

Advanced Engineering Mathematics, Seventh Edition
Dennis G. Zill
Chapter 1 Test Bank

Import Settings:
Base Settings: Brownstone Default
Information Field: Complexity
Information Field: Ahead
Information Field: Subject
Information Field: Title
Highest Answer Letter: D
Multiple Keywords in Same Paragraph: No
NAS ISBN13: 9781284207972, add to Ahead, Title tags

Chapter: Chapter 01 – Test Bank

Multiple Choice

1. Classify the following differential equation by order and linearity. $\frac{d^3y}{dx^3} = \left(\frac{d^2y}{dx^2}\right)^4$

- A) 3rd order nonlinear
- B) 8th order nonlinear
- C) 4th order linear
- D) 3rd order linear

Ans: A

Complexity: Easy
Ahead: Definitions and Terminology
Subject: Chapter 1
Title: Introduction to Differential Equations

2. Classify the following differential equation by order and linearity. $y'' = \sqrt{5x^6 + (\sin x)y^{(8)}}$

- A) 8th order nonlinear
- B) 4th order linear
- C) 2nd order linear
- D) 2nd order nonlinear

Ans: A

Complexity: Easy
Ahead: Definitions and Terminology
Subject: Chapter 1
Title: Introduction to Differential Equations

3. Classify the following differential equation by order and linearity. $(1 - e^y)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = \cos 2t$

- A) 2nd order linear
- B) 2nd order nonlinear
- C) 1st order linear
- D) 1st order nonlinear

Ans: A

Complexity: Easy

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

4. Classify the following differential equation by order and linearity.

$$\ddot{x} - (1 - x^3)\dot{x} + x = 2$$

A) 2nd order nonlinear

B) 2nd order linear

C) 1st order linear

D) 1st order nonlinear

Ans: A

Complexity: Easy

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

5. For the differential equation $y' = \frac{\tan(y)}{x}$, the Existence-Uniqueness Theorem (Theorem 1.2.1) guarantees the existence of a unique solution whose graph passes through:

A) $(\frac{\pi}{2}, \pi)$.

B) $(1, \frac{\pi}{2})$.

C) $(\pi, \frac{3\pi}{2})$.

D) $(0, \pi)$.

Ans: A

Complexity: Moderate

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

6. For the DE $y' = \frac{2y^2+5}{\ln(4y-3x)}$, the Existence-Uniqueness Theorem (Theorem 1.2.1) guarantees the existence of a unique solution whose graph passes through:

A) $(\frac{2}{3}, \frac{3}{2})$.

B) $(0, \frac{1}{4})$.

C) $(1, \frac{3}{4})$.

D) $(-7, -5)$.

Ans: A

Complexity: Moderate

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

7. For the DE $\frac{dy}{dx} = |\sin y| \sin^{-1}(x)$, the Existence-Uniqueness Theorem (Theorem 1.2.1) guarantees the existence of a unique solution whose graph passes through:

A) $(\frac{1}{2}, \frac{\pi}{4})$.

B) $(\frac{\pi}{2}, \frac{\pi}{3})$.

C) (0,0).

D) $(\frac{2}{3}, \pi)$.

Ans: A

Complexity: Moderate

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

8. $y = -x \ln |x| + cx$ is a one-parameter family of solutions to the DE $x \frac{dy}{dx} = y - x$. Which of the following is the largest interval of existence for a solution to the IVP $x \frac{dy}{dx} = y - x$, $y(2) = -7$?

A) $(0, \infty)$

B) $(-\infty, 0)$

C) $(-7, 2)$

D) $(2, \infty)$

Ans: A

Complexity: Easy

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

9. $y = \frac{1}{4}e^{3x} + ce^{-x}$ is a one-parameter family of solutions to the DE $\frac{dy}{dx} + y = e^{3x}$. Which of the following is the largest interval of existence for a solution to the IVP $\frac{dy}{dx} + y = e^{3x}$, $y(-4) = 15$?

A) $(-\infty, \infty)$

B) $(-\infty, 0)$

C) $(-4, 15)$

D) $(-4, \infty)$

Ans: A

Complexity: Easy

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

10. $x = c_1 \cos t + c_2 \sin t$ is a two-parameter family of solutions of the second-order DE $x'' + x = 0$. Find a solution of the second-order IVP consisting of this differential equation and the initial conditions $x(\pi/3) =$

$\frac{1}{2}$, $x'(\pi/3) = -\frac{\sqrt{3}}{2}$.

A) $x = \cos t$

B) $x = -\sin t$

C) $x = \cos t + \sin t$

D) $x = -\cos t + \frac{1}{2}\sin t$

Ans: A

Complexity: Moderate

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

11. A hot cup of coffee is initially at 180° F, sits in a room with a constant air temperature of 72° F, and cools according to Newton's law of cooling. Write an initial-value problem describing the temperature of the coffee at time t , $T(t)$.

A) $\frac{dT}{dt} = k(T - 72), \quad T(0) = 180$

B) $\frac{dT}{dt} = k(T - 80), \quad T(0) = 72$

C) $\frac{dT}{dt} = 72(T - k), \quad T(0) = 180$

D) $\frac{dT}{dt} = 180(72 - T), \quad T(0) = k$

Ans: A

Complexity: Easy

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

12. Suppose that the birth rate of a certain population is proportional to the current size of the population with proportionality constant b . Suppose that the death rate is also proportional to the current size of the population with proportionality constant d . If the change in size of the population depends solely on the births and deaths that occur, which of the following could represent the size of the population P as a function of time t ?

A) $P(t) = e^{(b-d)t + \ln(4)}$

B) $P(t) = 6e^{(d-b)t}$

C) $P(t) = e^{bt} - e^{dt}$

D) $P(t) = 6e^{(b+d)t}$

Ans: A

Complexity: Moderate

Ahead: Differential Equations as Mathematical Models

Subject: Chapter 1

Title: Introduction to Differential Equations

13. Suppose two particles A and B are moving along the x -axis. At time t , particle A is located at the point on the x -axis with coordinate $5 + 7t$. When $t = 0$, B is at the origin. B 's velocity is along the positive x -axis and is proportional to the distance between A and B with proportionality constant $-2k$. The initial value problem that models B 's position x on the axis as a function of t is:

A) $\frac{dx}{dt} = k(5 + 7t - x), \quad x(0) = 0.$

B) $\frac{dx}{dt} = k(x + 5 - 7t), \quad x(0) = 0.$

C) $\frac{dx}{dt} = 7, \quad x(0) = 5.$

D) $\frac{dx}{dt} = k(5 - x), \quad x(0) = 0.$

Ans: A

Complexity: Moderate

Ahead: Differential Equations as Mathematical Models

Subject: Chapter 1

Title: Introduction to Differential Equations

True/False

1. True or False? The following differential equation is linear: $\frac{1}{x}y''' + (3 - x^2)(y'' - y') + e^xy = 3y$.

Ans: True

Complexity: Easy

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

2. True or False? The following differential equation is linear: $\frac{d^3u}{dt^3} + 4^u = \sin t$.

Ans: False

Complexity: Easy

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

3. True or False? There is a constant solution to the differential equation: $2y'' - 3y' = y^2 - 6y + 10$.

Ans: False

Complexity: Easy

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

4. True or False? There is a constant solution to the differential equation: $(y^2 - 5)y'' = 2 - y'$.

Ans: False

Complexity: Easy

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

5. True or False? $y = 2e^{-x} + 3e^{4x}$ is a solution to the differential equation $y'' - 3y' - 4y = 0$.

Ans: True

Complexity: Easy

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

6. True or False? An implicit solution to a differential equation is a relation that defines one or more solutions to the differential equation.

Ans: True

Complexity: Easy

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

7. True or False? A singular solution to a differential equation is a solution that cannot be obtained by specializing any of the parameters in the family of solutions.

Ans: True

Complexity: Easy

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

8. True or False? The function $y = \frac{e^t}{1+e^t}$ is a solution to the IVP $y' - y + y^2 = 0$, $y(0) = \frac{1}{2}$.

Ans: True

Complexity: Moderate

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

9. True or False? The function $y = -x - 1 + \tan(x + 1)$ is a solution to the IVP $\frac{dy}{dx} = (x + y + 1)^2$, $y(-1) = 0$.

Ans: True

Complexity: Moderate

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

10. True or False? $y = c_1x^{-1} + c_2x - \ln x$ is a two-parameter family of solutions to the differential equation $x^2y'' + xy' - y = \ln x$.

Ans: True

Complexity: Easy

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

11. True or False? $y = \frac{x^3}{7} - \frac{x}{5} + \frac{72}{35}x^{-4}$ is a solution to the IVP $xy' + 4y = x^3 - x$, $y(1) = 2$.

Ans: True

Complexity: Moderate

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

12. True or False? Every initial-value problem has a unique solution.

Ans: False

Complexity: Easy

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

13. True or False? A solution to an initial-value problem is always defined on $(-\infty, \infty)$.

Ans: False

Complexity: Easy

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

14. True or False? Consider a population where the number of births and deaths are proportional to the size of the population with proportionality constants b and d respectively. Suppose also that the population experiences a constant net immigration rate of r . Then the differential equation that models the population P as a function of time t is $\frac{dP}{dt} = (b - d + r)P$.

Ans: False

Complexity: Moderate

Ahead: Differential Equations as Mathematical Models

Subject: Chapter 1

Title: Introduction to Differential Equations

Essay

1. Find values of m so that the function $y = e^{mx}$ is a solution of the differential equation $y'' + 2y' - 35y = 0$.

Ans: $m = -7, 5$

Complexity: Easy

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

2. Find values of m so that the function $y = x^m$ is a solution of the following differential equation.

$$xy'' + 5xy' - 12y = 0$$

Ans: $m = -6, 2$

Complexity: Moderate

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

3. Classify the following differential equation by order and linearity.

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^5 - \frac{d^4y}{dx^4} + \left(\frac{d^3y}{dx^3}\right)^2$$

Ans: 4th order nonlinear

Complexity: Easy

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

4. Describe how to identify the order of a differential equation.

Ans: The order of a differential equation is the order of the highest derivative in the equation.

Complexity: Easy

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

5. Give an example of a linear differential equation and an example of a nonlinear differential equation.

Clearly indicate which is which.

Ans: Answers will vary.

Complexity: Difficult

Ahead: Definitions and Terminology

Subject: Chapter 1

Title: Introduction to Differential Equations

6. $y = c_1 e^{-4x} + c_2 e^{2x}$ is a two-parameter family of solutions of the second-order differential equation $y'' + 2y' - 8y = 0$, $y(0) = 0$, $y'(0) = 12$. Find a solution of the second-order IVP $y'' + 2y' - 8y = 0$, $y(0) = 0$, $y'(0) = 12$.

Ans: $y = -2e^{-4x} + 2e^{2x}$

Complexity: Moderate

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

7. $y = c_1 \cos 3t + c_2 \sin 3t$ is a two-parameter family of solutions of the second-order differential equation $y'' + 9y = 0$. Find a solution of the second-order differential equation

$y'' + 9y = 0$, $y(0) = 1$, $y'\left(\frac{\pi}{3}\right) = 2$.

Ans: $y = \cos 3t - \frac{2}{3} \sin 3t$

Complexity: Moderate

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

8. $y = x^{-1} \ln x + cx^{-1}$ is a family of solutions to the differential equation $x^2y' + xy = 1$. Find a solution to the IVP $x^2y' + xy = 1$, $y(1) = 9$.

Ans: $y = x^{-1} \ln x + 9x^{-1}$

Complexity: Moderate

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

9. A certain island initially contains 500 inhabitants who communicate only by word of mouth. A boat containing 10 people from another country arrives on the island with news of the outside world. Determine a differential equation for the number of people $x(t)$ who have heard the news at time t if the rate at which the news spreads is proportional to the number of interactions between those who have heard the news and those who have not. Treating this as an initial-value problem, what is the initial condition? (Assume throughout the problem that the 10 outsiders stay indefinitely and become part of the islander's population.)

Ans: $\frac{dx}{dt} = kx(510 - x)$, $x(0) = 10$

Complexity: Difficult

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

10. Suppose a cylindrical tank with radius 3 meters and height 9 meters is resting on its circular base. Water is leaking from the tank through a circular hole of area A at its bottom. The volume of water leaving the tank per second is equal to $cA\sqrt{2gh}$, where h is the height of the water in the tank, g is the acceleration due to gravity, and c is an empirical constant with $0 < c < 1$. If the tank is full at time $t = 0$ and the radius of the hole is 0.01 meters, determine a differential equation and initial condition for the height h of water at time t .

Ans: $\frac{dh}{dt} = -c\left(\frac{01}{3}\right)^2\sqrt{2gh}$, $h(0) = 9$

Complexity: Difficult

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

11. Find the value of w , which makes $y = c_1 \sin wt + c_2 \cos wt$ a two-parameter family of solutions of the differential equation $y'' + 18y = 0$.

Ans: $w = 3\sqrt{2}$

Complexity: Moderate

Ahead: Initial-Value Problems

Subject: Chapter 1

Title: Introduction to Differential Equations

12. Suppose that a large mixing tank initially holds 500 gallons of water in which 75 pounds of salt have been dissolved. Pure water is pumped into the tank at a rate of 6 gal/min, and when the solution is well stirred, it is pumped out at the same rate. Determine a differential equation for the amount $A(t)$ of salt in the tank at time t . What is $A(0)$?

Ans: $\frac{dA}{dt} = \frac{-3A}{250}$, $A(0) = 75$

Complexity: Moderate

Ahead: Differential Equations as Mathematical Models

Subject: Chapter 1

Title: Introduction to Differential Equations

13. Suppose that a large mixing tank initially holds 500 gallons of water in which 75 pounds of salt have been dissolved. Water containing a salt concentration of $\frac{1}{2}(1 + 3 \sin t)$ lb/gal flows into the tank at a rate of 2 gal/min, and when the solution is well stirred, it is pumped out at the same rate. Determine a differential equation for the amount $A(t)$ of salt in the tank at time t . What is $A(0)$?

Ans: $\frac{dA}{dt} = 1 + \sin t - \frac{A}{250}$, $A(0) = 75$

Complexity: Moderate

Ahead: Differential Equations as Mathematical Models

Subject: Chapter 1

Title: Introduction to Differential Equations

14. Suppose that a large mixing tank initially holds 400 gallons of water in which 65 pounds of salt have been dissolved. Another brine solution is pumped into the tank at a rate of 6 gal/min, and when the solution is well stirred, it is pumped out at a slower rate of 5 gal/min. If the concentration of the solution entering is 3 lb/gal, determine a differential equation for the amount $A(t)$ of salt in the tank at time t . What is $A(0)$?

Ans: $\frac{dA}{dt} = 18 - \frac{5A}{400+t}$, $A(0) = 65$

Complexity: Moderate

Ahead: Differential Equations as Mathematical Models

Subject: Chapter 1

Title: Introduction to Differential Equations

15. Suppose that a large mixing tank initially holds 400 gallons of water in which 65 pounds of salt have been dissolved. Another brine solution is pumped into the tank at a rate of 6 gal/min, and when the solution is well stirred, it is pumped out at a faster rate of 8 gal/min. If the concentration of the solution entering is 3 lb/gal, determine a differential equation for the amount $A(t)$ of salt in the tank at time t .

Ans: $\frac{dA}{dt} = 18 - \frac{8A}{400-2t}$, $A(0) = 65$

Complexity: Moderate

Ahead: Differential Equations as Mathematical Models

Subject: Chapter 1

Title: Introduction to Differential Equations

16. Determine a differential equation for the velocity $v(t)$ of a falling body of mass m if air resistance is proportional to the square root of the cube of the instantaneous velocity.

$$\text{Ans: } m \frac{dv}{dt} = mg - kv^{3/2}$$

Complexity: Moderate

Ahead: Differential Equations as Mathematical Models

Subject: Chapter 1

Title: Introduction to Differential Equations

17. Suppose water is leaking from a spherical tank with radius 5 meters through a circular hole of area A at its bottom. The volume of water leaving the tank per second is equal to $cA\sqrt{2gh}$, where h is the height of the water in the tank, g is the acceleration due to gravity, and c is an empirical constant with $0 < c < 1$. If the tank is half-full at time $t = 0$ and the radius of the hole is 0.02 meters, determine a differential equation and initial condition for the height h of water at time t .

$$\text{Ans: } \frac{dh}{dt} = -c \left(\frac{.02}{10h-h^2} \right)^2 \sqrt{2gh}, \quad h(0) = 5$$

Complexity: Moderate

Ahead: Differential Equations as Mathematical Models

Subject: Chapter 1

Title: Introduction to Differential Equations

18. Suppose the temperature of a room fluctuates with time so that the temperature at time t is given by $R(t) = 79 + 11 \cos t$. A 200° F cup of hot tea is placed on a table in the room at time $t = 12$. Determine a differential equation for the temperature of the tea $T(t)$ if the magnitude of the constant of proportionality is $|k| = 0.3$.

$$\text{Ans: } \frac{dT}{dt} = -0.3(T - 79 - 11 \cos t), \quad T(12) = 200$$

Complexity: Moderate

Ahead: Differential Equations as Mathematical Models

Subject: Chapter 1

Title: Introduction to Differential Equations

19. Suppose a dog is chasing a rabbit along the y -axis, and the coordinate of the rabbit's position at time t is given by $\frac{1}{3}t^2 + 4t$. If the dog's speed is equal to one half the distance between the two animals, and the dog's coordinate at $t = 0$ is $y = -4$, determine a differential equation for the position of the dog.

$$\text{Ans: } \frac{dD}{dt} = \frac{1}{2} \left(\frac{1}{3}t^2 + 4t - D \right), \quad D(0) = -4$$

Complexity: Moderate

Ahead: Differential Equations as Mathematical Models

Subject: Chapter 1

Title: Introduction to Differential Equations

20. Consider a population where the number of births and deaths are proportional to the size of the population with proportionality constants b and d respectively. Suppose also that people immigrate to the country at a constant rate of r . If people emigrate from the country at a rate proportional to the population P with proportionality constant w , determine a differential equation that models P as a function of time t .

Advanced Engineering Mathematics, Seventh Edition

Dennis G. Zill

Chapter 1 Test Bank

Ans: $\frac{dP}{dt} = (b - d - w)P + r$

Complexity: Moderate

Ahead: Differential Equations as Mathematical Models

Subject: Chapter 1

Title: Introduction to Differential Equations

21. Water drains out of a tank according to Torricelli's law. If the upper surface of the water is always 5 times the area of the drain hole and the acceleration due to gravity is 32, then write a differential equation describing $h(t)$, the height of the water in the tank at time t .

Ans: $\frac{dh}{dt} = -\frac{1}{5}\sqrt{2 \cdot 32 \cdot h}$

Complexity: Moderate

Ahead: Differential Equations as Mathematical Models

Subject: Chapter 1

Title: Introduction to Differential Equations