205)(12)^2(0.008)(1.0) = \mathbf{0.76\ N/m} \qquad \text{(Ans.)}$$

1-4 Given, without proof, the Poiseuille-paraboloid laminar-pipe-flow formula from Chap. 3, $u = (C/\mu)(R^2 - r^2)$, find the wall shear stress if $u_{max} = 30$ m/s, $D = 1$ cm, and $\mu = 0.3$ kg/(m-s). [The exact analysis will be given in Sect. 3-3.1.]

Examining the formula, we see that the maximum velocity occurs on the centerline:

$$u_{mx} = u(r=0) = CR^2/\mu = 30\text{ m/s} = C(0.005)^2/(0.3)\text{, or: } C = 3.6\text{E}5\text{ N}/\left(\text{m}^2\text{-s}^2\right)$$

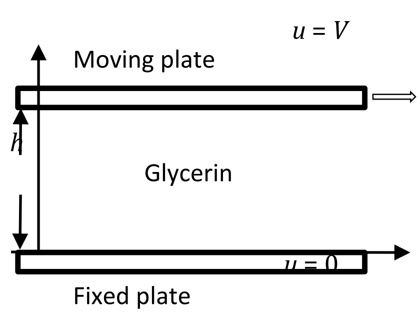
With C thus known for this data, we may evaluate wall shear stress by differentiation:

$$\tau_{wall} = \mu\left|\frac{\partial u}{\partial r}\right|_{r=0} = \mu\left(\frac{2RC}{\mu}\right) = 2RC = 2(0.005)(3.6\text{E}5) = \mathbf{3600\ Pa} \qquad \text{(Ans.)}$$

We should check the Reynolds number Re_D but we don't know the density. But "oil" is usually in the range $\rho \approx 900$ kg/m^3. Then $Re_D = \rho u_{max}D/\mu = (900)(30)(0.01)/(0.3) \approx 900$, which is well within the laminar-flow range.

1-5 Glycerin at 20°C is confined between two large parallel plates. One plate is fixed and the other moves parallel at 17 mm/s. The distance between the plates is 3 mm. Assuming no-slip, estimate the shear stress in the glycerin, in Pa.

Solution: Glycerin at 20°C is confined between two large parallel plates. One plate is fixed and the other moves parallel at $V = 17$ mm/s. The distance h between the plates is 3 mm.

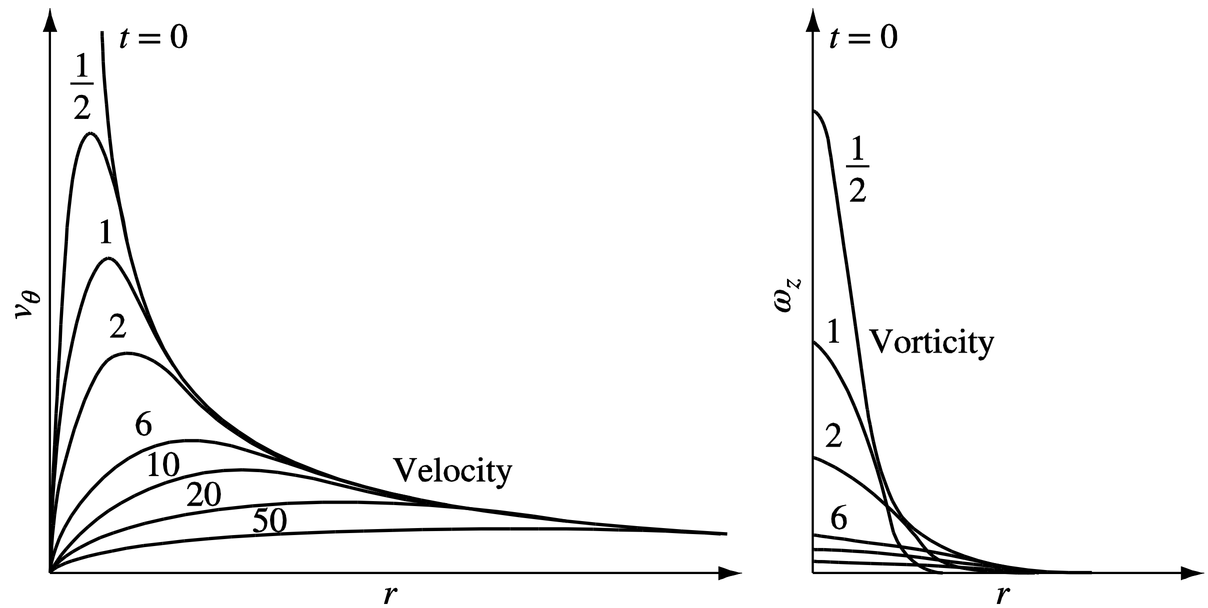


For glycerin at 20°C, the viscosity $\mu = 1.5$ kg/m·s.

Assuming no-slip, the shear stress $\tau = \frac{\mu V}{h} = \frac{(1.5\ \text{kg/m}\cdot\text{s})(17\times10^{-3}\ \text{m/s})}{(3\times10^{-3}\ \text{m/s})} = 8.5\ \text{Pa}$. (Ans.)

1-6 Given a plane unsteady viscous flow in polar coordinates:

$$v_r = 0;\ v_\theta = \frac{C}{r}\left[1-\exp\left(-\frac{r^2}{4\nu t}\right)\right]$$



Compute the vorticity and sketch some profiles of vorticity and velocity.

From Appendix B, the vorticity is

$$\omega_z = \frac{1}{r}\frac{\partial}{\partial r}(r v_\theta) = \frac{C}{2\nu t}\exp\left(-\frac{r^2}{4\nu t}\right)$$

The instantaneous velocity and vorticity profiles are plotted at top. At $t = 0$, the flow is a "line" vortex, irrotational everywhere except at the origin $(\omega = \infty)$.

1-7 Given the two-dimensional unsteady flow $u = x/(1+t)$, $v = y/(1+2t)$, find the equation for the streamlines which pass through the point (x_0, y_0) at time $(t = 0)$. From the geometric requirement for two-dimensional streamlines at any instant,

$$\left.\frac{dy}{dx}\right|_{\text{streamline}} = \frac{v}{u} = \frac{y(1+t)}{x(1+2t)}$$

Holding time t constant, we may separate the variables and integrate to obtain $y = C\,x^n$, where $n = (1+t)/(1+2t)$. To satisfy the initial condition, we must have $C = y_0/(x_0)^n$. The final result for the (unsteady) streamlines is

$$\frac{y}{y_0} = \left(\frac{x}{x_0}\right)^n, \quad n = \frac{1+t}{1+2t} \qquad \text{(Ans.)}$$

1-8 For the inviscid streamline approaching the forward stagnation point of the cylinder in Fig. 1-5, evaluate the strain rates and the time to go from $(2R, \pi)$ to (R, π)

From Eqs. (1-2), $v_r = U_\infty\left(1 - R^2/r^2\right)\cos\theta, \quad v_\theta = -U_\infty\left(1 + R^2/r^2\right)\sin\theta$

Then, from Appendix B, evaluate the normal and shear-strain rates along the line $\theta = \pi$:

$$\varepsilon_{rr} = \frac{\partial v_r}{\partial r} = \frac{2U_\infty R^2\cos\theta}{r^3}\Big|_{\theta=\pi} = -\frac{2U_\infty R^2}{r^3}$$

$$\varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r} = -\frac{2U_\infty R^2\cos\theta}{r^3}\Big|_{\theta=\pi} = +\frac{2U_\infty R^2}{r^3}$$

$$\varepsilon_{r\theta} = \frac{1}{r}\frac{\partial v_r}{\partial\theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} = \frac{4U_\infty R^2\sin\theta}{r^3}\Big|_{\theta=\pi} = 0 \qquad \text{(Ans. \textbf{a})}$$

The particle moving toward the stagnation point gets shorter in the "r" direction and fatter by the same amount in the "θ" direction, thus maintaining constant volume for this incompressible flow. The shear strain rate is *zero* because we are on a line of symmetry.

For part (b), by definition, the radial velocity along the stagnation line $(\theta = \pi)$ is

$$v_r = \frac{dr}{dt} = -U_\infty\left(1 - R^2/r^2\right)$$

We may separate the variables and integrate to find the time of travel between $(2R)$ and (R):

$$-U_\infty t = \int_{2R}^{R}\frac{r^2\,dr}{r^2 - R^2} = \left[r + \frac{R}{2}\ln\left(\frac{r-R}{r+R}\right)\right]_{2R}^{R} = -\infty \qquad \text{(Ans. \textbf{b})}$$

It takes infinite time to actually *reach* the stagnation point, where $V = 0$.

1-9 Use the approximate equation of state for water,

$$p/p_o \approx (A+1)(\rho/\rho_o)^n - A \quad \text{with } A \approx 3000,\ n = 7$$

to compute the following quantities for water, $p_o = 1$ atm, $\rho_o = 998$ kg/m^3:

(a) the pressure required to double the density of water:

$$p/p_o = (3000+1)(2.0)^7 - 3000 = 381128, \text{ or: } p = \mathbf{381{,}000\ atm} \qquad \text{(Ans. } \mathbf{a}\text{)}$$

(b) the bulk modulus K of water at 1 atm. By definition,

$$K = \rho\frac{dp}{d\rho}|_T = \rho p_o (A+1)\frac{n\rho^{n-1}}{\rho_o^n} = np_o(A+1) = 21007\ p_o \text{ at 1 atm.}$$

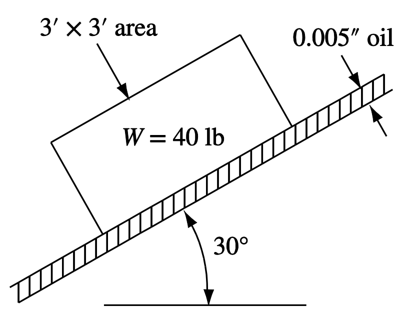
Thus the bulk modulus is $K \approx 21{,}007$ atm $\approx$ **2.13 E9 Pa** (Ans. **b**)

(c) the speed of sound at 1 atm:

$$a|_{1\ atm} = (K/\rho_o)^{1/2} = \left(\frac{2.13\ \text{E9 Pa}}{998\ \text{kg/m}^3}\right)^{1/2} = \mathbf{1460\ m/s} \qquad \text{(Ans. } \mathbf{c}\text{)}$$

These are accurate estimates of the measured compressibility and sound speed of water.

1-10 As shown, a plate slides down an incline on a film of oil of viscosity $\mu = 5\text{E-4}$ slug/ft-s



(a) Estimate the terminal sliding velocity V*:

Acceleration is zero, so

$$W\sin\theta = \tau A = \mu\frac{\Delta V}{\Delta y}A$$

where A is the plate area touching the oil film.

Assuming a linear velocity profile, $\Delta V = V^*$ and $\Delta y =$ the film thickness, hence

$$W \sin\theta = 40\sin(30°) = \mu \frac{\Delta V}{\Delta y} A = (0.0005)\left(\frac{V^*}{0.005/12}\right)(9)$$

in English units. Hence solve for $V^*(\text{terminal}) = \mathbf{1.85\ ft/s}$ (Ans. **a**)

(b) Estimate the time for the plate to accelerate from rest to 99% terminal velocity: If x is down the incline, then a dynamic force balance gives

$$\sum F_x = W \sin\theta - \mu \frac{V}{\Delta y} A = \frac{W}{g}\frac{dV}{dt},$$

$$\text{or:} \quad \frac{dV}{dt} + \left(\frac{g\mu A}{W\,\Delta y}\right) V = g \sin\theta$$

The solution to this first-order linear ordinary differential equation is

$$V = V^*\left[1 - \exp\left(-\frac{g\mu A}{W\Delta y}t\right)\right] = 0.99V^* \text{ if } t^* = \frac{4.605 W\,\Delta y}{g\mu A}$$

For our data, then, the time to reach 99% of terminal velocity is

$$t^* = \frac{4.605(40)(0.005/12)}{(32.2)(0.0005)(9.0)} = \mathbf{0.53\ sec}$$ (Ans. **b**)

1-11 Estimate the viscosity of nitrogen at 86 MPa and 49°C. From Appendix A-3, for N_2, read $T_c = 226°R = 126°K$, $p_c = 33.5$ atm, $\mu_c = 18.0$ E-6 Pa-s. At this high pressure, we cannot use "low density" formulas but rather must use Fig. 1-17. Compute ratios:

$$\frac{T}{T_c} = \frac{49+273}{126} = 2.55; \quad \frac{p}{p_c} = \frac{86E6}{33.5(101350)} = 25.3; \text{ Read } \frac{\mu}{\mu_c} \approx 2.5 \pm 0.1$$

Then our estimate is $\mu = 2.5\ \mu_c = 2.5(18.0) = \mathbf{45 \pm 2\ \mu Pa\text{-}s}$ (Ans.)

The agreement with the measured value (also 45 μPa-s) is excellent.

1-12 Estimate the thermal conductivity of helium at 420°C and 1 atm. This is truly "low-density", since $p < p_c$ and $T >> T_c$. A power-law estimate would be based on 0°C:

$$k \approx k_o (T/T_o)^n \approx (0.142 W/m\text{-}K)\left(\frac{420+273}{273}\right)^{072} \approx \mathbf{0.278\ W/m\text{-}K}$$

Alternately, we could use the kinetic theory formula, Eq. (1-41). From Appendix A-5 for helium, $\sigma = 2.551$ Å, $T_\varepsilon = 10.22°K$, and $M = 4.003$. First use Eq. (1-34) to compute

$$\Omega_v \approx 1.147\left(\frac{420+273}{10.22}\right)^{-0.145} + \left(\frac{420+273}{10.22}+0.5\right)^{.20} = 0.6226 \qquad \text{[check with Table 1-1]}$$

Our estimate from (1-41) then is

$$k = \frac{0.0833\sqrt{T}}{\sigma^2\Omega_v\sqrt{M}} = \frac{0.0833\sqrt{(693)}}{(2.551)^2(0.6226)\sqrt{4.003}} = \mathbf{0.27\ W/m\text{-}K}$$

The agreement with the experimental value of 0.28 W/m-K is good for both estimates.

1-13 According to Table C-5 and Fig. 1-15, at what pressure is the viscosity of CO_2 equal to approximately 30×10^{-5} Pa·s when the temperature is 800°R ?

Solution: $T = 800°R = 444.4444\ K$, $T_c = 548°R = 304.4444\ K$, $T_r = \frac{T}{T_c} = \frac{444.4444\ K}{304.4444\ K} = 1.46$

$\mu = 30\times10^{-5}$ Pa·s, $\mu_c = 3.43\times10^{-5}$ Pa·s, $\mu_r = \frac{\mu}{\mu_c} = \frac{30\times10^{-5}\ Pa\cdot s}{34.3\times10^{-6}\ Pa\cdot s} = 8.75$

Thus, $p_r = 25$ (from Fig. 1-15).

Then, pressure $p = p_c p_r = (72.9\ atm)\times 25 = 1822.5\ atm$. (Ans.)

1-14 Fit the given viscosity-vs-temperature data for ammonia gas to power-law and Sutherland-law formulas.

(a) The power-law is an excellent fit to this data. Taking $T_o = 300°K$, we obtain, by least-squares to a $\log(\mu)$ vs. $\log(T)$ plot,

$$\frac{\mu}{\mu_o} \approx \left(\frac{T}{T_o}\right)^{\mathbf{1.051}} \pm 0.3\% \text{ for } T_o = 300°K \qquad \text{(Ans. a)}$$

(b) The Sutherland-law is not an especially good fit to the data, which has μ rising with T at an increasing rate. It may be fit to least squares by minimizing the functional

$$\sum_{i=1}^{6}\left[\mu_i^* - \frac{\left(T_i^*\right)^{3/2}\left(1+S^*\right)}{T_i^* + S^*}\right]^2 \quad \text{where } \mu^* = \frac{\mu}{\mu_o}, T^* = \frac{T}{T_o}, S^* = \frac{S}{T_o}$$

for the given six data points. The minimum is found by differentiating the functional with respect to S^*, v with the result $S^* = 1.91$, or:

$$S_{\text{best fit}} \approx \mathbf{573°K} \qquad \text{(Ans. b)}$$

The error is $\pm 2.4\%$, or eight rimes more than the power-law fit.

1-15 Experimental data for the viscosity of helium at low pressure are as follows:

T, °C	0	100	200	300	400	500
μ, Pa·s	1.87 × 10^{-5}	2.32 × 10^{-5}	2.73 × 10^{-5}	3.12 × 10^{-5}	3.48 × 10^{-5}	3.48 × 10^{-5}

Fit these values to a suitable formula.

Solution: Experimental data for the viscosity of helium in Kelvin scale at low pressure are as follows:

T, K	273	373	473	573	673	773
μ, Pa·s	1.87 × 10^{-5}	2.32 × 10^{-5}	2.73 × 10^{-5}	3.12 × 10^{-5}	3.48 × 10^{-5}	3.48 × 10^{-5}

Using power law curve, $\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^n$.

For $\mu_0 = 1.87\times10^{-5}$ Pa·s and $T_0 = 273$ K

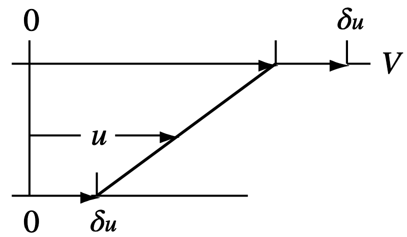
T	373 K	473 K	573 K	673 K	773 K
n	0.691	0.688	0.69	0.688	0.597

Therefore, the mean value $n = \frac{\sum_{i=1}^{5} n_i}{5} = 0.671$ (± 4 % accuracy for $250\text{ K} \le T \le 1000\text{ K}$). (Ans.)

1-16 Analyze newtonian flow between parallel plates (Fig. 1-15) with a finite *slip* velocity $\delta u = l(du/dy)$ at both walls.

The velocity profile is still linear, but with slip at both walls the slope is less, as shown in the sketch:

$$du/dy = (V - 2\delta u)/h$$



Introducing δu from the slip relation, we obtain

$$\frac{du}{dy} = \frac{V}{h + 2\ell} \quad \text{or:} \quad \tau_w = \frac{\mu V}{h + 2\ell} \quad \text{at both walls} \qquad \text{(Ans.)}$$

1-17 Derive Eq. (1-106) from a balance of forces on the differential surface-area alement shown in the problem.

Since the sliver of area is negligibly thin, it has no weight. The pressures act on a projected surface area $dS_x dS_y$. The surface tension forces are slanted slightly upward, at angles $(d\theta_x/2)$ and $(d\theta_y/2)$, respectively. The force balance is

$$2\mathcal{T} \qquad\qquad \mathcal{T} \qquad\qquad +(p - p_a)dS_x dS_y = 0$$

For differentially small angles, $\sin(d\theta) = d\theta.$ Clean up this equation and rearrange:

$$p = p_a - \mathcal{T}\left(\frac{d\theta_x}{dS_x} \qquad \frac{d\theta_y}{dS_y}\right) \qquad \mathcal{T}\left(\frac{1}{R_x} \qquad \frac{1}{R_y}\right) \qquad \text{(Ans.)}$$

since, by definition, $d\theta/dS = 1/R,$ where R is the radius of curvature.

1-18 Two bubbles of radii R_1 and R_2 coalesce isothermally into a single bubble R_3. Find the radius of the new (single) bubble.

Because of surface tension, the pressure inside a bubble (which has two surfaces) is higher than ambient, $p = p_o + 4\mathcal{T}$ Assuming that no interior-bubble air mass escapes during the coalescence, $m_1 + m_2 = m_3,$ or, for an ideal isothermal gas of temperature T,

$$\frac{p_o + 4\Im/R_1}{\Re T}\frac{4\pi}{3}R_1^3 + \frac{p_o + 4\Im/R_2}{\Re T}\frac{4\pi}{3}R_2^3 = \frac{p_o + 4\Im/R_3}{\Re T}\frac{4\pi}{3}R_3^3$$

where $\Re$ is the gas constant. Canceling common terms and cleaning up, we have

$$p_o R_3^3 + 4\Im R_3^2 = p_o\left(R_1^3 + R_2^3\right) + 4\Im\left(R_1^2 + R_2^2\right) \qquad (Ans.)$$

This must be solved numerically or algebraically for the new radius R_3.

1-19 In Prob. 1-1, if the temperature, sphere size, and velocity remain the same for air flow, at what air pressure will the Reynolds number $\mathrm{Re_D}$ be equal to 10,000?

Solution: From Prob. 1−1, $T = 293$ K, $D = 0.014$ m, $V = 18$ m/s, and $\mu = 1.81\text{E-5}$ Pa-s. Use the specified Reynolds number to compute the required air density:

$$\mathrm{Re}_D = \frac{\rho V D}{\mu} = \frac{\rho(18\ m/s)(0.014\ m)}{1.81E\text{-}5\ kg/m\text{-}s} = 10{,}000 \ \text{ Solve } \rho = 0.718\ kg/m^3$$

$$\text{Ideal gas: } \rho = 0.718 = \frac{p}{RT} = \frac{p}{(287)(293)}, \text{ Solve for } p = 60400\ Pa \approx \mathbf{60}\ kPa \qquad (Ans.)$$

1-20 A solid cylinder of mass *m*, radius *R*, and length *L* falls concentrically through a vertical tube of radius $R + \Delta R$, where $\Delta R << R$. The tube is filled with gas of viscosity μ and mean free path ℓ Neglect fluid forces on the front and back faces of the cylinder and consider only shear stress in the annular region, assuming a linear velocity profile. Find an analytic expression for the terminal velocity of fall, *V*, of the cylinder (*a*) for no-slip; (*b*) with slip, Eq. (1-91).

Solution: (*a*) For no-slip, the shear stress in the thin annular region between cylinders is

$$\tau = \mu\frac{\delta u}{\delta y} = \mu\frac{V}{\Delta R}, \text{ then } W = mg = F_{shear} = \tau A_{wall} = \left(\mu\frac{V}{\Delta R}\right)(2\pi RL)$$

$$\text{Solve for } V_{no\text{-}slip} = \frac{mg\Delta R}{2\pi RL} \qquad Ans.\ (a)$$

(*b*) For slip, modify the shear stress (see Prob. 1.16 for another example):

$$\delta u = \ell \quad \ell \quad \text{or: } \frac{du}{dy} = \frac{V}{\Delta R + 2\ell} \quad \mu V \quad \ell$$

As above *in part* (*a*), $mg = \tau A_w$, $V_{slip} = \dfrac{mg(\Delta R + 2\ell}{2\pi RL}$ *Ans*. (*b*)

1-21 Solve P1-20 for the terminal fall velocity for no-slip if the cylinder is aluminum, with diameter $4\ \text{cm}$ and length $10\ \text{cm}$. The tube has a diameter of $4.02\ \text{cm}$ and is filled with argon gas at 20° ⌣.

Solution: For no-slip, the shear stress τ in the thin annular region is $\tau = \mu \dfrac{\partial u}{\partial y} = \mu \dfrac{V}{\Delta R}$.

Then, $W = mg = F_{shear} = \tau A_{wall} = \left(\mu \dfrac{V}{\Delta R}\right)(2\pi RL)$.

$\rho_{Al} = 2710\ \text{kg/m}^3$, $R = 0.02\ \text{m}$, $L = 0.1\ \text{m}$, $g = 9.81\ \text{m/s}^2$; then, $mg = \rho_{Al}\pi R^2 L = 3.3408\ \text{N}$.

$\Delta R = (R + \Delta R) - R = 0.0201\ \text{m} - 0.02\ \text{m} = 0.0001\ \text{m}$

$\mu_{Ar} = 2.24 \times 10^{-5}\ \text{Pa}\cdot\text{s}$

Therefore, $V_{no-slip} = \dfrac{mg\Delta R}{2\pi RL\mu_{Ar}} = \dfrac{(3.3408\ \text{N})(0.0001\ \text{m})}{(0.01256\ \text{m}^2)(2.24\times10^{-5}\ \text{Pa}\cdot\text{s})} = 1187.4431\ \text{m/s}$ (Ans.)

1-22 In Fig. Pl-22 a disk rotates steadily inside a disk-shaped container filled with oil of viscosity μ. Assume linear velocity profiles with no-slip and neglect stress on the outer edges of the disk. Find a formula for the torque *M* required to drive the disk.

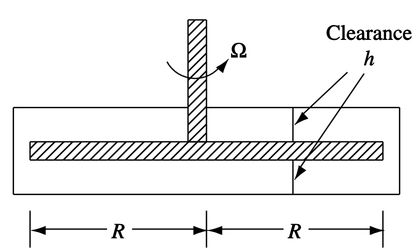


Fig. Pl-22

Solution: The disk tangential velocity varies with radius, $V = \Omega r$, hence the local shear stress is $\tau = \mu\,\Omega\, r/h$ on the top and bottom of the disk. The torque on a circular strip *dr* wide is

$$dM = (\tau dA)r(2\ sides) = 2r\left(\mu \frac{\Omega r}{h}\right)2\pi r\ dr$$

or: $M = 4\pi \dfrac{\mu\,\Omega}{h}\displaystyle\int_0^R r^3 dr = \dfrac{\pi\,\mu\,\Omega R^4}{h}$ *Ans.*

1-23 Show, from Eq. (1-86), that the coefficient of thermal expansion of a perfect gas is given by $\beta = 1/T$. Use this approximation to estimate β of ammonia gas (NH_3) at 20°C and 1 atm and compare with the accepted value from a data reference.

Solution: Introduce the ideal-gas law into the definition of β:

$$\beta = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p = -\frac{1}{\rho}\frac{\partial}{\partial T}\left(\frac{p}{RT}\right)_p = -\frac{1}{\rho}\left(\frac{-p}{RT^2}\right) = \frac{1}{\rho}\left(\frac{\rho}{T}\right) = \mathbf{\frac{1}{T}} \quad \textit{Ans.}$$

It doesn't matter what gas we are considering, ammonia or carbon dioxide or whatever, the ideal gas approximation predicts $\beta = 1/T = 1/293\text{K} = \mathbf{0.00341\ K^{-1}}$. *Ans.*

This estimate is very close to estimates for ammonia in the literature, e.g., White (1988).

1-24 The *rotating-cylinder viscometer* in Fig. P1-24 shears the fluid in a narrow clearance Δr, as shown. Assuming a linear velocity distribution in the gaps, if the driving torque *M* is measured, find an expression for μ by (*a*) neglecting, and (*b*) including the bottom friction.

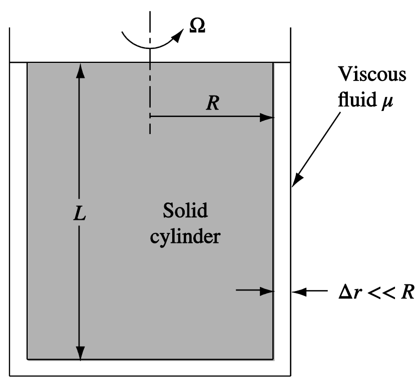


Fig. Pl-24

Solution: (*a*) Analyze the annular region only. The shear stress equals $\mu(\text{du/dy}) \approx \mu(\Omega R/\Delta R)$. The shear force on the cylinder side is normal to the radius, and the driving moment must be

$$M = \int R\,dF = \int R(\tau dA_w) = \int_0^{2\pi} R\left(\mu\frac{\Omega\,R}{\Delta R}\right)RL\,d\theta = 2\pi\mu\frac{\Omega\,R^3 L}{\Delta R}$$

$$\text{Solve for } \mu = \frac{M\Delta R}{2\pi\,\Omega\,R^3 L} \qquad \textit{Ans. (a)}$$

(*b*) On the bottom, the shear stress varies linearly with radius:

$$M_{\text{bottom}} = \int r\tau dA_w = \int_0^R r\left(\mu\frac{\Omega\,r}{\Delta R}\right)2\pi r dr = \frac{2\pi\,\Omega\,\mu}{\Delta R}\int_0^R r^3 dr = \frac{2\pi\,\Omega\,\mu R^4}{4\Delta R}$$

$$\text{Thus } M_{\text{total}} = \frac{2\pi\,\Omega\,\mu R^3 L}{\Delta R} + \frac{2\pi\,\Omega\,\mu R^4}{4\Delta R}, \ \text{ Solve } \mu = \frac{M\Delta R}{2\pi\,\Omega\,R^3\left(L + R/4\right)} \qquad \textit{Ans. (b)}$$

1-25 Consider 1 m^3 of a fluid at 20°C and 1 atm. For an isothermal process, calculate the final density and the energy, in joules, required to compress the fluid until the pressure is 10 atm, for (*a*) air; and (*b*) water. Discuss the difference in results.

Solution: (*a*) The work done is $-\int p d\upsilon$, where υ is the volume. From the ideal-gas law, $p\upsilon = mRT$. Thus

$$W_{1-2} = -\int p d\upsilon = -\int_1^2 \frac{mRT}{\upsilon} d\upsilon = -mRT \ln\left(\frac{\upsilon_2}{\upsilon_1}\right) = p_1\upsilon_1 \ln\left(\frac{p_2}{p_1}\right)$$

$$= (101350 Pa)(1 m^3)\ln\left(\frac{10}{1}\right) = \mathbf{233{,}000\ J} \qquad \textit{Ans. (a)}$$

(*b*) For water, we could use the compressed-liquid tables, but we can estimate the (very small) result from the bulk modulus $K = \rho(dp/d\rho)_T = 2.23\text{E}9$ Pa for water, Eq. (1-84). The change in volume of the water is very small when the change in pressure is only 9 atm:

$$\Delta\upsilon \approx -\frac{\upsilon\Delta p}{K} = -\frac{(1m^3)[(9)(101350 Pa)]}{2.23E9 Pa} \approx -0.0041 m^3$$

A slightly more accurate estimate from Prob. 1-9, or from the compressed-liquid tables, gives $\Delta\upsilon \approx -0.00042\ m^3$. Then the work required to compress water from 1 atm to 10 atm is

$$W_{1-2} = -\int p d\upsilon \approx -p_{avg}\Delta\upsilon = -[(5.5)(101350 Pa)](-0.00042 m^3) \approx \mathbf{230\ J} \qquad \textit{Ans. (b)}$$

This is 1000 times less than *Ans.*(*a*) for air above, since water is nearly incompressible.

1-26 Equal layers of two immiscible fluids are being sheared between a moving and a fixed plate, as in Fig. P1-26. Assuming linear velocity profiles, find an expression for the interface velocity U as a function of V, μ_1, and μ_2.

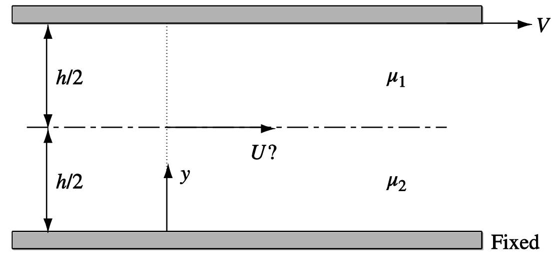


Fig. P1-26

Solution: The shear stress is the same in each layer:

$$\tau_1 = \mu_1 \frac{V-U}{h/2} = \tau_2 = \mu_2 \frac{U}{h/2}, \quad \text{solve for} \quad U = \frac{\mu_1}{\mu_1+\mu_2}V \qquad \textit{Ans.}$$

1-27 Utilize the inviscid-flow solution of flow past a cylinder, Eqs. (1-3), to (*a*) find the location and value of the maximum fluid acceleration a_{max} along the cylinder surface. Is your result valid for gases and liquids? (*b*) Apply your formula for a_{max} to air flow at 10 m/s past a cylinder of diameter 1 cm and express your result as a ratio compared to the acceleration of gravity. Discuss what your result implies about the ability of fluids to withstand acceleration.

Solution: Along the cylinder surface, $r = R$, and Eqs. (1-3) reduce to $v_r = 0$ and $v_\theta = -2U_\infty \sin\theta$. Thus, along the surface, the absolute velocity is $\mathrm{V} = 2U_\infty \sin(s/R)$, where s is the arc length along the surface, measured from the front stagnation point. There is a convective acceleration given by

$$a = V\frac{dV}{ds} = \left(2U_\infty \sin\frac{s}{R}\right)\left(\frac{2U_\infty}{R}\cos\frac{s}{R}\right)$$

(*a*) The acceleration is a maximum at $\theta = 135°$, or $s/R = \pi/4$. Thus $\boldsymbol{a_{max}} = \mathbf{2}\boldsymbol{U_\infty^2/R}$. *Ans*. (*a*)

This result is valid for all fluids, gases or liquids, in the inviscid approximation.

(*b*) For the given data, $R = 0.005$ m, $U_\infty = 10$ m/s, compute, independent of fluid properties,

$$a_{max} = 2(10 \text{ m/s})^2/(0.005 \text{ m}) = 40{,}000 \text{ m/s}^2 \approx \mathbf{4080\ g\text{'}s} \qquad \textit{Ans. (b)}$$

The lesson is that fluids have no fear of huge accelerations that would defeat a human being.

1-28 The coefficient of thermal expansion is defined as

$$\beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}\bigg|_p$$

Determine β for an ideal gas with $p = \rho RT$. Show your work in detail.

Solution: Given, the coefficient for thermal expansion is $\beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}\Big|_p$

Using $p = \rho RT$ for ideal gas,

$$\beta = -\frac{1}{\rho}\frac{\partial}{\partial T}\left(\frac{p}{RT}\right)\bigg|_p = -\frac{1}{\rho}\left(-\frac{p}{RT^2}\right) = -\frac{1}{\rho}\left(-\frac{\rho RT}{RT^2}\right) = \frac{1}{T} \qquad \text{(Ans.)}$$

1-29 Starting with Maxwell's low-density approximation of the viscosity, namely, $\mu \approx \frac{2}{3}\rho\ell$, and Newton's expression of the wall shear stress as a function of the velocity gradient, $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_w$, express Maxwell's slip velocity, $u_w = \ell\left(\frac{\partial u}{\partial y}\right)_w$,

(*a*) as a function of the shear stress, density, and speed of sound a;

(*b*) as a function of the Mach number, the mean-flow velocity U, and the skin friction coefficient, $C_f = \frac{2\tau_w}{\rho U^2}$.

Solution: Given, Maxwell's low-density approximation of viscosity is $\mu \approx \frac{2}{3}\rho\ell$, where $\rho =$ density, ℓ mean free path, and $a =$ speed of sound.

Newton's wall shear stress τ_w as a function of velocity gradient is $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_w$.

Slip velocity u_w is $u_w = \alpha\tau_w$, and constant $\alpha = \frac{\ell}{\mu}$.

Therefore, $u_w \approx \ell \left(\frac{\partial}{\partial y} \right)_w \quad \ell \quad \mu \quad \ell \left(\frac{}{3}\rho\ell \right) \quad \frac{3}{}\frac{\tau_w}{a}$.

(Ans.)

Dividing by mean-flow velocity U and arranging gives $\frac{u_w}{U} = \frac{3}{4}\frac{U}{a}\frac{2\tau_w}{\rho U^2}$.

Mach number is $Ma = \frac{U}{a}$, and skin friction coefficient is $C_f = \frac{2\tau_w}{\rho U^2}$.

Therefore, $u_w = \frac{3}{4} Ma \cdot U \cdot C_f$. (Ans.)

1-30 Consider a hydraulic lift with a $50\ \text{cm}$ diameter shaft sliding inside a housing with an inside diameter of $50.02\ \text{cm}$. If the shaft travels at $0.25\ \text{m/s}$, calculate the shaft resistance to motion per unit length. You may use water as the working fluid.

Solution: Shaft resistance F to motion is $F = \tau A_{wall} = \left(\mu \frac{V}{\Delta R} \right)(2\pi R L)$.

$R = 0.25\ \text{m}$, $L = 1\ \text{m}$, $V = 0.25\ \text{m/s}$

$\Delta R = (R + \Delta R) - R = 0.251\ \text{m} - 0.25\ \text{m} = 0.001\ \text{m}$

$\mu_{water} = 1.02 \times 10^{-5}\ \text{Pa} \cdot \text{s}$ (at 20°‿ and 1 atm)

$$F_{unit\text{-}length} = \frac{\left(1.02\times10^{-5}\ \text{Pa}\cdot\text{s}\right)\left(0.25\ \text{m/s}\right)2\pi\left(0.25\ \text{m}\right)\left(1\ \text{m}\right)}{\left(0.001\ \text{m}\right)} = 4.0055\times10^{-3}\ \text{N}$$ (Ans.)

1-31 Consider a thin air gap of $1\ \text{mm}$ that is formed between two parallel surfaces that are maintained at 20°‿ and 40°‿, respectively. In the case of a quiescent medium (say still air), calculate the heat transfer rate across the gap per unit area.

Solution: The rate of heat transfer per unit area (in one-dimensional space) is $q \approx -k \frac{T(x+\Delta x) - T(x)}{\Delta x}$.

The parallel plates are at $20^{\circ}\smile$ and $40^{\circ}\smile$. Then, k needs to be determined for $30^{\circ}\smile$.

Using power law curve, $k_{air} = k_0\left(\dfrac{T}{T_0}\right)^n = \left(0.0241\ \text{W/m}\cdot\text{K}\right)\left(\dfrac{303\ \text{K}}{273\ \text{K}}\right)^{0.81} = 0.0262\ \text{W/m}\cdot\text{K}$.

Therefore, $q \approx -\left(0.0262\ \text{W/m}\cdot\text{K}\right)\left(\dfrac{313\ \text{K} - 293\ \text{K}}{0.001\ \text{m}}\right) = 524\ \text{W/m}^2$. (Ans.)

1-32 In the presence of viscosity, the pressure drop associated with a fully developed laminar motion in a horizontal tube of length L and diameter D may be evaluated analytically. One finds:

$$\Delta p = p_1 - p_2 = \frac{128\mu LQ}{\pi D^4}$$

where μ stands for the dynamic viscosity and $Q = \dfrac{1}{4}\pi D^2 V$ denotes the volumetric flow rate. Show that the corresponding head loss may be written as

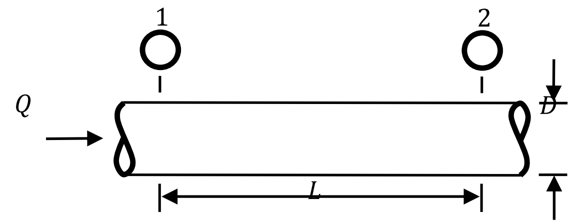
$$h_L = \frac{p_1 - p_2}{\rho g} = f_{\text{lam}}\frac{L}{D}\frac{V^2}{2g}$$

What value of f_{lam} do you obtain?

Solution: Consider fully developed flow and apply steady-flow energy equation between section 1 and section 2.

$$\left(\frac{p}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_2 = \left(\frac{p}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_1 + h_S - h_L$$

Use $z_1 = z_2$ (horizontal), $V_1 = V_2$ (constant cross-section), $\alpha_1 = \alpha_2$ (same velocity profile), $h_S = 0$ (no pump); to simplify as $h_L = \dfrac{p_1 - p_2}{\rho g} = \dfrac{\Delta p}{\rho g}$ … (1)



Empirical data on viscous losses in straight sections of pipe are correlated by the dimensionless

Darcy friction factor $f = \dfrac{\Delta p}{\frac{1}{2}\rho V^2}\dfrac{D}{L}$ … (2)

From equation (2), $\Delta p = f\dfrac{1}{2}\rho V^2\dfrac{L}{D}$ … (3)

Combining equations (1) and (3) gives $h_L = f_{\text{lam}}\dfrac{L}{D}\dfrac{V^2}{2g}$ (where $f = f_{\text{lam}}$, for laminar flow). (Ans.)

In the presence of viscosity, the pressure drop associated with a fully developed laminar motion in a horizontal tube of length L and diameter D is $\Delta p = p_1 - p_2 = \dfrac{128\mu LQ}{\pi D^4}$ … (4)

Dynamic viscosity is μ, and volumetric flow rate is $Q = \dfrac{1}{4}\pi D^2 V$ … (5)

For fully developed laminar flow, using equations (4) and (5) in equation (2) gives

$$f_{\text{lam}} \equiv \frac{128\mu L\left(\frac{1}{4}\pi D^2 V\right)}{\pi D^4}\frac{2}{\rho V^2}\frac{D}{L} = \frac{64}{\rho V D/\mu} = \frac{64}{\text{Re}_D} \qquad \text{(Ans.)}$$

1-33 A time-dependent, two-dimensional motion has three velocity components that are given by

$$u = \frac{x}{1+at} \qquad v = \frac{y}{1+bt} \qquad w = 0$$

where a and b are pure constants. The objective of this problem is to compare and contrast the streamlines in this flow with the pathlines of the fluid particles.

(*a*) Find the equations governing the streamline that passes through the point $(1,1)$ at time t.

(*b*) Calculate the path of a particle that starts at $r_0 = (x_0, y_0) = (1,1)$ at $t = 0$. Determine the location of a particle at $t = 1$, denoted as r_1.

(*c*) Use the results of part (*a*) to determine the condition under which the streamlines and pathlines coincide.

Solution: The geometric requirement for two-dimensional streamlines is as follows:

$$\frac{v}{u}=\frac{dy}{dx}=\frac{y(1+at)}{x(1+bt)}$$

By separating the variables, $\frac{dy}{y}=\left(\frac{1+at}{1+bt}\right)\frac{dx}{x}$.

By integrating, $\ln(y)=\left(\frac{1+at}{1+bt}\right)\ln(x)+\ln(C)$.

Solving this gives $y=Cx^{\left(\frac{1+at}{1+bt}\right)}$ (where C is the integration constant).

The condition of the streamline passing through the point $(1,\ 1)$ at time t is $C=1$ must be satisfied.

Therefore, the governing equation is $y=x^{\left(\frac{1+at}{1+bt}\right)}$ … (1) (Ans.)

The rate of change of x component of particle velocity with respect to time is $\frac{dx}{dt}=\frac{x}{1+at}$.

By separating the variables and integrating, $x=C_1(1+at)^{\frac{1}{a}}$ (where C_1 is the integration constant).

The rate of change of y component of particle velocity with respect to time is $\frac{dy}{dt}=\frac{y}{1+bt}$.

By separating the variables and integrating, $y=C_2(1+bt)^{\frac{1}{b}}$ (where C_2 is the integration constant).

At $t=0$, $x=x_0=1=C_1$ and $y=y_0=1=C_2$.

Therefore, the path of the particle is $y=\frac{(1+bt)^{\frac{1}{b}}}{(1+at)^{\frac{1}{a}}}x$ … (2) (Ans.)

Therefore, at $t=1$, $x=x_1=(1+a)^{\frac{1}{a}}$ and $y=y_1=(1+b)^{\frac{1}{b}}$.

Then, $r_1=(x_1,\ y_1)=\left((1+a)^{\frac{1}{a}},\ (1+b)^{\frac{1}{b}}\right)$. (Ans.)

Comparing equations (1) and (2), we can say that the condition under which the streamlines coincide with pathlines is $a=b=0$.
(Ans.)

1-34 A tornado may be simulated as a two-part circulating flow in cylindrical coordinates, with $v_r = v_z = 0$,

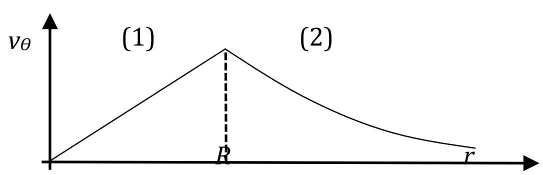
$$v_\theta = \begin{cases} \omega r & (r \le R) \\ \dfrac{\omega R^2}{r} & (r \ge R) \end{cases}$$

(*a*) Calculate the divergence of the velocity. Is the flow compressible or incompressible?

(*b*) Determine the vorticity. Is the flow rotational or irrotational?

(*c*) Determine the strain rates in each segment of the flow. What is the sum of the three normal strain rates?

Solution:



Divergence of velocity is $\nabla \cdot v = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$ (incompressible). (Ans.)

Vorticity is $\Omega = \nabla \times v = \left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z}\right)\hat{} \quad \left(\partial z \quad \partial r \right)\hat{} \quad \left(r \quad \partial r \quad r\, \partial\theta \right)\hat{}$.

Only nonzero component of vorticity is $\Omega_z = \frac{1}{r}\frac{\partial(r v_\theta)}{\partial r}$. (Ans.)

For segment (1), $\Omega_z = 2\omega$ (nonzero; thus rotational). (Ans.)

For segment (2), $\Omega_z = 0$ (irrotational). (Ans.)

Only nonzero component of tangential strain rate is $\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right)$. (Ans.)

For segment (1), $\varepsilon_{r\theta} = 0$. (Ans.)

For segment (2), $\varepsilon_{r\theta} = -\frac{\omega R^2}{r^2}$. (Ans.)

The sum of normal strain rate components is $\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz} = \frac{\partial v_r}{\partial r} + \left(\frac{v_r}{r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} \right) + \frac{\partial v_z}{\partial z} = 0$.

(Ans.)

1-35 In modeling the motion of an 8-meter diameter tornado rotating at an angular speed of ω at the point of maximum swirl, it is possible to use the Maicke–Majdalani profile (Maicke and Majdalani 2009) as a piecewise approximation for which $v_r = v_z = 0$ and the tangential velocity is given by

$$v_\theta(r) = \begin{cases} 16\omega r\left[1-\ln\left(r^2\right)\right] & 0 < r \le 1 \quad \text{(inner, forced vortex segment)} \\ \dfrac{16\omega}{r} & r > 1 \quad \text{(outer, free vortex segment)} \end{cases}$$

(*a*) State whether the flow is 1D, 2D, or 3D; steady or unsteady; and specify $v_\theta(r)$ as $r \to \infty$.

(*b*) Calculate the divergence of the velocity. Is the flow compressible or incompressible?

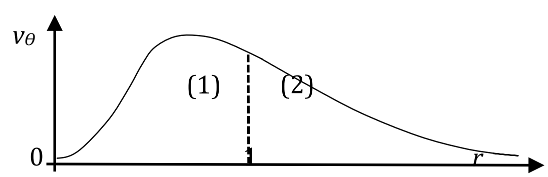
(*c*) Determine the vorticity. Is the flow rotational or irrotational?

(*d*) Determine the strain rates and the shear stresses in the inner and outer flow segments.

(*e*) What is the limit of $v_\theta(r)$ as $r \to 0$? Hint: In taking the limit, it is helpful to remember that $(\ln u)' = \frac{u'}{u}$ and that, in the inner segment, the tangential velocity can be rewritten as

$$v_\theta = 16\omega \frac{\left[1-\ln\left(r^2\right)\right]}{r^{-1}}$$

Solution:



The velocity varies with respect to the radial distance r from the centerline and is independent of the axial distance z or of the angular position θ. This represents a typical one-dimensional flow. (Ans.)

Since $\frac{\partial v_r}{\partial t} = \frac{\partial v_\theta}{\partial t} = \frac{\partial v_z}{\partial t} = 0$, the flow is time invariant (steady). (Ans.)

As $r \to \infty$, $v_\theta(r) \to 0$. (Ans.)

Divergence of velocity is $\nabla \cdot v = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$ (incompressible). (Ans.)

Vorticity is $\Omega = \nabla \times v = \left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z} \right)\hat{}\quad \left(\quad \partial z \quad \partial r \right)\hat{}\quad \left(r \quad \partial r \quad r\,\partial \theta \right)\hat{}$.

Only nonzero component of vorticity is $\Omega_z = \frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}$.

For segment (1), $\Omega_z = 32\omega \ln\left(\frac{1}{r^2}\right)$ (rotational). (Ans.)

For segment (2), $\Omega_z = 0$ (irrotational). (Ans.)

Only nonzero component of tangential strain rate is $\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right)$.

For segment (1), $\varepsilon_{r\theta} = -16\omega$. (Ans.)

For segment (2), $\varepsilon_{r\theta} = -\frac{16\omega}{r^2}$.

(Ans.)

Then, shear stress for segment (1) is $\tau_{r\theta} = -32\mu\omega$ and segment (2) is $\tau_{r\theta} = -\frac{32\mu\omega}{r^2}$. (Ans.)

As $r \to 0$, $v_\theta(r) \to 0$.

(Ans.)

1-36 The Taylor profile, which has been used to describe the bulk gaseous motion in planar, slab rocket chambers (Maicke and Majdalani 2008), corresponds to a self-similar profile in porous channels that bears symmetry with respect to the chamber's midsection plane. Using normalized Cartesian coordinates, the streamfunction may be written as $\psi = x \sin\left(\frac{1}{2}\pi y\right)$; $0 \le y \le 1$, and $0 \le x \le l$, where l represents the aspect ratio of the chamber (i.e., the length of the porous surface normalized by the chamber half height). In this problem, the velocity vector, normalized by the wall injection speed, may be expressed as $\mathrm{V}(x, y) = u\mathrm{i} + v\mathrm{j}$.