Chapter 1

1-1 Write equations that define each of the following laws: Fick's, Fourier's, Newton's, and Ohm's. What is the conserved quantity in each of these laws? Can you represent all of these laws by one general expression? If so, does this mean that all of the processes represented by these laws are always analogous? If they aren't, why not?

Solution:

(a) Fick: 1–D: $n_{Ay} = -D_{AB}\frac{dC_A}{dy}$, 3–D: $\vec{n}_A = -D_{AB}\vec{\nabla}C_A$

Fourier: 1–D: $q_y = -\alpha\frac{d(\rho C_v T)}{dy} = -k\frac{dT}{dy}$, 3-D: $\vec{q} = -k\vec{\nabla}T$

$\alpha = \frac{k}{\rho C_v}$ = Thermal Diffusivity

Newton: 1–D: $(\tau_{yx})_m = -\nu\frac{d(\rho v_x)}{dy} = -\mu\frac{dv_x}{dy}$, 3-D: $\vec{\vec{\tau}}_m = -\mu\left(\vec{\nabla}\vec{v} + (\vec{\nabla}\vec{v})^+\right)$

ν = Kinematic viscosity

(3–D form is not analogous to other laws)

Ohm: 1–D: $i_y = -k_e\frac{de}{dy}$, 3-D: $\vec{i} = -k_e\vec{\nabla}e$

(b) What is the conserved quantity?

Fick: Mass of species A

Fourier: Heat

Newton: *x*-momentum

Ohm: Charge

(c) General Expression: $Q_y = -k_T\frac{dC_Q}{dy}$ applies to 1-D forms of all laws

3–D: $\boxed{\vec{Q} = -k_T\vec{\nabla}C_Q}$ applies to all except momentum

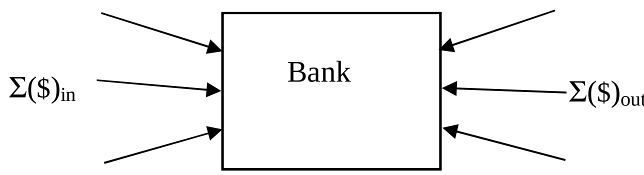
d) All but momentum equations are analogous. (in 3–D)

1-2 The general conservation law for any conserved quantity Q can be written in the form of Eq. (1-12). We have said that this law can also be applied to ‘‘dollars’’ as the conserved quantity Q. If the ‘‘system’’ is your bank account,

(a) Identify specific ‘‘rate in,’’ ‘‘rate out,’’ and ‘‘rate of accumulation’’ terms in this equation relative to the system (i.e., each term corresponds to the rate at which dollars are moving into or out of your account).

(b) Identify one or more ‘‘driving force’’ effects that are responsible for the magnitude of each of these rate terms, i.e., things that influence how fast the dollars go in or out. Use this to define corresponding ‘‘transport constants’’ for each ‘‘in’’ and ‘‘out’’ term relative to the appropriate ‘‘driving force’’ for each term.

Solution:



Balance:

$$\sum\begin{pmatrix}\text{Rate of}\\ \$\text{in}\end{pmatrix}-\begin{pmatrix}\text{Rate of}\\ \$\text{out}\end{pmatrix}+\begin{pmatrix}\text{Rate of}\\ \text{Generation of }\$\end{pmatrix}=\begin{pmatrix}\text{Rate of}\\ \text{Accumulation of }\$\end{pmatrix}$$

$$\sum\begin{pmatrix}\text{Rate of}\\ \$\text{in}\end{pmatrix}=\text{Paycheck}+\text{Gifts}+\text{Investment Income}+....$$

$$\sum\begin{pmatrix}\text{Rate of}\\ \$\text{out}\end{pmatrix}=\text{School}+\text{Rent}+\text{Food}+\text{Transport}+\text{Clothing}+\text{Entertainment}+...$$

$$\sum\begin{pmatrix}\text{Rate of}\\ \text{Genetation of }\$\end{pmatrix}=\text{Interest}\left(\text{This can also be considered a "Rate in" term}\right)$$

$$\text{Rate of X}=k_x\cdot(\text{DF})_x,\quad \text{DF}=\text{driving force}$$

$$(\text{DF})_{\text{Paycheck}}\cong k_1\left(\text{Education}+\text{Ingenuity}+\text{Experence}+...\right)$$

$$(\text{DF})_{\text{Gifts}}\cong k_2\left(\text{Generosity of Parents}\right)$$

$$(\text{DF})_{\text{Investment}}\cong k_3\left(\text{Amount of Savings / Portfolio}\right)$$

$$(\text{DF})_{\text{School}}\cong k_4\left(\text{Where}+\text{no. hours}\right)$$

$$(\text{DF})_{\text{Rent}}\cong k_5\left(\text{Size, Loation of Apt.}\right)$$

$$(\text{DF})_{\text{Food}}\cong k_6\left(\text{"Taste", e.g. Steak vs. Hamburger}\right)$$

$$(\mathrm{DF})_{\mathrm{Transportation}} \cong \mathrm{k}_7(\text{Cost of car} \rightarrow \text{"image"})$$

$$(\mathrm{DF})_{\mathrm{Clothing}} \cong \mathrm{k}_8(\text{"Style"})$$

$$(\mathrm{DF})_{\mathrm{Entertainment}} \cong \mathrm{k}_9(\text{"Life style"}, \text{Cost of Beer})$$

$$(\mathrm{DF})_{\mathrm{Interest}} \cong \mathrm{k}_{10}(\text{Size of Account}, \text{Interst rate})$$

1-3 A dimensionless group called the Reynolds number is defined for flow in a pipe or tube

$$N_{\mathrm{Re}} = \frac{DV\rho}{\mu} = \frac{\rho V^2}{\mu V/D}$$

where V is the average velocity in the pipe, ρ is the fluid density, μ is the fluid viscosity, D is the tube diameter. The second form of the group indicates that it is a ratio of the convective (turbulent) momentum flux to the molecular (viscous) momentum flux, or the ratio of inertial forces (which are destabilizing) to viscous forces (which are stabilizing). When viscous forces dominate over inertial forces, the flow is laminar and fluid elements flow in smooth, straight streamlines, whereas when inertial forces dominate, the flow is unstable and the flow pattern break up into random fluctuating eddies. It is found that laminar flow in a pipe occurs as long as the value of the Reynolds number is less than about 2000.

Calculate the maximum velocity and the corresponding flow rate (in cm^3/s) at which laminar flow of water is possible in tubes with the following diameters:

D = 0.25, 0.5, 1.0, 2.0, 4.0, 6.0, 10.0 in.

<u>Solution</u>:

Reynolds No: $N_{Re} = \frac{\rho VD}{\mu} = \frac{\rho V^2}{\mu V/D}$

Water: ρ = 1 gm/cc, μ = 1 cP = 0.01 P =0.01 g/(cm s)

Laminar: $N_{Re} \leq 2000 = \frac{\rho VD}{\mu}$, $V = \frac{2000\,\mu}{D\rho}$, $Q = \frac{\pi D^2}{4} V$

For, D = 0.25 in :

$$V = \frac{N_{Re}\mu}{D\rho} = \frac{2000\left(0.01\ \mathrm{g/cm\ s}\right)}{\left(25 \times 2.54\,\mathrm{cm}\right)\left(1\ \mathrm{g/cc}\right)} = 31.5\ \mathrm{cm/s}$$

$$Q = \frac{\pi D^2 V}{4} = \frac{\pi\left(0.25 \times 2.54\,\mathrm{cm}\right)^2}{4}\left(31.5\ \mathrm{cm/s}\right) = 9.97\ \mathrm{cc/s}$$

Similarly, for air at standard temperature and pressure (see Appx.A):

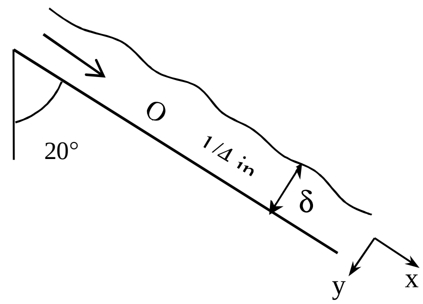
ρ = 1.205 kg/m^3, μ = 1.983 $\times 10^{-5}$ Pa s (i.e.1 Pas = Ns/m^2 = kg/(m s)

D(in)	Water		Air	
	V (cm/s)	*Q*(cc/s)	*V* (cm/s)	*Q*(cc/s)
0.25	31.5	9.97	5.18	164
0.5	15.7	19.9	2.59	328
1	7.87	39.9	1.30	657
2	3.94	79.8	0.648	1310
4	1.97	160	0.324	2630
6	1.31	239	0.216	3940
10	0.787	399	0.130	6570

(Note: Although the fluid properties are given to 4 digits, the number of digits in N_{Re} = 2000 is undefined, so that we give the solution to only 3 digits. See Ch. 2 for details on Units and Dimensions.)

1-4 A layer of water is flowing down a flat plate that is inclined at an angle of 20° to the vertical. If the depth of the layer is 1/4 in., what is the shear stress exerted by the plate on the water? (Remember: Stress is a dyad, with two directions associated with each component.)

Solution:



$$\tau_{yx} = \frac{F_x}{A_y} \text{ at plate, } \left(\tau_{yx}\right)_w = \frac{\left(F_x\right)_w}{A_y}$$

F_x = force in x-direction on water

A_y = area of water contacting plate

Force in x direction on plate = x-component of weight of water

$$= \rho g \ \nabla \cos 20°, \quad \nabla = \delta WL \ (W = \text{width}, L = \text{length})$$

$A_y = WL$

F_x on water = – F_x on plate = – ρg WL cos 20°

$$\left(\tau_{yx}\right)_w = \frac{-\rho g \, WL \cos 20^o}{WL} = -\rho g \ \cos 20^o$$

This is (–), since wall exerts a force in – x direction on water.

(Note: The "system" is the water. τ_{yx} is the force on the water in the x-direction acting on the surface A_y. (the $+A_y$ surface <u>outward</u> normal is in + y direction, with +y direction from the surface inward).

$$\therefore A_y \text{ surface of water at plate is (+); } \left(\tau_{yx}\right)_m = -\tau_{yx} = \frac{F_x}{A_y} = \tau_{yx} = \frac{F_x}{A_y} \text{ is } (-)$$

(the stress exerted <u>by</u> the plate, <u>on</u> the water is (–))

1-5 The relationship between the forces and fluid velocity can easily be illustrated by considering the simple one-dimensional steady flow between two parallel plates in Fig. 1.1. The area of each plate $A_y = 1\ \mathrm{m}^2$ and the gap between the two plates is 0.1 mm (very thin). Fluids of different viscosities, as listed below, are contained between the two plates. A tangential force of one Newton is applied tangential to the top plate in the x direction at a fixed velocity V_0 , while the bottom plate is held stationary. Calculate the value of V_0 for each of the following fluids:

Substance	**Viscosity (cP)**
Olive oil	100
Castor oil	600
Pure Glycerine (293K)	1500
Honey	10^4
Corn Syrup	10^5
Bitumen	10^{11}
Molten glass	10^{15}

In view of these results, what is the meaning of the phrase from the Bible: "the *mountains flowed before the Lord"* and the significance of the phrase: "*Everything flows*"!

$$\tau_{yx}\left(\mathrm{N}/\mathrm{m}^2\right) = \frac{F_x\left(\mathrm{N}\right)}{A_y\left(\mathrm{m}^2\right)} = 1\ \mathrm{Pa} \quad = \mu\left(\frac{\mathrm{N\,s}}{\mathrm{m}^2}\right)\frac{\left(V_o - 0\right)\left(\mathrm{m}/\mathrm{s}\right)}{0.1\times10^{-3}\left(\mathrm{m}\right)}$$

$$1\ \mathrm{Pa} = \frac{\mu\left(\mathrm{Pa\ s}\right)V_o\left(\mathrm{m}/\mathrm{s}\right)}{10^{-4}\left(\mathrm{m}\right)} \quad \text{or} \quad V_o = \frac{10^{-4}\ \mathrm{Pa\ m}}{\mu\ \left(\mathrm{Pa\ s}\right)}\ \left(=\mathrm{m/s}\right)$$

Substance	**μ (Pa. s)**	**V_o (m/s)**
Olive oil	100×10^{-3}	1×10^{-3} *or* 1 mm/s
Castor oil	600×10^{-3}	1.67×10^{-4} *or* 0.167 mm/s
Pure Glycerine (293 K)	1500×10^{-3}	6.67×10^{-5} *or*66.7 μm/s
Honey	$10^4\times10^{-3}$	1×10^{-5} *or* 10 μm/s
Corn Syrup	$10^5\times10^{-3}$	1×10^{-6} *or* 1 μm/s
Bitumen	$10^{11}\times10^{-3}$	1×10^{-12} *i.e.*, 86.4 nm/day
Molten Glass	$10^{15}\times10^{-3}$	1×10^{-16} *i.e.*, 8.64×10^{-3} nm/day

In contrast to the velocity of 1 mm/s attained by the top plate with olive oil as the working fluid, the corresponding value is only 3.15 nm/year for the molten glass! Thus, everything flows but the relevant question to ask is at what time scale. Thus, "the Mountains flowed before the Lord" implies that even mountains flow over timescales much longer than we ordinary human beings can observe since Gods are immortal and only they can see the mountains flow! In other words the distinction between solids and fluids is not as sharp as we think!

1-6 A slider bearing consists of a sleeve surrounding a cylindrical shaft that is free to move axially within the sleeve. A lubricant (e.g., grease) is in the gap between the sleeve and the shaft to isolate the metal surfaces and support the stress resulting from the shaft motion. The diameter of the shaft is 1 in., and the sleeve has an inside diameter of 1.02 in. and a length of 2 in.

(a) If you want to limit the total force on the sleeve to less than 0.5 lb_f when the shaft is moving at a velocity of 20 ft/s, what should the viscosity of the grease be? What is the magnitude of the flux of momentum in the gap, and which direction is the momentum being transported?

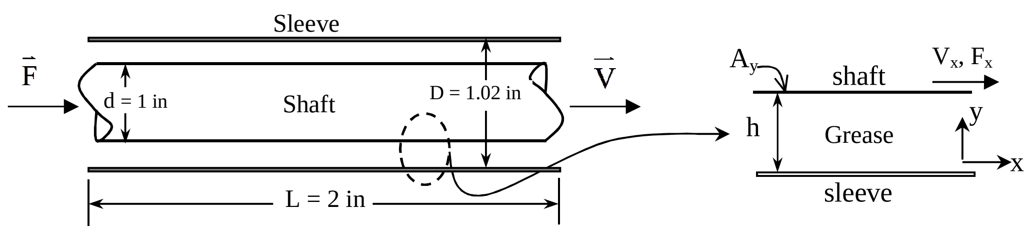
(b) If the lubricant is grease with a viscosity of 400 cP (centipoise), what is the force exerted on the sleeve when the shaft is moving at 20 ft/s?

(c) The sleeve is cooled to a temperature of 150°F, and it is desired to keep the shaft temperature below 200°F. What is the cooling rate (i.e., the rate at which heat must be removed by the coolant), in Btu/hr, to achieve this? Properties of the grease may be assumed to be: specific heat = 0.5 Btu/(lb_m°F); SG (specific gravity) = 0.85; thermal conductivity = 0.06 Btu/(hr ft °F).

(d) If the grease becomes contaminated, it could be corrosive to the shaft metal. Assume that this occurs and the surface of the shaft starts to corrode at a rate of 0.1 μm/yr. If this corrosion rate is constant, determine the maximum concentration of metal ions in the grease when the ions from the shaft just reach the sleeve. Properties of the shaft metal may be assumed to be MW = 65; SG = 8.5; diffusivity of metal ions in grease = 8.5 x 10^{-5} cm^2/s.

Solution :

Slider Bearing



(a) F_{max} on sleeve = 0.5 lb_f when V_x = 20 ft/s , μ = ?

$$(F_x)_{shaft} = (-F_x)_{sleeve}$$

assume grease is Newtonian: $\tau_{yx} = \mu \frac{dv_x}{dy} = \frac{F_x}{A_y} \quad (A_y)_{shaft} = (-A_y)_{sleeve}$

A_y is +ve (surface of grease in +y direction)

$F_x = F$ on grease in x-direction

$A_y = \pi DL$

$$\frac{dV_x}{dy} = \frac{\Delta V_x}{\Delta y} = \frac{V_x}{h} \quad \text{so} \quad \frac{F_x}{A_y} = \mu \frac{V_x}{h}$$

$$\mu = \frac{F_x h}{A_y V_x} = \frac{0.5\,lb_f\,(0.01in.)\left(4.45\times10^5\,{dyn}/{lb_f}\right)}{\pi(1.02\,in.)(2\,in.)\left(20\,{ft}/{s}\right)\left(30.48\,{cm}/{ft}\right)\left(2.54\,{cm}/{in.}\right)} = 0.224\ dyn\ s/cm^2$$

$$= 0.224\ Poise = 22.4\ cP$$

$$\text{Momentum flux} = (\tau_{yx})_m = -\tau_{yx} = -\frac{F_x}{A_y} = -\frac{0.5\,lb_f\left(4.45\times10^5\,{dyn}/{lb_f}\right)}{\pi(1.02)\left(2\,in.^2\right)\left(2.54\,{cm}/{in.}\right)^2} = -5.38\times10^3\,{dyn}/{cm^2}$$

i.e. Momentum flux is (–) w.r.t coordinates specified. (+ve x-momentum transferred in –y direction), i.e. momentum goes from faster to slower fluid - from shaft to sleeve.

(b) Grease: μ = 400 cP

What is F on sleeve if V_{shaft} = 200 ft /s ?

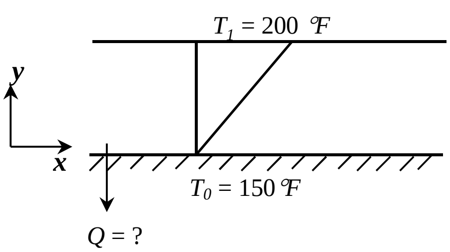
$$F_x = \frac{A_y \mu V_x}{h} = \frac{\pi(1.02\,in.)(2\,in.)\left(4\,{dyn}/{cm^2}\right)\left(20\,{ft}/{s}\right)\left(30.54\,{cm}/{ft}\right)\left(2.54\,{cm}/{in.}\right)^2}{(0.01in.)\left(2.54\,{cm}/{in.}\right)}$$

$$= \frac{3.97\times10^6\,dyn}{4.45\times10^5\,{dyn}/{lb_f}} = 8.92\ lb_f$$

(c) Sleeve T = 150°F want shaft $T \le 200$°F   Cooling rate = ?

C_v = 0.5 $BTU/lb_m\,°F$, SG = 0.85, k = 0.06 $BTU/\ hr\ ft\ °F$

Fourier's Law: $q_y = -k\frac{dT}{dy} \simeq -k\frac{(T_1 - T_0)}{h}$

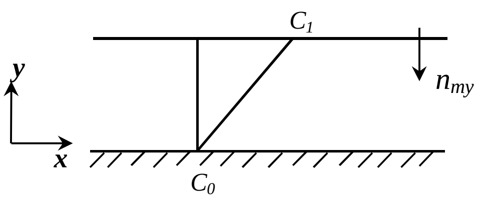


$$q_y = -\left(0.06\frac{Btu}{hr\ ft\ ^oF}\right)\left(\frac{(200-150)^\circ F}{0.01\text{in}\left(12\frac{in}{ft}\right)}\right) = -25\frac{Btu}{hr\,in.^2}$$

(i.e., heat is transfer in -y direction to cool shaft)

$$Q_y = q_y A_y = -25\frac{Btu}{hr\,in.^2}(\pi)(1.02\,in.)(2in.) = -160\frac{Btu}{hr} = -0.0445\frac{Btu}{s}$$

(d) Shaft corrodes at rate of 0.1 $\mu m/year$. What is maximum concentration of ions in grease when diffusing ions first reach the shaft? MW = 65, SG_{metal} = 8.5, D_{mG} = 8.5×10^{-5} cm^2/s



Fick's Law: $n_{my} = -D_{mG}\dfrac{dC}{dy} \simeq -D_{mG}\dfrac{C_1}{h}$

$$n_{my} = \text{flux of ions} = \frac{\text{rate of corrosion}}{A_y} = \frac{\left(-0.1\times10^{-4}\ cm/yr\right)\left(8.5\ g/cm^3\right)}{\left(65\ g/mol\right)} = -1.308\times10^{-6}\ mol/cm^2\ yr$$

(dissolved ions diffuse in -y direction)

Maximum C_1 occurs when C_0 = 0 (before concentration builds up in grease)

$$n_{my} \simeq -D_{mG}\frac{C_1}{h}$$

$$C_1 = \frac{-n_{my}h}{D_{mG}} = \frac{\left(1308\times10^{6}\ mol/cm^2\ yr\right)(0.01\text{in})\left(2.54\ cm/in.\right)}{\left(8.5\times10^{-5}\ cm^2/s\right)\left(3600\ s/hr\right)\left(24\ hr/day\right)\left(365\ day/yr\right)}$$

$$C_1 = 1.24\times10^{-11}\ mol/cm^3 = 1.24\times10^{-8}\ mol/l$$

(Note: Corrosion rate of "$x\ cm$" is equivalent to "$x\ cm^3/cm^2$")

1-7 By making use of the analogies between the molecular transport of the various conserved quantities, describe how you would set up an experiment to solve each of the following problems by making electrical measurements (e.g., describe the design of the experiment, how and where you would measure voltage and current, and how the measured quantities are related to the desired quantities).

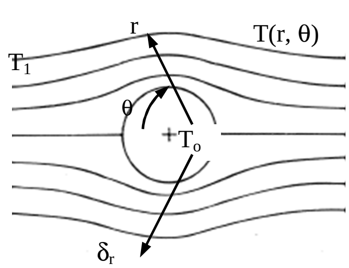
(a) Determine the rate of heat transfer from a long cylinder to a fluid flowing normal to the cylinder axis if the surface of the cylinder is at temperature T_0 and the fluid far away from the cylinder is at temperature T_1. Also determine the temperature distribution within the fluid and the cylinder.

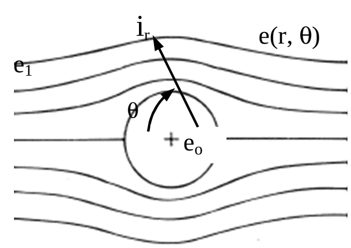
(b) Determine the rate at which a (spherical) mothball evaporates when it is immersed in stagnant air, and also the concentration distribution of the evaporating compound in the air.

(c) Determine the local stress as a function of position on the surface of a wedge-shaped body immersed in a fluid stream that is flowing slowly parallel to the surface. Also, determine the local velocity distribution in the fluid as a function of position in the fluid.

<u>Solution</u>:

(a) Heat transfer from cylinder to fluid flowing ⊥ to axis. Cylinder at T_o, fluid at T_1. Also want $T = f(r,\theta)$.





Fourier's Law: $\vec{q}=-k\vec{\nabla}T$

Ohm's Law: $\vec{i}=-k_e\vec{\nabla}e$

Measure: $e(r,\theta)\rightarrow T(r,\theta),\quad \vec{i}=k_e\vec{\nabla}\left(\dfrac{e(r)-e_o}{e_1-e_o}\right)(e_1-e_o)$

$$\vec{i}(r,\theta)\rightarrow \delta(r,\theta),\quad \vec{q}=k\vec{\nabla}\left(\frac{T-T_o}{T_1-T_o}\right)(T_1-T_o)$$

<u>Experiment</u>: Put a highly conducting disk in a medium of lower material (e.g., a fluid). Apply a voltage (e_o-e_1) between the cylinder and the fluid far from the cylinder, using electrodes.

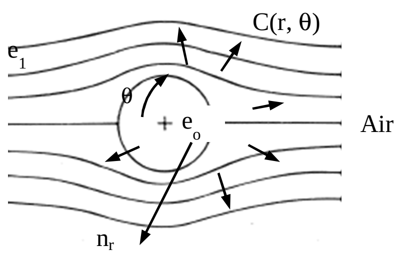
Measure: Voltage at various points in the medium i.e., $e(r, \theta)$. Can also measure (or calculate) voltage differences (e.g., gradients) at various points: $\frac{\partial e}{\partial r}, \frac{\partial e}{\partial \theta}$.

Knowing electrical conductivity of medium (or measure it if current flowing between 2 points is measured), and the thermal conductivity of the fluid:

apply $(e_o\text{-}e_1) \Rightarrow (T_o\text{-}T_1)$, measure i_w, calculate $q_w = \frac{k}{k_e} i_{wall}$:

$$q_w = i_w \frac{k}{k_e} \frac{(T_1\text{-}T_o)}{(e_1\text{-}e_o)}$$

(b) Evaporating mothball:



Fick's Law: $\overline{n_A} = -D_A \overline{\nabla} C_A$

Ohm's Law: $\vec{i} = -k_e \overline{\nabla} e$

Use highly conducting ball in a lower conductivity medium.

Apply voltage $(e_o\text{-}e_1)$ between ball and medium far from ball. Measure $i_w \sim n_{Aw}$.

$e_o \Rightarrow e_1$ (concentration at surface)

(Must know solubility of compound in air, C_o)

$(C_o - 0) \propto (e_o - e_1)$

Conductivity of medium is k_e (or measure it)

$$n_r = -D_A \frac{d}{dr}\left(\frac{C_A}{C_o}\right) C_o,\ i_r = -k_e \frac{d}{dr}\left(\frac{e\text{-}e_1}{e_o\text{-}e_1}\right)(e_o\text{-}e_1)$$

$$\frac{n_{Aw}}{D_A C_o} \equiv G_c, \qquad \frac{i_r}{k_e (e_o\text{-}e_1)} \equiv G_e$$

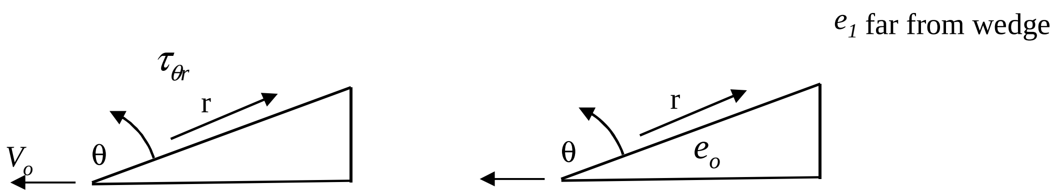
1. $G_c \Rightarrow G_e$, $\frac{dC_A}{dr} = \frac{(e_o\text{-}e_1)}{C_o} \frac{de}{dr}$

2. $n_{Ar} = i_r \frac{D_A C_o}{k_e (e_o\text{-}e_1)} = Rate\,of\ evaporation/area$

1. Relates concentration distribution $(C_A(r,\theta))$ to the voltage distribution $(e(r,\theta))$.

2. Relates the diffusion flux, n_{Ar} (rate of evaporation/area) to the current density, i_r (current through wall/area)

(c) Wedge moving at velocity V_o in still fluid is equivalent to fluid moving at V_o past still wedge. Is 1-D in polar coordinates (r, θ):



Flux of *r-momentum* in θ - direction: $\tau_{\theta r} = -\dfrac{\mu}{r}\dfrac{dV_r}{d\theta}$

Current in θ - direction: $i_e = -\dfrac{k_e}{r}\dfrac{de}{d\theta}$

$\tau_{\theta r}(r) \Rightarrow i_\theta(r), \quad V_r(\theta) \Rightarrow e(\theta)$

Apply e_o voltage to conducting wedge, e_1 at boundary far from wedge.

$V_1 = 0 \quad (V_o - 0) \Rightarrow (e_o - e_1)$

At surface: $\tau_w = -\mu\left(\dfrac{dV_r}{rd\theta}\right)_w \; i_w = -k_e\left(\dfrac{d(e-e_1)}{rd\theta}\right)_w$

$V_r(\theta) = \left(e(\theta) - e_1\right) \Rightarrow \textit{velocity distribution}$

$\tau_w = \dfrac{\mu}{k_e}\left(\dfrac{V_o}{e_o - e_1}\right) \Rightarrow \textit{wall stress related to current through wall}$

1-8 What is the “system” in Example 1-3?

<u>Answer:</u> The System is the grease between the plates.

1-9 Explain the "fruit salad law", and how does it apply to the formulation of equations describing engineering systems?

<u>Answer:</u> The “fruit salad law” states that *“You can’t add apples and oranges unless you want fruit salad.”* This means that every term in any equation must have the same dimensions (and units) – either “apples” <u>or</u> “oranges” - , if the equation is to be valid.