

## Chapter 2: Determinants

### Multiple Choice Questions

1. What is  $M_{23}$  for the following matrix?

$$\begin{bmatrix} 3 & 4 & 0 \\ -1 & 2 & 7 \\ -2 & -4 & 4 \end{bmatrix}$$

- (A) 0 (B) -4 (C) 21 (D) 26

2. Compute.

$$\begin{vmatrix} 1 & -2 & 3 & -4 \\ 0 & 5 & -6 & 7 \\ 0 & 0 & -8 & 9 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

- (A) -40 (B) -1 (C) 6 (D) 0

3. Let  $A$  be a square matrix. If  $\det(A) = 5$ , what is  $\det(A^T)$ ?

- (A)  $-\frac{1}{5}$  (B)  $\frac{1}{5}$  (C) -5 (D) 5

4. Evaluate the determinant of the following matrix by using signed elementary products.

$$\begin{bmatrix} 0 & -1 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

- (A) -16 (B) 16 (C) -9 (D) 9

5. If  $A$  is a  $3 \times 3$  matrix and  $\det(A) = 7$ , what is  $\det(2A)$ ?

- (A)  $\frac{7}{2}$  (B) 14 (C) 42 (D) 56

6. If  $A$  is an invertible matrix and  $\det(A) = 7$ , what is  $\det(A^{-1})$ ?

- (A) 7 (B) -7 (C)  $\frac{1}{7}$  (D)  $-\frac{1}{7}$

### Free Response Questions

1. Evaluate.

$$\begin{vmatrix} 1 & 3 & -2 \\ 4 & 2 & 5 \\ 0 & 1 & 1 \end{vmatrix}$$

2. Find all values of  $\lambda$  for which

$$\begin{vmatrix} 1 - \lambda & 1 & 0 \\ 0 & 2 - \lambda & 0 \\ 0 & -1 & 4 - \lambda \end{vmatrix} = 0$$

3. Compute the determinant of the following matrix. Simplify completely.

$$\begin{bmatrix} \sin(\alpha) \cos(\beta) & r \cdot \cos(\alpha) \cos(\beta) & -r \cdot \sin(\alpha) \sin(\beta) \\ \sin(\alpha) \sin(\beta) & r \cdot \cos(\alpha) \sin(\beta) & r \cdot \sin(\alpha) \cos(\beta) \\ \cos(\alpha) & -r \cdot \sin(\alpha) & 0 \end{bmatrix}$$

4. If  $\det(A) = -4$ , find all possible values of  $k$ , where

$$A = \begin{bmatrix} 1 & k \\ k & 3k \end{bmatrix}.$$

5. Justify, without evaluating, that the determinant of the following matrix is zero.

$$\begin{bmatrix} 1 & 0 & 2 & 4 \\ -2 & 3 & 8 & 6 \\ -1 & 3 & 10 & 10 \\ 6 & 6 & -3 & 7 \end{bmatrix}$$

6. Evaluate the determinant of the following matrix by reducing to row-echelon form.

$$\begin{bmatrix} 1 & 2 & 8 \\ 2 & 5 & -3 \\ 8 & 19 & 5 \end{bmatrix}$$

7. Let  $E$  be a  $3 \times 3$  elementary matrix that interchanges two rows of  $I_3$ . Prove that  $\det(E) = -1$ .

8. If the system  $A\mathbf{x} = \mathbf{b}$  is inconsistent and  $A$  is a square matrix, what is  $\det(A^T)$ ?

9. Use Cramer's Rule to solve the following system of equations.

$$\begin{aligned} x + y - z &= 2 \\ 3x - y + z &= 5 \\ 3x + 2y + 4z &= 0 \end{aligned}$$

10. Use the determinant to determine whether the following matrix is invertible.

$$\begin{bmatrix} 7 & 1 & -1 \\ 2 & 4 & 3 \\ -14 & -2 & 2 \end{bmatrix}$$

11. Compute the inverse of the following matrix using the adjoint matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \\ 3 & 0 & 3 \end{bmatrix}$$

12. Prove that if  $AA^T = I$ , then  $\det(A) = \pm 1$ .

13. Let  $A, B$ , and  $P$  be  $n \times n$  matrices where at least  $P$  is invertible. Prove that if  $A = P^{-1}BP$ , then  $\det(A) = \det(B)$ .

**Answers***Multiple Choice Answers*

1. (B)
2. (A)
3. (D)
4. (B)
5. (D)
6. (C)

*Free Response Answers*

1.  $-23$
2.  $\lambda = 1, 2, 4$
3.  $r^2 \cdot \sin(\alpha)$
4.  $k = 4$  or  $k = -1$
5. The third row is a linear combination of the first two rows.
6.  $-2$
8.  $0$
9.  $x = \frac{7}{4}, y = -\frac{17}{24}, z = -\frac{23}{24}$
10. The determinant equals 0. Not invertible.

$$11. A^{-1} = -\frac{1}{6} \begin{bmatrix} -3 & -6 & 3 \\ -6 & -6 & 6 \\ 3 & 6 & -5 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 & -\frac{1}{2} \\ 1 & 1 & -1 \\ -\frac{1}{2} & -1 & \frac{5}{6} \end{bmatrix}$$