Chapter 2

Discrete-Time Signals and Systems

1. (i)

$$x(n) = \begin{cases} 3^n & n < 0 \\ 0.7^n & n \geq 0 \end{cases}$$

$$\Rightarrow X(z) = \sum_{n=-\infty}^{-1} 3^n z^{-n} + \sum_{n=0}^{\infty} 0.7^n z^{-n} = \sum_{n=1}^{\infty} (3^{-1}z)^n + \sum_{n=0}^{\infty} (0.7z^{-1})^n$$
$$= \frac{z}{3-z} + \frac{z}{z-0.7}$$

And region of convergence is

$$0.7 < |z| < 3$$

(ii)

$$x(n) = \begin{cases} 2^n & n < 0 \\ 0.7^n & 0 \leq n < 5 \\ 0.5^n & n \geq 5 \end{cases}$$

and thus

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=-\infty}^{-1} 2^n z^{-n} + \sum_{n=0}^{4} 0.7^n z^{-n} + \sum_{n=5}^{\infty} 0.5^n z^{-n}$$
$$= \sum_{n=1}^{\infty} (2^{-1}z)^n + \sum_{n=0}^{4} (0.7z^{-1})^n + \sum_{n=5}^{\infty} (0.5z^{-1})^n$$
$$= \frac{z}{2-z} + \frac{1-(0.7z^{-1})^5}{1-0.7z^{-1}} + \frac{(0.5z^{-1})^5}{1-0.5z^{-1}}$$

for

$$0.5 < |z| < 2$$

2. By method of partial fraction,

$$H(z) = \frac{1}{(1-0.5z^{-1})(1-2z^{-1})} = \frac{A}{1-0.5z^{-1}} + \frac{B}{1-2z^{-1}},$$

where $A = -\frac{1}{3}$ and $B = \frac{4}{3}$.

(i) When the region of convergence is I, both terms on the right hand side are z-transform of a left-sided sequence. We thus get

$$\frac{A}{1-0.5z^{-1}} = -A \sum_{n=-\infty}^{-1} 0.5^n z^{-n},$$

$$\frac{B}{1-2z^{-1}} = -B \sum_{n=-\infty}^{-1} 2^n z^{-n}$$

and

$$h(n) = \begin{cases} 0 & n \geq 0 \\ \frac{1}{3} \times 0.5^n - \frac{4}{3} \times 2^n & n < 0. \end{cases}$$

(ii) When the region of convergence is II, the first corresponds to a right-sided sequence and the second term to a left-sided sequence. We thus get

$$\frac{A}{1-0.5z^{-1}} = A \sum_{n=0}^{\infty} 0.5^n z^{-n},$$

$$\frac{B}{1-2z^{-1}} = \frac{-B \times 0.5z}{1-0.5z} = -B \times 0.5z \sum_{n=0}^{\infty} 0.5^n z^n$$

$$= -B \sum_{n=1}^{\infty} 0.5^n z^n = -B \sum_{n=-\infty}^{-1} 0.5^{-n} z^{-n} = -B \sum_{n=-\infty}^{-1} 2^n z^{-n}$$

and

$$h(n) = \begin{cases} -\frac{1}{3} \times 0.5^n & n \geq 0 \\ \frac{4}{3} \times 2^n & n < 0. \end{cases}$$

(iii) When the region of the convergence is III, both terms corresponds to a right sided sequence. We thus get

$$\frac{A}{1-0.5z^{-1}} = A \sum_{n=0}^{\infty} 0.5^n z^{-n},$$

$$\frac{B}{1-2z^{-1}} = B \sum_{n=0}^{\infty} 2^n z^{-n}$$

and

$$h(n) = \begin{cases} -\frac{1}{3} \times 0.5^n + \frac{4}{3} \times 2^n & n \geq 0 \\ 0 & n < 0. \end{cases}$$

3. From (2.43),

$$\phi_{xx}(k) = r_{xx}(k) + |m_x|^2.$$

Taking z-transform from both sides,

$$\Phi_{xx}(z) = \sum_{k=-\infty}^{\infty} r_{xx}(k) z^{-k} + |m_x|^2 \sum_{k=-\infty}^{\infty} z^{-k}.$$

We note that

$$\sum_{k=-\infty}^{\infty} z^{-k} = \sum_{k=-\infty}^{-1} z^{-k} + \sum_{k=0}^{\infty} z^{-k}$$

and for this to be convergent, $|z| > 1$ for the first term on the right side, and $|z| < 1$ for the second term. Clearly there is no region of z for which $\sum_{k=-\infty}^{\infty} z^{-k}$ is convergent.

4. Applying (2.38),

$$\begin{aligned}\phi_{xx}(k) &= \phi_{vv}(k) + E[\sin(\omega_0 n + \theta)\sin(\omega_0(n-k)+\theta)] \\ &= \sigma_v^2 \delta(k) + E[\frac{1}{2}\cos(\omega_0 k) - \frac{1}{2}\cos(\omega_0(2n-k)+2\theta)] = \sigma_v^2\delta(k) + \frac{1}{2}\cos\omega_0 k\end{aligned}$$

where $\sigma_v^2 = E[|v(n)|^2]$ Hence,

$$\Phi_{xx}(z) = \sigma_v^2 + \frac{1}{2}\sum_{k=-\infty}^{\infty} \cos\omega_0 k z^{-k}$$

and

$$\frac{1}{2}\sum_{k=-\infty}^{\infty} \cos\omega_0 k z^{-k} = \frac{1}{4}[\sum_{k=-\infty}^{\infty} e^{j\omega_0 k} z^{-k} + \sum_{k=-\infty}^{\infty} e^{-j\omega_0 k} z^{-k}]$$

In a similar way to the proof given in Problem 2.5, we can show that here also there is no region of z-plane in which either of summation on the right side of the above equation may converge.

5. (i) From (2.43),

$$\phi_{xx}(k) = r_{xx}(k) + |m_x|^2.$$

Taking z-transform from both sides,

$$\Phi_{xx}(z) = \sum_{k=-\infty}^{\infty} r_{xx}(k) z^{-k} + |m_x|^2 \sum_{k=-\infty}^{\infty} z^{-k}.$$

We note that

$$\sum_{k=-\infty}^{\infty} z^{-k} = \sum_{k=-\infty}^{-1} z^{-k} + \sum_{k=0}^{\infty} z^{-k}$$

and for this to be convergent, $|z| > 1$ for the first term on the right side, and $|z| < 1$ for the second term. Clearly there is no region of z for which $\sum_{k=-\infty}^{\infty} z^{-k}$ is convergent.

(ii) Since $x(n)$ is a stationary process with $m_x \neq 0$, we can separate it into two parts

$$x(n) = m_x + y(n)$$

In the above equation $y(n)$ is a stationary process with $m_y = 0$. Taking a Fourier transform from the above equation we have

$$X(e^{j\omega}) = m_x\delta(\omega) + Y(e^{j\omega})$$

$$y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)e^{-j\omega n}$$

In the vicinity of $\omega = 0$, $Y(e^{j\omega})$ becomes

$$\lim_{\omega\to 0} Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)$$

Since $m_y = 0$, we can expect that the above equation would also be zero., and thus in vicinity of $\omega = 0$, we have

$$X(e^{j\omega}) = m_x\delta(\omega)$$

(iii) As in previous part

$$x(n) = m_x + y(n)$$

$y(n)$ is a stationary process with $m_y = 0$. So we have

$$\phi_{xx}(k) = m_x^2 + \phi_{yy}(k)$$

$$\Phi_{xx}(e^{j\omega}) = m_x^2\delta(\omega) + \Phi_{yy}(z)$$

$$\Phi_{yy}(z) = \sum_{k=-\infty}^{\infty} \phi_{yy}(k)e^{-j\omega k}$$

In vicinity of $\omega = 0$ we have

$$\Phi_{yy}(z) = \sum_{k=-\infty}^{\infty} \phi_{yy}(k)$$

$$= \sum_{k=-\infty}^{\infty} E[y(n)y^*(n-k)]$$

$$= E\left[y(n)\sum_{k=-\infty}^{\infty} y^*(n-k)\right] = 0$$

Accordingly,

$$\Phi_{xx}(e^{j\omega}) = m_x^2\delta(\omega)$$

6. Solution to this problem follows the same line of derivations as Problem 2.5, with m_x replaced by $Ae^{j\omega_0 n}$.

7. Starting from (2.54), we arrived at (2.55) as follows

$$\begin{aligned}\Phi_{xx}(z) &= \sum_{k=-\infty}^{\infty} \phi_{xx}(k)z^{-k} = (\sum_{k=-\infty}^{\infty} \phi_{xx}^*(k)(z^*)^{-k})^* \\ &= (\sum_{k=-\infty}^{\infty} \phi_{xx}(-k)(\frac{1}{z^*})^{k})^* \qquad \text{(from (2.48))} \\ &= (\sum_{k=-\infty}^{\infty} \phi_{xx}(k)(\frac{1}{z^*})^{-k})^* \qquad (k \to -k) \\ &= (\Phi_{xx}(\frac{1}{z^*}))^* \\ &= \Phi_{xx}^*(\frac{1}{z^*})\end{aligned}$$

Equation (2.56) is also proved in the same way.

8. (i) From (2.57)

$$\Phi_{vv}(z) = H^*(\frac{1}{z^*})\Phi_{vv}(z) = \sigma_v^2 \times (\frac{1}{1-az^*})^* = \sigma_v^2 \times \frac{1}{1-a^*z}$$

From (2.80),

$$\Phi_{uu}(z) = H(z)H^*(\frac{1}{z^*})\Phi_{vv}(z) = \frac{\sigma_v^2}{(1-az^{-1})(1-a^*z)}$$

(ii) For $|a| < 1$,

$$\frac{1}{1-a^*z} = \sum_{k=0}^{\infty}(a^*)^k a^k = \sum_{k=-\infty}^{0} (a^*)^{-k} z^{-k}$$

Since $\phi_{vu}(k)$ is the inverse z-transform of $\Phi_{vu}(z)$ and $\phi_{vv}(z) = \sum_{k=-\infty}^{\infty} \phi_{vu}(k)z^{-k}$, it follows that

$$\phi_{vu}(k) = \begin{cases} 0 & k > 0 \\ \sigma_v^2(a^*)^{-k}, & k \le 0 \end{cases}$$

By method of partial fraction,

$$\Phi_{uu}(z) = \frac{\sigma_v^2}{(1-a^*z)(1-az^{-1}) = \frac{-(\sigma_v^2/a^*)z}{(z-1/a^*)(z-a)}}$$

$$= \frac{A}{z-1/a^*} + \frac{B}{z-a}$$

Where

$$A = \left[-\frac{\sigma_v^2}{a^*}\frac{z}{z-a}\right]_{z=1/a^*} = -\frac{\sigma_v^2}{a^*}\frac{1}{1-|a|^2}$$

$$B = \left[-\frac{\sigma_v^2}{a^*}\frac{z}{z-1/a^*}\right]_{z=a} = \frac{\sigma_v^2 a}{1-|a|^2}$$

The first term on the right hand side corresponds to a left sided sequence,

$$\frac{A}{z-1/a^*} = -\frac{Aa^*}{1-a^*z} = \frac{\sigma_v^2}{1-|a|^2}\frac{1}{1-a^*z}$$
$$= \frac{\sigma_v^2}{1-|a|^2}\sum_{k=0}^{\infty}(a^*)^k z^k = \frac{\sigma_v^2}{1-|a|^2}\sum_{k=-\infty}^{0}(1/a^*)^k z^{-k}$$

and the second term to a right sided sequence

$$\frac{B}{z-a} = \frac{Bz^{-1}}{1-az^{-1}} = Bz^{-1}\sum_{k=0}^{\infty}a^k z^{-k}$$
$$= \frac{\sigma_v^2}{1-|a|^2}\sum_{k=1}^{\infty}a^{k-1}z^{-k}$$

Using the above results, and since $\Phi_{uu}(z) = \sum_{k=-\infty}^{\infty}\phi_{uu}z^{-k}$, we get

$$\phi_{uu}(k) = \begin{cases} \frac{\sigma_v^2}{1-|a|^2}a^{k-1} & k > 0 \\ \frac{\sigma_v^2}{1-|a|^2}(a^*)^{-k} & k \le 0 \end{cases}$$

(iii)

$$\sigma_u^2 = \phi_{uu}(0) = \frac{\sigma_v^2}{1-|a|^2}$$

9. (i) When $a \neq b$,

$$\Phi_{vu}(z) = H^*(\frac{1}{z^*})\Phi_{vv}(z) = \frac{\sigma_v^2}{(1-a^*z)(1-b^*z)}$$

$$\Phi_{uu}(z) = H(z)H^*(\frac{1}{z^*})\Phi_{vv}(z) = \frac{\sigma_v^2}{(1-az^{-1})(1-bz^{-1})(1-a^*z)(1-b^*z)}$$

When $a = b$,

$$\Phi_{vu}(z) = H^*(\frac{1}{z^*})\Phi_{vv}(z) = \frac{\sigma_v^2}{(1-a^*z)^2}$$

$$\Phi_{uu}(z) = H(z)H^*(\frac{1}{z^*})\Phi_{vv}(z) = \frac{\sigma_v^2}{(1-az^{-1})^2(1-a^*z)^2}$$

(ii) When $a \neq b$

$$\Phi_v u(z) = \frac{\sigma_v^2}{a^* - b^*}\left[\frac{a^*}{1-a^*z} - \frac{b^*}{1-b^*z}\right]$$

Thus, we have

$$\phi_{vu}(k) = \begin{cases} 0 & k > 0 \\ \frac{\sigma_v^2}{a^*-b^*}\left[(a^*)^{-k+1} - (b^*)^{-k+1}\right] & k \leq 0 \end{cases}$$

Similarly,

$$\Phi_{uu}(z) = \frac{A}{(z-a)} + \frac{B}{(z-b)} + \frac{C}{(z-1/a^*)} + \frac{D}{(z-1/b^*)}$$

Where

$$A = \left[\frac{\sigma_v^2 z^2}{a^*b^*(z-b)(z-1/a^*)(z-1/b^*)}\right]_{z=a} = \frac{\sigma_v^2 a^2}{(a-b)(1-|a|^2)(1-ab^*)}$$

$$B = \left[\frac{\sigma_v^2 z^2}{a^*b^*(z-a)(z-1/a^*)(z-1/b^*)}\right]_{z=b} = \frac{-\sigma_v^2 a^2}{(a-b)(1-|b|^2)(1-ba^*)}$$

$$C = \left[\frac{\sigma_v^2 z^2}{a^*b^*(z-a)(z-b)(z-1/b^*)}\right]_{z=1/a^*} = \frac{-\sigma_v^2}{(a^*-b^*)(1-|a|^2)(1-ba^*)}$$

$$D = \left[\frac{\sigma_v^2 z^2}{a^*b^*(z-a)(z-b)(z-1/a^*)}\right]_{z=1/b^*} = \frac{\sigma_v^2 a^2}{(a^*-b^*)(1-|b|^2)(1-ab^*)}$$

Hence,

$$\Phi_{uu}(z) = \frac{Az^{-1}}{(1-az^{-1})} + \frac{Bz^{-1}}{(1-bz^{-1})} - \frac{Ca^*}{(1-a^*z)} - \frac{Da^*}{(1-a^*z)}$$

$$= \sum_{k=1}^{\infty} Aa^{k-1}z^{-k} + \sum_{k=1}^{\infty} Ba^{k-1}z^{-k} - \sum_{k=-\infty}^{0} C(a^*)^{-k+1}z^{-k} - \sum_{k=-\infty}^{0} D(b^*)^{-k+1}z^{-k}$$

and

$$\phi_{vu}(k) = \begin{cases} Aa^{k-1} + Bb^{k-1} & k > 0 \\ -C(a^*)^{-k+1} - D(b^*)^{-k+1} & k \leq 0 \end{cases}$$

When $a = b$,

$$\phi_{vu}(k) = \frac{1}{2\pi j}\oint_C \Phi_{vu}(z)z^{k-1}dz = \frac{1}{2\pi j}\oint_C \frac{\sigma_v^2 z^{k-1}}{(1-a^*z)^2}dz = \frac{1}{2\pi j}\oint_C \frac{\sigma_v^2 z^{k-1}}{(a^*)^2(z-1/a^*)^2}dz$$

Where C is a contour in $|a| < |z| < |a|^{-1}$. For $k \geq 1$, there is no poles inside C implies that $\phi_{vu}(k) = 0$. For $k < 1$, substituting $z = 1/p$ and noting that $p = a^*$ is the only pole in C,

$$\phi_{vu}(k) = \frac{1}{2\pi j}\oint_C \Phi_{vu}(1/p)p^{-k-1}dp = \frac{1}{2\pi j}\oint_C \frac{\sigma_v^2 p^{-k+1}}{(p-a^*)^2}dp$$

$$= Residue\left[\frac{\sigma_v^2 p^{-k+1}}{(p-a^*)^2}\right]_{p=a^*} = \left[\frac{d}{dp}\sigma_v^2 p^{-k+1}\right]_p = a^* = \sigma_v^2(1-k)(a^*)^{-k}$$

Hence,

$$\phi_{vu}(k) = \begin{cases} 0 & k > 0 \\ \sigma_v^2(1-k)(a^*)^{-k} & k \le 0 \end{cases}$$

Similarly, applying the residue method to find inverse z-transform of $\Phi_{uu}(z)$,

$$\phi_{uu}(k) = \begin{cases} \frac{\sigma_v^2}{(1-|a|^2)^3}[(1-|a|^2)(1+k)+2|a|^2]a^k & k \ge -1 \\ \frac{\sigma_v^2}{(1-|a|^2)^3}[(1-|a|^2)(1-k)+2|a|^2](a^*)^{-k} & k < -1 \end{cases}$$

(iii)

$$\sigma_u^2 = \phi_{uu}(0)$$

10. (i) Using (2.75),

$$\Phi_{vu}(z) = H^*(1/z^*)\Phi_{vv}(z) = \left(\sum_{n=0}^{N-1} h(n)(z^*)^n\right)^* \sigma_v^2$$

$$\sigma_v^2 \sum_{n=0}^{N-1} h^*(n)z^n = \sigma_v^2 \sum_{n=-(N-1)}^{0} h^*(-n)z^{-n}$$

Using (2.80),

$$\Phi_{uu}(z)H(z)H^*(1/z^*)\Phi_{vv}(z) = \sigma_v^2 \sum_{n=0}^{N-1} h(n)z^{-n} \sum_{m=0}^{N-1} h^*(m)z^m$$

$$\sigma_v^2 \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} h(n)h^*(m)z^{-(n-m)}$$

With $n-m=k$, we obtain

$$\Phi_{uu}(z) = \sigma_v^2 \sum_{k=-(N-1)}^{N-1} \alpha_h(k)z^{-k},$$

where $\alpha_h(k) = \sum_{m=max(0,-k)}^{min(N-1,N-1-k)} h(m+k)h^*(m)$.

(ii) From the result of Part (a), we immediately obtain

$$\phi_{vu}(k) = \begin{cases} \sigma_v^2 h^*(-k) & -(N-1) \le k \le 0 \\ 0 & otherwise \end{cases}$$

and

$$\phi_{uu}(k) = \begin{cases} \sigma_v^2 \alpha_h(k) & -(N-1) \le k \le N-1 \\ 0 & otherwise \end{cases}$$

$= \sigma_v^2\alpha_h(k)$ (since $\alpha_h(k) = 0$ for $k \le -N$ and $k \ge N$)

11. From (2.65),

$$E[|x_N(e^{j\omega})|^2] = \sum_{n=-N}^{N}\sum_{m=-N}^{N} E[x(n)x^*(m)]e^{-j\omega(n-m)} = \sum_{n=-N}^{N}\sum_{m=-N}^{N} \phi_{xx}(n-m)e^{j\omega(n-m)}$$

Next, we note that $k = n - m$ can take values in the range $-2N$ to $2N$ and for any $k = n - m$, there are $2N + 1 - |k|$ similar terms added together. Thus we obtain

$$E[|x_N(e^{j\omega})|^2] = \sum_{k=-2N}^{2N} (2N+1-|k|)\phi_{xx}(k)e^{-j\omega k}$$

Dividing both sides of the result by $2N + 1$, gives (2.66).

12. From (2.75), $\phi_{xy}(z) = H^*(1/z^*)\Phi_{xx}(z)$. Using (2.55) and (2.56), we get

$$\Phi^*_{yx}(1/z^*) = H^*(1/z^*)\Phi_{xx}(z)$$

Hence, $\Phi_{yx}(z) = H(z)\Phi^*_{xx}(1/z^*) = H(z)\Phi_{xx}(z)$ From (2.90), $\Phi_d y(z) = H^*(1/z^*)\Phi_{dx}(z)$. Again using (2.56), we get

$$\Phi^*_{yd}(1/z^*) = H^*(1/z^*)\Phi^*_{xd}(1/z^*)$$

Hence, $\Phi_{yd}(z) = H(z)\Phi_{xd}(z)$.

13. Using $\star$ to denote convolution,

    (i) $\phi_{yx}(n) = h(n) \star \phi_{xx}(n)$,
    (ii) $\phi_{xy}(n) = h^*(-n) \star \phi_{xx}(n)$,
    (iii) $\phi_{yd}(n) = h(n) \star \phi_{xd}(n)$,
    (iv) $\phi_{dy}(n) = h^*(-n) \star \phi_{dx}(n)$

14. Let $y(n) = y_1(n) + y_2(n)$, where $y_1(n)$ and $y_2(n)$ are the outputs of $H(z)$ and $G(z)$, respectively.

    (i) Since $u(n)$ and $v(n)$ are uncorrelated,

    $$\phi_{uy}(n) = \phi_{uy1}(n) + \phi_{uy2}(n) = \phi_{uy1}(n)$$

    Hence,
    $$\Phi_{uy}(z) = \Phi_{uy1}(z) = H^*(1/z^*)\Phi_{uu}(z) = \sigma_u^2 H^*(1/z^*).$$

    (ii) Similar to Part (b), $\Phi_{vy}(z) = \sigma_v^2 G^*(1/z^*)$.

    (iii) Noting that $y_1(n)$ and $y_2(n)$ are originating from uncorrelated source, $u(n)$ and $v(n)$, respectively, we obtain $\Phi_{yy}(z) = \Phi_{y1y1}(z) + \Phi_{y2y2}(z) = \sigma_u^2 H(z)H^*(1/z^*) + \sigma_v^2 G(z)G^*(1/z^*)$.

15. (i) Using $\Phi_{xx}(z) = H(z)H^*(1/z^*)\Phi_{vv}(z)$, with $H(z) = \frac{1}{1-0.5z^{-1}}$ and $\Phi_{vv}(z) = 1$, we obtain

$$\Phi_{xx}(z) = \frac{1}{1-0.5z^{-1}}\left[\frac{1}{1-0.5(1/z^*)^{-1}}\right]^* = \frac{1}{(1-0.5z^{-1})(1-0.5z)}$$

(ii) Since $\Phi_{yy}(z) = G(z)G^*(1/z^*)\Phi_{zz}(z)$, where $G(z) = 1-2z^{-1}$, we obtain

$$\Phi_{yy}(z) = \frac{1-2z^{-1}}{1-0.5z^{-1}} \times \left[\frac{1-2(1/z^*)^{-1}}{1-0.5(1/z^*)^{-1}}\right]^* \Phi_{vv}(z) = \frac{1-2z^{-1}}{1-0.5z^{-1}} \times \frac{1-2z}{1-0.5z}$$

$$= \frac{1-2(z+z^{-1})+4}{1-0.5(z+z^{-1})+1/4} = 4$$

(iii) Using $\Phi_{vy}(z) = H^*(1/z^*)G^*(1/z^*)\Phi_{vv}(z)$,

$$\Phi_{vy}(z) = \left[\frac{1-2(1/z^*)^{-1}}{1-0.5(1/z^*)^{-1}}\right]^* \Phi_{vv}(z) = \frac{1-2z}{1-0.5z}.$$

16. (i) Since $x(n) = \sum_k h(k)v(n-k)$ and $y(n) = \sum_k g(k)v(n-k)$,

$$\phi_{xy}(m) = E[x(n)y^*(n-m)] = E[\sum_l \sum_k h(k)g^*(l)v(n-k)v^*(n-m-l)]$$

$$= \sum_l \sum_k h(k)g^*(l)E[v(n-k)v^*(n-m-l)] = \sum_l \sum_k h(k)g^*(l)\delta(-k+l+m).$$

But $\delta(-k+l+m) = 1$ for $k = l+m$ and zero elsewhere. Using this in the above result, we obtain $\phi_{xy}(m) = \sum_l h(l+m)g^*(l)$.

(ii) Since $\Phi_{xy}(z)$ is the z-transform of $\phi_{xy}(m)$,

$$\Phi_{xy}(z) = \sum_m \phi_{xy}(m)z^{-m} = \sum_m \sum_l h(l+m)g^*(l)z^{-m}$$

$$\sum_m \sum_l h(l+m)z^{-(m+l)}g^*(l)z^l = \sum_l g^*(l)z^l \sum_m h(l+m)z^{-(m+l)}$$

$$= G^*(1/z^*)H(z) = H(z)G^*(1/z^*).$$

17. (i) Since $u(n) = x(n) + y(n)$, we have

$$\begin{aligned}\phi_{uu}(k) &= E[(x(n)+y(n))(x^*(n+k)+y^*(n+k))] \\ &= E[x(n)x^*(n+k)] + E[y(n)y^*(n+k)] + E[x(n)y^*(n+k)] + E[y(n)x^*(n+k)] \\ &= \phi_{xx}(k) + \phi_{yy}(k) + \phi_{xy}(k) + \phi_{yx}(k)\end{aligned}$$

Hence,

$$\Phi_{uu}(z) = \Phi_{xx}(z) + \Phi_{yy}(z) + \Phi_{xy}(z) + \Phi_{yx}(z) = \Phi_{xx}(z) + \Phi_{yy}(z) + \Phi_{xy}(z) + \Phi^*_{xy}(1/z^*)$$

(ii) Since $u(n)$ is the output of a system with input v(n) and transfer function $H(z) + G(z)$,

$$\Phi_{uu}(z) = (H(z) + G(z))(H^*(1/z^*) + G^*(1/z^*))\Phi_{vv}(z)$$

$$= H(z)H^*(1/z^*)\Phi_{vv}(z) + G(z)G^*(1/z^*)\Phi_{vv}(z) + H(z)G^*(1/z^*)\Phi_{vv}(z) + H^*(1/z^*)G(z)\Phi_{vv}(z)$$

Since $v(n)$ in a zero-mean unit-variance white noise process,

$$\Phi_{vv}(z) = 1$$

and we have

$$\Phi_{uu}(z) = \Phi_{xx}(z) + \Phi_{yy}(z) + \Phi_{xy}(z) + \Phi_{yx}(z) = \Phi_{xx}(z) + \Phi_{yy}(z) + \Phi_{xy}(z) + \Phi^*_{xy}(1/z^*)$$