Chapter 1 Solutions

Problem 1.3-1

$L = 4.25m \quad q_0 = 175\frac{N}{m} \qquad P = 225N \qquad M_0 = 410N{\cdot}m$

Reactions

$$\Sigma F_x = 0 \qquad B_x = \frac{3}{5}\cdot P = 135\cdot N$$

$$\Sigma M_A = 0 \qquad B_y = \frac{1}{L}\cdot\left[-M_0 + \left(\frac{1}{2}\cdot q_0\right)\cdot L\cdot\left(\frac{2\cdot L}{3}\right) + \frac{4}{5}\cdot P\cdot\left(L + \frac{L}{2}\right)\right] = 421.446\cdot N$$

$$\Sigma F_y = 0 \qquad A_y = \left(\frac{1}{2}\cdot q_0\right)\cdot L + \frac{4}{5}\cdot P - B_y = 130.429\cdot N$$

N, V and M at midspan of AB - LHFB is used below

$$N_{mid} = 0$$

$$V_{mid} = A_y - \frac{1}{2}\cdot\frac{q_0}{2}\cdot\frac{L}{2} = 37.46\cdot N$$

$$M_{mid} = -M_0 + A_y\cdot\frac{L}{2} - \frac{1}{2}\cdot\frac{q_0}{2}\cdot\frac{L}{2}\cdot\left(\frac{1}{3}\cdot\frac{L}{2}\right) = -198.691\cdot N\cdot m$$

Problem 1.3-2

$$L = 4m \qquad q_0 = 160\frac{N}{m} \qquad P = 200 \cdot N \qquad M_0 = 380 N \cdot m$$

Reactions

$$\Sigma F_x = 0 \qquad B_x = \frac{-3}{5} \cdot P = -120\,N$$

$$\Sigma M_A = 0 \qquad B_y = \frac{1}{L} \cdot \left[M_0 + \left(\frac{1}{2} \cdot q_0\right) \cdot L \cdot \left(\frac{L}{3}\right) - \frac{4}{5} \cdot P \cdot \left(L + \frac{L}{2}\right)\right] = -38.333 \cdot N$$

$$\Sigma F_y = 0 \qquad A_y = \left(\frac{1}{2} \cdot q_0\right) \cdot L - \frac{4}{5} \cdot P - B_y = 198.333 \cdot N$$

N, V and M at midspan of AB - LHFB is used below

$$N_{mid} = 0$$

$$V_{mid} = A_y - \frac{1}{2} \cdot \left(\frac{q_0}{2} + q_0\right) \cdot \frac{L}{2} = -41.667 \cdot N$$

$$M_{mid} = M_0 + A_y \cdot \frac{L}{2} - \frac{1}{2} \cdot q_0 \cdot \frac{L}{2} \cdot \left(\frac{2}{3} \cdot \frac{L}{2}\right) - \frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} \cdot \left(\frac{1}{3} \cdot \frac{L}{2}\right) = 510 \cdot N \cdot m$$

Check using RHFB

$$N_{mid} = B_x + \frac{3}{5} \cdot P = 0\,N \qquad V_{mid} = \frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} - B_y - \frac{4}{5} \cdot P = -41.667\,N$$

$$M_{mid} = \frac{-1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} \cdot \left(\frac{1}{3} \cdot \frac{L}{2}\right) + B_y \cdot \frac{L}{2} + \frac{4}{5} \cdot P \cdot \left(\frac{L}{2} + \frac{L}{2}\right) = 510 \cdot N \cdot m$$

Problem 1.3-3

(a) APPLY LAWS OF STATICS

$\Sigma F_x = 0 \qquad C_x = N_A - N_2 = 220\text{ N}$

FBD of BC: $\quad \Sigma M_B = 0 \qquad C_y = \frac{1}{L_1}(0) = 0$

Entire FBD: $\quad \Sigma M_A = 0 \qquad B_y = \frac{1}{L_2}(-M_B) = -22.667\text{ N}$

$\Sigma F_y = 0 \qquad A_y = -B_y = 22.667\text{ N}$

Reactions are $\boxed{A_y = 22.7\text{ N}}$ $\boxed{B_y = -22.7\text{ N}}$ $\boxed{C_x = 220\text{ N}}$ $\boxed{C_y = 0}$

(b) INTERNAL STRESS RESULTANTS N, V, AND M AT $x = 4.5$ m

Use FBD of segment from A to $x = 4.5$ m.

$\Sigma F_x = 0 \qquad \boxed{N_x = N_A - N_2 = 220\text{ N}}$

$\Sigma F_y = 0 \qquad \boxed{V_x = A_y = 22.7\text{ N}}$

$\Sigma M = 0 \qquad \boxed{M_x = A_y(4.5\text{ m}) = 102\text{ N}\cdot\text{m}}$

Problem 1.3-4

(a) Apply laws of statics

$\Sigma F_x = 0 \qquad A_x = 0$

FBD of $AB \qquad \Sigma M_B = 0 \qquad M_A = 0$

Entire FBD $\qquad \Sigma M_C = 0 \qquad D_y = \frac{1}{3\text{ m}}\left[200\text{ N}\cdot\text{m} - \frac{1}{2}(80\text{ N/m})\,4\text{ m}\left(\frac{2}{3}\right)4\text{ m}\right] = -75.556\text{ N}$

$\Sigma F_y = 0 \qquad C_y = \frac{1}{2}(80\text{ N/m})\,4\text{ m} - D_y = 235.556\text{ N}$

Reactions are $\boxed{M_A = 0} \qquad A_x = 0 \qquad \boxed{C_y = 236\text{ N}} \qquad \boxed{D_y = -75.6\text{ N}}$

(b) Internal stress resultants N, V, and M at $x = 5$ m

Use FBD of segment from A to $x = 5$ m; ordinate on triangular load at $x = 5$ m is $\frac{3}{4}(80\text{ N/m}) = 60\text{ N/m}$.

$\Sigma F_x = 0 \qquad N_x = -A_x = 0$

$\Sigma F_y = 0 \qquad V = \frac{-1}{2}[(80\text{ N/m} + 60\text{ N/m})\,1\text{ m}] = -70\text{ N} \qquad \boxed{V = -70\text{ N}}$ Upward

$\Sigma M = 0 \qquad M = -M_A - \frac{1}{2}(80\text{ N/m})\,1\text{ m}\left(\frac{2}{3}1\text{ m}\right) - \frac{1}{2}(60\text{ N/m})\,1\text{ m}\left(\frac{1}{3}1\text{ m}\right) = -36.667\text{ N}\cdot\text{m}$

(break trapezoidal load into two triangular loads in moment expression)

$\boxed{M = -36.7\text{ N}\cdot\text{m}}$ CW

(c) Replace roller support at C with spring support

Structure remains statically determinate so all results above in (a) and (b) are unchanged.

Problem 1.3-5

(a) Statics

FBD of AB (cut through beam at pin): $\Sigma M_B = 0 \qquad A_y = \frac{1}{3\text{ m}}(-M_A) = -\frac{200\text{ N}}{3} = -66.667\text{ N}$

Entire FBD: $\Sigma M_D = 0 \qquad L_1 = 3\text{ m}$

$$C_y = \frac{1}{3\text{ m}}\left[\frac{4}{5}P(1.5\text{ m}) + \frac{1}{2}q_2L_1\left(L_1 + \frac{L_1}{3}\right) + \frac{1}{2}q_1L_1\left(L_1 + \frac{2}{3}L_1\right) - M_A - A_y(3L_1)\right] = 464.333\text{ N}$$

$$\Sigma F_y = 0 \quad D_y = \frac{4}{5}P + \frac{1}{2}(q_1 + q_2)L_1 - A_y - C_y = -87.167\text{ N} \quad \text{so} \quad D_x = \frac{-D_y}{\tan(60°)} = 50.326\text{ N}$$

$$\Sigma F_x = 0 \quad A_x = \frac{3}{5}P - D_x = 57.674\text{ N}$$

CHECK

$A_x = 57.7\text{ N} \qquad A_y = -66.7\text{ N} \qquad C_y = 464.3\text{ N} \qquad A_x + D_x = 108\text{ N} \qquad P\frac{3}{5} = 108\text{ N}$

$D_x = 50.3\text{ N} \qquad D_y = -87.2\text{ N} \qquad A_y + C_y + D_y = 310.5\text{ N} \qquad \frac{4}{5}P + \frac{1}{2}(q_1 + q_2)L_1 = 310.5\text{ N}$

(b) Use FBD of AB only; moment at pin is zero

$F_{Bx} = -A_x \qquad F_{Bx} = -57.674\text{ N} \qquad F_{By} = -A_y \qquad F_{By} = 66.667\text{ N} \qquad \boxed{\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 88.2\text{ N}}$

(c) Add rotational spring at A & remove roller at C; apply equations of statical equilibrium

Use FBD of BCD: $\Sigma M_B = 0 \qquad D_y = \frac{1}{2L_1}\left[\frac{1}{2}q_2L_1\left(\frac{2}{3}L_1\right) + \frac{1}{2}q_1L_1\left(\frac{1}{3}L_1\right) + \frac{4}{5}P\left(\frac{3}{2}L_1\right)\right] = 145\text{ N}$

so $\qquad D_x = \frac{-D_y}{\tan(60°)} = -83.716\text{ N}$

Use entire FBD: $\Sigma F_y = 0 \qquad A_y = \frac{1}{2}(q_1 + q_2)L_1 + \frac{4}{5}P - D_y = 165.5\text{ N}$

$\Sigma F_x = 0 \qquad A_x = \frac{3}{5}P - D_x = 191.716\text{ N}$

Use FBD of AB: $\Sigma M_B = 0 \qquad M_{Az} = M_A + A_yL_1 = 696.5\text{ N}\cdot\text{m}$

So reactions are $\boxed{A_x = 191.7\text{ N}} \quad \boxed{A_y = 165.5\text{ N}} \quad \boxed{M_{Az} = 697\text{ N}\cdot\text{m}} \quad \boxed{D_x = -83.72\text{ N}} \quad \boxed{D_y = 145\text{ N}}$

Resultant force in pin connection at B

$F_{Bx} = -A_x \qquad F_{By} = -A_y \qquad \boxed{\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 253\text{ N}}$

Problem 1.3-6

(a) STATICS

$\Sigma F_y = 0 \qquad R_{3y} = 20\text{ N} - 45\text{ N} = -25\text{ N}$

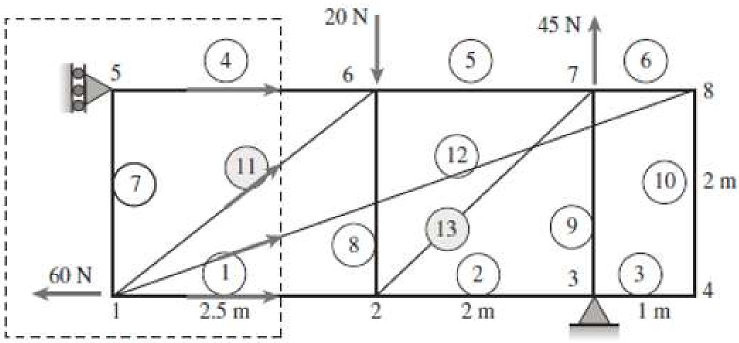
$\Sigma M_3 = 0 \qquad R_{5x} = \frac{1}{2\text{ m}}(20\text{ N} \times 2\text{ m}) = 20\text{ N}$

$\Sigma F_x = 0 \qquad R_{3x} = -R_{5x} + 60\text{ N} = 40\text{ N}$

(b) MEMBER FORCES IN MEMBERS 11 and 13

Number of unknowns: $m = 13 \qquad r = 3 \qquad m + r = 16$

Number of equations: $j = 8 \qquad 2j = 16$ So statically determinate



Section cut for left FBD

TRUSS ANALYSIS

(1) $\Sigma F_V = 0$ at joint 4 so $F_{10} = 0$

(2) $\Sigma F_V = 0$ at joint 8 so $F_{12} = 0$

(3) $\Sigma F_H = 0$ at joint 5 so $F_4 = -R_{5x} = -20\text{ N}$

(4) Cut vertically through 4, 11, 12, and 1; use left FBD; sum moments about joint 2

$F_{11V} = \frac{1}{2.5\text{ m}}(R_{5x} - F_4)$ so $\boxed{F_{11} = 0}$

(5) Sum vertical forces at joint 3; $F_9 = -R_{3y}$

$F_9 = 25\text{ N}$

(6) Sum vertical forces at joint 7 $\qquad F_{13V} = 45\text{ N} - F_9 = 20\text{ N} \qquad \boxed{F_{13} = \sqrt{2}F_{13V} = 28.3\text{ N}}$

Problem 1.3-7

(a) STATICS $\quad P = 4.5\text{ kN} \quad L_1 = 3\text{ m}$

$\Sigma F_x = 0 \quad A_x = 0$

$\Sigma M_A = 0 \quad E_y = \frac{1}{6\text{ m}}[3P(3\text{ m}) + 2P(6\text{ m}) + P(9\text{ m})] = 22.5\text{ kN}$

$\Sigma F_y = 0 \quad A_y = 3P + 2P + P - E_y = 4.5\text{ kN}$

(b) MEMBER FORCE IN MEMBER *FE*

Number of unknowns: $\quad m = 11 \quad r = 3 \quad m + r = 14$

Number of equations: $\quad j = 7 \quad 2j = 14 \quad$ so statically determinate

TRUSS ANALYSIS

(1) Cut vertically through *AB*, *GC*, and *GF*; use left FBD; sum moments about *C*.

$$F_{GFx}(4.5\text{ m}) - F_{GFy}(6\text{ m}) = A_y(6\text{ m}) = 27\text{ kN·m} \qquad F_{GFx} = F_{GF}\frac{3\text{ m}}{\sqrt{(0.5\text{ m})^2 + (3\text{ m})^2}}$$

$$F_{GFy} = F_{GF}\frac{0.5\text{ m}}{\sqrt{(0.5\text{ m})^2 + (3\text{ m})^2}}$$

$$\text{so} \quad F_{GF} = \frac{A_y(6\text{ m})}{4.5\text{ m}\frac{3\text{ m}}{\sqrt{(0.5\text{ m})^2 + (3\text{ m})^2}} - 6\text{ m}\frac{0.5\text{ m}}{\sqrt{(0.5\text{ m})^2 + (3\text{ m})^2}}} = 7.821\text{ kN} \quad \text{and}$$

$$F_{GFx} = F_{GF}\frac{3\text{ m}}{\sqrt{(0.5\text{ m})^2 + (3\text{ m})^2}} = 7.714\text{ kN}$$

(2) Sum horizontal forces at joint *F*: $\quad F_{FEx} = F_{GFx} = 7.714\text{ kN} \qquad F_{FE} = \frac{\sqrt{(3\text{ m})^2 + (1\text{ m})^2}}{3\text{ m}}F_{FEx} = 8.132\text{ kN}$

$$\boxed{F_{FE} = 8.13\text{ kN}}$$

Problem 1.3-8

(a) Statics

$\Sigma F_x = 0 \qquad F_x = 0$

$\Sigma M_F = 0 \qquad D_y = \dfrac{1}{6\text{ m}}\left[3\text{ kN}(6\text{ m}) + 6\text{ kN}(3\text{ m})\right] = 6\text{ kN}$

$\Sigma F_y = 0 \qquad F_y = 9\text{ kN} + 6\text{ kN} + 3\text{ kN} - D_y = 12\text{ kN}$

(b) Member force in member FE

Number of unknowns: $m = 11 \qquad r = 3 \qquad m + r = 14$

Number of equations: $j = 7 \qquad 2j = 14$ So statically determinate

Truss analysis

(1) Cut vertically through AB, GD, and GF; use left FBD; sum moments about D to get $F_{GF} = 0$

(2) Sum horizontal forces at joint $F \qquad F_{FEx} = -F_x = 0$ so $\boxed{F_{FE} = 0}$

Problem 1.3-9

$c = 2.5m \quad P = 90kN$

$$a = \frac{\sin(60deg)}{\sin(80deg)} \cdot c = 2.198 \cdot m \qquad b = \frac{\sin(40deg)}{\sin(80deg)} \cdot c = 1.632 \cdot m$$

$$\Sigma M_A = P \cdot \frac{c}{2} - P \cdot b \cdot \cos(60deg) - 2P \cdot b \cdot \sin(60deg) + B_y \cdot c = 0$$

$$B_y = \frac{P \cdot b \cdot \cos(60deg) + 2P \cdot b \cdot \sin(60deg) - P \cdot \frac{c}{2}}{c} = 86.118 \cdot kN \qquad A_y = -B_y = -86.118 \cdot kN$$

$$A_x = 0$$

Joint A

$$F_{AC} = \frac{-A_y}{\sin(60deg)} = 99.441 \cdot kN \qquad F_{AD} = -F_{AC} \cdot \cos(60deg) - A_x = -49.72 \cdot kN$$

Joint B

$$F_{BC} = \frac{-B_y}{\sin(40deg)} = -133.976 \cdot kN$$

$$F_{BD} = -F_{BC} \cdot \cos(40deg) = 102.632 \cdot kN$$

$$CD = \sqrt{b^2 + \left(\frac{c}{2}\right)^2 - 2 \cdot b \cdot \frac{c}{2} \cdot \cos(60deg)} = 1.478 \cdot m$$

Joint D

$$ACD = \operatorname{asin}\left(\frac{\sin(60deg)}{CD} \cdot \frac{c}{2}\right) = 47.077 \cdot deg$$

$$F_{DC} = \frac{-P}{\cos(90deg - 72.923deg)} = -94.151 \cdot kN$$

$$180deg - 60deg - ACD = 72.923 \cdot deg$$

$$BCD = \operatorname{asin}\left(\frac{\sin(40deg)}{CD} \cdot \frac{c}{2}\right) = 32.923 \cdot deg$$

$$\frac{a \cdot \sin(BCD)}{\sin(ACD)} = 1.632 \cdot m$$

$$ACD + BCD = 80 \cdot deg$$

Problem 1.3-10

Geometry $b = 3m$ $P = 80kN$

$$a = \sin(60deg)\cdot\left(\frac{b}{\sin(40deg)}\right) = 4.042\,m$$

$$L_{AB} = \sin(80deg)\cdot\left(\frac{b}{\sin(40deg)}\right) = 4.596\,m$$

$$L_{DB} = \sqrt{\left(\frac{b}{2}\right)^2 + L_{AB}^2 - 2\cdot\left(\frac{b}{2}\right)\cdot(L_{AB})\cdot\cos(60deg)} = 4.06\,m$$

$$\frac{L_{DB}}{\sin(60deg)} = \frac{\frac{b}{2}}{\sin(DBA)} \quad \text{so} \quad DBA = \operatorname{asin}\left(\frac{\frac{b}{2}}{L_{DB}}\cdot\sin(60deg)\right) = 18.662\cdot deg$$

and $CBD = 40deg - DBA = 21.338\cdot deg$ $ADB = 180deg - 60deg - DBA = 101.338\cdot deg$

$CDB = 180deg - ADB = 78.662\cdot deg$

Reactions

$\Sigma F_x = 0$ $A_x = -2\cdot P + 2\cdot P = 0\,N$

$$\Sigma M_A = 0 \qquad B_y = \frac{1}{L_{AB}}\cdot\left[-2\cdot P\cdot\left(\frac{b}{2}\cdot\sin(60deg)\right) + P\cdot(b\cdot\cos(60deg)) + 2\cdot P\cdot(b\cdot\sin(60deg))\right] = 71.329\cdot kN$$

$\Sigma F_y = 0$ $A_y = P - B_y = 8.671\cdot kN$

MoJ to find member forces

Joint A $AD = \dfrac{-A_y}{\sin(60deg)} = -10.013\cdot kN$ $AB = -A_x - AD\cdot\cos(60deg) = 5.006\cdot kN$

Joint D - sum forces normal to & along line ADC $DB = \dfrac{2\cdot P\cdot\sin(60deg)}{\cos(90deg - CDB)} = 141.322\cdot kN$

$$DC = AD + 2\cdot P\cdot(\cos(60deg)) - DB\cdot\cos(CDB) = 42.204\cdot kN$$

Joint C $CB = \dfrac{1}{\cos(40deg)}\cdot(-2\cdot P + DC\cdot\cos(60deg)) = -181.319\cdot kN$

Joint B check $-AB - DB\cdot\cos(DBA) - CB\cdot\cos(40deg) = 0\,N$

$$DB\cdot\sin(DBA) + CB\cdot\sin(40deg) + B_y = 0\,N$$

Problem 1.3-11

Reactions $c = 2.5m \; P = 90kN$

$$a = \frac{\sin(60deg)}{\sin(80deg)} \cdot c = 2.198 \cdot m \qquad b = \frac{\sin(40deg)}{\sin(80deg)} \cdot c = 1.632 \cdot m$$

$$\Sigma M_A = P \cdot \frac{c}{2} - P \cdot b \cdot \cos(60deg) - 2P \cdot b \cdot \sin(60deg) + B_y \cdot c = 0$$

$$B_y = \frac{P \cdot b \cdot \cos(60deg) + 2P \cdot b \cdot \sin(60deg) - P \cdot \frac{c}{2}}{c} = 86.118 \cdot kN \qquad A_y = -B_y = -86.118 \cdot kN$$

AC: MoS - cut through AC and AD, use LHFB

$$\Sigma M_D = 0 \qquad -A_y \cdot \frac{c}{2} - AC \cdot \sin(60deg) \cdot \frac{c}{2} = 0$$

$$AC = \frac{-A_y}{\sin(60deg)} = 99.441 \cdot kN$$

BD: MoS - cut through BC andf BD, use RHFB $\qquad b = \frac{\sin(40deg)}{\sin(80deg)} \cdot c = 1.632 \cdot m$

$$\Sigma M_C = 0 \qquad B_y \cdot (c - b \cdot \cos(60deg)) - BD \cdot (b \cdot \sin(60deg)) = 0$$

$$BD = \frac{B_y \cdot (c - b \cdot \cos(60deg))}{b \cdot \sin(60deg)} = 102.632 \cdot kN$$

Problem 1.3-12

<u>Reactions</u> $b = 3m$ $P = 80kN$

$A_x = 0$ $A_y = 8.671kN$ $B_y = 71.329kN$

<u>AB: MoS - cut through AD and AB, use LHFB</u>

$\Sigma M_D = 0$ $AB \cdot \frac{b}{2} \cdot \sin(60deg) + A_x \cdot \frac{b}{2} \cdot \sin(60deg) - A_y \cdot \frac{b}{2} \cdot \cos(60deg) = 0$

$$AB = \frac{-\left(A_x \cdot \frac{b}{2} \cdot \sin(60deg) - A_y \cdot \frac{b}{2} \cdot \cos(60deg)\right)}{\left(\frac{b}{2} \cdot \sin(60deg)\right)} = 5.006 \cdot kN$$

<u>DC: MoS - cut through DC and CB, use upper FBD</u> $a = \sin(60deg) \cdot \left(\frac{b}{\sin(40deg)}\right) = 4.042\,m$

$DC_x = DC \cdot \cos(60deg)$ $DC_y = DC \cdot \sin(60deg)$

$\Sigma M_B = 0$ $-(-DC_x + 2 \cdot P) \cdot (a \cdot \sin(40deg)) + (DC_y + P) \cdot (a \cdot \cos(40deg)) = 0$

$$-(-DC \cdot \cos(60deg) + 2 \cdot P) \cdot (a \cdot \sin(40deg)) + (DC \cdot \sin(60deg) + P) \cdot (a \cdot \cos(40deg)) = 0$$

Collect and simplify, solve for DC

$$DC = \frac{1.0 \cdot (80.0 \cdot kN \cdot \cos(40.0 \cdot deg) - 160.0 \cdot kN \cdot \sin(40.0 \cdot deg))}{\cos(60.0 \cdot deg) \cdot \sin(40.0 \cdot deg) + \sin(60.0 \cdot deg) \cdot \cos(40.0 \cdot deg)} = 42.204 \cdot kN$$

Problem 1.3-13

(a) FIND REACTIONS USING STATICS $\quad m = 3 \quad r = 9 \quad m + r = 12 \quad j = 4 \quad 3j = 12$

$m + r = 3j$ so truss is statically determinate

$$r_{AQ} = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} \quad r_{OA} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \quad e_{AQ} = \frac{r_{AQ}}{|r_{AQ}|} = \begin{pmatrix} 0.8 \\ -0.6 \\ 0 \end{pmatrix} \quad P_A = P e_{AQ} = \begin{pmatrix} 0.8P \\ -0.6P \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

$\Sigma M = 0$

$$M_O = r_{OA} \times P_A + r_{OC} \times \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} + r_{OB} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 4C_z + 3.0P \\ 4.0P - 2B_z \\ 2B_y - 4C_x \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \text{ gives } C_z = \frac{-3}{4}P$$

$$\Sigma M_y = 0 \text{ gives } \boxed{B_z = 2P}$$

$\Sigma F = 0$

$$R_O = P_A + \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} = \begin{pmatrix} B_x + C_x + O_x + 0.8P \\ B_y + C_y + O_y + -0.6P \\ O_z + \frac{5P}{4} \end{pmatrix} \quad \text{so} \quad \Sigma M_z = 0 \text{ gives } \boxed{O_z = \frac{-5}{4}P}$$

METHOD OF JOINTS $\quad$ Joint O $\quad \Sigma F_x = 0 \quad O_x = 0 \quad \Sigma F_y = 0 \quad O_y = 0$

Joint B $\quad \Sigma F_y = 0 \quad B_y = 0$

Joint C $\quad \Sigma F_x = 0 \quad C_x = 0$

For entire structure: $\Sigma F_x = 0$ gives $\boxed{B_x = -0.8P}$ $\quad \Sigma F_y = 0 \quad C_y = 0.6P - B_y = O_y \quad C_y = 0.6P$

(b) FORCE IN MEMBER AC

$$\Sigma F_z = 0 \text{ at joint } C: \quad F_{AC} = \frac{\sqrt{4^2 + 5^2}}{5}|C_z| = \frac{3\sqrt{41}\,|P|}{20} \qquad \boxed{F_{AC} = \frac{3\sqrt{41}}{20}P} \quad \text{tension} \qquad \frac{3\sqrt{41}}{20} = 0.96$$

Problem 1.3-14

(a) FIND REACTIONS USING STATICS $m = 4 \quad r = 8 \quad m + r = 12 \quad j = 4 \quad 3j = 12$

$m + r = 3j$ so truss is statically determinate

$$r_{OA} = \begin{pmatrix} 0 \\ 0 \\ 0.8L \end{pmatrix} \quad r_{OB} = \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 0.6L \\ 0 \end{pmatrix} \quad F_A = \begin{pmatrix} A_x \\ A_y \\ P \end{pmatrix} \quad F_B = \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} \quad F_c = \begin{pmatrix} C_x \\ -2P \\ 0 \end{pmatrix} \quad F_O = \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix}$$

$\Sigma M = 0$

Resultant moment at O

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B + r_{OC} \times F_C = \begin{pmatrix} -0.8A_yL \\ 0.8A_xL - B_zL \\ B_yL - 0.6C_xL \end{pmatrix} \quad so \quad \Sigma M_x = 0 \text{ gives } A_y = 0$$

$\Sigma F = 0$

Resultant force at O

$$R_O = F_O + F_A + F_B + F_C = \begin{pmatrix} A_x + C_x + O_x \\ A_y + B_y + O_y - 2P \\ B_z + O_z + P \end{pmatrix}$$

METHOD OF JOINTS Joint O $\Sigma F_z = 0 \quad O_z = 0$

so from $\Sigma F_z = 0$ $\boxed{B_z = -P}$ and $\Sigma M_y = 0$ $\boxed{A_x = \frac{B_z}{0.8} = -1.25P}$

Joint B $\Sigma F_y = 0$ $\boxed{B_y = 0}$

Joint C $\Sigma F_x = 0 \quad C_x = 0$

(b) FORCE IN MEMBER AB

$\Sigma F_z = 0$ at joint B $F_{AB} = \frac{\sqrt{(0.8L)^2 + L^2}}{0.8L}|B_z| \quad |B_z| = |P| \quad \frac{\sqrt{(0.8L)^2 + L^2}}{0.8L} = 1.601$

$\boxed{F_{AB} = 1.601P}$ tension

Problem 1.3-15

(a) FIND REACTIONS USING STATICS $m = 3 \quad r = 6 \quad m + r = 9 \quad j = 3 \quad 3j = 9$

$m + r = 3j$ So truss is statically determinate

$$r_{OA} = \begin{pmatrix} 3L \\ 0 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 0 \\ 4L \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 2L \\ 4L \end{pmatrix} \quad F_A = \begin{pmatrix} -2P \\ A_y \\ A_z \end{pmatrix} \quad F_B = \begin{pmatrix} B_x \\ B_y \\ 3P \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ C_y \\ P \end{pmatrix}$$

$\Sigma M = 0$

Resultant moment at O

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B + r_{OC} \times F_C = \begin{pmatrix} 14LP - 4C_yL \\ 4C_xL - 3A_zL \\ 3A_yL - 4B_xL - 2C_xL \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \quad \text{gives} \quad C_y = \frac{14}{4}P$$

$\Sigma F = 0$

Resultant force at O

$$R_O = F_A + F_B + F_C = \begin{pmatrix} B_x + C_x - 2P \\ A_y + B_y + C_y \\ A_z + 4P \end{pmatrix} \quad \text{so} \quad \Sigma F_z = 0 \quad \text{gives} \quad \boxed{A_z = -4.0P}$$

METHOD OF JOINTS

Joint A $\quad \Sigma F_z = 0 \quad F_{ACz} = -A_z = 4.0P \quad$ so $\quad F_{ACy} = \frac{2}{4}F_{ACz} = 2.0P \quad F_{ACx} = \frac{3}{4}F_{ACz} = 3.0P$

$\Sigma F_x = 0 \quad F_{ABx} = -2P - F_{ACx} = -3.0P - 2P \quad$ so $\quad F_{ABy} = \frac{4}{3}F_{ABx} = -4.0P - \frac{8P}{3}$

$\Sigma F_y = 0 \quad A_y = -(F_{ABy} + F_{ACy}) = \frac{8P}{3} + 4.0P + -2.0P \quad \boxed{A_y = 4.67P}$

(b) FORCE IN MEMBER AB

$$F_{AB} = \sqrt{F_{ABx}^2 + F_{ABy}^2} \quad F_{AB} = -\sqrt{5^2 + \left(\frac{20}{3}\right)^2}\,P = -\frac{25P}{3} \quad \frac{25}{3} = 8.33$$

$\boxed{F_{AB} = -8.33P}$ compression

Problem 1.3-16

(a) FIND REACTIONS USING STATICS $m = 3$ $r = 6$ $m + r = 9$ $j = 3$ $3j = 9$

$m + r = 3j$ so truss is statically determinate

$L = 2\text{ m}$ $P = 5\text{ kN}$

$$r_{OA} = \begin{pmatrix} 3L \\ 0 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 0 \\ 4L \\ 2L \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 0 \\ 4L \end{pmatrix} \quad F_A = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \quad F_B = \begin{pmatrix} B_x \\ 0 \\ P \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ C_y \\ -P \end{pmatrix}$$

$\Sigma F = 0$

Resultant force at O $\quad R_O = F_A + F_B + F_C = \begin{pmatrix} A_x + B_x + C_x \\ A_y + C_y \\ A_z \end{pmatrix}$ so $\Sigma F_z = 0$ gives $A_z = 0$

RESULTANT MOMENT AT A

$$r_{AC} = \begin{pmatrix} -3L \\ 0 \\ 4L \end{pmatrix} \quad e_{AC} = \frac{r_{AC}}{|r_{AC}|} = \begin{pmatrix} -0.6 \\ 0 \\ 0.8 \end{pmatrix} \quad r_{AB} = \begin{pmatrix} -3L \\ 4L \\ 2L \end{pmatrix}$$

$$M_A = r_{AB} \times F_B + r_{AC} \times F_C = \begin{pmatrix} 40\cdot\text{kN}\cdot\text{m} - 8\cdot C_y\cdot\text{m} \\ 4\cdot B_x\cdot\text{m} + 8\cdot C_x\cdot\text{m} \\ -8\cdot B_x\cdot\text{m} - 6\cdot C_y\cdot\text{m} \end{pmatrix} \quad M_A e_{AC} = -6.4\,B_x - 24.0\text{ kN} \quad \text{so} \quad \boxed{B_x = \frac{-24}{6.4}\text{kN} = -3.75\text{ kN}}$$

(b) FORCE IN MEMBER AB

Method of joints at B $\quad \Sigma F_x = 0 \quad F_{ABx} = -B_X \quad \boxed{F_{AB} = \frac{\sqrt{29}}{3}F_{ABx} = 6.73\text{ kN}}$

Problem 1.3-17

(a) APPLY LAWS OF STATICS

$\Sigma M_x = 0$ $\quad$ $\boxed{T_A = T_1 - T_2 = 1270\ \text{N}\cdot\text{m}}$

(b) INTERNAL STRESS RESULTANT T AT TWO LOCATIONS

Cut shaft at midpoint between A and B at $x = L_1/2$ (use left FBD). $\quad$ $\Sigma M_x = 0$ $\quad$ $\boxed{T_{AB} = -T_A = -1270\ \text{N}\cdot\text{m}}$

Cut shaft at midpoint between B and C at $x = L_1 + L_2/2$ (use right FBD). $\quad$ $\Sigma M_x = 0$ $\quad$ $\boxed{T_{BC} = T_2 = 1130\ \text{N}\cdot\text{m}}$

Problem 1.3-18

(a) Reaction torque at A $\quad L_1 = 0.75$ m $\quad L_2 = 0.75$ m $\quad t_1 = 3100$ N·m/m $\quad T_2 = 1100$ N·m

Statics $\quad \Sigma M_x = 0 \quad T_A = -t_1 L_1 + T_2 = -1225$ N·m $\quad \boxed{T_A = -1225 \text{ N·m}}$

(b) Internal torsional moments at two locations

Cut shaft between A and B (use left FBD) $\quad T_1(x) = -T_A - t_1 x \quad \boxed{T_1\left(\frac{L_1}{2}\right) = 62.5 \text{ N·m}}$

Cut shaft between B and C (use left FBD) $\quad T_2(x) = -T_A - t_1 L_1 \quad \boxed{T_2\left(L_1 + \frac{L_2}{2}\right) = -1100 \text{ N·m}}$

Problem 1.3-19

(a) Statics

$$\Sigma F_H = 0 \qquad A_x = \frac{-1}{2}\left(1300\ \frac{\text{N}}{\text{m}}\right)3.70\ \text{m} = -2405\ \text{N}$$

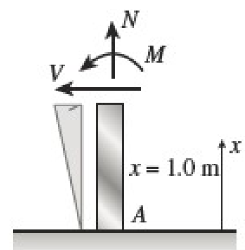
$$\Sigma F_V = 0 \qquad A_y + C_y = 0$$

$$\Sigma M_{FBDBC} = 0 \qquad C_y = \frac{680\ \text{N}\cdot\text{m}}{2.75\ \text{m}} = 247\ \text{N} \qquad A_y = -C_y = -247\ \text{N}$$

$$\Sigma M_A = 0 \qquad M_A = 680\ \text{N}\cdot\text{m} + \frac{1}{2}\left(1300\ \frac{\text{N}}{\text{m}}\right)3.70\ \text{m}\left(\frac{2}{3}\,3.70\ \text{m}\right) - C_y 2.75\ \text{m} = 5932\ \text{N}\cdot\text{m}$$

$$A_x = -2405\ \text{N} \qquad A_y = -247\ \text{N} \qquad M_A = 5932\ \text{N}\cdot\text{m} \qquad C_y = 247\ \text{N} \qquad \leftarrow$$

(b) Internal stress resultants



$$N_x = -A_y = 247\ \text{N}$$

$$V_x = -A_x - \frac{1}{2}\left(\frac{1.0}{3.70}\,1300\ \frac{\text{N}}{\text{m}}\right)1.0\ \text{m} = 2229\ \text{N}$$

$$M_x = -M_A - A_x 1.0\ \text{m} - \frac{1}{2}\left(\frac{1.0}{3.70}\,1300\ \frac{\text{N}}{\text{m}}\right)1.0\ \text{m}\left(\frac{1}{3}\,1.0\ \text{m}\right) = -3586\ \text{N}\cdot\text{m}$$

$$N_x = 247\ \text{N} \qquad V_x = 2229\ \text{N} \qquad M_x = -3586\ \text{N}\cdot\text{m} \qquad \leftarrow$$

Problem 1.3-20

(a) STATICS

$\Sigma F_x = 0 \qquad A_x = \frac{3}{5}(200\text{ N}) + \frac{1}{2}(80\text{ N/m})\,4\text{ m} = 280\text{ N}$

$\Sigma M_{BRHFB} = 0 \quad D_y = \frac{1}{3\text{ m}}\left[\frac{4}{5}(200\text{ N})(1.5\text{ m}) + \frac{1}{2}(80\text{ N/m})\,4\text{ m}\left(\frac{1}{3}4\text{ m}\right)\right]$

$= 151.1\text{ N}$ < use right hand FBD (*BCD* only)

$\Sigma F_y = 0 \qquad A_y = -D_y + \frac{4}{5}(200\text{ N}) = 8.89\text{ N}$

$\Sigma M_A = 0 \qquad M_A = \frac{4}{5}(200\text{ N})(1.5\text{ m}) - \frac{3}{5}(200\text{ N})(4\text{ m}) - D_y\,3\text{ m} - \frac{1}{2}(80\text{ N/m})\,4\text{ m}\left(\frac{2}{3}4\text{ m}\right) = -1120\text{ N·m}$

(b) RESULTANT FORCE IN PIN AT *B*

LEFT HAND FBD (SEE FIGURE)

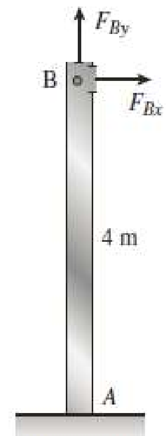
$F_{Bx} = -A_x = -280\text{ N} \qquad F_{By} = -A_y = -8.89\text{ N}$

RIGHT HAND FBD

$F_{Bx} = \frac{3}{5}(200\text{ N}) + \frac{1}{2}(80\text{ N/m})\,4\text{ m} = 280\text{ N}$

$F_{By} = \frac{4}{5}(200\text{ N}) - D_y = 8.89\text{ N}$

$\boxed{\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 280\text{ N}}$



Left hand FBD

Problem 1.3-21

$$L = 4.25m \; q_o = 175\frac{N}{m} \quad P = 225N \quad M_o = 400N \cdot m$$

$$\Sigma M_D = -M_o + \frac{1}{2} \cdot q_o \cdot L \cdot \frac{L}{3} - \frac{4}{5} \cdot P \cdot \frac{L}{2} + \frac{3}{5} P \cdot L - \frac{1}{2} \cdot q_o \cdot L \cdot \frac{2L}{3} - A_y \cdot L = 0$$

$$A_y = \frac{-M_o + \frac{1}{2} \cdot q_o \cdot L \cdot \frac{L}{3} - \frac{4}{5} \cdot P \cdot \frac{L}{2} + \frac{3}{5} P \cdot L - \frac{1}{2} \cdot q_o \cdot L \cdot \frac{2L}{3}}{L} = -173.076 \cdot N$$

$$D_y = -A_y + \frac{1}{2} \cdot q_o \cdot L + \frac{4}{5} \cdot P = 724.951 \cdot N$$

$$D_x = \frac{-1}{2} \cdot q_o \cdot L + \frac{3}{5} \cdot P = -236.875 \cdot N$$

$$V_{midAB} = A_y - \frac{1}{2} \cdot \frac{L}{2} \cdot \frac{q_o}{2} = -266.045 \cdot N \qquad N_{mid} = 0$$

$$N_{midAB} = 0$$

$$M_{midAB} = M_o + A_y \cdot \frac{L}{2} - \frac{1}{2} \cdot \frac{q_o}{2} \cdot \frac{L}{2} \cdot \frac{\frac{L}{2}}{3} = -33.639 \cdot N \cdot m$$

Problem 1.3-22

$$L = 4m \qquad q_0 = 160\cdot\frac{N}{m} \qquad P = 200N \qquad M_0 = 380\cdot N\cdot m$$

Reactions

$$\Sigma F_x = 0 \qquad A_x = 2\cdot\left(\frac{3}{5}\cdot P\right) - \frac{1}{2}\cdot q_0\cdot L = -80\,N$$

$$\Sigma M_A = 0 \qquad D_y = \frac{1}{L}\cdot\left[M_0 + \frac{4}{5}\cdot P\cdot\frac{L}{2} + \frac{4}{5}\cdot P\cdot\frac{3\cdot L}{2} - \frac{1}{2}\cdot q_0\cdot L\cdot\left(\frac{L}{3}\right)\right] = 308.333\,N$$

$$\Sigma F_y = 0 \qquad A_y = -D_y + 2\cdot\left(\frac{4}{5}\cdot P\right) = 11.667\,N$$

Column BD internal forces and moment at mid-height - cut through column, use lower FBD (D on your left)

$$N_{mid} = -D_y = -308.333\,N \qquad V_{mid} = \frac{-1}{2}\cdot\frac{q_0}{2}\cdot\frac{L}{2} = -80\,N \qquad M_{mid} = -\left(\frac{1}{2}\cdot\frac{q_0}{2}\cdot\frac{L}{2}\right)\cdot\left(\frac{1}{3}\cdot\frac{L}{2}\right) = -53.333\cdot N\cdot m$$

Problem 1.3-23

(a) STATICS

$\Sigma M_A = 0 \qquad C_y = \dfrac{1}{L_{AC}}\left(1900\text{ N}\,\dfrac{1}{2}\,\dfrac{3}{5}\,0.75\text{ m}\right) = 570\text{ N} \qquad C_x = \dfrac{-1}{2}C_y = -285\text{ N} \qquad \leftarrow$

$\Sigma F_x = 0 \qquad A_x = -C_x = 285\text{ N} \qquad \leftarrow$ (resultant of C_x and C_y acts along line of strut)

$\Sigma F_y = 0 \qquad A_y = 1900\text{ N} - C_y = 1330\text{ N} \qquad \leftarrow$

(b) INTERNAL STRESS RESULTANTS N, V, M (SEE FIGURE)

Distributed weight of door in $-y$ direction: $\quad w = \dfrac{1900\text{ N}}{0.75\text{ m}} = 2533.333\ \dfrac{\text{N}}{\text{m}}$

Components of w along and perpendicular to door:

$$w_a = \frac{4}{5}w = 2026.667\ \frac{\text{N}}{\text{m}} \qquad w_p = \frac{3}{5}w = 1520\ \frac{\text{N}}{\text{m}}$$

$$N_x = w_a(0.5\text{ m}) - \frac{3}{5}A_x - \frac{4}{5}A_y = -221.667\text{ N}$$

$$V_x = -w_p(0.5\text{ m}) - \frac{4}{5}A_x + \frac{3}{5}A_y = -190\text{ N}$$

$$M_x = -w_p(0.5\text{ m})\,\frac{0.5\text{ m}}{2} - \frac{4}{5}A_x(0.5\text{ m}) + \frac{3}{5}A_y(0.5\text{ m}) = 95\text{ N}\cdot\text{m}$$

$\boxed{N_x = -222\text{ N}} \quad \boxed{V_x = -190\text{ N}} \quad \boxed{M_x = 95\text{ N}\cdot\text{m}}$

Problem 1.3-24

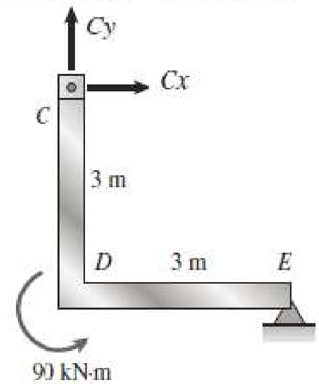
(a) Statics

$\Sigma M_A = 0$

$$10\text{ kN}(6\text{ m}) - 10\text{ kN}\left(\frac{1}{\sqrt{2}}\right)(6\text{ m}) + 90\text{ kN}\cdot\text{m} + E_y(6\text{ m}) - E_x 3\text{ m} = 6E_y\text{m} - 3E_x\text{m} + 150\text{ kN}\cdot\text{m} - 30\sqrt{2}\text{ kN}\cdot\text{m}$$

so $\quad 6E_y\text{m} - 3E_x\text{m} + 150\text{ kN}\cdot\text{m} - 30\sqrt{2}\text{ kN}\cdot\text{m} = 0$

or $\quad -E_x + 2E_y = \dfrac{-(150\text{ kN}\cdot\text{m} - 30\sqrt{2}\text{ kN}\cdot\text{m})}{3\text{ m}} = -35.858\text{ kN}$



$\Sigma M_{CRHFB} = 0$ < right hand FBD (*CDE*) - see figure.

$(E_x + E_y)3\text{ m} = -90\text{ kN}\cdot\text{m} \qquad E_x + E_y = \dfrac{-90\text{ kN}\cdot\text{m}}{3\text{ m}} = -30\text{ kN}$

Solving $\quad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -35.858\text{ kN} \\ -30\text{ kN} \end{pmatrix} = \begin{pmatrix} -8.05 \\ -21.95 \end{pmatrix}\text{ kN}$

$\boxed{E_x = -8.05\text{ kN}} \quad \boxed{E_y = -22\text{ kN}}$

$\Sigma F_x = 0 \qquad A_x = -E_x + 10\text{ kN} - 10\text{ kN}\left(\dfrac{1}{\sqrt{2}}\right) = 10.98\text{ kN} \qquad \boxed{A_x = 10.98\text{ kN}}$

$\Sigma F_y = 0 \qquad A_y = -E_y + 10\text{ kN}\left(\dfrac{1}{\sqrt{2}}\right) = 29.07\text{ kN} \qquad \boxed{A_y = 29.1\text{ kN}}$

(b) Right hand FBD $\qquad C_x = -E_x = 8.05\text{ kN} \qquad C_y = -E_y = 22\text{ kN}$

$\boxed{\text{Resultant}_C = \sqrt{C_x^2 + C_y^2} = 23.4\text{ kN}}$

Problem 1.3-25

(a) STATICS

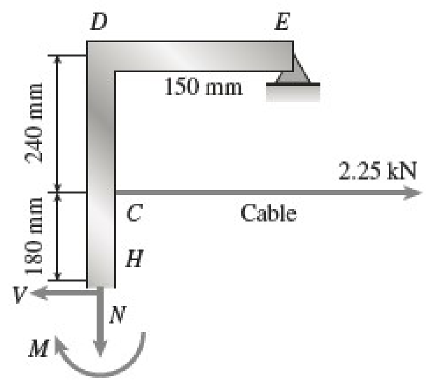
$\Sigma F_x = 0 \qquad E_x = 0$

$\Sigma M_E = 0 \qquad A_y = \frac{1}{300\text{ mm}}[-2250\text{ N}(750\text{ mm})] = -5625\text{ N}$

$\Sigma F_y = 0 \qquad E_y = 2250\text{ N} - A_y = 7875\text{ N}$

$A_y = -5625\text{ N} \qquad E_x = 0 \qquad E_y = 7875\text{ N} \qquad \leftarrow$

(b) USE UPPER (SEE FIGURE BELOW) OR LOWER FBD TO FIND STRESS RESULTANTS N, V, AND M AT H



$\Sigma F_x = 0 \qquad V_x = E_x + 2250\text{ N} = 2250\text{ N}$

$\Sigma F_y = 0 \qquad N_x = E_y = 7875\text{ N}$

$\Sigma M_H = 0$

$M_H = -(180\text{ mm})(2250\text{ N}) - E_x(180\text{ mm} + 240\text{ mm}) + E_y(150\text{ mm})$

$= 776\text{ N}\cdot\text{m}$

$N_x = 7875\text{ N} \qquad V_x = 2250\text{ N} \qquad M_x = 776\text{ N}\cdot\text{m} \qquad \leftarrow$

Problem 1.3-26

(a) STATICS

$$\Sigma F_x = 0 \qquad A_x = \frac{4}{5}(400\text{ N}) = 320\text{ N} \qquad \boxed{A_x = 320\text{ N}}$$

Use left hand FBD (cut through pin just left of C)

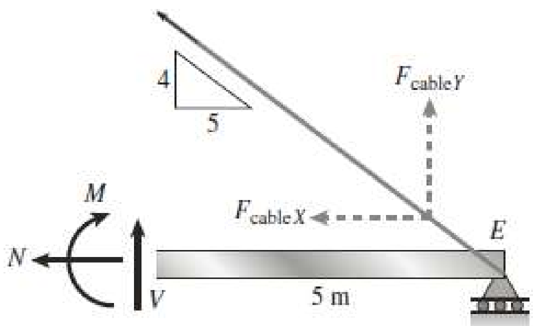
$$\Sigma M_C = 0 \qquad A_y = \frac{1}{7\text{ m}}\left[\left[\frac{-3}{5}(400\text{ N}) - \frac{4}{5}(400\text{ N})\right](3\text{ m})\right] = -240\text{ N} \qquad \boxed{A_y = -240\text{ N}}$$

Use entire FBD $\qquad \Sigma M_C = 0 \qquad E_y = \frac{1}{5\text{ m}}\left[A_y(7\text{ m}) + \left(\frac{3}{5}400\text{ N}\right)(3\text{ m})\right] = -192\text{ N} \qquad \boxed{E_y = -192\text{ N}}$

$$\Sigma F_y = 0 \qquad C_y = -A_y - E_y - \frac{3}{5}(400\text{ N}) = 192\text{ N} \qquad \boxed{C_y = 192\text{ N}}$$

(b) N, V, AND M JUST RIGHT OF C; USE RIGHT HAND FBD

$$F_{cableX} = 400\text{ N}\left(\frac{5}{\sqrt{4^2+5^2}}\right) = 312.348\text{ N}$$



$$F_{cableY} = \frac{4}{5}F_{cableX} = 249.878\text{ N}$$

$$\Sigma F_x = 0 \qquad \boxed{N_x = -F_{cableX} = -312\text{ N}}$$

$$\Sigma F_y = 0 \qquad \boxed{V = -F_{cableY} - E_y = -57.9\text{ N}}$$

$$\Sigma M_C = 0 \qquad M = \left(F_{cableY} + E_y\right)(5\text{ m}) = \boxed{289\text{ N·m}}$$

(c) RESULTANT FORCE IN PIN JUST LEFT OF C; USE LEFT HAND FBD $\qquad A_x = 320\text{ N}$

$$F_{Cx} = -A_x + \left(\frac{4}{5} - \frac{3}{5}\right)400\text{ N} = -240\text{ N} \qquad F_{Cy} = -A_y - \left(\frac{3}{5} + \frac{4}{5}\right)400\text{ N} = -320\text{ N}$$

$$\boxed{\text{Res}_C = \sqrt{{F_{Cx}}^2 + {F_{Cy}}^2} = 400\text{ N}}$$

Problem 1.3-27

(a) STATICS $W = 650\text{ N}$

$$\Sigma M_A = 0 \qquad B_x(0.6\text{ m} + 1.2\text{ m}\sin(30^\circ)) + W\left[\frac{1.2\text{ m}}{2}(\cos(30^\circ))\right] = 0 \text{ solve, } B_x = \frac{650.0\text{ N}\cos(30.0^\circ)}{2.0\sin(30.0^\circ) + 1.0}$$

$$\text{so} \qquad B_x = -\frac{650.0\text{ N}\cos(30.0^\circ)}{2.0\sin(30.0^\circ) + 1.0} = -281.458\text{ N}$$

$$\Sigma F_X = 0 \qquad -A\sin(30^\circ) + B_x + T\cos(30^\circ) + T\cos\left(\arctan\left(\frac{0.9 + 0.6 + 1.2\sin(30^\circ)}{1.2\cos(30^\circ)}\right)\right) = 0$$

$$\Sigma F_y = 0 \qquad A\cos(30^\circ) + T\sin(30^\circ) + T\sin\left(\arctan\left(\frac{0.9 + 0.6 + 1.2\sin(30^\circ)}{1.2\cos(30^\circ)}\right)\right) = W$$

$$\begin{pmatrix} A \\ T \end{pmatrix} = \begin{pmatrix} -\sin(30^\circ) & \cos(30^\circ) + \cos\left(\arctan\left(\frac{0.9 + 0.6 + 1.2\sin(30^\circ)}{1.2\cos(30^\circ)}\right)\right) \\ \cos(30^\circ) & \sin(30^\circ) + \sin\left(\arctan\left(\frac{0.9 + 0.6 + 1.2\sin(30^\circ)}{1.2\cos(30^\circ)}\right)\right) \end{pmatrix}^{-1} \begin{pmatrix} -B_x \\ W \end{pmatrix}$$

$$\begin{pmatrix} A \\ T \end{pmatrix} = \begin{pmatrix} 250.09 \\ 310.412 \end{pmatrix}\text{ N}$$

SUPPORT REACTIONS

$\boxed{B_x = -281.5\text{ N}}$ $\boxed{A = 250.1\text{ N}}$

$A_x = -A\sin(30^\circ) = -125\text{ N} \qquad A_y = A\cos(30^\circ) = 216.6\text{ N}$

$\sqrt{A_x^2 + A_y^2} = 250.09\text{ N}$

(b) CABLE FORCE IS T (NEWTONS) FROM ABOVE SOLUTION $\boxed{T = 310.4\text{ N}}$

Problem 1.3-28

(a) STATICS

RIGHT-HAND FBD

$\Sigma M_{pin} = 0 \qquad E_y = \frac{1}{6\text{ m}}\left[\frac{1}{2}(3\text{ kN/m})\,4\text{ m}\left(\frac{1}{3}4\text{ m}\right)\right] = 1.333\text{ kN} \qquad \boxed{E_y = 1.333\text{ kN}}$

ENTIRE FBD

$\Sigma M_A = 0 \quad C_y = \frac{1}{6\text{ m}}\left[-E_y 12\text{ m} + (16\text{ kN})4\text{ m} + (1.5\text{ kN/m})\,6\text{ m}(3\text{ m}) - \frac{1}{2}(3\text{ kN/m})\,4\text{ m}\left(\frac{2}{3}4\text{ m}\right)\right] = 9.833\text{ kN}$

$\boxed{C_y = 9.83\text{ kN}}$

$\Sigma F_y = 0 \qquad A_y = -C_y - E_y + (1.5\text{ kN/m})\,6\text{ m} = -2.167\text{ kN} \qquad \boxed{A_y = -2.17\text{ kN}}$

$\Sigma F_x = 0 \qquad A_x = -16\text{ kN} + \frac{1}{2}(3\text{ kN/m})\,4\text{ m} = -10\text{ kN} \qquad \boxed{A_x = -10\text{ kN}}$

(b) RESULTANT FORCE IN PIN; USE EITHER RIGHT HAND OR LEFT HAND FBD (CUT THROUGH PIN EXPOSING PIN FORCES F_{Dx} AND F_{Dy}) THEN SUM FORCES IN x AND y DIRECTIONS FOR EITHER FBD

LHFB:

$F_{Dx} = -16\text{ kN} - A_x = -6\text{ kN}$

$F_{Dy} = -A_y + (1.5\text{ kN/m})\,6\text{ m} = 11.167\text{ kN}$

$\text{Resultant}_D = \sqrt{F_{Dx}^2 + F_{Dy}^2} = 12.68\text{ kN}$

RHFB:

$F_{Dx} = \frac{1}{2}(3\text{ kN/m})\,4\text{ m} = 6\text{ kN}$

$F_{Dy} = -C_y - E_y = -11.167\text{ kN}$

$\boxed{\text{Resultant}_D = 12.68\text{ kN}}$

Problem 1.3-29

(a) STATICS $\quad P_1 = 220\ \text{N} \quad P_2 = 180\ \text{N}$

$\Sigma F_x = 0 \quad O_x = -P_1\cos(15°) = -212.5\ \text{N} \quad \Sigma F_y = 0 \quad O_y = P_2 = 180\ \text{N}$

$\Sigma F_z = 0 \quad O_z = P_1\sin(15°) = 56.94\ \text{N}$

$\Sigma M_x = 0 \quad M_{Ox} = P_2\,150\ \text{mm} + P_1\sin(15°)(178\ \text{mm}) = 37.1\ \text{N·m}$

$\Sigma M_y = 0 \quad M_{Oy} = P_1\sin(15°)(200\ \text{mm}\sin(15°)) + P_1\cos(15°)(150\ \text{mm} + 200\ \text{mm}\cos(15°))$

$M_{Oy} = 75.9\ \text{N·m}$

$\Sigma M_z = 0 \quad M_{Oz} = P_1\cos(15°)(178\ \text{mm}) = 37.8\ \text{N·m}$

$O_x = -213\ \text{N} \quad O_y = 180\ \text{N} \quad O_z = 56.9\ \text{N}$

$M_{Ox} = 37.1\ \text{N·m} \quad M_{Oy} = 75.9\ \text{N·m} \quad M_{Oz} = 37.8\ \text{N·m} \quad \leftarrow$

(b) INTERNAL STRESS RESULTANTS AT MIDPOINT OF *OA*

$N_x = -O_y = -180\ \text{N}$

$V_x = -O_x = 212.5\ \text{N} \quad V_z = -O_z = -56.9\ \text{N} \quad V_{res} = \sqrt{V_x^2 + V_z^2} = 220\ \text{N}$

$T_x = -M_{Oy} = -75.9\ \text{N·m}$

$M_x = -M_{Ox} = -37.1\ \text{N·m} \quad M_z = -M_{Oz} = -37.8\ \text{N·m} \quad M_{res} = \sqrt{M_x^2 + M_z^2} = 53\ \text{N·m}$

$N_x = -180\ \text{N} \quad V_{res} = 220\ \text{N}$

$T_x = -75.9\ \text{N·m} \quad M_{res} = 53\ \text{N·m} \quad \leftarrow$

Problem 1.3-30

FORCES

$P_x = 60\text{ N} \quad P_z = -45\text{ N} \quad M_y = 120\text{ N·m} \quad q_0 = 75\text{ N/m}$

$$F_C = \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} = \begin{pmatrix} 60 \\ 0 \\ -45 \end{pmatrix}\text{N} \qquad R_A = \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix}$$

VECTOR ALONG MEMBER CD

$$r_{EC} = \begin{bmatrix} 1.5 - 2.5 \\ 2 - 0 \\ 0 - (-0.5) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0.5 \end{bmatrix} \qquad |r_{EC}| = 2.291 \qquad e_{EC} = \frac{r_{EC}}{|r_{EC}|} = \begin{pmatrix} -0.436 \\ 0.873 \\ 0.218 \end{pmatrix}$$

(a) STATICS (FORCE AND MOMENT EQUILIBRIUM)

$$\Sigma F = 0 \qquad \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} + \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = 0 \qquad \text{resultant of triangular load:} \quad R_T = \frac{1}{2}q_0(2\text{ m}) = 75\text{ N}$$

$$\text{where} \qquad \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = D\,e_{EC}$$

SOLVING ABOVE THREE EQUATIONS:

$\Sigma F_x = 0 \qquad D_x = -P_x \text{ so} \qquad D = \dfrac{-P_x}{e_{EC_1}} \qquad D = 137.477\text{ N} \qquad \boxed{D_x = -60\text{ N}}$

$\Sigma F_y = 0 \qquad D_y = e_{EC_2}D \qquad \boxed{D_y = 120\text{ N}} \qquad \boxed{A_y = -D_y = -120\text{ N}}$

$\Sigma F_z = 0 \qquad D_z = e_{EC_3}D \qquad \boxed{D_z = 30\text{ N}} \qquad \sqrt{D_x^2 + D_y^2 + D_z^2} = 137.477\text{ N}$

$\text{so} \quad A_z = -D_z - R_T - P_z \qquad \boxed{A_z = -60\text{ N}}$

$\Sigma M_A = 0$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} + r_{AE} \times D + r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} 0 \\ M_y \\ 0 \end{pmatrix} + r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} = 0$$

$$r_{AE} = \begin{pmatrix} 2.5 - 0 \\ 0 - 0 \\ -0.5 - 0 \end{pmatrix}\text{m} \qquad D = \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} \qquad D = \begin{pmatrix} -60 \\ 120 \\ 30 \end{pmatrix}\text{N} \qquad |D| = 137.477\text{ N} \qquad r_{AE} \times D = \begin{pmatrix} 60 \\ -45 \\ 300 \end{pmatrix}\text{N·m}$$

$$r_{AC} = \begin{pmatrix} 1.5 - 0 \\ 2 - 0 \\ 0 - 0 \end{pmatrix}\text{m} \qquad r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} = \begin{pmatrix} -90 \\ 67.5 \\ -120 \end{pmatrix}\text{J} \qquad r_{cg} = \begin{pmatrix} 0 \\ \frac{2}{3}(2\text{ m}) \\ 0 \end{pmatrix} \qquad r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} = \begin{pmatrix} 100 \\ 0 \\ 0 \end{pmatrix}\text{N·m}$$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} = -\left[r_{AE} \times D + r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} 0 \\ M_y \\ 0 \end{pmatrix} + r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} \right] = \begin{pmatrix} -70 \\ -142.5 \\ -180 \end{pmatrix}\text{N·m} \qquad \boxed{\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} = \begin{pmatrix} -70 \\ -142.5 \\ -180 \end{pmatrix}\text{N·m}}$$