1/1

(a) $m = \frac{W}{g} = \frac{3500}{32.2} = \underline{108.7 \text{ slugs}}$

(b) $W = 3500 \text{ lb} \left[\frac{4.4482 \text{ N}}{\text{lb}}\right] = \underline{15\,570 \text{ N}}$

(c) $m = \frac{W}{g} = \frac{15\,570}{9.81} = \underline{1587 \text{ kg}}$

1/2 For a 180-lb person:

$W = mg: \quad 180\text{ lb} = m\,(32.2\text{ ft/sec}^2)$

$m = \underline{5.59\text{ slugs}}$

$180\text{ lb}\left(\dfrac{4.4482\text{ N}}{\text{lb}}\right) = \underline{801\text{ N}}$

$W = mg: \quad 801\text{ N} = m\,(9.81\text{ m/s}^2)$

$m = \underline{81.6\text{ kg}}$

1/3

$$\mathbf{V}_1 = 15\left(\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}\right) = 12\mathbf{i} + 9\mathbf{j}$$

$$\mathbf{V}_2 = 12\left(-\cos 60^\circ \mathbf{i} + \sin 60^\circ \mathbf{j}\right) = -6\mathbf{i} + 10.39\mathbf{j}$$

$$V_1 + V_2 = 15 + 12 = \underline{27}$$

$$\mathbf{V}_1 + \mathbf{V}_2 = (12-6)\,\mathbf{i} + (9+10.39)\,\mathbf{j} = \underline{6\mathbf{i} + 19.39\mathbf{j}}$$

$$\mathbf{V}_1 - \mathbf{V}_2 = (12-(-6))\,\mathbf{i} + (9-10.39)\,\mathbf{j} = \underline{18\mathbf{i} - 1.392\mathbf{j}}$$

$$\mathbf{V}_1 \times \mathbf{V}_2 = (12\mathbf{i} + 9\mathbf{j}) \times (-6\mathbf{i} + 10.39\mathbf{j})$$

$$= (124.7 + 54)\,\mathbf{k} = \underline{178.7\,\mathbf{k}}$$

$$\mathbf{V}_2 \times \mathbf{V}_1 = -(\mathbf{V}_1 \times \mathbf{V}_2) = \underline{-178.7\,\mathbf{k}}$$

$$\mathbf{V}_1 \cdot \mathbf{V}_2 = (12\mathbf{i} + 9\mathbf{j}) \cdot (-6\mathbf{i} + 10.39\mathbf{j})$$

$$= 12(-6) + 9\,(10.39) = \underline{21.5}$$

1/4 The weight of an average apple is

$$W = \frac{5 \text{ lb}}{12 \text{apples}} = 0.417 \text{ lb}$$

Mass in slugs is $m = \frac{W}{g} = \frac{0.417}{32.2} = \underline{0.01294 \text{ slugs}}$

Mass in kg is $m = 0.01294 \text{ slugs} \left(\frac{14.594 \text{ kg}}{1 \text{ slug}} \right)$

$$= \underline{0.1888 \text{ kg}}$$

Weight in N is $W = mg = 0.1888(9.81) = \underline{1.853 \text{ N}}$

These apples weigh closer to 2 N each than to the rule of 1 N each!

1/5 Mass of iron sphere $m = \rho V$

$$= \left(7210 \ \frac{kg}{m^3}\right)\left(\frac{4}{3}\pi (0.050)^3\right) = 3.78 \ kg$$

Force of mutual attraction : $\dfrac{Gm^2}{d^2}$

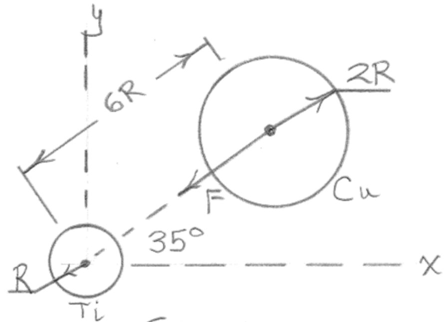
Weight of each sphere : $\dfrac{Gm_e m}{r^2}$

$$\frac{Gm^2}{d^2} = \frac{Gm_e m}{r^2}, \quad r = d\sqrt{\frac{m_e}{m}}$$

$$= 0.1\sqrt{\frac{5.976 \times 10^{24}}{3.78}} \ \frac{1}{10^3}$$

$$= \underline{1.258 (10^8) \ km}$$

1/6



$$F = \frac{G m_{Ti} m_{Cu}}{d^2} = \frac{G\left[\frac{4}{3}\pi R^3 \rho_{Ti}\right]\left[\frac{4}{3}\pi (2R)^3 \rho_{Cu}\right]}{(6R)^2}$$

$$= \frac{32}{81}\pi^2 G \rho_{Ti} \rho_{Cu} R^4$$

$$= \frac{32}{81}\pi^2 (6.673\cdot 10^{-11})(4510)(8910)(0.040)^4$$

$$= 2.68(10^{-8}) \text{ N}$$

Force is a vector quantity, so

$$\underline{F} = F\underline{n} = 2.68(10^{-8})\left[-\cos 35°\,\underline{i} - \sin 35°\,\underline{j}\right]$$

$$= \underline{(-2.19\underline{i} - 1.535\underline{j})\,10^{-8} \text{ N}}$$

1/7

$$\cancel{m}g = \frac{1}{3}\cancel{m}g_{h=0}$$

$$\frac{R^2}{(R+h)^2}g_0 = \frac{1}{3}g_0$$

Solve for h: $h = (\sqrt{3}-1)R = \underline{0.732R}$

1/8

$g_{rel} = 9.780\,327\left(1 + 0.005\,279\sin^2\gamma + 0.000\,023\sin^4\gamma \cdots\right)$

At $\gamma = 35^\circ$, $g_{rel} = 9.797\,337 \text{ m/s}^2$

$$g_{abs} = g_{rel} + 0.03382\cos^2\gamma$$
$$= 9.797\,337 + 0.03382\cos^2 35^\circ$$
$$= 9.820\,031 \text{ m/s}^2$$

$W_{abs} = mg_{abs} = 60(9.820\,031) = \underline{589 \text{ N}}$

$W_{rel} = mg_{rel} = 60\,(9.797\,337) = \underline{588 \text{ N}}$

(More precise values : $W_{abs} = 589.2$ N
$W_{rel} = 587.8$ N)

1/9

$$g_h = \frac{Gm_e}{(R+h)^2}$$

$$= \frac{(3.439 \cdot 10^{-8})(4.095 \cdot 10^{23})}{[(3959+200)(5280)]^2} = \underline{29.2 \text{ ft/sec}^2}$$

Mass of passenger $m = \frac{W}{g} = \frac{180}{32.174}$

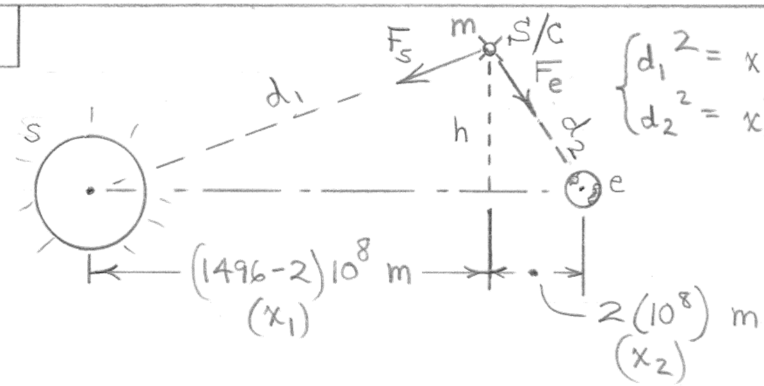
$= 5.59$ slugs

Absolute weight at $h = 200$ mi:

$W_h = mg_h = (5.59)(29.2) = \underline{163.4 \text{ lb}}$

The terms "zero-g" and "weightless" are <u>absolutely</u> (!) misnomers in this case.

1/10



$$\begin{cases} d_1^2 = x_1^2 + h^2 \\ d_2^2 = x_2^2 + h^2 \end{cases}$$

$$F_s = \frac{G m m_s}{d_1^2}, \quad F_e = \frac{G m m_e}{d_2^2}$$

For equal force magnitudes, $F_s = F_e$

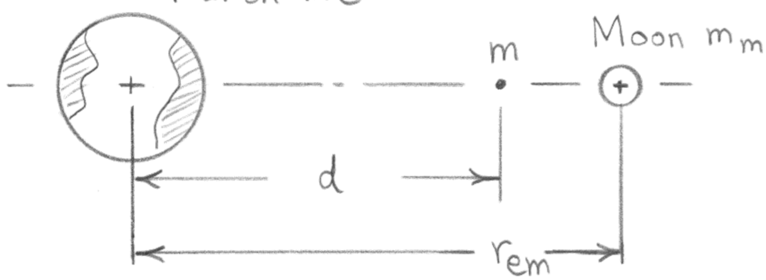
$$\Rightarrow \frac{m_s}{d_1^2} = \frac{m_e}{d_2^2} \quad \text{or} \quad \frac{m_s}{x_1^2 + h^2} = \frac{m_e}{x_2^2 + h^2}$$

$$h = \left[\frac{m_e x_1^2 - m_s x_2^2}{m_s - m_e}\right]^{1/2}$$

With $m_e = 5.976(10^{24})$ kg, $m_s = 333\,000\, m_e$, and x_1 and x_2 as above:

$$h = 1.644(10^8) \text{ m} \quad \text{or} \quad \underline{1.644(10^5) \text{ km}}$$

1/11



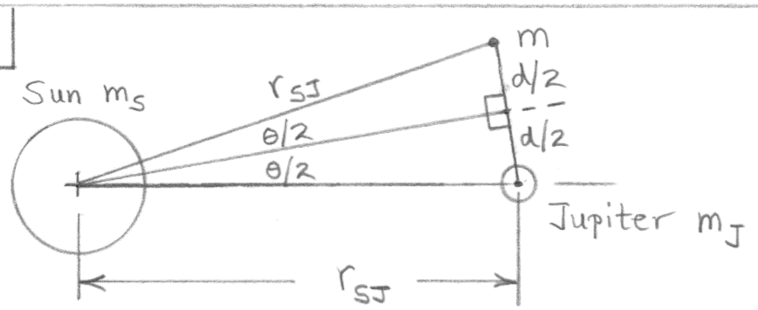
Newton's Law of Universal Gravitation:

$$\frac{G m_e m}{d^2} = \frac{G m_m m}{(r_{em}-d)^2} \Rightarrow m_m d^2 = m_e (r_{em}-d)^2$$

With $m_m = 0.0123\, m_e$ and $r_{em} = 384\,398$ km,

$$\begin{cases} \underline{d = 346\,000 \text{ km}} & \text{(between the earth and the moon)} \\ \underline{d = 432\,000 \text{ km}} & \text{(to the right of the moon)} \end{cases}$$

1/12



Newton's Law of Universal Gravitation:

$$\frac{Gmm_S}{r_{SJ}^2} = \frac{Gmm_J}{d^2}$$

$$d = r_{SJ}\sqrt{\frac{m_J}{m_S}} = 778(10^6)\sqrt{\frac{317.8}{333000}}$$

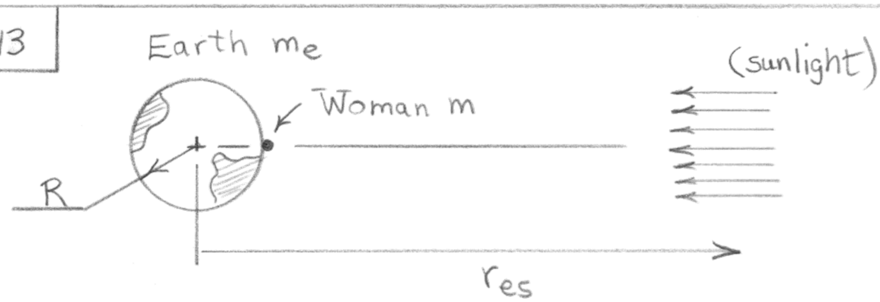
$$= 24.0\,(10^6)\text{ km}$$

Then $\sin\frac{\theta}{2} = \frac{d/2}{r_{SJ}} = \frac{24.0\,(10^6)/2}{778\,(10^6)}$

$$\underline{\theta = 1.770^\circ}$$

(We could have used the small-angle approach $d = r_{SJ}\theta$!)

1/13

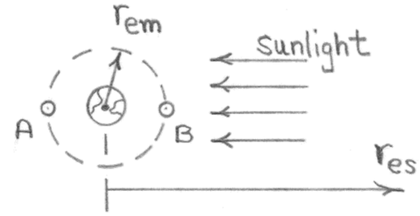


$$F_e = \frac{G m_e m}{R^2} \qquad F_s = \frac{G m_s m}{(r_{es} - R)^2}$$

$$\text{Ratio } R_{es} = \frac{F_e}{F_s} = \frac{m_e (r_{es} - R)^2}{m_s R^2}$$

$$= \frac{1}{333\,000} \; \frac{[149.6(10^6) - 6371]^2}{[6371]^2} = \underline{1656}$$

1/14



Force exerted by earth on moon:

$$F_e = \frac{G m_e m_m}{r_{em}^2} = \frac{(6.673\times10^{-11})(5.976\times10^{24})^2(1)(0.0123)}{(3.84398\times10^{8})^2}$$

$$= 1.984\times10^{20}\text{ N}$$

Forces exerted by sun on moon:

$$F_{S_A} = \frac{G m_s m_m}{(r_{es}+r_{em})^2} = \frac{(6.673\times10^{-11})(5.976\times10^{24})^2(333{,}000)(0.0123)}{(1.496\times10^{11}+3.84398\times10^{8})^2}$$

$$= 4.34\times10^{20}\text{ N}$$

$$F_{S_B} = \frac{G m_s m_m}{(r_{es}-r_{em})^2} = 4.38\times10^{20}\text{ N}$$

Ratios:

$R_A = 2.19$

$R_B = 2.21$

1/15

$$E = \int_{t_1}^{t_2} mgr \, dt$$

$$[E] = (M)(L/T^2)(L)(T) = ML^2/T$$

SI: $\underline{[E] = kg \cdot m^2/s}$ (base ✓)

U.S.: $[E] = slugs \cdot ft^2/sec$ (not base)

$$= \frac{lb\text{-}sec^2}{ft} \cdot ft^2/sec = \underline{lb\text{-}ft\text{-}sec}$$

1/16

$$Q = \frac{1}{2}\rho v^2$$

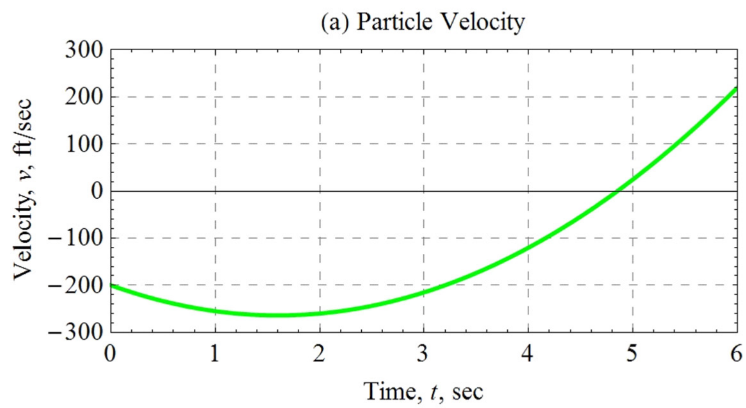
$$[Q] = \frac{M}{L^3}\left(\frac{L}{T}\right)^2 = \underline{ML^{-1}T^{-2}}$$

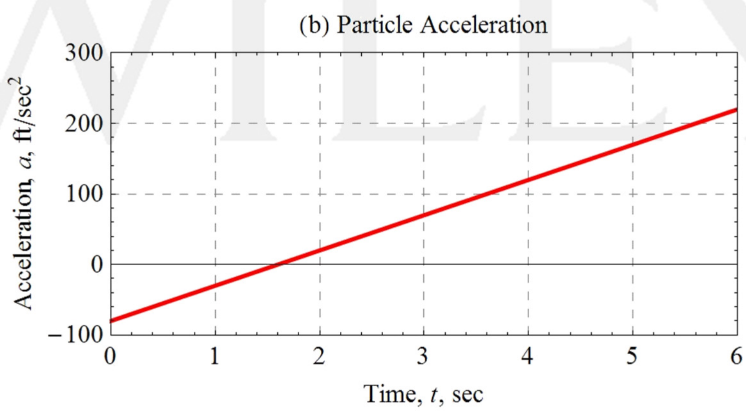
2/1

$$\left.\begin{aligned} v &= 25t^2 - 80t - 200 \\ a &= \frac{dv}{dt} = 50t - 80 \end{aligned}\right\}$$ See plots

$a = 0$: $50t - 80 = 0$, $t = 1.6$ sec

At $t = 1.6$ sec, $v = 25(1.6)^2 - 80(1.6) - 200 = \underline{-264 \frac{ft}{sec}}$





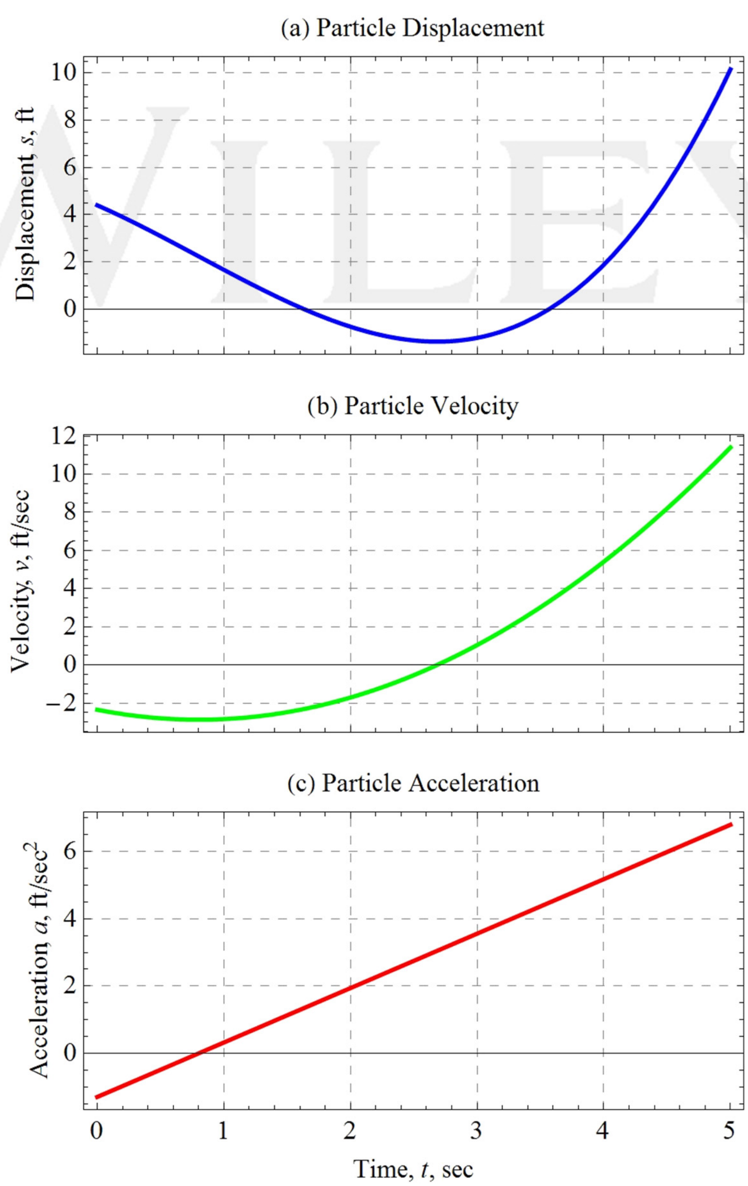
2/2

$$S = 0.27t^3 - 0.65t^2 - 2.35t + 4.4 \quad ft$$

$$\begin{cases} v = \dfrac{ds}{dt} = 0.81t^2 - 1.3t - 2.35 \quad ft/sec \\ \\ a = \dfrac{dv}{dt} = 1.62t - 1.3 \quad ft/sec^2 \end{cases}$$

FOR A CHANGE IN DIRECTION, $v = 0$ TO CHANGE SIGN.

$$v = 0 = 0.81t^2 - 1.3t - 2.35 \longrightarrow \begin{cases} t = -1.080 \text{ sec} \\ \underline{t = 2.69 \text{ sec}} \end{cases}$$



2/3

$$v = 2 - 4t + 5t^{3/2}$$

$$a = \frac{dv}{dt} = -4 + \frac{15}{2} t^{1/2}$$

$$\frac{ds}{dt} = 2 - 4t + 5t^{3/2}$$

$$\int_{s_0=3}^{s} ds = \int_{0}^{t} (2 - 4t + 5t^{3/2})\, dt$$

$$s = 3 + 2t - 2t^2 + 2t^{5/2}$$

At $t = 3\,s$: $\begin{cases} s = 22.2 \text{ m} \\ v = 15.98 \text{ m/s} \\ a = 8.99 \text{ m/s}^2 \end{cases}$

2/4 $\quad s = (-2+3t)e^{-0.5t}$

$$v = \frac{ds}{dt} = 3e^{-0.5t} + (-2+3t)(-0.5)e^{-0.5t}$$

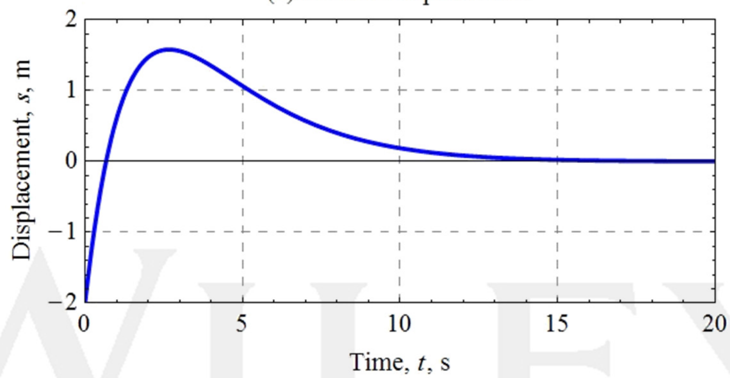
$$= \underline{(4-1.5t)\,e^{-0.5t}}$$

$$a = \frac{dv}{dt} = -1.5e^{-0.5t} + (4-1.5t)(-0.5)e^{-0.5t}$$

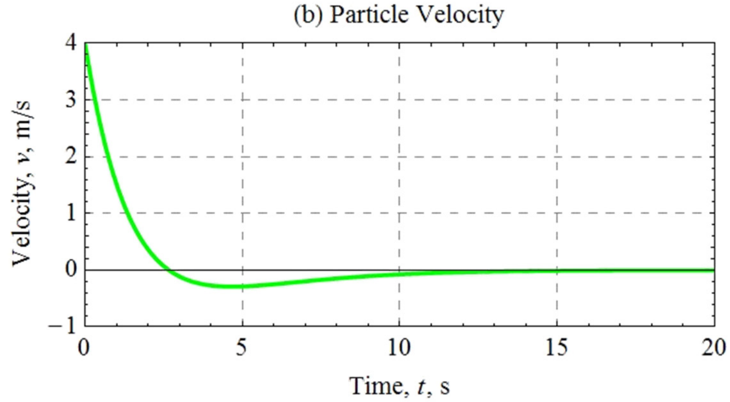
$$= \underline{(-3.5+0.75t)\,e^{-0.5t}}$$

$a=0:\quad (-3.5+0.75t)e^{-0.5t} = 0, \quad \underline{t = 4.67 \text{ s}}$

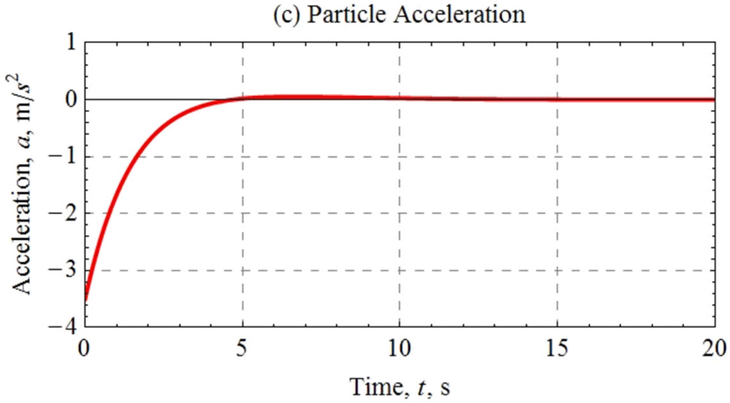
(a) Particle Displacement



(b) Particle Velocity



(c) Particle Acceleration



2/5

$$a = \frac{dv}{dt} = 2t - 10$$

$$\int_{v_0 = 3}^{v} dv = \int_0^t (2t - 10)\,dt, \quad \underline{v = 3 - 10t + t^2} \text{ (m/s)}$$

$$\frac{ds}{dt} = 3 - 10t + t^2$$

$$\int_{s_0 = -4}^{s} ds = \int_0^t (3 - 10t + t^2)\,dt$$

$$\underline{s = -4 + 3t - 5t^2 + \frac{1}{3}t^3} \text{ (m)}$$

2/6

AT $t=0,\ v_0 = 12\ m/s$

$$a = -kt^2 = \frac{dv}{dt} \longrightarrow \int_0^t -kt^2\,dt = \int_{v_0}^{v} dv$$

$$v = v_0 - \frac{1}{3}kt^3 \qquad \text{AND AT } t=6,\ v=0 = 12 - \frac{1}{3}k(6)^3$$

SO... $\underline{k = \frac{1}{6}}$

FOR UNITS...

$$[a] = -[k][t^2] \longrightarrow \frac{L}{T^2} = [k]T^2 \longrightarrow [k] = \frac{L}{T^4} \text{ OR } \underline{\frac{m}{s^4}}$$

SO... $v = 12 - \frac{1}{18}t^3 = \frac{ds}{dt}$

$$\int_{S_0}^{S_6} ds = \int_{t_0=0}^{t=6} \left(12 - \frac{1}{18}t^3\right)dt \longrightarrow S_6 - S_0 = \Delta s = 12t - \frac{1}{72}t^4 \Big|_0^6$$

$\underline{\Delta s = 54\ m}$

2/7

$$a = v\frac{dv}{ds} = -ks^2$$

$$\int_{v_0}^{v} v\,dv = -\int_{s_0}^{s} ks^2\,ds \Rightarrow v^2 = v_0^2 - \frac{2}{3}k(s^3 - s_0^3)$$

Taking positive sign: $\underline{v = \left[v_0^2 - \frac{2}{3}k\left(s^3 - s_0^3\right)\right]^{1/2}}$

Numbers: $v = \left[10^2 - \frac{2}{3}(0.1)(5^3 - 3^3)\right]^{1/2}$

$= \underline{9.67 \text{ m/s}}$

2/8

AT $t=0$, $s=s_0$ AND $v=v_0$

$$a = c_1 + c_2 v = \frac{dv}{dt} \longrightarrow \int_{t_0=0}^{t} dt = \int_{v_0}^{v} \frac{dv}{c_1 + c_2 v} \quad \begin{cases} u = c_1 + c_2 v \\ du = c_2\, dv \end{cases}$$

$$t = \frac{1}{c_2}\int \frac{du}{u} = \frac{\ln u}{c_2} = \left.\frac{\ln(c_1 + c_2 v)}{c_2}\right|_{v_0}^{v} = \frac{1}{c_2}\ln\left(\frac{c_1 + c_2 v}{c_1 + c_2 v_0}\right)$$

SOLVING...
$$v = \frac{1}{c_2}\left[(c_1 + c_2 v_0)e^{c_2 t} - c_1\right] = \frac{ds}{dt}$$

SO...
$$\int_{s_0}^{s} ds = \frac{1}{c_2}\int_{t_0=0}^{t}\left[(c_1 + c_2 v_0)e^{c_2 t} - c_1\right]dt = \left.\frac{1}{c_2^2}(c_1 + c_2 v_0)e^{c_2 t} - \frac{c_1}{c_2}t\right|_0^t$$

$$\underline{s = s_0 + \frac{c_1 + c_2 v_0}{c_2^2}\left(e^{c_2 t} - 1\right) - \frac{c_1}{c_2}t}$$

$$a = c_1 + c_2 v = \frac{v\,dv}{ds} \longrightarrow \int_{s_0}^{s} ds = \int_{v_0}^{v} \frac{v\,dv}{c_1 + c_2 v}$$

FROM APPENDIX C/10...
$$\int_{v_0}^{v} \frac{v\,dv}{c_1 + c_2 v} = \left.\frac{1}{c_2^2}\left[c_1 + c_2 v - c_1 \ln(c_1 + c_2 v)\right]\right|_{v_0}^{v}$$

$$s - s_0 = \frac{1}{c_2^2}\left[c_1 + c_2 v - c_1\ln(c_1 + c_2 v)\right] - \frac{1}{c_2^2}\left[c_1 + c_2 v_0 - c_1 \ln(c_1 + c_2 v_0)\right]$$

$$\underline{s = s_0 + \frac{1}{c_2^2}\left[c_2(v - v_0) + c_1 \ln\left(\frac{c_1 + c_2 v_0}{c_1 + c_2 v}\right)\right]}$$

2/9

For $a =$ constant, $v^2 = v_0^2 + 2as$

$$\left[\frac{180\,(5280)}{3600}\right]^2 = 0^2 + 2a\,(300)$$

$a = 116.2\ \text{ft/sec}^2$

or $a = 116.2/32.2 = \underline{3.61g}$

2/10 $k = 0.2\ mm^{1/2} s^{-1}$, $v_0 = 3\ mm/s$

$$v = ks^{1/2} = \frac{ds}{dt} \longrightarrow \int_{t_0=0}^{t} k\,dt = \int_{s_0}^{s} \frac{ds}{s^{1/2}} \longrightarrow kt = 2(s^{1/2} - s_0^{1/2})$$

SOLVING... $s = \frac{1}{4}(kt + 2s_0^{1/2})^2$

$$v_0 = k s_0^{1/2} \longrightarrow 3 = 0.2 s_0^{1/2} \longrightarrow s_0 = 225\ mm$$

SO... $\underline{s = \frac{1}{4}(0.2t + 30)^2 \quad mm}$

$$\begin{cases} v = \dfrac{ds}{dt} \longrightarrow v = \dfrac{k}{2}(kt + 2s_0^{1/2}) \quad \text{OR...} \quad \underline{v = \dfrac{1}{10}(0.2t + 30)\ mm/s} \\ a = \dfrac{dv}{dt} \longrightarrow a = \dfrac{1}{2}k^2 \quad \text{OR...} \quad \underline{a = 0.02\ mm/s^2} \end{cases}$$

FOR $v = 15\ mm/s$...

$$\begin{cases} 15 = \frac{1}{10}(0.2t + 30) \longrightarrow \underline{t = 600\ s} \\ 15 = 0.2 s^{1/2} \longrightarrow \underline{s = 5620\ mm} \end{cases}$$