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## Chapter 1

# Introduction

### 1.1 Problems

- 1-1. Compute  $\lambda_D$  and  $N_D$  for the following cases: (a) A glow discharge,  $N_e = 10^{16} \text{ m}^{-3}$ ,  $k_B T_e = 2 \text{ eV}$ , (b) The Earth's ionosphere,  $N_e = 10^{12} \text{ m}^{-3}$ ,  $k_B T_e = 0.1 \text{ eV}$ , (c) A fusion machine,  $N_e = 10^{23} \text{ m}^{-3}$ ,  $k_B T_e = 9 \text{ keV}$ .

**Solution:**

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{N_0 q_e^2}}; \quad N_D = N_0 \left[ \frac{4\pi \lambda_D^3}{3} \right]$$

The formulas are straight forward but students often make the mistake of not using SI units for the temperature which requires a conversion from eV to K using  $1 \text{ eV} = 11600 \text{ K}$  and then multiplying this temperature by  $k_B = 1.38 \times 10^{-23} \text{ Joule/K}$ .

(a)

$$\lambda_D = 1.05 \times 10^{-4} \text{ m}; \quad N_D = 48738$$

(b)

$$\lambda_D = 0.00235 \text{ m}; \quad N_D = 54532$$

(c)

$$\lambda_D = 2.23 \times 10^{-6} \text{ m}; \quad N_D = 4.656 \times 10^6$$

- 1-2. Calculate the average velocity of nitrogen molecules at room temperature assuming three degrees of freedom.

**Solution:**

$$E_{av} = \frac{3}{2} k_B T \text{ where } T = 293 \text{ K}$$

$E_{av} = 6.065 \times 10^{-21} \text{ J}$ ; Now use the fact that  $E_{av} = \frac{1}{2} m u^2$  where  $m$  is the mass of the Nitrogen molecule. Nitrogen gas is diatomic, the molecular mass is  $28 \text{ g/mol}$ . So

$$m = \frac{28 \text{ g}}{\text{mol}} \cdot \frac{1 \text{ mol}}{6.02 \times 10^{23}} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} = 4.65 \times 10^{-26} \text{ kg}$$

$$u = \sqrt{\frac{2E_{av}}{m}} = 510 \text{ m/s}$$

- 1-3. Calculate and plot the electrostatic potential and electric field of a test particle of charge  $+Q$  in free space and in a plasma of number density  $N_0$  and temperature  $T$ . Label the distance axis of your plot in units of Debye length.

**Solution:**

This problem involves plotting the functions

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

and

$$\Phi(r) = \left[ \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \right] e^{-r/\lambda_D}$$

Figure 1.1 shows what the plots should look like. The plots shown were obtained using the code shown in Figure 1.2 executed in the MathWorks MATLAB software package.

- 1-4. A metal sphere of radius,  $r = a$ , with charge,  $Q$ , is placed in a neutral plasma with number density,  $N_0$  and temperature,  $T$ . Calculate the effective capacitance of the system. Compare this with the capacitance of the same sphere placed in free space.

**Solution:**

Capacitance  $C$  is the ratio of stored charge  $Q$  to potential  $V$ :  $C = Q/V$ . In free space, the potential from a charge  $Q$  is given by

$$V(r) = \frac{Q}{4\pi\epsilon_0 r}$$

So the capacitance of the sphere in free space will be

$$C = 4\pi\epsilon_0 a \text{ Farads}$$

In a plasma, the potential from the same charge  $Q$  will be modified by the Debye shielding effect:

$$V(r) = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r} e^{-r/\lambda_D}$$

where

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{N_0 q_e^2}}$$

Thus the capacitance of the sphere in the plasma will be:

$$C = 4\pi\epsilon_0 a e^{-a/\lambda_D} \text{ Farads}$$

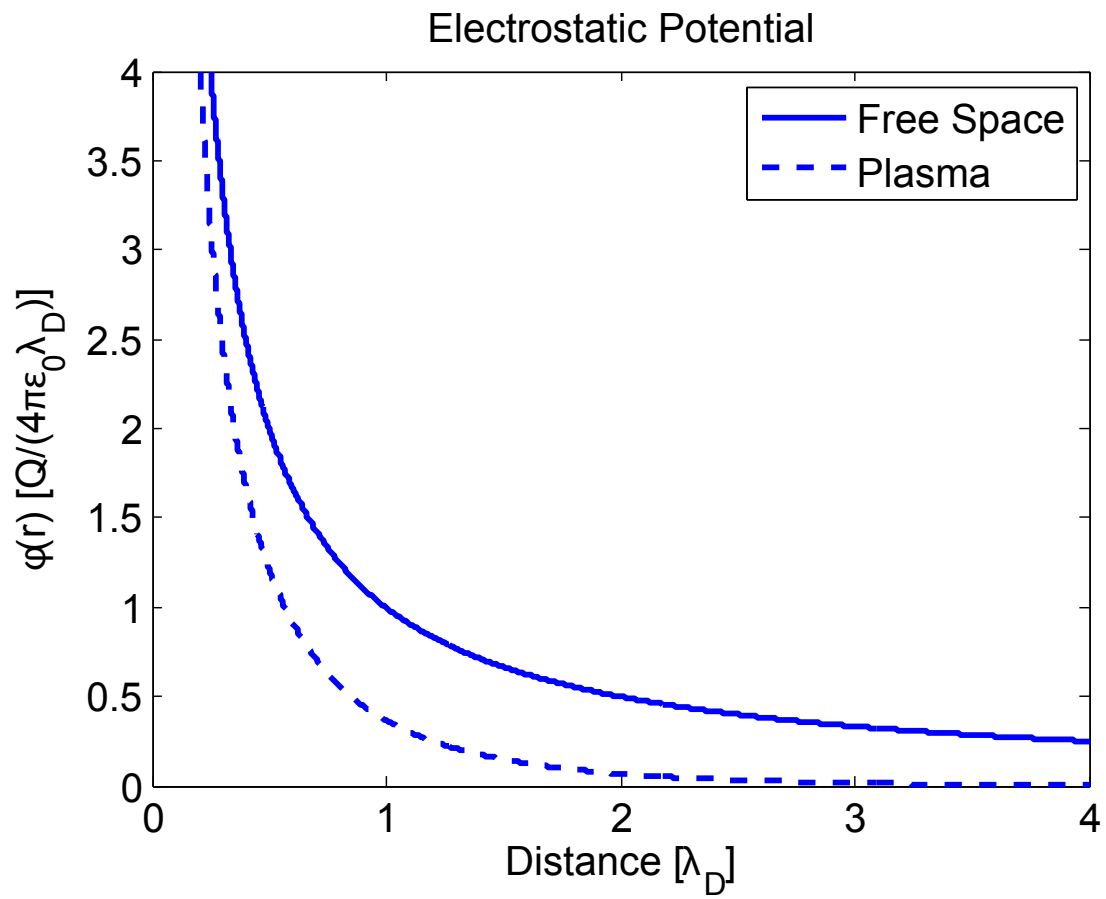


Figure 1.1: Plot for Problem 1-3.

```

%Problem 1-3
%define constants and variables
clear all;
eps0=8.85e-12;
kB=1.38e-23;
qe=1.6e-19;
T=0.2*11600; %Temperature and Density arbitrary for this problem
N0=10^11;
Q=8.65e-12; %Charge also arbitrary

%First calculate the Debye length
LambdaD=sqrt(eps0*kB*T./(N0*qe.^2));

%Define r vector in terms of Debye length, we will not start at zero
%because the potential goes to infinity at r=0
r=[LambdaD./100:LambdaD./1000:4*LambdaD];

%Define the two potential functions
Phi_freespace=1./(4*pi*eps0).*Q./r;

Phi_plasma=1./(4*pi*eps0).*Q./r.*exp(-r./LambdaD);

%Make the plots, normalize axes to Debye Length and Q/(4*pi*eps0*lambdaD)
figure(1)
plot(r./LambdaD, Phi_freespace.*(4*pi*eps0*LambdaD)/Q, 'LineWidth',2); hold on;
plot(r./LambdaD, Phi_plasma.*(4*pi*eps0*LambdaD)/Q, '--', 'LineWidth',2);
hold off

ylim([0 4])
%xlim([0 5e-4])
%make the font size bigger, 'gca' means 'get current axis'
set(gca, 'FontSize', 14)

legend('Free Space', 'Plasma')

%Add title and labels
title('Electrostatic Potential')
xlabel('Distance [\lambda_D]')
ylabel('\phi(r) [Q/(4\pi\epsilon_0\lambda_D)]')

```

Figure 1.2: MATLAB Code for plot shown in Figure 1.1.

- 1-5. Consider two infinite, parallel plates located at  $x = \pm d$ , kept at a potential of  $\Phi = 0$ . The space between the plates is uniformly filled with a gas of density  $N$  of particles of charge  $q$ . (a) Using Poisson's equation, show that the potential distribution between the plates is  $\Phi(x) = [Nq/(2\epsilon_0)](d^2 - x^2)$ . (b) Show that for  $d > \lambda_D$ , the energy needed to transport a particle from one of the plates to the mid-point (i.e.,  $x = 0$ ) is greater than the average kinetic energy of the particles. (Assume a Maxwellian distribution of particle speeds.)

**Solution:**

(a) We start with Poisson's equation:

$$\nabla^2 \Phi = -\rho/\epsilon_0$$

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{-Nq}{\epsilon_0}$$

Since the two plates are infinite, this is a one dimensional problem  $\frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial z^2} = 0$ , Thus:

$$\frac{\partial^2 \Phi}{\partial x^2} = \frac{-Nq}{\epsilon_0}$$

$$\frac{\partial \Phi}{\partial x} = \frac{-Nq}{\epsilon_0}x + C_1$$

$$\Phi(x) = -\frac{Nqx^2}{2\epsilon_0} + C_1x + C_2$$

We now use the boundary condition that the potential at the plates is zero  $\Phi(-d) = \Phi(d) = 0$  to get

$$\Phi(x) = \frac{Nq}{2\epsilon_0}(d^2 - x^2)$$

(b) The energy  $E_p$  needed to transport a particle from the wall to the center can be calculated from the difference in potential between these two locations:

$$E_p = q(\Phi(0) - \Phi(d))$$

$$E_p = q \cdot \frac{Nq}{2\epsilon_0} \cdot d^2 = \frac{Nq^2 d^2}{2\epsilon_0}$$

The average kinetic energy  $E_k$  of the particle, assuming a 1-D system is :

$$E_k = \frac{1}{2}k_B T$$

We can relate  $E_p$  to  $E_k$  using the fact that  $d > \lambda_D$ :

$$\begin{aligned}d > \lambda_D &= \sqrt{\frac{\epsilon_0 k_B T}{N q^2}} \\d^2 &> \frac{\epsilon_0 k_B T}{N q^2} \\ \frac{N q^2 d^2}{2 \epsilon_0} &> \frac{k_B T}{2} \\ E_p &> E_k\end{aligned}$$