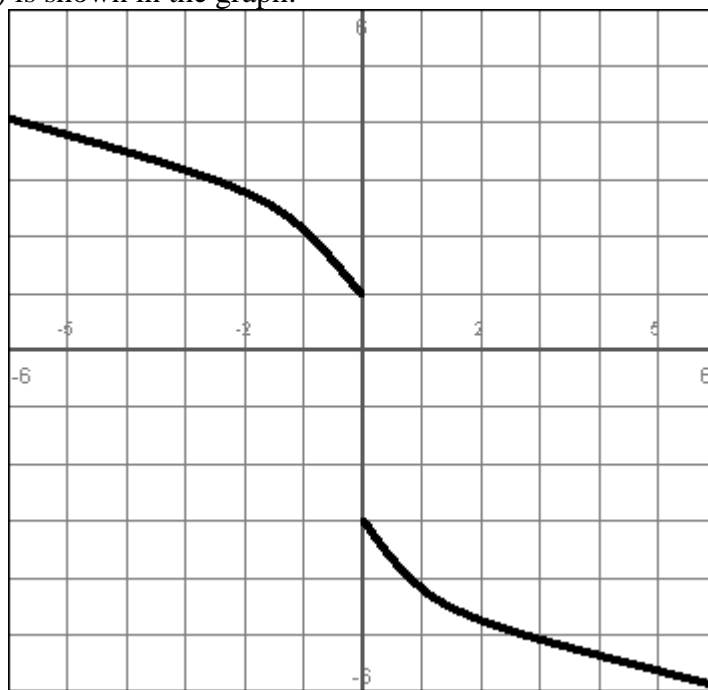


## Table of Contents

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ch1_sec1	2
ch1_sec2	11
ch1_sec3	21
ch1_sec4	30
ch1_sec5	36
ch1_sec6	47

1. The function  $f(x)$  is shown in the graph:



Find  $\lim_{x \rightarrow 0^+} f(x) =$

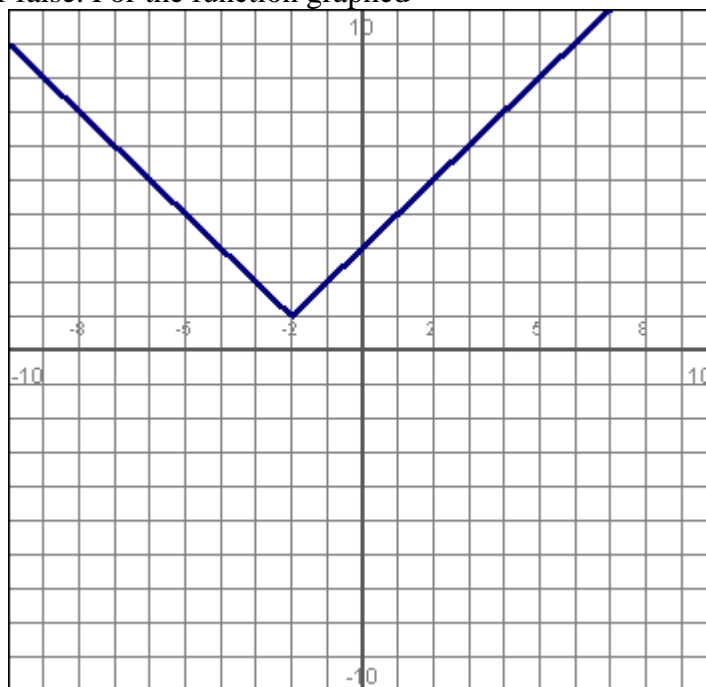
A) 2    B) -3    C) 3    D) -2    E) 0

Ans: B

Difficulty: Easy

Section: 1.1

2. Answer true or false. For the function graphed



$\lim_{x \rightarrow 2} f(x)$  is undefined

Ans: False

Difficulty: Easy

Section: 1.1

3. Approximate the

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} \quad \text{by evaluating} \quad f(x) = \frac{x^2 - 25}{x - 5}$$

at  $x = 4.5, 4.9, 4.99, 4.999, 5.5, 5.1, 5.01, \text{ and } 5.001$ .

A) 5   B) 2.5   C) 11   D) 10   E) 12

Ans: D

Difficulty: Medium

Section: 1.1

4. Answer true or false. If  $\lim_{x \rightarrow 8^+} f(x) = 9$  and  $\lim_{x \rightarrow 8^-} f(x) = 9$  then  $\lim_{x \rightarrow 8} f(x) = 0$

Ans: False

Difficulty: Easy

Section: 1.1

5. Approximate the  $\lim_{x \rightarrow -12^-} \frac{x}{x+12}$  by evaluating  $f(x) = \frac{x}{x+12}$  at appropriate values of  $x$ .  
 A) 1    B) 0    C)  $\infty$     D)  $-\infty$     E) 12

Ans: C

Difficulty: Easy

Section: 1.1

6. Approximate the limit by evaluating  $f(x) = \frac{9x}{\sin x}$  at appropriate values of  $x$ .

$$\lim_{x \rightarrow 0^-} \frac{9x}{\sin x} =$$

- A) 1    B) 9    C) -9    D)  $\infty$     E) undefined

Ans: B

Difficulty: Medium

Section: 1.1

7. Approximate the limit by evaluating  $f(x) = \frac{\sin x}{9x}$  at appropriate values of  $x$ .  $\lim_{x \rightarrow 0} \frac{\sin x}{9x} =$

- A) 1    B) -9    C)  $\frac{1}{9}$     D)  $\infty$     E) undefined

Ans: C

Difficulty: Medium

Section: 1.1

8. Approximate the limit by evaluating  $f(x) = \frac{\sqrt{x+49}-7}{x}$  at appropriate values of  $x$ .

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{x+49}-7}{x} =$$

- A)  $\frac{1}{14}$     B)  $\frac{1}{49}$     C)  $\frac{1}{7}$     D) 0    E)  $\infty$

Ans: A

Difficulty: Medium

Section: 1.1

9. Approximate the horizontal asymptote of  $f(x) = \frac{2-11x}{8+x}$ .

- A) -8    B) 8    C) -11    D) 2    E) 11

Ans: C

Difficulty: Medium

Section: 1.1

$$10. \lim_{x \rightarrow 0} \frac{\sin(35x)}{\sin(5x)} =$$

- A) 35    B)  $\infty$     C) 0    D) It does not exist    E) 7

Ans: E

Difficulty: Hard

Section: 1.1

11. Find the equation of the tangent line to the graph of  $y = 3x^5$  at  $(-1, -3)$ .

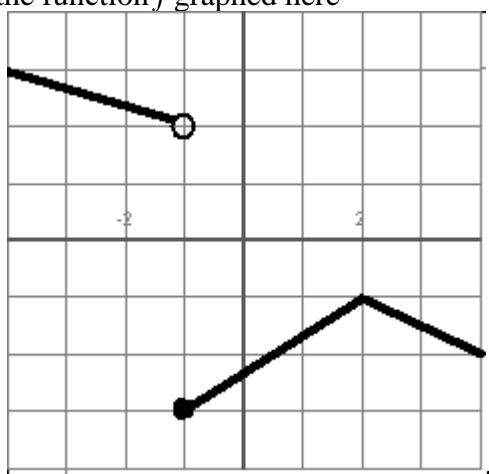
- A)  $y = 15x + 12$     B)  $y = 15x$     C)  $y = 12x$     D)  $y = 12x + 15$     E)  $y = 5x + 12$

Ans: A

Difficulty: Hard

Section: 1.1

12. For the function  $f$  graphed here



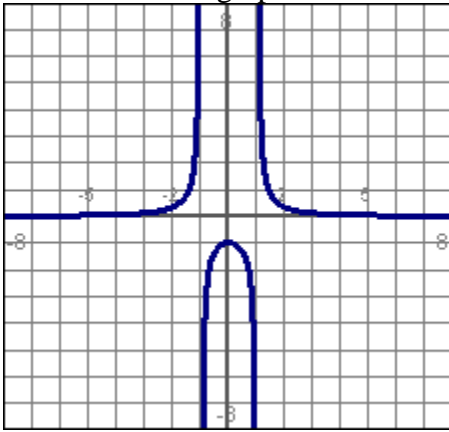
find  $\lim_{x \rightarrow -1} f(x)$

Ans: Does not exist

Difficulty: Easy

Section: 1.1

13. For the function graphed here



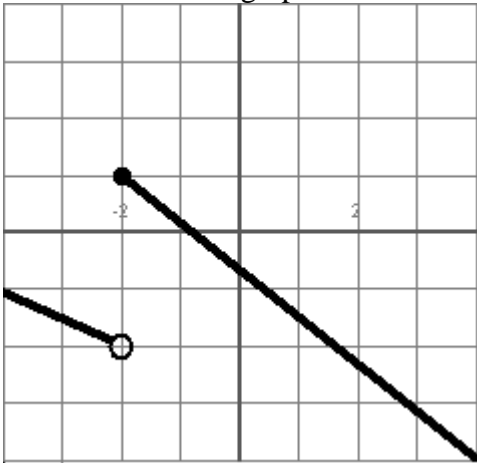
Find  $\lim_{x \rightarrow -1^-} f(x)$ .

Ans:  $+\infty$

Difficulty: Easy

Section: 1.1

14. For the function  $h$  graphed here



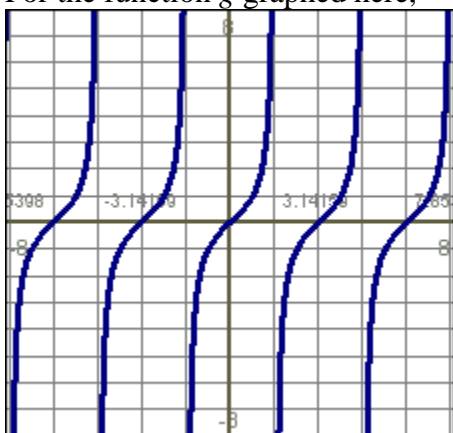
find  $\lim_{x \rightarrow -2^-} f(x)$

Ans: -2

Difficulty: Easy

Section: 1.1

15. For the function  $g$  graphed here,



find  $\lim_{x \rightarrow -\frac{\pi}{2}^+} g(x)$  and identify the function.

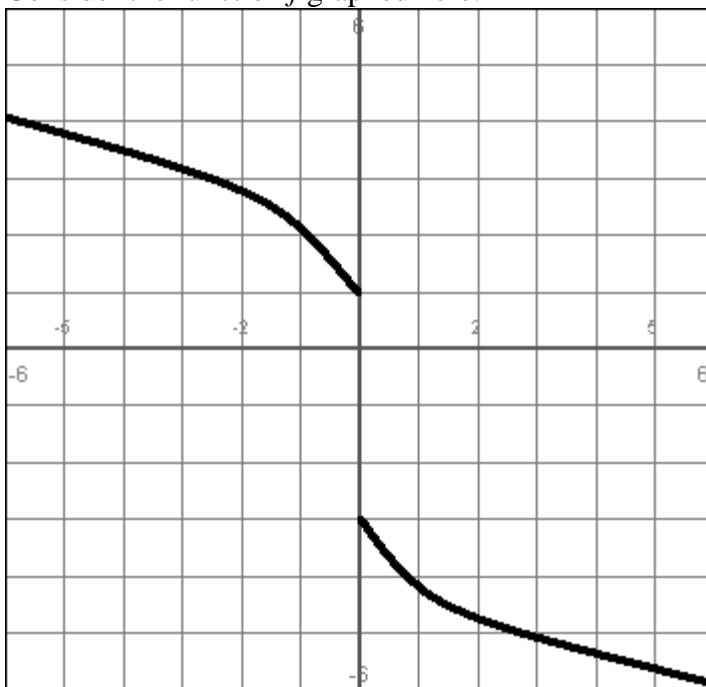
Ans:  $-\infty$

$$g(x) = \tan x$$

Difficulty: Easy

Section: 1.1

16. Consider the function  $f$  graphed here:



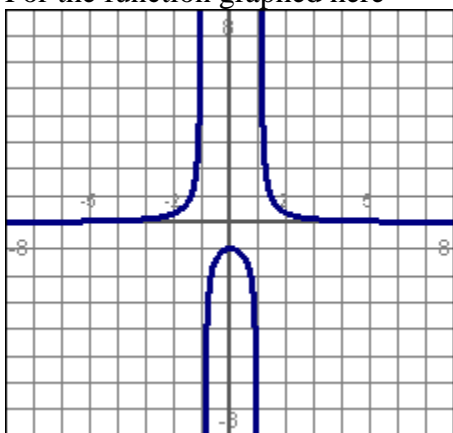
For what values of  $a$  does  $\lim_{x \rightarrow a} f(x)$  exist?

Ans:  $(-\infty, 0) \cup (0, +\infty)$

Difficulty: Medium

Section: 1.1

17. For the function graphed here



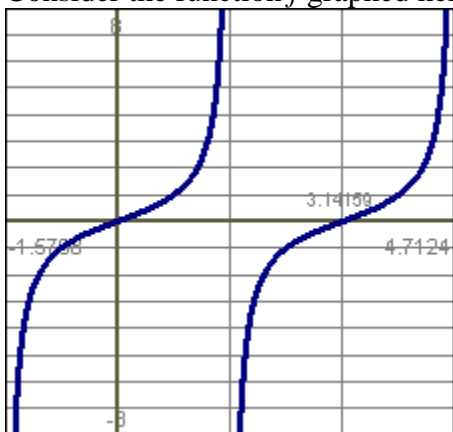
Find  $\lim_{x \rightarrow +\infty} f(x)$ .

Ans: 0

Difficulty: Easy

Section: 1.1

18. Consider the function  $f$  graphed here:



For what values of  $a$  does  $\lim_{x \rightarrow a} f(x)$  not exist?

Ans:  $\frac{\pi}{2}$  or  $\pm \frac{(2n-1)\pi}{2}$ , for  $n = 1, 2, 3, \dots$

Difficulty: Medium

Section: 1.1

19. Consider the function  $f$  graphed here:



For what values of  $a$  does  $\lim_{x \rightarrow a} f(x)$  exist?

Ans:  $(-\infty, 0) \cup (0, 2) \cup (2, +\infty)$

Difficulty: Medium

Section: 1.1

20. Approximate  $\lim_{x \rightarrow 3} -5x^4$  by evaluating  $-5x^4$  at appropriate values of  $x$ .

Ans:

$x$	$-5x^4$
2.5	-195.31
2.9	-353.640
2.99	-399.627
3.01	-410.427
3.1	-461.760
3.5	-750.31

The limit is -405

Difficulty: Medium

Section: 1.1

21. Approximate  $\lim_{x \rightarrow 5} \frac{2x}{\sin x}$  by evaluating  $\frac{2x}{\sin x}$  at appropriate values of  $x$ .

Ans:

x	$\frac{2x}{\sin x}$
4.5	-9.2069
4.9	-9.9750
4.99	-10.3773
5.001	-10.4335
4.999	-10.4232
5.01	-10.4807
5.1	-11.0173
5.5	-15.5909

The limit is -10.4284

Difficulty: Hard

Section: 1.1

22. Use the substitution  $t = \frac{1}{x}$  to express the limit as an equivalent limit in which  $t \rightarrow 0^+$ .

$$\lim_{x \rightarrow +\infty} \frac{3-x}{6+x}$$

Ans:  $\lim_{t \rightarrow 0^+} \frac{3t-1}{6t+1}$

Difficulty: Hard

Section: 1.1

1. Given that  $\lim_{x \rightarrow a} f(x) = 5$  and  $\lim_{x \rightarrow a} g(x) = 4$ , find, if it exists,  $\lim_{x \rightarrow a} (2f(x) - 9g(x))^2$ .  
A) -676    B) 676    C) 1    D) -26    E) It does not exist

Ans: B

Difficulty: Easy

Section: 1.2

2.  $\lim_{x \rightarrow 6} -7 =$   
A) 6    B) -6    C) -7    D) -42    E) 0

Ans: C

Difficulty: Easy

Section: 1.2

3. Answer true or false.  $\lim_{x \rightarrow 5} 9x = 45$

Ans: True

Difficulty: Easy

Section: 1.2

4.  $\lim_{x \rightarrow -8} \frac{x^2 - 64}{x + 8} =$   
A) 8    B) -16    C) 64    D) -8    E) -64

Ans: B

Difficulty: Medium

Section: 1.2

5.  $\lim_{x \rightarrow -7} \frac{14}{x + 7} =$   
A)  $+\infty$     B)  $-\infty$     C) 0    D) It does not exist.    E) 14

Ans: D

Difficulty: Hard

Section: 1.2

6. Find  $\lim_{x \rightarrow 0^+} f(x)$  where  $f(x) = \frac{|4x|}{x}$ .  
A) 4    B) 0    C) -4    D)  $\infty$     E) It does not exist.

Ans: A

Difficulty: Medium

Section: 1.2

7.  $\lim_{x \rightarrow 10^+} \frac{x}{x-10} =$

- A) 10    B) 0    C)  $\infty$     D)  $-\infty$     E) It does not exist.

Ans: C

Difficulty: Medium

Section: 1.2

8.  $\lim_{x \rightarrow 3^-} \frac{15}{x-3} =$

- A) 3    B) 0    C)  $\infty$     D)  $-\infty$     E) It does not exist.

Ans: D

Difficulty: Medium

Section: 1.2

9.  $\lim_{x \rightarrow 12} \frac{3x-6}{x-12} =$

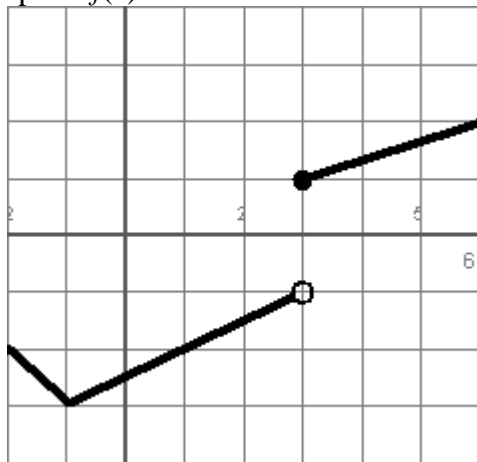
- A) 12    B) 0    C)  $\infty$     D)  $-\infty$     E) It does not exist.

Ans: E

Difficulty: Medium

Section: 1.2

10. Use this graph of  $f(x)$



to find  $\lim_{x \rightarrow 3} f(x) =$

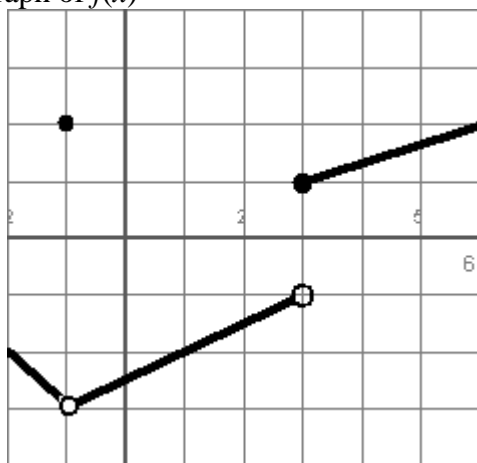
- A) 3    B) -1    C) It does not exist.    D) 0    E) 1

Ans: C

Difficulty: Easy

Section: 1.2

11. Use this graph of  $f(x)$



to find  $\lim_{x \rightarrow -1} f(x) =$

- A)  $\infty$    B) 0   C) It does not exist.   D) 2   E) -3

Ans: E

Difficulty: Medium

Section: 1.2

12. Let

$$f(x) = \begin{cases} x^4, & x \leq 6 \\ x-1, & x > 6 \end{cases}, \quad \text{find } \lim_{x \rightarrow 6^+} f(x) =$$

- A) 1,296   B) 4   C) 5   D) 6   E) 0

Ans: C

Difficulty: Hard

Section: 1.2

13. Let

$$f(x) = \begin{cases} x^2 - 2, & x < 3 \\ x^4, & x \geq 3 \end{cases}, \quad \text{then } \lim_{x \rightarrow 3} f(x) =$$

- A) 7   B) 81   C) 9   D) 0   E) It does not exist.

Ans: E

Difficulty: Medium

Section: 1.2

14.  $\lim_{x \rightarrow 64} \frac{64-x}{\sqrt{x}-8} =$

- A) 64   B) -16   C) 8   D) -64   E) -8

Ans: B

Difficulty: Hard

Section: 1.2

15. Answer true or false.  $\lim_{x \rightarrow 0^+} \frac{\sqrt{x^2 + 36} - 6}{x} = \frac{1}{5}$   
 Ans: False  
 Difficulty: Hard  
 Section: 1.2

16. Find  $\lim_{x \rightarrow 0} (x^3 + 3x^2 - 1)$ .  
 Ans: -1  
 Difficulty: Easy  
 Section: 1.2

17. Find  $\lim_{x \rightarrow 7} 2^x$ .  
 Ans: 4  
 Difficulty: Easy  
 Section: 1.2

18. Find  $\lim_{x \rightarrow 1} \frac{x^2 + 2}{x^2 - 3}$ .  
 Ans:  $-\frac{3}{2}$   
 Difficulty: Easy  
 Section: 1.2

19. Find  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + x - 6}$ .  
 Ans:  $\frac{4}{5}$   
 Difficulty: Medium  
 Section: 1.2

20. Find  $\lim_{x \rightarrow 0} \frac{x^2 + 12x}{3x - 3x^2}$ .  
 Ans: 4  
 Difficulty: Medium  
 Section: 1.2

21. Find  $\lim_{x \rightarrow 2} \frac{11 - x^2}{x^2 + 7x - 3}$ .

Ans:  $\frac{7}{15}$

Difficulty: Easy

Section: 1.2

22. Find  $\lim_{x \rightarrow 5} \frac{x^2 + x - 30}{x^2 - 12x + 35}$ .

Ans:  $-\frac{11}{2}$  or  $-5\frac{1}{2}$

Difficulty: Medium

Section: 1.2

23. Find  $\lim_{x \rightarrow b} \frac{x^2 - b^2}{x - b}$ .

Ans:  $2b$

Difficulty: Medium

Section: 1.2

24. Find  $\lim_{x \rightarrow 5} \frac{x^3 - 125}{x - 5}$ .

Ans:  $75$

Difficulty: Hard

Section: 1.2

25. Find  $\lim_{x \rightarrow 3} \frac{x^3 - 7x^2 + 12x}{x - 3}$ .

Ans:  $-3$

Difficulty: Hard

Section: 1.2

26. Find  $\lim_{h \rightarrow 4} \frac{h^3 - 16h}{h^3 - 4h^2}$ .

Ans:  $2$

Difficulty: Hard

Section: 1.2

$$27. \text{ Find } \lim_{x \rightarrow c} \frac{\frac{8}{x} - \frac{8}{c}}{x - c} .$$

$$\text{Ans: } -\frac{8}{c^2}$$

Difficulty: Hard

Section: 1.2

$$28. \text{ Find } \lim_{h \rightarrow 0} \frac{\frac{7}{2+h} - \frac{7}{2}}{h} .$$

$$\text{Ans: } -\frac{7}{4}$$

Difficulty: Hard

Section: 1.2

$$29. \text{ Find } \lim_{x \rightarrow -b} \frac{x^3 + b^3}{x + b} .$$

$$\text{Ans: } 3b^2$$

Difficulty: Hard

Section: 1.2

$$30. \text{ Find } \lim_{x \rightarrow 6} \frac{x - 6}{x^3 - 216} .$$

$$\text{Ans: } \frac{1}{108}$$

Difficulty: Hard

Section: 1.2

$$31. \text{ Find } \lim_{x \rightarrow 3} \frac{1 - \frac{9}{x^2}}{1 - \frac{3}{x}} .$$

$$\text{Ans: } 2$$

Difficulty: Hard

Section: 1.2

32. Find  $\lim_{x \rightarrow 3} \frac{|x-4|-4}{x}$   
Ans: -1  
Difficulty: Easy  
Section: 1.2

33. Find  $\lim_{x \rightarrow 10^-} \frac{x-10}{|x-10|}$ .  
Ans: -1  
Difficulty: Medium  
Section: 1.2

34. Find  $\lim_{x \rightarrow 7^+} \frac{x-7}{|x-7|}$ .  
Ans: 1  
Difficulty: Medium  
Section: 1.2

35.  $\lim_{x \rightarrow -6} 5 =$   
Ans: 5  
Difficulty: Easy  
Section: 1.2

36.  $\lim_{x \rightarrow 3} \frac{2x}{|x|} =$   
Ans: 2  
Difficulty: Easy  
Section: 1.2

37. Find  $\lim_{x \rightarrow 8^-} \frac{|x-8|}{x-8}$ .  
Ans: -1  
Difficulty: Medium  
Section: 1.2

38. Find  $\lim_{x \rightarrow 3^+} \frac{|x-3|}{x-3}$ .  
Ans: 1  
Difficulty: Medium  
Section: 1.2

39. Find  $\lim_{x \rightarrow 11} \frac{|x-11|}{x-11}$ .

Ans: Does not exist.

$$\lim_{x \rightarrow 11^-} \frac{|x-11|}{x-11} \neq \lim_{x \rightarrow 11^+} \frac{|x-11|}{x-11}$$

Difficulty: Medium

Section: 1.2

40.  $\lim_{x \rightarrow 6} \frac{x^2 - 36}{x - 6} =$

Ans: 12

Difficulty: Medium

Section: 1.2

41. Find  $\lim_{x \rightarrow 1} f(x)$  where  $f(x) = \begin{cases} \frac{1}{x+7}, & x < 1 \\ 1-7x, & x > 1 \end{cases}$ .

Ans: The limit does not exist

$$\text{because } \lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$$

Difficulty: Medium

Section: 1.2

42. Express the right hand limit at  $x = 2$  for  $f(x) = \begin{cases} 4-x, & x > 2 \\ 12, & x = 2 \\ 4+x, & x < 2 \end{cases}$ .

$$\lim_{x \rightarrow 2} 4 - x = 2$$

Ans:  $x \rightarrow 2$

Difficulty: Medium

Section: 1.2

43. Find the left hand limit at  $x = 4$  for  $f(x) = \begin{cases} x^3 - 11, & x \geq 4 \\ x + 11, & x < 4 \end{cases}$ .

$$\lim_{x \rightarrow 4^-} x + 11 = 15$$

Ans:  $x \rightarrow 4^-$

Difficulty: Medium

Section: 1.2

44. Find  $\lim_{x \rightarrow 2} f(x)$  where  $f(x) = \begin{cases} x^2 - 3, & x < 2 \\ (x-1)^3, & x > 2 \end{cases}$ .

Ans: 1

Difficulty: Medium

Section: 1.2

45.  $\lim_{x \rightarrow 2^-} \frac{7}{|x-2|} =$

Ans:  $+\infty$

Difficulty: Medium

Section: 1.2

46.  $\lim_{x \rightarrow 2^+} \frac{7}{|x-2|} =$

Ans:  $+\infty$

Difficulty: Medium

Section: 1.2

47.  $\lim_{x \rightarrow 4} 4x =$

Ans: 16

Difficulty: Easy

Section: 1.2

48.  $\lim_{x \rightarrow 1} \frac{x^2 - 6x + 5}{x - 2} =$

Ans: 0

Difficulty: Medium

Section: 1.2

49. Let  $f(x) = \begin{cases} x+3, & x \leq 3 \\ x-6, & x > 3 \end{cases}$ . Then  $\lim_{x \rightarrow 3^-} f(x) =$

Ans: 6

Difficulty: Medium

Section: 1.2

50. Let  $f(x) = \begin{cases} x+7, & x \leq 4 \\ x-12, & x > 4 \end{cases}$ . Then  $\lim_{x \rightarrow 4^+} f(x) =$

Ans: -8

Difficulty: Medium

Section: 1.2

51. Let  $f(x) = \begin{cases} x+4, & x \leq 4 \\ x-6, & x > 4 \end{cases}$ . Find  $\lim_{x \rightarrow 4} f(x)$ .

Ans: It does not exist.

Difficulty: Medium

Section: 1.2

52. First rationalize the numerator, then find the limit.  $\lim_{x \rightarrow 0} \frac{\sqrt{x+64}-8}{x}$

Ans:  $\frac{1}{16}$

Difficulty: Hard

Section: 1.2

1. Answer true or false.  $\lim_{x \rightarrow +\infty} \frac{6x^2 - 3x + 2}{2x^2 + 3x - 3} = 0$   
Ans: False  
Difficulty: Medium  
Section: 1.3

2. Answer true or false.  $\lim_{x \rightarrow -\infty} \frac{3x^3 + 3x^2 - 12x + 3}{x^3 + 7} = -3$   
Ans: False  
Difficulty: Medium  
Section: 1.3

3. Answer true or false.  $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 1}}{x - 2} = 2$   
Ans: False  
Difficulty: Hard  
Section: 1.3

4. Answer true or false.  $\lim_{x \rightarrow -\infty} \frac{\sqrt{36x^2 + 1}}{x + 8} = -6$   
Ans: True  
Difficulty: Hard  
Section: 1.3

5. Answer true or false.  $\lim_{x \rightarrow -\infty} \frac{1}{8x - 11} = 0$   
Ans: True  
Difficulty: Easy  
Section: 1.3

6.  $\lim_{x \rightarrow +\infty} \frac{4x - 9}{x^5 - 1}$   
A) 0   B) 4   C) 1   D)  $\infty$    E) It does not exist.  
Ans: A  
Difficulty: Easy  
Section: 1.3

$$7. \lim_{x \rightarrow -\infty} \frac{10x^4 - 1}{2x^4 + 5} =$$

A) 0    B)  $-\infty$     C)  $\infty$     D) 5    E) It does not exist.

Ans: D

Difficulty: Medium

Section: 1.3

$$8. \lim_{x \rightarrow -1} \frac{9x^2}{x^6 - 2x^2 - x} =$$

A) 0    B)  $+\infty$     C)  $-\infty$     D) 9    E) It does not exist.

Ans: B

Difficulty: Medium

Section: 1.3

$$9. \lim_{x \rightarrow 25} \frac{x+2}{\sqrt{x}-5} =$$

A)  $-\frac{2}{5}$     B)  $+\infty$     C)  $-\infty$     D) 25    E) It does not exist.

Ans: E

Difficulty: Hard

Section: 1.3

$$10. \lim_{x \rightarrow -\infty} \sqrt{\frac{18x^8 - 2x^4 + 5}{2x^8 + 9x^4 - 4}}$$

A)  $+\infty$     B)  $-\infty$     C) 9    D) 3    E) It does not exist.

Ans: D

Difficulty: Hard

Section: 1.3

$$11. \lim_{x \rightarrow +\infty} (-7x^6 - 4x^4 - x^3 + 7x)$$

A)  $-\infty$     B)  $\infty$     C) 0    D) 7    E) It does not exist.

Ans: A

Difficulty: Easy

Section: 1.3

$$12. \lim_{x \rightarrow +\infty} \frac{\sqrt[5]{1,024x^5}}{16x}$$

- A)  $+\infty$    B)  $-\infty$    C) 0   D)  $\frac{1}{4}$    E) It does not exist.

Ans: D

Difficulty: Medium

Section: 1.3

$$13. \lim_{x \rightarrow +\infty} \frac{\sqrt[4]{81x^4}}{3x}$$

- A)  $+\infty$    B)  $-\infty$    C) 0   D) 1   E) It does not exist.

Ans: D

Difficulty: Medium

Section: 1.3

$$14. \text{ Answer true or false. } \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 - 7} + 5}{x} \text{ does not exist.}$$

Ans: False

Difficulty: Medium

Section: 1.3

$$15. \lim_{x \rightarrow -\infty} \frac{4x^2 + 8}{2x^2 - 4} =$$

- A) -2   B) 2   C) 0   D)  $-\infty$    E)  $+\infty$

Ans: B

Difficulty: Medium

Section: 1.3

$$16. \text{ Answer true or false. } \lim_{x \rightarrow -\infty} \frac{3-x}{8+x} \text{ is equivalent to } \lim_{x \rightarrow -\infty} \left( \frac{\frac{3}{x} - 1}{\frac{8}{x} + 1} \right)$$

Ans: True

Difficulty: Medium

Section: 1.3

$$17. \lim_{x \rightarrow +\infty} 8 =$$

Ans: 8

Difficulty: Easy

Section: 1.3

$$18. \lim_{x \rightarrow -\infty} \frac{|x|}{x} =$$

Ans: -1

Difficulty: Easy

Section: 1.3

$$19. \text{ If } \lim_{x \rightarrow +\infty} f(x) = 5 \text{ and } \lim_{x \rightarrow +\infty} g(x) = 5, \text{ what is } \lim_{x \rightarrow +\infty} |8f(x) - g(x)| \text{ ?}$$

Ans: 35

Difficulty: Medium

Section: 1.3

$$20. \lim_{x \rightarrow -\infty} (4 - 3x) =$$

Ans:  $+\infty$

Difficulty: Easy

Section: 1.3

$$21. \lim_{x \rightarrow -\infty} (1 + 6x) =$$

Ans:  $-\infty$

Difficulty: Easy

Section: 1.3

$$22. \lim_{x \rightarrow -\infty} \left( 5 - \frac{9}{x} \right) =$$

Ans: 5

Difficulty: Easy

Section: 1.3

$$23. \lim_{x \rightarrow -\infty} \sqrt{9 - x} =$$

Ans:  $+\infty$

Difficulty: Medium

Section: 1.3

$$24. \lim_{x \rightarrow +\infty} \sqrt{1 + 6x} =$$

Ans:  $+\infty$

Difficulty: Easy

Section: 1.3

$$25. \lim_{x \rightarrow -\infty} \frac{8}{9x+1} =$$

Ans: 0

Difficulty: Easy

Section: 1.3

$$26. \lim_{x \rightarrow +\infty} \frac{4x^2+3}{3x^2-6} =$$

Ans:  $\frac{4}{3}$

Difficulty: Medium

Section: 1.3

$$27. \lim_{x \rightarrow +\infty} \frac{9x^2+1x}{2x^2-4x} =$$

Ans:  $\frac{9}{2}$

Difficulty: Medium

Section: 1.3

$$28. \lim_{x \rightarrow +\infty} \frac{\sqrt{13x^2+2}}{x-11} =$$

Ans:  $\sqrt{13}$

Difficulty: Hard

Section: 1.3

$$29. \lim_{x \rightarrow +\infty} \frac{\sqrt[4]{11x^8-4x}}{x^2+10} =$$

Ans:  $\sqrt[4]{11}$

Difficulty: Hard

Section: 1.3

$$30. \lim_{x \rightarrow +\infty} \left( \sqrt{x^2+14} - x \right) =$$

Ans: 0

Difficulty: Hard

Section: 1.3

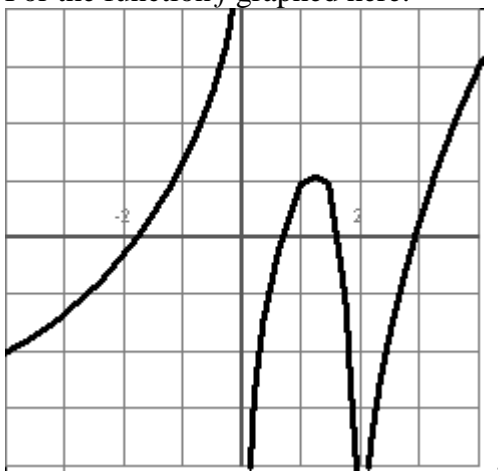
31. Find  $\lim_{x \rightarrow -\infty} f(x)$  where  $f(x) = \begin{cases} x^2, & x < 4 \\ -8x + 3, & x \geq 4 \end{cases}$

Ans:  $+\infty$

Difficulty: Easy

Section: 1.3

32. For the function  $f$  graphed here:



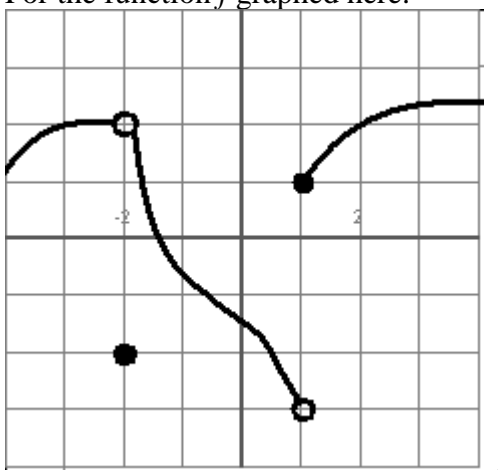
find  $\lim_{x \rightarrow 2^-} f(x)$ .

Ans:  $-\infty$

Difficulty: Easy

Section: 1.3

33. For the function  $f$  graphed here:



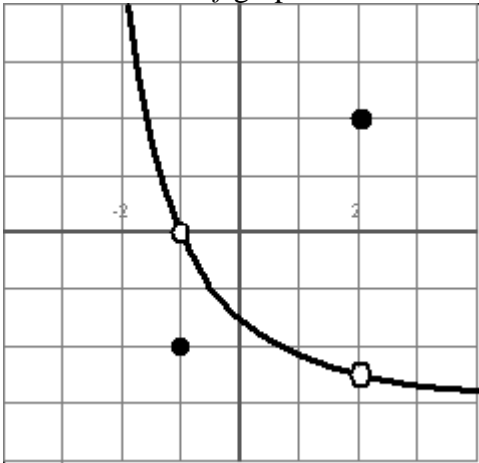
find  $\lim_{x \rightarrow -2^-} f(x)$ .

Ans: 2

Difficulty: Easy

Section: 1.3

34. For the function  $f$  graphed here:



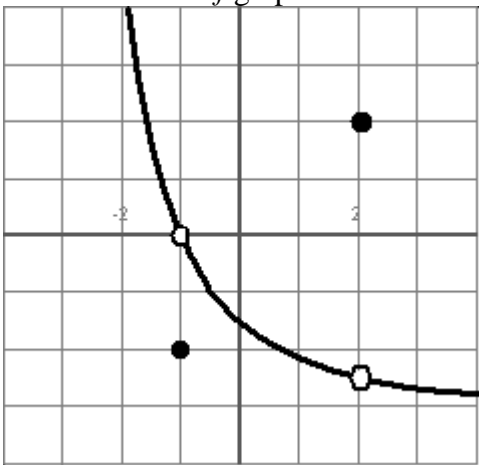
find  $\lim_{x \rightarrow 2} f(x)$

Ans: -2.5

Difficulty: Easy

Section: 1.3

35. For the function  $f$  graphed here:



find  $\lim_{x \rightarrow +\infty} f(x)$

Ans: -3

Difficulty: Easy

Section: 1.3

36. For the function  $f$  graphed here:



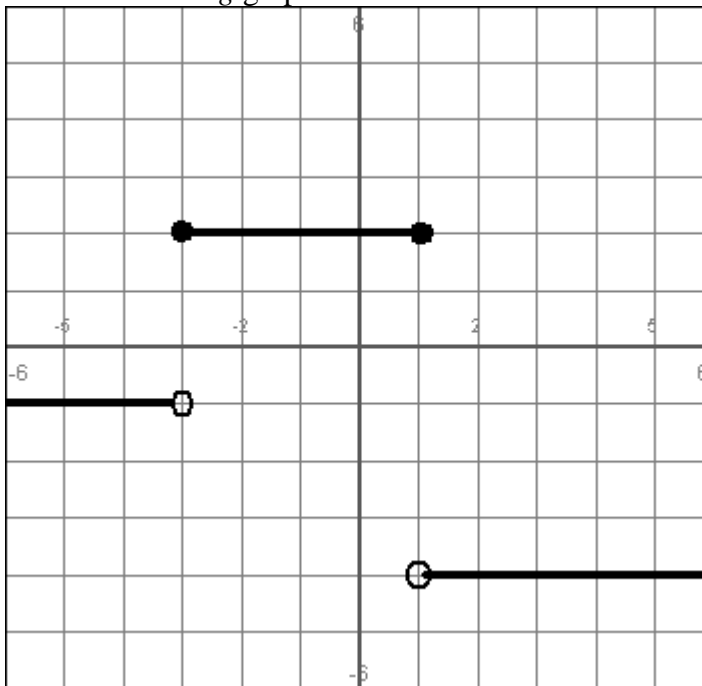
find  $\lim_{x \rightarrow -2} f(x)$ .

Ans: It does not exist.

Difficulty: Easy

Section: 1.3

37. For the function  $g$  graphed here:



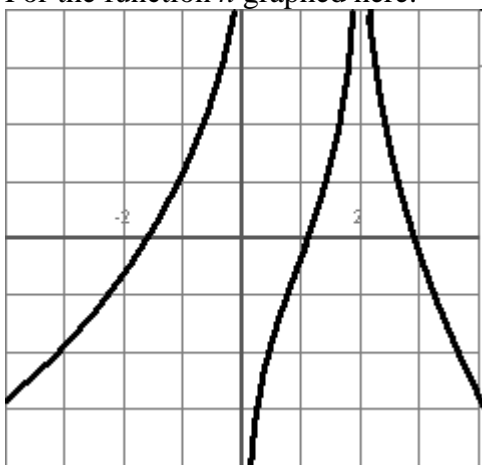
find  $\lim_{x \rightarrow -3^-} g(x)$ .

Ans: -1

Difficulty: Easy

Section: 1.3

38. For the function  $h$  graphed here:



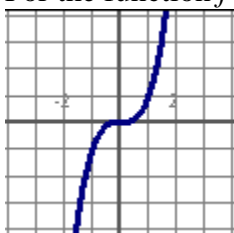
find  $\lim_{x \rightarrow 0^+} h(x)$

Ans:  $-\infty$

Difficulty: Easy

Section: 1.3

39. For the function  $f$  graphed here:



find  $\lim_{x \rightarrow -1^-} f(x)$

Ans:  $-1$

Difficulty: Medium

Section: 1.3

40. Convert  $\lim_{x \rightarrow +\infty} \frac{9+4x}{x}$  into a limit as  $x$  nears 0.

Ans:  $\lim_{x \rightarrow 0^+} \left( \frac{9}{x} + 4 \right)$

Difficulty: Medium

Section: 1.3

1. Find a least number  $\delta$  such that  $|f(x) - L| < \varepsilon$  if  $0 < |x - a| < \delta$ .  $\lim_{x \rightarrow 7} 11x = 77$  ;  $\varepsilon = 0.187$   
 A) 1    B) 0.187    C) 0.017    D) 0.014    E) 0.021

Ans: C

Difficulty: Medium

Section: 1.4

2. Find a least number  $\delta$  such that  $|f(x) - L| < \varepsilon$  if  $0 < |x - a| < \delta$ .  $\lim_{x \rightarrow 11} 2x - 2 = 20$  ;  $\varepsilon = 0.005$   
 A) 0    B) 0.0025    C) 0.0028    D) 0.0035    E) 0.0039

Ans: B

Difficulty: Medium

Section: 1.4

3. Answer true or false. It can be shown that if  $|f(x) - L| < \varepsilon$  when  $0 < |x - a| < \delta$ , then  $|x^2 - 9| < \varepsilon$  if  $|x - 3| < \delta$  for arbitrarily small positive  $\varepsilon$ .

Ans: True

Difficulty: Easy

Section: 1.4

4. Find a least number  $\delta$  such that  $|f(x) - L| < \varepsilon$  if  $0 < |x - a| < \delta$ .  $\lim_{x \rightarrow -7} \frac{x^2 - 49}{x + 7} = -14$  ;  $\varepsilon = 0.005$ .

A) 0.002    B) 0.004    C) 0.005    D) 0.007    E) 0.008

Ans: C

Difficulty: Medium

Section: 1.4

5. Find a least positive number  $N$  such that  $|f(x) - L| < \varepsilon$  if  $x > N$ .  $\lim_{x \rightarrow +\infty} \frac{10}{x} = 0$  ;  $\varepsilon = 0.05$   
 A) 400    B) 20    C) 40    D) 100    E) 200

Ans: E

Difficulty: Medium

Section: 1.4

6. Answer true or false. It is possible to prove that  $\lim_{x \rightarrow +\infty} \frac{2}{x^2 + 1} = 0$ .

Ans: True

Difficulty: Easy

Section: 1.4

7. Answer true or false. It is possible to prove that  $\lim_{x \rightarrow +\infty} \frac{2x^2}{6x^2 + 4} = 0$ .

Ans: False

Difficulty: Easy

Section: 1.4

8. Answer true or false. It is possible to prove that  $\lim_{x \rightarrow 6} \frac{1}{x^2 - 36} = +\infty$ .

Ans: False

Difficulty: Medium

Section: 1.4

9. To prove that  $\lim_{x \rightarrow 4} (x - 4) = 0$  a reasonable relationship between  $\delta$  and  $\varepsilon$  would be

A)  $\delta = 4\varepsilon$    B)  $\delta = \varepsilon$    C)  $\delta = \sqrt{\varepsilon}$    D)  $\delta = \frac{1}{\varepsilon}$    E)  $\delta = \varepsilon - 4$

Ans: B

Difficulty: Medium

Section: 1.4

10. Answer true or false. To use a  $\delta$ - $\varepsilon$  approach to show that  $\lim_{x \rightarrow 0^+} \frac{10}{x^6} = +\infty$ , a reasonable

first step would be to change the limit to  $\lim_{x \rightarrow +\infty} 10x^6 = +\infty$ .

Ans: True

Difficulty: Easy

Section: 1.4

11. Answer true or false. It is possible to show that  $\lim_{x \rightarrow 4} \frac{-5}{|x - 4|} = -\infty$ .

Ans: True

Difficulty: Medium

Section: 1.4

12. To prove that  $\lim_{x \rightarrow 2} f(x) = 8$ , where

$$f(x) = \begin{cases} 4x, & x < 2 \\ x+6, & x \geq 2 \end{cases}, \text{ a reasonable relationship between } \delta \text{ and } \varepsilon \text{ would be}$$

A)  $\delta = 4\varepsilon$     B)  $\delta = \frac{\varepsilon}{4}$     C)  $\delta = 4\varepsilon + 6$     D)  $\delta = 2\varepsilon$     E)  $\delta = \frac{\varepsilon}{2}$

Ans: B

Difficulty: Hard

Section: 1.4

13. Answer true or false. It is possible to show that  $\lim_{x \rightarrow 0} \frac{x}{6} = 0$ .

Ans: True

Difficulty: Easy

Section: 1.4

14. If  $\lim_{x \rightarrow 4} \sqrt{x} + 3 = 5$  and  $\varepsilon = 0.006$ , find a largest positive number  $\delta$  such that  $|f(x) - 5| < 0.006$  if  $0 < |x - 4| < \delta$ . (Round to 3 decimal places.)

A) 0.021    B) 0.022    C) 0.023    D) 0.024    E) 0.025

Ans: D

Difficulty: Hard

Section: 1.4

15. If  $\lim_{x \rightarrow 4} 4x = 16$  and  $\varepsilon = 0.0368$ , find a smallest positive number  $\delta$  such that  $|f(x) - 16| < 0.0368$  if  $0 < |x - 4| < \delta$ .

A) 0.0072    B) 0.0082    C) 0.0092    D) 0.092    E) 0.0368

Ans: C

Difficulty: Easy

Section: 1.4

16. Find a number  $\delta$  such that  $|f(x) - L| < \varepsilon$  if  $|x - 4| < \delta$ .  $\lim_{x \rightarrow 4} 3x = 12$ .

Ans:  $\frac{\varepsilon}{3}$

Difficulty: Easy

Section: 1.4

17. Use  $\delta$  and  $\varepsilon$  to prove  $\lim_{x \rightarrow 2} 3x+7=13$ .

Ans: Show that  $\forall \varepsilon, \exists \delta$  such that if  $0 < |x-2| < \delta$  then  $|f(x)-L| < \varepsilon$   
 $|3x+7-L| < \varepsilon$ ,  $|(3x+7)-13| < \varepsilon$  (when  $0 < |x-2| < \delta$ ),  $|3|x-2|+7-L| < \varepsilon$ ,  
 $3|x-2| < \varepsilon$ ,  $|x-2| < \frac{\varepsilon}{3}$ .  
 So if  $\delta = \frac{\varepsilon}{3}$ ,  $|f(x)-L| < \varepsilon$ , when  $\lim_{x \rightarrow 2} 3x+7=13$

Difficulty: Medium

Section: 1.4

18. Use  $\delta$  and  $\varepsilon$  to prove  $\lim_{x \rightarrow 0} \sqrt[5]{x} = 0$ .

Ans: Show  $|\sqrt[5]{x}-0| < \varepsilon$  if  $0 < |x-0| < \delta$ .  
 $|\sqrt[5]{x-0}-0| < \varepsilon \rightarrow |\sqrt[5]{|x-0|}| < \varepsilon \rightarrow |x-0| < \varepsilon^5$ ,  $\delta = \varepsilon^5$

Difficulty: Medium

Section: 1.4

19. Use  $\delta$  and  $\varepsilon$  to prove  $\lim_{x \rightarrow 25} \sqrt{x} = 5$ .

Ans:  $|\sqrt{x}-5| < \varepsilon$  if  $0 < |x-25| < \delta$ . If  $0 < |(\sqrt{x}-5)(\sqrt{x}+5)| < \delta$  Then  
 $0 < k|\sqrt{x}-5| < \delta$  for some k.

Since  $k < 25$ ,  $\delta = \frac{\varepsilon}{25}$ .

Difficulty: Hard

Section: 1.4

20. Use  $\delta$  and  $\varepsilon$  to prove  $\lim_{x \rightarrow 0} x^5 = 0$ .

Ans: Show  $|x^5-0| < \varepsilon$  if  $0 < |x-0| < \delta$ .  $x^5 < \varepsilon$  when  $x < \sqrt[5]{\varepsilon}$ . So  $\delta = \sqrt[5]{\varepsilon}$

Difficulty: Medium

Section: 1.4

21. Use  $\delta$  and  $\varepsilon$  to prove  $\lim_{x \rightarrow 81} \sqrt[4]{x} = 3$ .

Ans: Show  $|\sqrt[4]{x} - 3| < \varepsilon$  if  $0 < |x - 81| < \delta$ .  
 $|\sqrt[4]{x-81} - 3| < \varepsilon$  for  $x > 1$ ,  $|\sqrt[4]{x} - 3| \leq |x - 81| < \varepsilon^4$

$$\text{So, } \delta = \varepsilon^4$$

Difficulty: Medium

Section: 1.4

22. Prove that  $\lim_{x \rightarrow +\infty} \frac{7}{x} = 0$ .

Ans: Show that for  $\varepsilon > 0$  there exists  $N > 0$  such that if  $|\frac{7}{x} - 0| < \varepsilon$  if  $x > N$ . So,  
 $\frac{7}{x} < \varepsilon$ , when  $N = x > \frac{7}{\varepsilon}$ .

Difficulty: Medium

Section: 1.4

23. Prove that  $\lim_{x \rightarrow +\infty} \frac{1}{x^2} = 0$ .

Ans: Show that  $|\frac{1}{x^2} - 0| < \varepsilon$  if  $x > N$ .  $\frac{1}{x^2} < \varepsilon \rightarrow N = x > \sqrt{\frac{1}{\varepsilon}}$

Difficulty: Medium

Section: 1.4

24. Prove that  $\lim_{x \rightarrow +\infty} \frac{1}{x^3} = 0$ .

Ans:  $|\frac{1}{\sqrt[3]{x}} - 0| < \varepsilon$  if  $x > N$ .  $1x^{-1/3} < \varepsilon$ .  $N = x > \left(\frac{\varepsilon}{1}\right)^{-3}$

Difficulty: Medium

Section: 1.4

25. Find  $\delta$  if  $\lim_{x \rightarrow 3} 3x = 9$  and  $\varepsilon = 0.03$ .

Ans:  $\delta = \frac{\varepsilon}{3} = \frac{0.03}{3} = 0.01$

Difficulty: Medium

Section: 1.4

26. Find  $\delta$  if  $\lim_{x \rightarrow 2} \frac{x^2 - 64}{x - 8} = 10$  and  $\varepsilon = 0.02$ .

Ans:  $\delta = 0.02$

Difficulty: Medium

Section: 1.4

27. Find the smallest integer  $N$  such that  $\lim_{x \rightarrow +\infty} \frac{5}{x^4} = 0$  and  $\varepsilon = 0.000099$ .

Ans:  $N = 15$

Difficulty: Medium

Section: 1.4

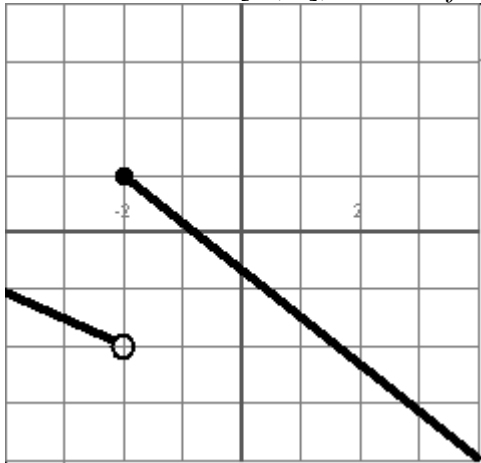
28. Prove that  $\lim_{x \rightarrow 7} x^2 = 49$ .

Ans:  $|f(x) - L| = |x^2 - 49| = |(x - 7)(x + 7)| < \varepsilon$ ,  $k|x - 7| < \varepsilon$ ,  $\delta = \frac{\varepsilon}{k}$

Difficulty: Hard

Section: 1.4

1. On the interval of  $[-4, 4]$ , where is  $f$  not continuous?



- A) -2    B) 0 and 2    C) -3 and 1    D) 3    E) Nowhere

Ans: A

Difficulty: Easy

Section: 1.5

2. Answer true or false.  $f(x) = x^5 - 5x^4 + 9$  has no point of discontinuity.

Ans: True

Difficulty: Medium

Section: 1.5

3. Answer true or false.  $f(x) = |x^2 - 4|$  has points of discontinuity at  $x = -2$  and  $x = 2$ .

Ans: False

Difficulty: Medium

Section: 1.5

4. Find the  $x$ -coordinates for all points of discontinuity for

$$f(x) = \frac{x-6}{x^2-8x+12}$$

- A) 6 and 2    B) 6    C) -2 and -6    D) -2    E) 2

Ans: A

Difficulty: Hard

Section: 1.5

5. Find the  $x$ -coordinates for all points of discontinuity for

$$f(x) = \frac{25x^2+100}{|5x+10|}$$

- A) 0    B) -2 and 2    C) -2    D) 2    E) 5

Ans: C

Difficulty: Medium

Section: 1.5

$$f(x) = \begin{cases} x^2 + 8, & x \leq 5 \\ -7, & x > 5 \end{cases}$$

6. Find the  $x$ -coordinates for all points of discontinuity for

- A) 5   B) 7   C) -5   D) 0 and 7   E) None exists.

Ans: A

Difficulty: Easy

Section: 1.5

7. Find the value of  $k$ , if possible, that will make the function continuous.

$$f(x) = \begin{cases} x + 2k, & x \leq 3 \\ kx^2 + x - 14, & x > 3 \end{cases}$$

- A) -2   B) 6   C) -6   D) 2   E) None exists.

Ans: D

Difficulty: Hard

Section: 1.5

8. Answer true or false. The function  $f(x) = \frac{x+2}{x-5}$  has a removable discontinuity at  $x = 5$ .

Ans: False

Difficulty: Medium

Section: 1.5

9. Answer true or false. The function  $f(x) = \begin{cases} x^3, & x \leq 3 \\ x^2 + 18, & x > 3 \end{cases}$  is continuous everywhere.

Ans: True

Difficulty: Hard

Section: 1.5

10. Answer true or false. If  $f$  and  $g$  are each continuous at  $c$ ,  $\frac{f}{g}$  may be discontinuous at  $c$ .

Ans: True

Difficulty: Easy

Section: 1.5

11. Answer true or false. The Intermediate-Value Theorem can be used to approximate the

locations of all discontinuities for  $f(x) = \frac{-7x^2 + 4x + 12}{x}$ .

Ans: True

Difficulty: Easy

Section: 1.5

12. Answer true or false.  $f(x) = x^2 - 6x + 5 = 0$  has at least one solution on the interval  $[0, 2]$ .

Ans: True

Difficulty: Medium

Section: 1.5

13. Answer true or false.  $f(x) = x^4 - 2x^2 + 11 = 0$  has at least one solution on the interval  $[0, 9]$ .

Ans: False

Difficulty: Hard

Section: 1.5

14. Use the fact that  $\sqrt[4]{2}$  is a solution of  $x^4 - 2 = 0$  to approximate  $\sqrt[4]{2}$  with an error of at most 0.005.

A) 1.169    B) 1.179    C) 1.189    D) 1.199    E) 1.209

Ans: C

Difficulty: Easy

Section: 1.5

15.  $f(x) = \begin{cases} 3x+1, & x \leq 3 \\ kx+4, & x > 3 \end{cases}$ . Find the value for the constant  $k$  that will make the function continuous everywhere.

A) 2    B) 3    C) 4    D) 5    E) 6

Ans: A

Difficulty: Medium

Section: 1.5

16.  $f(x) = \begin{cases} x^2 - 6, & x \leq 5 \\ 5x - k, & x > 5 \end{cases}$ . Find the value for the constant  $k$  that will make the function continuous everywhere.

A) 5    B) 6    C) 7    D) 8    E) 9

Ans: B

Difficulty: Medium

Section: 1.5

17. Answer true or false.  $f(x) = \tan(x^4 - 1)$  has no point of discontinuity.

Ans: False

Difficulty: Medium

Section: 1.5

18. A point of discontinuity of  $f(x) = \frac{-3}{|0.707 - \sin x|}$  is at
- A)  $\frac{\pi}{2}$     B)  $\frac{\pi}{3}$     C)  $\frac{\pi}{6}$     D)  $\frac{\pi}{4}$     E)  $2\pi$

Ans: D

Difficulty: Hard

Section: 1.5

19. Find any points of discontinuity for  $f(x) = \frac{x-3}{x^2-9}$ .

Ans:  $x = \pm 3$

Difficulty: Easy

Section: 1.5

20. Find any points of discontinuity for  $f(x) = \frac{x-2}{x^2+4}$ .

Ans:  $f$  is continuous everywhere since  $x^2 + 4 \neq 0$ .

Difficulty: Easy

Section: 1.5

21. Show that  $f(x) = \frac{x^4-6}{x-\sqrt{2}}$  is not a continuous function.

Ans:  $f$  is not continuous because  $f(\sqrt{2})$  is not defined.

Difficulty: Medium

Section: 1.5

22. Redefine  $f(x) = \frac{x^3+125}{x+5}$  so that it will be continuous everywhere.

$$f(x) = \begin{cases} \frac{x^3+125}{x+5}, & x \neq -5 \\ 75, & x = -5 \end{cases}$$

Ans:

Difficulty: Medium

Section: 1.5

23. Define  $g(x) = \frac{x^2 + x - 42}{x - 6}$  so that it will be continuous everywhere.

$$g(x) = \begin{cases} \frac{x^2 + x - 42}{x - 6}, & x \neq 6 \\ 13, & x = 6 \end{cases}$$

Ans:

Difficulty: Medium

Section: 1.5

24. Prove that  $f(x) = \sqrt{x^2 + 7x}$  is continuous on  $[0, +\infty)$ .

Ans: For  $c$  in the interval  $(0, \infty)$ ,

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} \sqrt{x^2 + 7x} = \sqrt{c^2 + 7c} = f(c)$$

So  $f$  is continuous on  $[0, +\infty)$ .

Difficulty: Medium

Section: 1.5

25. Assign a value to the constant  $k$  which will make  $g$  continuous.

$$g(x) = \begin{cases} \frac{x + 6}{x^3 + 6x^2 + x + 6}, & x \neq -6 \\ k, & x = -6 \end{cases}$$

$$k = \frac{1}{37}$$

Ans:

Difficulty: Medium

Section: 1.5

26. Assign a value to the constant  $k$  which will make  $g$  continuous.

$$g(x) = \begin{cases} \frac{x^3 + 8x^2 + x + 8}{x + 8}, & x \neq -8 \\ k, & x = -8 \end{cases}$$

Ans:  $k = 65$

Difficulty: Medium

Section: 1.5

27. Assign a value to the constant  $k$  which will make  $f$  continuous.

$$f(x) = \begin{cases} \frac{x^2 - 11x + 30}{x - 6}, & x \neq 6 \\ k, & x = 6 \end{cases}$$

Ans:  $k = 1$

Difficulty: Medium

Section: 1.5

28. Show that  $f(x) = \begin{cases} \frac{x^2 - x - 42}{x + 6}, & x > -6 \\ 2x + 7, & x \leq -6 \end{cases}$  is not continuous at  $x = -6$  but is continuous from the right at  $x = -6$ .

$$\text{Ans: } \lim_{x \rightarrow -6^+} f(x) = \lim_{x \rightarrow -6^+} \frac{x^2 - x - 42}{x + 6} = \lim_{x \rightarrow -6^+} \frac{(x-7)(x+6)}{x+6} = \lim_{x \rightarrow -6^+} (x-7) = -13$$

which =  $f(-6)$

So  $f(x)$  is continuous from the right.

$$\lim_{x \rightarrow -6^-} f(x) = \lim_{x \rightarrow -6^-} (2x + 7) = -5$$

So  $f(x)$  is not continuous at  $x = -6$  since the limits are not the same.

Difficulty: Medium

Section: 1.5

$$h(x) = \begin{cases} \frac{5x^2 + 29x - 186}{x + 6}, & x < -6 \\ \frac{|x|}{x}, & -6 \leq x < 0 \\ 5x, & x \geq 0 \end{cases}$$

29. Given \_\_\_\_\_, determine if  $h$  is continuous at  $-6$ .

$$\text{Ans: } \lim_{x \rightarrow -6^-} f(x) = \lim_{x \rightarrow -6^-} \frac{5x^2 + 29x - 186}{x + 6} = \lim_{x \rightarrow -6^-} (5x - 1) = -31$$

$$\lim_{x \rightarrow -6^+} f(x) = \lim_{x \rightarrow -6^+} \frac{|x|}{x} = -1$$

So  $f(x)$  is not continuous at  $x = -6$

Difficulty: Medium

Section: 1.5

$$h(x) = \begin{cases} \frac{4x^2 + 4x}{x + 1}, & x < -1 \\ \frac{|x|}{x}, & -1 \leq x < 0 \\ 4x, & x \geq 0 \end{cases}$$

30. Given \_\_\_\_\_, determine if  $h$  is continuous at  $0$ .

$$\text{Ans: } \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 4x = 0$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{|x|}{x} = -1$$

So  $f(x)$  is **not** continuous at  $x = 0$

Difficulty: Medium

Section: 1.5

$$h(x) = \begin{cases} \frac{5x^2 + 29x - 6}{x + 6}, & x < -6 \\ \frac{|x|}{x}, & -6 \leq x < 0 \\ 5x, & x \geq 0 \end{cases}$$

31. Given  $h(x)$ , determine if  $h$  is continuous from the right at 0.

Ans:  $h$  is continuous from the right at  $x = 0$

Difficulty: Medium

Section: 1.5

$$g(x) = \begin{cases} \frac{4x + 5}{20 + 21x + 4x^2}, & x < -\frac{5}{4} \\ \frac{1}{x + 4}, & x \geq -\frac{5}{4} \end{cases}$$

32. Given  $g(x)$ , determine if  $g$  is continuous at  $x = -\frac{5}{4}$ .

$$\lim_{x \rightarrow -\frac{5}{4}^-} f(x) = \lim_{x \rightarrow -\frac{5}{4}^-} \frac{4x + 5}{20 + 21x + 4x^2} = \lim_{x \rightarrow -\frac{5}{4}^-} \frac{1}{x + 4} = \frac{1}{\frac{11}{4}} = \frac{4}{11}$$

Ans:

$$\lim_{x \rightarrow -\frac{5}{4}^+} \frac{1}{x + 4} = \frac{1}{\frac{11}{4}} = \frac{4}{11}$$

So  $g$  is continuous at  $x = -\frac{5}{4}$ .

Difficulty: Medium

Section: 1.5

$$g(x) = \begin{cases} |x + 10|, & x \leq -5 \\ x + 10, & -5 < x < 10 \\ \sqrt{x + 4}, & 10 \leq x \leq 11 \\ \frac{11}{12 - x}, & 11 < x < 12 \\ 11, & x \geq 12 \end{cases}$$

33. Given  $g(x)$ , is  $g$  continuous at  $x = 10$ .

Ans: No

Difficulty: Medium

Section: 1.5

34. Find the discontinuities in  $f(x) = \frac{x-6}{x(x+6)}$  and state whether each is removable or nonremovable.

Ans:  $x = 0$  is nonremovable  
 $x = -6$  is nonremovable

Difficulty: Medium

Section: 1.5

35. Find the point of discontinuity in  $f(x) = \frac{6}{(x-7)^5}$  and state whether it is removable.

Ans:  $x = 7$  is nonremovable.

Difficulty: Easy

Section: 1.5

36. Show that the equation  $f(x) = x^2 + 3x - 4$  has at least one solution in the interval  $[-5,0]$ .

Ans:  $f(0) = -4$

$f(-5) = 6$

Since  $f(x)$  is continuous, the intermediate value theorem states that  $f(x)$  must equal 0 somewhere on the interval  $[-5,0]$ .

Difficulty: Medium

Section: 1.5

37. Show that the equation  $f(x) = x^3 + 8x + 3$  has at least one solution in the interval  $[-2,2]$ .

Ans:  $f(-2) = -21$

$f(2) = 27$

Since  $f(x)$  is continuous, the intermediate value theorem states that  $f(x)$  must equal 0 somewhere on the interval  $[-2,2]$ .

Difficulty: Medium

Section: 1.5

38. Determine the interval for which  $f(x) = \frac{2}{\sqrt{7-x}}$  is a continuous function.

Ans:  $(-\infty,7)$

Difficulty: Medium

Section: 1.5

39. Show that  $f(x) = \begin{cases} \frac{|8x|}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$  cannot be made continuous for any assigned value of the constant  $k$ .

$$\text{Ans: } \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{|8x|}{x} = -8$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{|8x|}{x} = 8$$

So the limit  $\lim_{x \rightarrow 0} f(x)$  does not exist.

Difficulty: Medium

Section: 1.5

40. Find the values of  $x$  (if any) at which  $f$  is not continuous.  $f(x) = (x + 8)^7$ .

Ans:  $f$  is continuous everywhere

Difficulty: Easy

Section: 1.5

41. Determine if the discontinuity at  $x = 0$  in the function  $f(x) = \frac{5x}{|x|}$  is removable.

$$\text{Ans: } \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{5x}{|x|} = -5$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{5x}{|x|} = 5$$

So the limit  $\lim_{x \rightarrow 0} f(x)$  does not exist and the discontinuity is not removable.

Difficulty: Medium

Section: 1.5

42. Determine if the discontinuity at  $x = 2$  in the function  $f(x) = \frac{x^2 - 4}{x - 2}$  is removable.

$$\text{Ans: } \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x + 2)(x - 2)}{x - 2} = \lim_{x \rightarrow 2} x + 2$$

So it is removable.

Difficulty: Medium

Section: 1.5

43. Determine if the discontinuity at  $x = 9$  in the function  $f(x) = \frac{x-9}{|x-9|}$  is removable.

Ans:  $\lim_{x \rightarrow 9^-} f(x) = \lim_{x \rightarrow 9^-} \frac{x-9}{|x-9|} = -1$

$$\lim_{x \rightarrow 9^+} f(x) = \lim_{x \rightarrow 9^+} \frac{x-9}{|x-9|} = 1$$

So the limit  $\lim_{x \rightarrow 9} f(x)$  does not exist and the discontinuity is not removable.

Difficulty: Medium

Section: 1.5

44. In  $f(x) = \frac{6}{x-2}$ , determine if the discontinuity at  $x = 2$  is removable.

Ans: No, it is not removable.

Difficulty: Medium

Section: 1.5

1. Find the limit.  $\lim_{x \rightarrow +\infty} \left( \cos\left(\frac{8}{x}\right) \sin\left(\frac{11}{x}\right) \right) =$   
A) 0   B) 1   C) -1   D)  $+\infty$    E)  $-\infty$   
Ans: A  
Difficulty: Easy  
Section: 1.6

2. Find the limit  $\lim_{x \rightarrow 0^-} \frac{6 \sin^2 x}{6x^2} =$   
A) 0   B) -1   C) 1   D)  $+\infty$    E)  $-\infty$   
Ans: C  
Difficulty: Medium  
Section: 1.6

3. Find the limit.  $\lim_{x \rightarrow 0} \frac{\sin(8x)}{\sin(11x)} =$   
A) 0   B)  $\frac{11}{8}$    C)  $\frac{8}{11}$    D) 1   E)  $+\infty$   
Ans: C  
Difficulty: Medium  
Section: 1.6

4. Find the limit.  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{6} =$   
A) 1   B) 6   C) 0   D)  $+\infty$    E)  $-\infty$   
Ans: C  
Difficulty: Medium  
Section: 1.6

5. Find the limit.  $\lim_{x \rightarrow 0} \frac{\cos x}{9} =$   
A) 9   B)  $\frac{1}{9}$    C) 0   D)  $+\infty$    E)  $-\infty$   
Ans: B  
Difficulty: Easy  
Section: 1.6

6. Find the limit  $\lim_{x \rightarrow 0} \frac{\sin x}{3} =$   
 A) 0    B) 1    C) 3    D)  $+\infty$     E)  $-\infty$

Ans: A

Difficulty: Easy

Section: 1.6

7. Find the limit  $\lim_{x \rightarrow 0} \frac{\sin^2 8x}{\tan^2 11x} =$   
 $\frac{8}{11}$   
 A) 0    B) 1    C)  $\frac{8}{11}$     D)  $+\infty$     E)  $-\infty$

Ans: C

Difficulty: Medium

Section: 1.6

8. Find the limit.  $\lim_{x \rightarrow 0} \frac{\sin 9x}{\sin(-11x)} =$   
 $-\frac{9}{11}$      $\frac{9}{11}$   
 A) 0    B)  $-\frac{9}{11}$     C)  $\frac{9}{11}$     D)  $+\infty$     E)  $-\infty$

Ans: B

Difficulty: Medium

Section: 1.6

9. Find the limit.  $\lim_{x \rightarrow 0^-} \tan \frac{8}{x} =$   
 A) 0    B) -1    C) 1    D)  $\infty$     E) Does not exist

Ans: E

Difficulty: Medium

Section: 1.6

10. Find the limit.  $\lim_{x \rightarrow 0} \frac{x^5}{3 \sin x} =$   
 A) 0    B) -1    C) 1    D)  $+\infty$     E)  $-\infty$

Ans: A

Difficulty: Hard

Section: 1.6

11. Answer true or false. The value of  $k$  that makes  $f$  continuous for

$$f(x) = \begin{cases} \frac{3 \sin x}{x}, & x \leq 0 \\ \cos x + k, & x > 0 \end{cases} \quad \text{is 2}$$

Ans: False

Difficulty: Hard

Section: 1.6

12. Answer true or false. The value of  $k$  that makes  $f$  continuous for

$$f(x) = \begin{cases} \frac{8 \sin x}{x}, & x < 0 \\ \cos x + k, & x \geq 0 \end{cases} \quad \text{is 7}$$

Ans: True

Difficulty: Hard

Section: 1.6

13. Answer true or false. The fact that  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  and that  $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} = 1$  guarantees that

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{x} = 1 \quad \text{by the Squeeze Theorem.}$$

Ans: False

Difficulty: Medium

Section: 1.6

14. Answer true or false. The Squeeze Theorem can be used to show  $\lim_{x \rightarrow 0} x + 3 = 3$  utilizing

$$\lim_{x \rightarrow 0} x = 0 \quad \text{and} \quad \lim_{x \rightarrow 0} 3 = 3$$

Ans: False

Difficulty: Medium

Section: 1.6

15. Answer true or false. The Intermediate-Value Theorem can be used to show that the equation  $x^5 = \cos x$  has at least one solution on the interval  $[-5\pi/6, 5\pi/6]$ .

Ans: True

Difficulty: Medium

Section: 1.6

16.  $\lim_{x \rightarrow 0} \left( \frac{\sin x}{9x} + 3 \left( \frac{x}{9 \sin x} \right) \right) =$

- A) 0   B) -1   C) 1   D)  $\frac{4}{9}$    E)  $\infty$

Ans: D

Difficulty: Hard

Section: 1.6

17. Find all points of discontinuity, if any, for  $f(x) = \frac{8}{5 - 3 \sin x}$

- A) 0   B)  $\pi$    C)  $2\pi$    D)  $\frac{\pi}{2}$    E) None exist

Ans: E

Difficulty: Hard

Section: 1.6

18.  $\lim_{x \rightarrow 0} \frac{\sin 5x}{x} =$

- A) 0   B) 1   C) 5   D)  $\infty$    E) It does not exist.

Ans: C

Difficulty: Medium

Section: 1.6

19.  $\lim_{x \rightarrow 0} \frac{e^x - 1}{7x} =$

- A) 0   B) 1   C)  $\frac{1}{7}$    D) 7   E)  $\infty$

Ans: C

Difficulty: Hard

Section: 1.6

20.  $\lim_{x \rightarrow 0} \sin x = \lim_{x \rightarrow 0} \tan x$

Ans: True

Difficulty: Easy

Section: 1.6

21. Find  $\lim_{x \rightarrow 0} \frac{\tan x}{x}$ .

Ans:

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x \cos x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) \left( \frac{1}{\cos x} \right) = \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) \lim_{x \rightarrow 0} \left( \frac{1}{\cos x} \right) = 1 \times 1 = 1$$

Difficulty: Easy

Section: 1.6

22. Find  $\lim_{x \rightarrow 0} \frac{\sin 2x}{\tan x}$ .

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\tan x} = \lim_{x \rightarrow 0} \frac{\frac{\sin 2x}{x}}{\frac{\tan x}{x}} = \lim_{x \rightarrow 0} \frac{2 \sin 2x}{\tan x} = \lim_{x \rightarrow 0} \frac{2}{1} = 2$$

Ans:

Difficulty: Medium

Section: 1.6

23. Find  $\lim_{x \rightarrow 0} \frac{\sin x - \tan x}{\sin^3 x}$ .

$$\lim_{x \rightarrow 0} \frac{\sin x - \tan x}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{\sin x - \frac{\sin x}{\cos x}}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{\cos x \sin x - \sin x}{\cos x \sin^3 x} = \lim_{x \rightarrow 0} \frac{\sin x (\cos x - 1)}{\cos x \sin^3 x} =$$

$$\lim_{x \rightarrow 0} \frac{(\cos x - 1)}{\cos x \sin^2 x} = \lim_{x \rightarrow 0} \frac{(\cos x - 1)}{\cos x (1 - \cos^2 x)} = \lim_{x \rightarrow 0} \frac{(\cos x - 1)}{\cos x (1 - \cos x)(1 + \cos x)} =$$

$$\lim_{x \rightarrow 0} \frac{-1}{\cos x (1 + \cos x)} = \lim_{x \rightarrow 0} \frac{-1}{1(1+1)} = -\frac{1}{2}$$

Ans:

Difficulty: Hard

Section: 1.6

24. Find  $\lim_{x \rightarrow 0} x \cot 5x$ .

$$\lim_{x \rightarrow 0} x \cot 5x = \lim_{x \rightarrow 0} \frac{x \cos 5x}{\sin 5x} = \lim_{x \rightarrow 0} \frac{x}{\sin 5x} = \lim_{x \rightarrow 0} \frac{5x}{5 \sin 5x} = \frac{1}{5}$$

Ans:

Difficulty: Medium

Section: 1.6

25. Find  $\lim_{x \rightarrow 0} \frac{\sin \sqrt{7x}}{\sqrt{x}}$ .

Ans:  $\lim_{x \rightarrow 0} \frac{\sin \sqrt{7x}}{\sqrt{x}} = \lim_{x \rightarrow 0} \frac{\sqrt{7} \sin \sqrt{7x}}{\sqrt{7x}} = \lim_{x \rightarrow 0} \sqrt{7} = \sqrt{7}$

Difficulty: Medium

Section: 1.6

26. Find  $\lim_{x \rightarrow 0} \frac{x^2}{\sin 3x^2}$ .

Ans:  $\lim_{x \rightarrow 0} \frac{x^2}{\sin 3x^2} = \lim_{x \rightarrow 0} \frac{3x^2}{3 \sin 3x^2} = \lim_{x \rightarrow 0} \frac{1}{3} = \frac{1}{3}$

Difficulty: Medium

Section: 1.6

27. Find  $\lim_{x \rightarrow 0} \frac{9}{x \csc x}$ .

Ans:  $\lim_{x \rightarrow 0} \frac{9}{x \csc x} = \lim_{x \rightarrow 0} \frac{9}{x \frac{1}{\sin x}} = \lim_{x \rightarrow 0} \frac{9 \sin x}{x} = 9$

Ans:

Difficulty: Medium

Section: 1.6

28. Find  $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 8x}$ .

Ans:  $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 8x} = \lim_{x \rightarrow 0} \frac{\frac{3 \sin 3x}{3x}}{\frac{8 \sin 8x}{8x}} = \frac{3}{8}$

Ans:

Difficulty: Medium

Section: 1.6

29. Find  $\lim_{x \rightarrow 0} \frac{2x}{\cos x}$ .

Ans: 0

Difficulty: Easy

Section: 1.6

30. Find  $\lim_{x \rightarrow 0} \left( \frac{12x^2}{1 - \cos^2 x} \right)$ .

Ans: 12

Difficulty: Medium

Section: 1.6

31. Find  $\lim_{x \rightarrow 0} \frac{5x}{\cos 4x}$ .

Ans: 0

Difficulty: Easy

Section: 1.6

32. Find  $\lim_{x \rightarrow 0} \frac{\sin^5 x}{\tan x}$ .

Ans: 0

Difficulty: Medium

Section: 1.6

33. Find  $\lim_{x \rightarrow 0} \frac{\sin x}{x^2 + 4x}$ .

Ans:  $\lim_{x \rightarrow 0} \frac{\sin x}{x^2 + 4x} = \lim_{x \rightarrow 0} \frac{\sin x}{x(x+4)} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{1}{x+4} = 1 \cdot \frac{1}{4} = \frac{1}{4}$

Difficulty: Medium

Section: 1.6

34. Find  $\lim_{x \rightarrow 0} \frac{2x^2 + \sin 12x}{x}$ .

Ans:  $\lim_{x \rightarrow 0} \frac{2x^2 + \sin 12x}{x} = \lim_{x \rightarrow 0} \left( \frac{2x^2}{x} + \frac{\sin 12x}{x} \right) = \lim_{x \rightarrow 0} \left( \frac{2x^2}{x} + \frac{12 \sin 12x}{12x} \right) = 0 + 12 = 12$

Difficulty: Hard

Section: 1.6

35. Find  $\lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{7}}{x^2}$ .

$$\lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{7}}{x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{7}}{49 \frac{x^2}{49}} = \lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{7}}{49 \left(\frac{x}{7}\right)^2} = \frac{1}{49}$$

Ans:

Difficulty: Hard

Section: 1.6

36. Find  $\lim_{x \rightarrow 0} \frac{\cos\left(\frac{9\pi}{2} + x\right)}{9x}$ .

$$\text{Ans: } \lim_{x \rightarrow 0} \frac{\cos\left(\frac{9\pi}{2} + x\right)}{9x} = \lim_{x \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} + x\right)}{9x} = \lim_{x \rightarrow 0} \frac{-\sin x}{9x} = -\frac{1}{9}$$

Ans:

Difficulty: Hard

Section: 1.6

37. Find a value for the constant  $k$  so that  $f(t) = \begin{cases} \frac{t}{\sin 6t}, & t \neq 0 \\ k, & t = 0 \end{cases}$  will be continuous at  $t = 0$ .

Ans:  $\frac{1}{6}$

Difficulty: Medium

Section: 1.6

38. Find a value for the constant  $k$  so that  $f(t) = \begin{cases} \frac{5t}{\sin 3t}, & t \neq 0 \\ k, & t = 0 \end{cases}$  will be continuous at  $t = 0$ .

Ans:  $\frac{5}{3}$

Difficulty: Medium

Section: 1.6

39. Find a value for the constant  $k$  so that  $f(t) = \begin{cases} \frac{\sin 11t}{5t}, & t \neq 0 \\ k, & t = 0 \end{cases}$  will be continuous at  $t = 0$ .

Ans:  $\frac{11}{5}$

Difficulty: Medium

Section: 1.6

40. Find a value for the constant  $k$  so that  $f(t) = \begin{cases} \frac{\sin 7t}{t}, & t \neq 0 \\ k, & t = 0 \end{cases}$  will be continuous at  $t = 0$ .

Ans: 7

Difficulty: Medium

Section: 1.6

41. Find a value for the constant  $k$  so that  $f(\theta) = \begin{cases} \frac{\tan 2\theta}{5\theta}, & \theta \neq 0 \\ k, & \theta = 0 \end{cases}$  will be continuous at  $\theta = 0$ .

Ans:  $\frac{2}{5}$

Difficulty: Medium

Section: 1.6

42. Find a value for the constant  $k$  so that  $f(\theta) = \begin{cases} \frac{4\theta}{\tan 3\theta}, & \theta \neq 0 \\ k, & \theta = 0 \end{cases}$  will be continuous at  $\theta = 0$ .

Ans:  $\frac{4}{3}$

Difficulty: Medium

Section: 1.6

43. Find  $\lim_{x \rightarrow 0} \frac{x}{8x + \sin x}$ .

$$\lim_{x \rightarrow 0} \frac{x}{8x + \sin x} = \lim_{x \rightarrow 0} \frac{1}{8 + \frac{\sin x}{x}} = \frac{1}{8+1} = \frac{1}{9}$$

Ans:

Difficulty: Hard

Section: 1.6

44.  $\lim_{x \rightarrow 0} \frac{\sin^5 x}{x^4} =$

Ans:  $\lim_{x \rightarrow 0} \frac{\sin^5 x}{x^4} = \lim_{x \rightarrow 0} \left( \frac{\sin^4 x}{x^4} \right) \sin x = \lim_{x \rightarrow 0} \sin x = 0$

Ans:

Difficulty: Easy

Section: 1.6